Simultaneous Global Analysis of Polarized and Unpolarized PDFs and Fragmentation Functions

Nobuo Sato
Old Dominion University

CTEQ Workshop
Parton Distributions as a Bridge from Low to High Energies
Jefferson Lab, 2018
Mapping the parton structure of the nucleon

- **Challenges:**
  - **Quantitative** limits of $x, Q^2, z, ...$ where factorization theorems are applicable
  - Universality of non perturbative objects → **predictive power**
  - QCD analysis framework that extracts **simultaneously** all non-perturbative objects (including TMDs)
  - Framework with the same theory assumptions
Mapping the parton structure of the nucleon

- Need for a **reliable** Bayesian likelihood analysis:
  - Retire maximum likelihood methods that can lead to biased results (CT, CJ, MMHT, DSSV, ...)
  - Embrace likelihood analysis via MC methods (JAM, NNPDF)
  - Faithful representation of uncertainties consistent with Bayes’ theorem
Bayesian likelihood analysis

- Inclusion of modern data analysis techniques
  
  - Bayesian theorem
    \[ \mathcal{P}(f|\text{data}) = \mathcal{L}(\text{data}, f) \pi(f) \]
  
  - Estimation of expectation values and variances:
    
    o data resampling
    o partition and cross validation
    o iterative Monte Carlo (IMC)
    o nested sampling
JAM15: \( \Delta \)PDFs

NS, Melnitchouk, Kuhn, Ethier, Accardi (PRD)

- Inclusion of all JLab 6 GeV data \( \rightarrow 0.1 < x < 0.7 \)
- Non vanishing twist 3 quark distributions
- Residual twist 4 contributions consistent with zero
JAM15: $d_2$ matrix element
NS, Melnitchouk, Kuhn, Ethier, Accardi (PRD)

Existing measurements of $d_2$ are in the resonance region → quark-hadron duality

$$d_2(Q^2) \equiv \int_0^1 dx x^2 \left[ 2g_1^r(x, Q^2) + 3g_2^r(x, Q^2) \right]$$
SU2, SU3 constraints imposed

What determines the sign of $\Delta s^+$?
JAM16: FFs
NS, Ethier, Melnitchouk, Hirai, Kumano, Accardi (PRD)

- $\pi$ and $K$ Belle, BaBar up to LEP energies
- JAM and DSS $D_{s+}^{K}$ consistent
JAM17: $\Delta$PDF + FF
Ethier, NS, Melnitchouk (PRL)

- No SU(3) constraints
- Sea polarization consistent with zero
- Precision of $\Delta$SIDIS is not sufficient to determine sea polarization
Flat priors that gives flat $a_8$ in order to have an unbiased extraction of $a_8$.

Data prefers smaller values for $a_8 \rightarrow 25\%$ larger total spin carried by quarks.

$a_3$ which is in a good agreement with values from $\beta$ decays within 2%.

<table>
<thead>
<tr>
<th></th>
<th>JAM15</th>
<th>JAM17</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_A$</td>
<td>1.269(3)</td>
<td>1.24(4)</td>
</tr>
<tr>
<td>$g_8$</td>
<td>0.586(31)</td>
<td>0.46(21)</td>
</tr>
<tr>
<td>$\Delta \Sigma$</td>
<td>0.28(4)</td>
<td>0.36(9)</td>
</tr>
<tr>
<td>$\Delta \bar{u} - \Delta \bar{d}$</td>
<td>0</td>
<td>0.05(8)</td>
</tr>
</tbody>
</table>
Present
Goals

+ Extract PDFs, $\Delta$PDFs and FFs simultaneously
  - DIS, SIDIS($\pi, K$), DY
  - $\Delta$DIS, $\Delta$SIDIS($\pi, K$)
  - $e^+ e^-(\pi, K)$
+ Consistent extraction of $s$ and $\Delta s$

Likelihood analysis (first steps)

+ Use maximum likelihood to find a candidate solution
+ Use resampling to check for stability and estimate uncertainties
+ 80 shape parameters and 91 data normalization parameters:
  171 dimensional space
SIDIS is in agreement with DY’s $\bar{d} - \bar{u}$

- $\bar{d} - \bar{u}$ constrained mainly by DY
- $s - \bar{s} \neq 0$
Comparison with other groups

- dashed: MMHT14
- dashed-dotted: CT14
- dotted: CJ15
- dot-dot-dash: ABMP16

Big differences for $s, \bar{s}$ distributions
For CJ and CT, $s = \bar{s}$

MMHT uses neutrino DIS

SIDIS favors a strange suppression

and a larger $s$, $\bar{s}$ asymmetry
Recall no SU2, SU3 imposed

- $\Delta s, \Delta \bar{u}, \Delta \bar{d}$ are much better known than $\Delta \bar{s}$

- It means, most of the uncertainty on $\Delta s^+$ is from $\Delta \bar{s}$
JAM18: IMC runs (preliminary)

\[ \text{flat priors} \]

\[ \text{DIS no HERA} \]

\[ \text{DIS with HERA} \]

\[ \text{DIS with HERA + DY} \]
... and beyond
SIDIS+Lattice analysis of nucleon tensor charge
Lin, Melnitchouk, Prokudin, NS, Shows (PRL)

- Extraction of transversity and Collins FFs from SIDIS $A_{UT+Lattice} g_T$

- In the absence of Lattice, SIDIS has no significant constraints on $g_T$
First global Monte Carlo analysis of pion PDFs
Barry, NS, Melnitchouk, Ji (PRL)
First global Monte Carlo analysis of pion PDFs
Barry, NS, Melnitchouk, Ji (PRL)

- How to probe pion structure

\[ + \pi + A \to \bar{\ell} + X \text{ (Drell-Yan)} \]

\[ + \pi + A \to \gamma + X \text{ (prompt photons)} \]

\[ + e + p \to e' + n + X \text{ (SIDIS) } \to \text{ small } x_\pi \text{ gluon PDF} \]
First global Monte Carlo analysis of pion PDFs

\[
\frac{d\sigma}{dx dQ^2 dy} \sim f_{p \rightarrow \pi + n}(y) \times \sum_q \int_{x/y}^1 \frac{d\xi}{\xi} C(\xi) q\left(\frac{x/y}{\xi}, Q^2\right)
\]
Our new analysis extends previous pion PDF analysis down to $x \sim 0.001$

The OPE+pQCD can describe the HERA data **simultaneously** with the DY data

\[
F_2^{\mathrm{LN}(3)}(x, Q^2, y) = 2f_{p \rightarrow \pi + n}(y)F_2^\pi(x_\pi, Q^2)
\]

\[
r(x, Q^2, y) = \frac{d^3\sigma^{\mathrm{LN}}/dxdQ^2dy}{d^2\sigma^{\mathrm{inc}}/dxdQ^2}\Delta y
\]
First global Monte Carlo analysis of pion PDFs

- **Significant reduction of the uncertainties**
- **Non-overlapping uncertainties** → tensions among the data
- **Accuracy will be improved with future TDIS (JLab12/EIC)**
Constraints from HERA significantly increase $\langle x^g_\pi \rangle$. The role of the glue is more important than suggested by DY alone.

In contrast, the strength of the sea is reduced.

Due to momentum sum rule $\langle x^\text{valence}_\pi \rangle$ decreases.
We performed an additional analysis of LN+DY+E866 → good description of E866 data except for large $x$. 
Backup
### EFT setup

- The splitting function \( f_{p \rightarrow \pi + n}(y) \)

\[
f_{p \rightarrow \pi + n}(y) = \frac{g_A^2 M^2}{(4\pi f_\pi)^2} \int dk_\perp \frac{y(k_\perp^2 + y^2 M^2)}{(1 - y)^2 D^2_{\pi N}} |F|^2
\]

- UV regulators used in the literature

\[
F = \begin{cases} 
[1 - \frac{(t-m_\pi^2)^2}{(t-\Lambda^2)^2}]^{1/2} & \text{Pauli-Villars} \\
(\Lambda^2 - m_\pi^2)/(\Lambda^2 - t) & \text{t-dependent monopole} \\
\exp[(t-m_\pi^2)/\Lambda^2] & \text{t-dependent exponential} \\
\exp[(M^2 - s)/\Lambda^2] & \text{s-dependent exponential} \\
y^{-\alpha}(t) \exp[(t-m_\pi^2)/\Lambda^2] & \text{Regge exponential},
\end{cases}
\]
pQCD setup

- $\pi^- + W \rightarrow l\bar{l} + X$ (Drell-Yan)

$$\frac{d\sigma}{dxFdQ^2} = \sum_{a,b} \int d\xi d\zeta \ C_{a,b}(\xi, \zeta) \ f_{a/\pi^-}(\xi) f_{b/W}(\zeta)$$

- $e + p \rightarrow e' + n + X$ (LN)

$$\frac{d\sigma}{dxdQ^2dy} \sim f_{p\rightarrow\pi^+n}(y) \times \sum_q \int_{x/y}^1 \frac{d\xi}{\xi} \ C(\xi) \ f_{q/\pi^+}\left(\frac{x/y}{\xi}\right)$$

- We parametrize PDFs in $\pi^-$

  + Valence: $\bar{u}_v = d_v$
  + Sea: $u = \bar{d} = s = \bar{s}$
  + Gluon: $g$