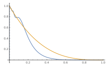

Scalar handbag and nonperturbative evolution in inclusive DIS

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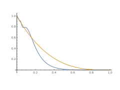
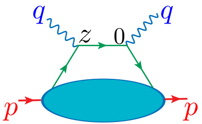


Handbag diagram

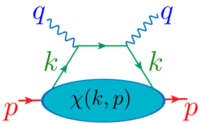


- Starting point of DIS analysis: virtual Compton amplitude
- Lowest approximation: handbag diagram

- To avoid inessential complications related to spin, consider a scalar handbag diagram
- In the coordinate representation

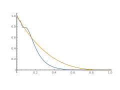
$$T(p,q) = \int d^4z\, e^{-i(qz)}\, D(z)\, \langle p|\phi(0)\phi(z)|p\rangle$$

- $D(z) = -i/4\pi^2 z^2$ is the scalar massless propagator



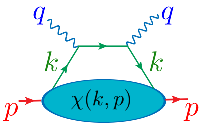
- In momentum space

$$T(p,q) = \int \frac{d^4k}{(q+k)^2}\, \chi_p(k)$$



Neglecting k_-

$$T(p,q) = \int \frac{d^4k}{(q+k)^2}\, \chi_p(k) = \int \frac{d^4k}{q^2 + 2(qk) + k^2}\, \chi_p(k)$$

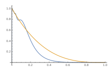


- Take the frame where p and q are purely longitudinal, i.e. $q = (q_+, q_- = -Q^2/q_+, q_\perp = 0)$, $p = (p_+, p_- = p^2/2p_+, p_\perp = 0)$, and $k = (k_+ = xp_+, k_-, k_\perp)$. Then

$$T(p,q) = \int \frac{d^2k_\perp dk_+ dk_-}{-Q^2 + 2(qk) + 2k_+k_- - k_\perp^2}\, \chi_p(k)$$

- To concentrate on $k_\perp$ effects, take $p^2 = 0$
- Then $p_- = 0$ and $q_+ = -x_{\rm Bj} p_+$, where $x_{\rm Bj} = Q^2/2(pq)$
- Neglecting k_- in the hard propagator (equivalent to assuming that k^2 equals $-k_\perp^2$) we get

$$T(p,q)|_{k_-=0} = \int \frac{dk_\perp^2\, dx}{Q^2(x/x_{\rm Bj} - 1) - k_\perp^2}\, \mathcal{F}(x, k_\perp^2)$$



Neglecting k_-, cont.

$$T(p,q)|_{k_-=0} = \int \frac{dk_\perp^2\, dx}{Q^2(x/x_{\rm Bj}-1)-k_\perp^2}\,\mathcal{F}(x,k_\perp^2)$$

- Transverse momentum dependent distribution

$$\mathcal{F}(x,k_\perp^2) = \frac{1}{p_+}\int dk_-\chi_p(k)$$

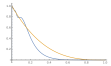
- Because of rotational invariance in $k_\perp$ plane TMD depends on $k_\perp^2$ only
- Taking imaginary part gives

$$\operatorname{Im} T(p,q)|_{k_-=0} = \frac{1}{2(pq)}\int dk_\perp^2\,\mathcal{F}(x=x_{\rm Bj}(1+k_\perp^2/Q^2),k_\perp^2)\ .$$

- Since $x<1$, we have restriction $x_{\rm Bj}(1+k_\perp^2/Q^2)<1$ or $k_\perp^2 < /Q^2(1/x_{\rm Bj}-1)$ which is $W^2=(q+p)^2$ for a massless hadron
- We may write

$$F(x_{\rm Bj},Q^2)|_{k_-=0} = \int_0^{W^2} dk_\perp^2\,\mathcal{F}(x(k_\perp^2),k_\perp^2)$$

with $x(k_\perp^2)=x_{\rm Bj}(1+k_\perp^2/Q^2)$



Virtuality distributions

- Can we get a description in terms of x and k_T without neglecting k_-?
- For any Feynman diagram, for arbitrary z and arbitrary p

$$\langle p|\phi(0)\phi(z)|p\rangle = \int_0^\infty d\sigma \int_{-1}^{1} dx\, \Phi(x,\sigma)\; e^{-ix(pz)-i\sigma(z^2-i\epsilon)/4}$$

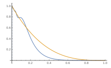
- *Parton virtuality distribution* (PVD) $\Phi(x,\sigma)$
- Support: $-1 \le x \le 1$ and $\sigma \ge 0$
- On the light cone $z^2 = 0$ and we get that the lowest moment

$$\int_0^\infty \Phi(x,\sigma)\, d\sigma = f(x)$$

  of PVD $\Phi(x,\sigma)$ gives the usual twist-2 parton density $f(x)$
- The first moment

$$\int_0^\infty \sigma\, \Phi(x,\sigma)\, d\sigma = i\Lambda^2 f_1(x)$$

  involves the function $f_1(x)$ that describes the x-distribution of the average parton virtuality, etc.



- Define “transverse” by requiring that the hadron momentum p is purely longitudinal, becoming a pure “plus” in the $p^2 \to 0$ limit
- Project matrix element onto $z_+ = 0$ by choosing z that has z_- and $z_\perp$ components only, and write

$$\langle p|\phi(0)\phi(z)|p\rangle|_{z_+=0} = \int_{-1}^{1} \mathcal{P}(x, z_\perp^2)\, e^{-ix(pz_-)}\, dx$$

- The function $\mathcal{P}(x, z_\perp)$ is the *impact parameter distribution* function
- Relation with PVD

$$\mathcal{P}(x, z_\perp^2) = \int_0^\infty d\sigma\, \Phi(x, \sigma)\; e^{i\sigma(z_\perp^2 + i\epsilon)}$$

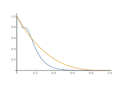
- Introducing TMD through a Fourier transform

$$\mathcal{P}(x, z_\perp^2) = \frac{1}{\pi} \int \mathcal{F}(x, k_\perp^2)\, e^{i(k_\perp z_\perp)}\, d^2k_\perp$$

- Its PVD representation is

$$\mathcal{F}(x, \kappa^2) = i \int_0^\infty \frac{d\sigma}{\sigma}\, \Phi(x, \sigma)\; e^{-i(\kappa^2 - i\epsilon)/\sigma}$$

- Note that $\mathcal{F}(x, \kappa^2)$ is defined not only for positive κ^2 but also for negative and complex values of κ^2



Integrated TMDs

- Another important function is *integrated TMD* $f(x,\mu^2)$

$$f(x,\mu^2) \equiv \int^{\mu^2} d\kappa^2 \mathcal{F}(x,\kappa^2)$$

- In terms of the PVD it can be written as

$$f(x,\mu^2) - f(x,\mu_0^2) = \int_0^\infty d\sigma\, \Phi(x,\sigma) \left[e^{-i(\mu_0^2 - i\epsilon)/\sigma} - e^{-i(\mu^2 - i\epsilon)/\sigma} \right]$$

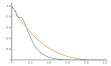
- Assuming that $f(x,\mu^2)$ is a regular function for $\mu^2 = 0$, we may view

$$f(x,\mu^2) - f(x,0) \equiv \int_0^{\mu^2} d\kappa^2 \mathcal{F}(x,\kappa^2)$$

  for real positive μ^2 as a scale-dependent parton distribution
- The hard $\sim 1/\kappa^2$ component of the TMD $\mathcal{F}(x,\kappa^2)$ results in a familiar logarithmic evolution dependence of $f(x,\mu^2)$
- For remaining $\mathcal{F}^{\rm soft}(x,\kappa^2)$ component, the integral over κ^2 converges at the upper limit, so that

$$f^{\rm soft}(x,+\infty) - f^{\rm soft}(x,0) = f^{\rm soft}(x)$$

- Still, integrating the soft part of $\mathcal{F}(x,\kappa^2)$ (say, $\sim e^{-\kappa^2/\Lambda^2}$) to a finite μ^2 value, results in a μ^2-dependence (e.g. in a form like $[1 - e^{-\mu^2/\Lambda^2}]$)
- This gives *nonperturbative* evolution in μ



Calculating DIS handbag diagram

$$T(p,q) = \int d^4z\, e^{-i(qz)}\, D(z)\, \langle p|\phi(0)\phi(z)|p\rangle \quad , \quad D(z) = -i/4\pi^2 z^2$$

- To integrate over d^4z, we combine

$$\langle p|\phi(0)\phi(z)|0\rangle = \int_0^\infty d\sigma \int_{-1}^1 dx\, \Phi(x,\sigma)\; e^{ix(pz)-i\sigma(z^2-i\epsilon)/4}$$

$$\text{and} \qquad \frac{4}{z^2 - i\epsilon} = i\int_0^\infty e^{-i\alpha z^2/4-\epsilon\alpha}\, d\alpha$$

- Obtain forward Compton amplitude in terms of PVD

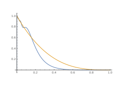
$$T(p,q) = i\int_0^1 d\xi \int_{-1}^1 dx \int_0^\infty \frac{d\sigma}{\sigma}\, \Phi(x,\sigma) e^{i\xi[(q+xp)^2+i\epsilon]/\sigma}$$

- In terms of TMDs

$$T(p,q) = \int_0^1 d\xi \int_{-1}^1 dx\, \mathcal{F}(x, -\xi[(q+xp)^2 + i\epsilon])$$

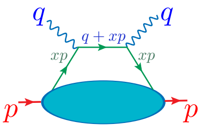
$$= \int_{-1}^1 dx\, \frac{f(x,0) - f(x,-(q+xp)^2 - i\epsilon))}{(q+xp)^2 + i\epsilon}$$

where $f(x,\mu^2)$ is the integrated TMD



Structure of forward Compton amplitude

$$T(p,q) = \int_0^1 dx \, \frac{f(x,0) - f(x, -(q+xp)^2 - i\epsilon))}{(q+xp)^2 + i\epsilon}$$

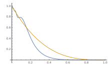


- Forward Compton amplitude is given by hard propagator $1/(q+xp)^2$ multiplied by the TMD $\mathcal{F}(x,\kappa^2)$ integrated over κ^2 up to $\mu^2 = -(q+xp)^2$
- Note: p is actual external momentum
- For spacelike $(q+xp)^2$, when $\mu^2 > 0$, we deal with a TMD integrated over transverse momentum up to μ
- Given the large scale Q^2 involved in $(q+xp)^2$, one may propose to take the $Q^2 \to \infty$ limit in the numerator to get, for the soft component,

$$T^{\rm soft}(p,q)\Big|_{Q^2\to\infty} \overset{?}{\Rightarrow} -\int_0^1 dx \, \frac{f^{\rm soft}(x)}{(q+xp)^2 + i\epsilon}$$

- For the imaginary part, this gives

$$\frac{1}{\pi} {\rm Im}\, T^{\rm soft}(p,q)\Big|_{Q^2\to\infty} \overset{?}{\Rightarrow} \int_0^1 dx \, f^{\rm soft}(x) \delta[(q+xp)^2]$$



Unexpected ξ-scaling

$$\frac{1}{\pi} \mathrm{Im}\, T^{\mathrm{soft}}(p,q)\Big|_{Q^2\to\infty} \overset{?}{\Rightarrow} \int_0^1 dx\, f^{\mathrm{soft}}(x)\delta[(q+xp)^2]$$

- Since p is the actual external momentum with $p^2 = M^2$, we have $(q+xp)^2 = -Q^2 + 2x(qp) + x^2M^2$, and

$$F(x_{\mathrm{Bj}}, Q^2) \overset{?}{=} 2(pq) \int_0^1 dx\, f(x)\delta(Q^2(1 - x/x_{\mathrm{Bj}}) - x^2M^2)$$

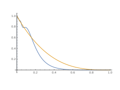
$$= \frac{1}{\sqrt{1 + 4x_{\mathrm{Bj}}^2 M^2/Q^2}} \int_0^1 dx\, f(x)\delta(x - \xi_{\mathrm{N}})$$

$$= \frac{f(\xi_{\mathrm{N}})}{\sqrt{1 + 4x_{\mathrm{Bj}}^2 M^2/Q^2}}$$

where ξ_{N} is the (twist-2) Nachtmann variable

$$\xi_{\mathrm{N}} = \frac{2x_{\mathrm{Bj}}}{1 + \sqrt{1 + 4x_{\mathrm{Bj}}^2 M^2/Q^2}}$$

- Note: we did not even discuss the twist decomposition



Spectral representation

$$T(p,q) = \int_0^1 dx\, \frac{f(x,0) - f(x, -(q+xp)^2 - i\epsilon))}{(q+xp)^2 + i\epsilon}$$

- Using $\delta[(q+xp)^2]$ in the calculation of imaginary part invalidates the argumentation that $(q+xp)^2$ is large
- If $f(x,\mu^2)$ is regular for $\mu^2 = 0$, then $T(p,q)$ cannot have imaginary part from the explicit $1/(q+xp)^2$ pole, because numerator cancels it
- But *we have no doubt* that the handbag diagram for $T(p,q)$, i.e.

$$\frac{f(x,0) - f(x, -(q+xp)^2 - i\epsilon))}{(q+xp)^2 + i\epsilon} \,,$$

must have imaginary part. Thus, we write a spectral representation

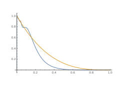
$$\frac{f(x,\mu^2) - f(x,0)}{\mu^2} = \int_{s_0}^{\infty} ds\, \frac{\rho(x,s)}{\mu^2 + s - i\epsilon}$$

- Imaginary part of $f(x,\mu^2)$ for $\mu^2 < -s_0$ is given by

$$\mathrm{Im}\, f(x,-s) = \pi s \rho(x,s)$$

- For the soft component, we can take the $\mu^2 \to +\infty$ limit to get

$$f^{\rm soft}(x) = \int_{s_0}^{\infty} ds\, \rho^{\rm soft}(x,s)$$



Running Nachtmann variable

$$f(x,\mu^2) - f(x,0) = \mu^2 \int_{s_0}^{\infty} ds\, \frac{\rho(x,s)}{\mu^2 + s - i\epsilon}$$

- Differentiating with respect to μ^2 gives a spectral representation for TMD:

$$\mathcal{F}(x,\kappa^2) = \int_{s_0}^{\infty} ds\, \frac{s\,\rho(x,s)}{(\kappa^2 + s - i\epsilon)^2}$$

- Spectral representation for DIS structure function

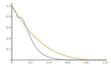
$$F(x_{\mathrm{Bj}},Q^2) = \int_{s_0}^{W^2} ds\, \frac{\rho(\xi(s),s)}{\sqrt{1+4x_{\mathrm{Bj}}^2 M^2(1+s/Q^2)/Q^2}}\ ,$$

where $W^2 = (p+q)^2 = Q^2(1/x_{\mathrm{Bj}} - 1) + M^2$

- Generalized Nachtmann variable $\xi(s)$ depends on the parameter s

$$\xi(s) = \frac{2x_{\mathrm{Bj}}(1+s/Q^2)}{1+\sqrt{1+4x_{\mathrm{Bj}}^2(1+s/Q^2)M^2/Q^2}}$$

- W^2-restriction on s comes from the requirement $\xi(s) < 1$



Massless target

- $\xi(s)$ reflects both dynamic transverse momentum and kinematic target-mass effects
- To concentrate on dynamic effects, take $M^2 = 0$ to get

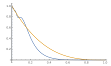
$$F_0(x_{\rm Bj}, Q^2) = \int_0^{W^2} ds\, \rho(x(s), s)\ ,$$

where $x(s) = x_{\rm Bj}(1 + s/Q^2)$

- Formula similar to that by Accardi and Qiu (2008) derived as a “jet-mass correction”, based a quark propagator with some effective mass $\sqrt{s}$
- In our approach, we do not change quark propagator, keeping it massless
- Compare to result obtained by neglecting k_- in the hard part

$$F(x_{\rm Bj}, Q^2)|_{k_-=0} = \int_0^{W^2} dk_\perp^2\, \mathcal{F}(x(k_\perp^2), k_\perp^2)\ ,$$

- Main difference is that the integrand is now the density $\rho(x(s), s)$ rather than TMD $\mathcal{F}(x(k_\perp^2), k_\perp^2)$



Models for soft TMDs

- For simplicity, assume a factorized form, $\rho^{\rm soft}(x,s) = f^{\rm soft}(x)\rho^{\rm soft}(s)$
- Density $\rho(s)$ is normalized by

$$\int_{s_0}^{\infty} ds\, \rho(s) = 1$$

- Use relation to TMD. For a factorized Ansatz,

$$\mathcal{F}(x,k_\perp^2) = f(x)\int_{s_0}^{\infty} ds\, \frac{s\,\rho(s)}{(k_\perp^2 + s - i\epsilon)^2} \equiv f(x)\,\mathcal{F}(k_\perp^2)$$

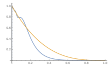
- Simplest function: $\rho(s) = \delta(s-m^2)$. Then

$$\mathcal{F}(x,k_\perp^2) = f(x)\,\frac{m^2}{(k_\perp^2+m^2)^2}$$

- Has a $\sim 1/k_\perp^4$ asymptotic behavior. The integrated TMD is given by

$$f(x,\mu^2) - f(x,0) = f(x)\left(1 - \frac{m^2}{\mu^2+m^2}\right),$$

with a $\sim m/\mu^2$ rate of approach to the asymptotic value
- IPD is given by $\mathcal{P}(x,b^2) = bmK_1(bm)$, with an exponential falloff for large b, and a finite unity value at the origin $b=0$



Models for exponentially decreasing TMDs

- The $\rho(s) = \delta(s - m^2)$ model gives $1/k_\perp^4$ behavior for large $k_\perp$
- In general, the coefficient of $1/k_\perp^4$ is given by

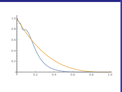
$$\int_0^\infty ds\ s\,\rho(s)$$

- Usual expectation: soft TMD decreases faster than any power of $1/k_\perp^2$
- Decrease faster than $1/k_\perp^4$ is realized only if the integral of $s\rho(s)$ vanishes, which is impossible for a positive-definite $\rho(s)$
- Let us try to build a model for $\mathcal{F}(x, k_\perp^2)$ that vanishes faster than any power of $1/k_\perp^2$ and is given by the PVD representation
- This "mild" requirement excludes a popular Gaussian $e^{-k_\perp^2/\Lambda^2}$ factor
- For orientation: propagator of a scalar particle with mass m is given by

$$D(z, m) = \frac{1}{(4\pi)^2} \int_0^\infty e^{-i\sigma z^2/4 - i(m^2 - i\epsilon)/\sigma}\, d\sigma\ .$$

- Falls off exponentially $\sim e^{-|z|m}$ for large space-like distances
- For small z^2, propagator $D(z, m)$ has $1/z^2$ singularity
- We want $\langle p|\phi(0)\phi(z)|p\rangle$ to be finite for $z = 0$ and add $(-1/\Lambda^2)$ to z^2. PVD:

$$\Phi_m(x, \sigma) = f(x)\,\frac{e^{i\sigma/4\Lambda^2 - im^2/\sigma}}{4im\Lambda K_1(m/\Lambda)}$$



Models for exponentially decreasing TMDs, cont.

- Results in the impact parameter distribution that is finite for $b_\perp = 0$

$$\mathcal{P}_m(x, b_\perp) = f(x) \frac{K_1\left(m\sqrt{1/\Lambda^2 + b_\perp^2}\right)}{K_1(m/\Lambda)\sqrt{1+\Lambda^2 b_\perp^2}}$$

  and has an exponential $\sim e^{-m|b_\perp|}$ fall-off for large $b_\perp$
- It corresponds to a TMD that is finite for $k_\perp = 0$

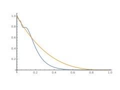
$$\mathcal{F}_m(x, k_\perp^2) = f(x) \frac{K_0\left(\sqrt{k_\perp^2 + m^2}/\Lambda\right)}{2m\Lambda K_1(m/\Lambda)}$$

  and exponentially decreases (like $e^{-k_\perp/\Lambda}$) for large $k_\perp$
- The scalar density in this case is

$$s\rho_m(x, s) = \frac{f(x)}{2}\sqrt{s - m^2}\,\frac{J_1(\sqrt{s-m^2}/\Lambda)}{mK_1(m/\Lambda)}\theta(s - m^2)$$

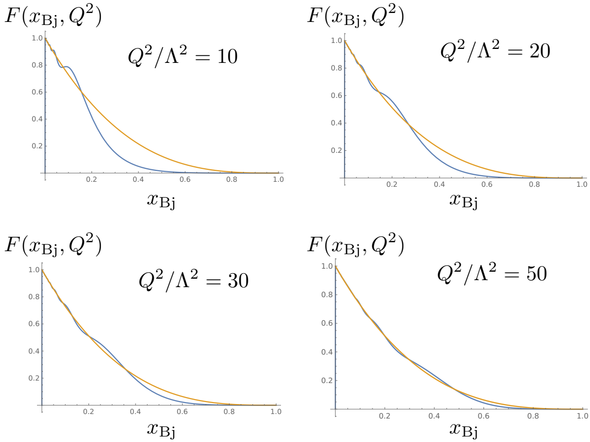
- In the spin-1/2 quark case one deals with the doubly integrated TMD which corresponds to the density

$$s^2\rho_m(x, s) = f(x)(s - m^2)\,\frac{J_2(\sqrt{s-m^2}/\Lambda)}{mK_1(m/\Lambda)}\theta(s - m^2)$$

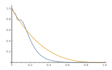


Modeling structure functions

- Take simplest model with $m = 0$ ($M = 0$ implied) and $f(x) = (1-x)^3$



- For small Q^2, expect that data are dual to a curve that is lower than pQCD evolution extrapolation from high Q^2



Summary

- Demonstrated that scalar handbag diagram can be expressed in terms of TMDs without approximations
- Introduced spectral representation for integrated TMDs
- Derived generalized Nachtmann variable depending on the target mass M and also on the spectral parameter s
- Proposed simple (but nontrivial) models for TMDs and spectral densities
- Demonstrated a pattern of nonperturbative evolution of structure functions