TMDs at small-x

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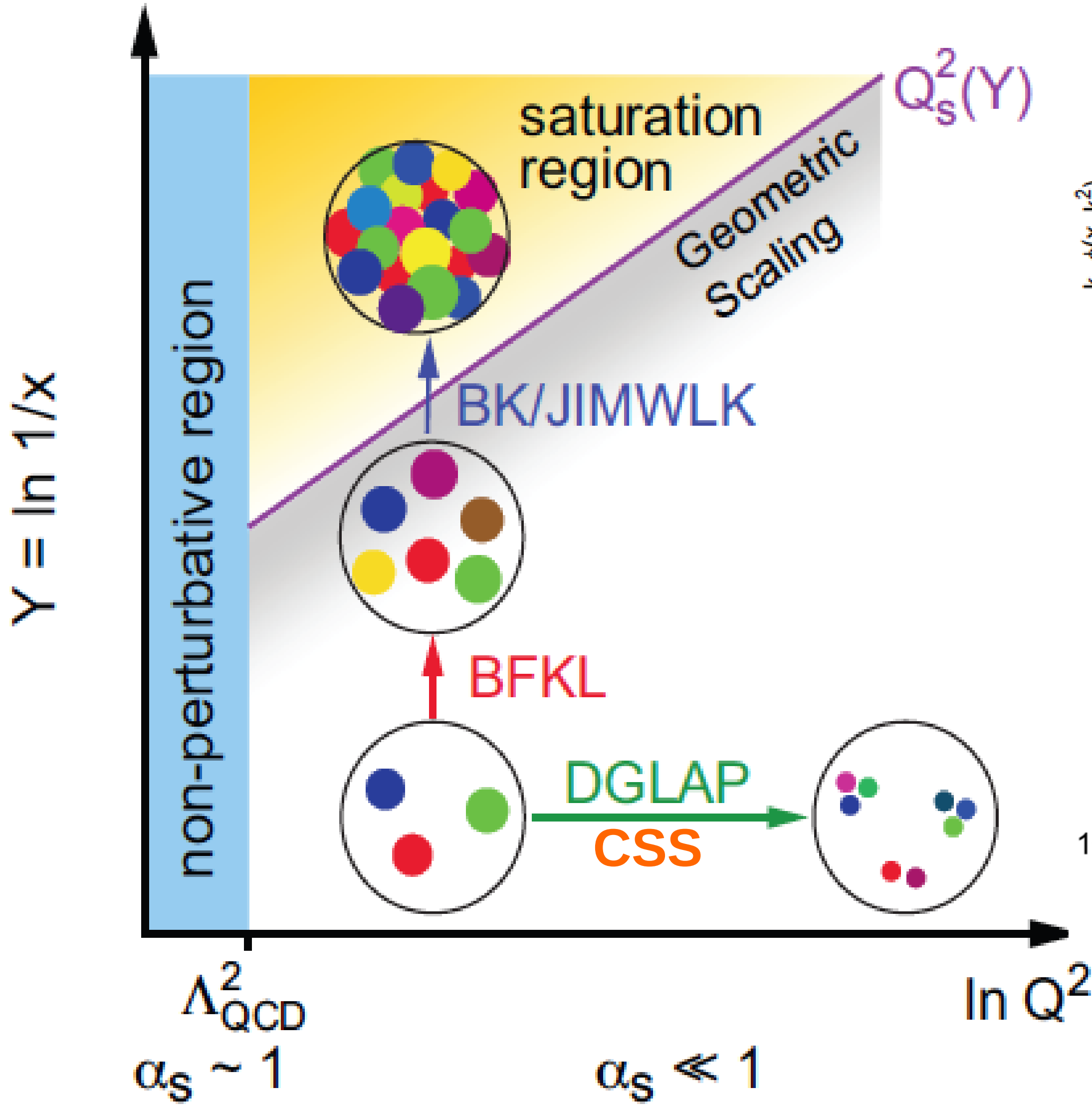
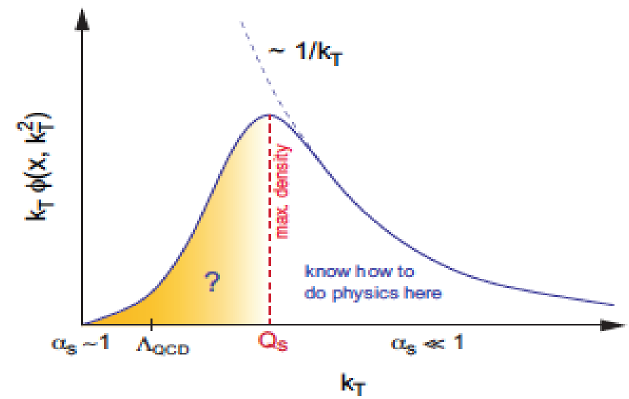
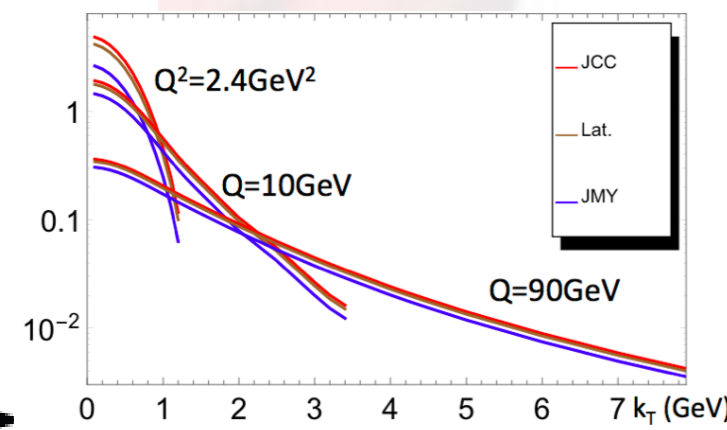
Transverse momentum distributions: A unified picture







Alexei's talk
Bin Yan's talk

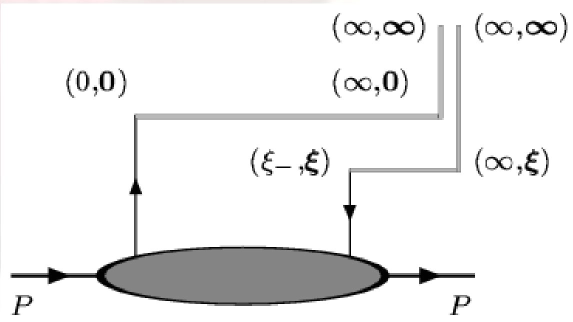
TMDs: Conventional gluon distribution

- Collins-Soper, 1981

$$xG^{(1)}(x,k_\perp) = \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - ik_\perp\cdot\xi_\perp}$$
$$\times \langle P|F^{+i}(\xi^-,\xi_\perp)\mathcal{L}_\xi^\dagger \mathcal{L}_0 F^{+i}(0)|P\rangle$$

  - Gauge link in the adjoint representation

$$\mathcal{L}_\xi = \mathcal{P}\exp\{-ig\int_{\xi^-}^{\infty} d\zeta^- A^+(\zeta,\xi_\perp)\}$$
$$\mathcal{P}\exp\{-ig\int_{\xi_\perp}^{\infty} d\zeta_\perp\cdot A_\perp(\zeta^-=\infty,\zeta_\perp)\}$$



Physical interpretation

- Choosing light-cone gauge, with certain boundary condition (either one, but not the principal value) $A_\perp(\zeta^- = \infty) = 0$
- Gauge link contributions can be dropped
- Number density interpretation, and can be calculated from the wave functions of nucleus
  - McLerran-Venugopalan
  - Kovchegov-Mueller

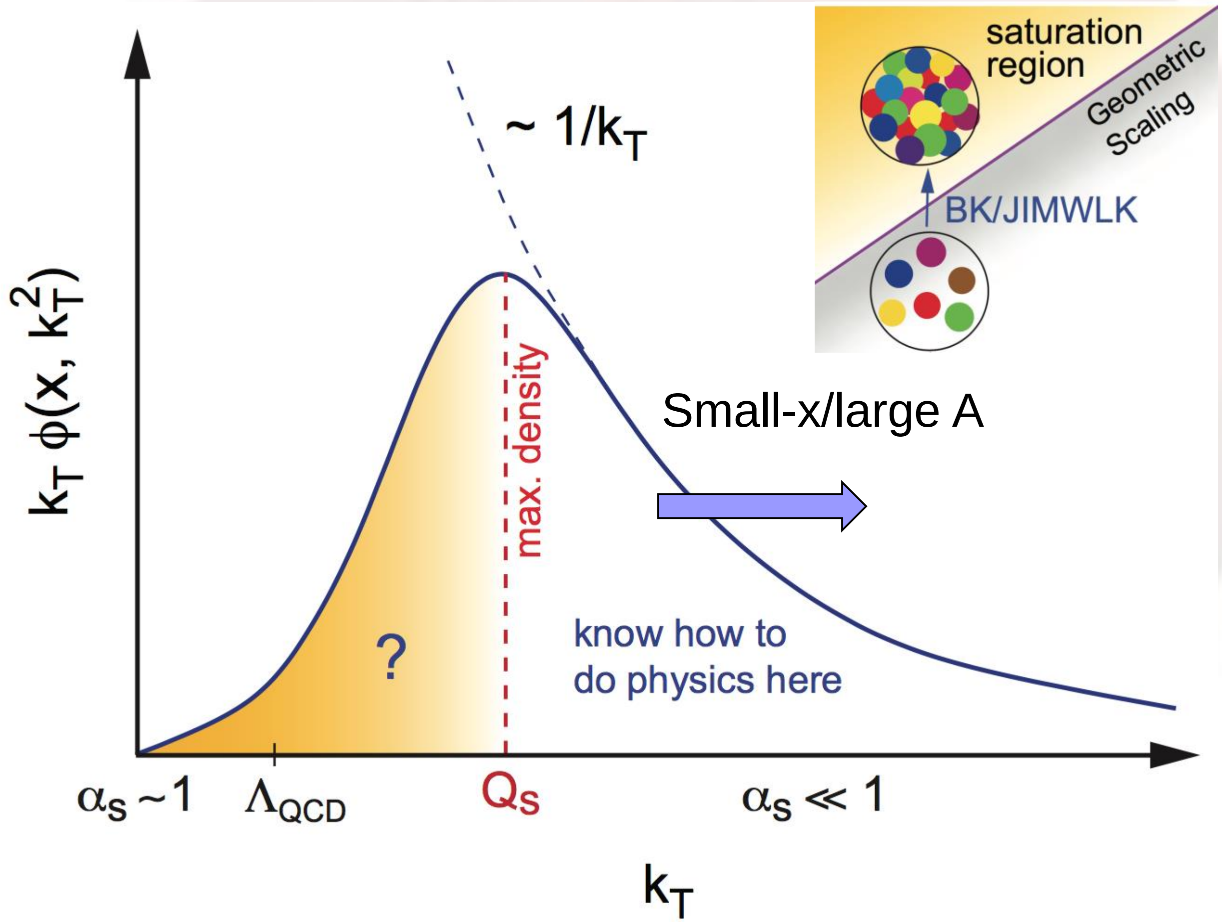
Classic YM theory

- McLerran-Venugopalan

$$xG^{(1)}(x,k_\perp) = \frac{S_\perp}{\pi^2\alpha_s}\frac{N_c^2-1}{N_c}\int\frac{d^2r_\perp}{(2\pi)^2}\frac{e^{-ik_\perp\cdot r_\perp}}{r_\perp^2}\left(1-e^{-\frac{r_\perp^2 Q_s^2}{4}}\right)$$

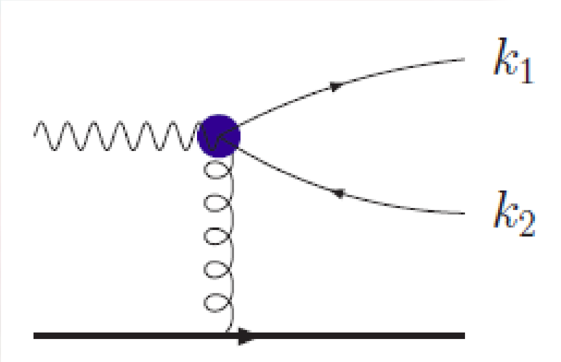
  - See also, Kovchegov-Mueller
  - We can reproduce this gluon distribution using the TMD definition with gauge link contribution, following BJY 02, BHPS 02
  - WW gluon distribution is the conventional one

Saturation at small-x/large A



DIS dijet probes **WW** gluons

$$\gamma_T^* A \to q(k_1) + \bar{q}(k_2) + X$$



- Hard interaction includes the gluon attachments to both quark and antiquark
- The q_t dependence is the gluon distribution w/ gauge link contribution

Dominguez-Marquet-Xiao-Yuan 2011

Golden channel for an EIC

- Directly probe the Weizsacker-Williams gluon distribution in nucleus
  - Non-Abelian manifest
- Factorization is very clear

- Various channels within DIS processes
  - Heavy flavor
  - Real/virtual photon

Among recent developments

- Spin-dependent TMD gluon at small-x
  - Related to the spin-dependent odderon, Boer-Echevarria-Mulders-Zhou, PRL 2016
  - Gluon/quark helicity distributions, Kovchegov-Pitonyak-Sievert, 2016, 2017, 2018
- Subleading power corrections in the TMD gluon/quark distributions
  - Balitsky-Tarasov, 2017, 2018
- Sudakov resummation for small-x TMDs
  - Mueller-Xiao-Yuan, PRL110, 082301 (2013); Xiao-Yuan-Zhou, NPB921, 104 (2017); Zhou 2018
  - Balitsky-Tarasov, JHEP1510,017 (2015)

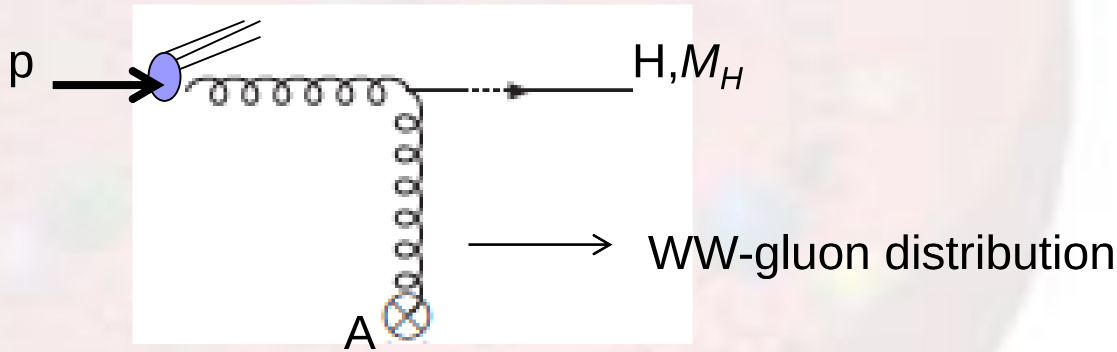
QCD evolution at high energy

- BFKL/BK-JIMWLK (small-x)
- Sudakov (TMD)

Mueller-Xiao-Yuan 2013
Balistky-Tarasov 2014
Xiao-Yuan-Zhou 2016

Sudakov resummation at small-x

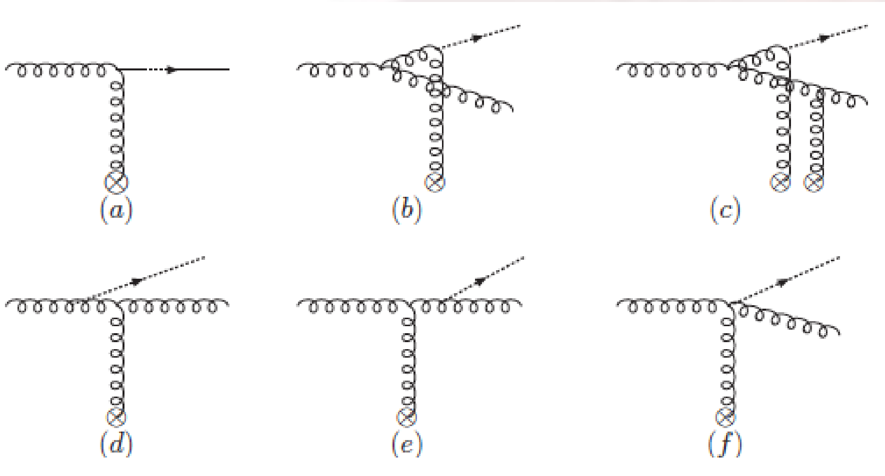
- Take massive scalar particle production *p+A->H+X* as an example to demonstrate the double logarithms, and resummation



$$\frac{d\sigma^{(\mathrm{LO})}}{dyd^2k_\perp} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp \cdot r_\perp} x_0 g_p(x_0) S^{(WW)}(x_\perp, x'_\perp)$$

$$S_Y^{WW}(x_\perp, y_\perp) = -\left\langle \mathrm{Tr}\left[\partial_\perp^\beta U(x_\perp) U^\dagger(y_\perp) \partial_\perp^\beta U(y_\perp) U^\dagger(x_\perp)\right]\right\rangle_Y$$

Explicit one-loop calculations



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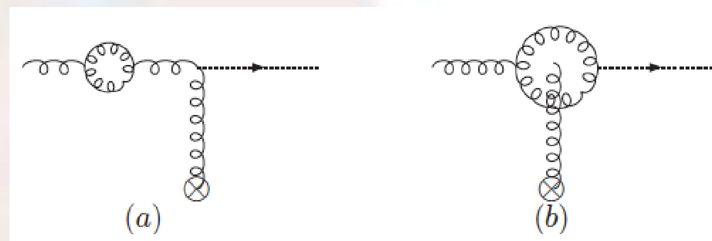


$$x_0 g_p(x_0) \int \frac{d\xi}{\xi} \mathbf{K}_{DMMX} \otimes S^{WW}(x_\perp, y_\perp) + \left(-\frac{1}{\epsilon}\right) S^{WW}(x_\perp, y_\perp) \mathcal{P}_{g/g} \otimes x_0 g(x_0) \ ,$$

- Collinear divergence ➔ DGLAP evolution
- Small-x divergence ➔ BK-type evolution

Jalilian-Marian-Kovchegov, 2004;Dominguiz-Mueller-Munier-Xiao, 2011

BERKELEY LAB

Soft vs Collinear gluons

- Radiated gluon momentum

$$k_g = \alpha_g p_1 + \beta_g p_2 + k_{g\perp} \ ,$$

- Soft gluon, $\alpha \sim \beta \ll 1$
- Collinear gluon, $\alpha \sim 1$, $\beta \ll 1$
- Small-x collinear gluon, $1-\beta \ll 1$, $\alpha \to 0$
  - Rapidity divergence

Final result

- Double logs at one-loop order

$$\frac{d\sigma^{(\mathrm{LO+NLO})}}{dyd^2k_\perp}|_{k_\perp \ll Q} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp \cdot r_\perp} S^{WW}_{Y=\ln 1/x_a}(x_\perp, x'_\perp) x g_p(x, \mu^2 = \frac{c_0^2}{r_\perp^2})$$
$$\left\{1 + \frac{\alpha_s}{\pi} C_A \left[\beta_0 \ln \frac{Q^2 r_\perp^2}{c_0^2} - \frac{1}{2}\left(\ln \frac{Q^2 r_\perp^2}{c_0^2}\right)^2 + \frac{\pi^2}{2}\right]\right\} ,$$

- Include both BFKL (BK) and Sudakov

$$\frac{d\sigma^{(\mathrm{resum})}}{dyd^2k_\perp}|_{k_\perp \ll Q} = \sigma_0 \int \frac{d^2x_\perp d^2x'_\perp}{(2\pi)^2} e^{ik_\perp \cdot r_\perp} e^{-S_{sud}(Q^2, r_\perp^2)} S^{WW}_{Y=\ln 1/x_a}(x_\perp, x'_\perp)$$
$$\times x g_p(x, \mu^2 = c_0^2/r_\perp^2) \left[1 + \frac{\alpha_s}{\pi} \frac{\pi^2}{2} N_c\right] ,$$

Mueller-Xiao-Yuan 2012

Sudakov leading double logs+small-x logs in hard processes

- Each incoming parton contributes to a half of the associated color factor in Sudakov
  - Initial gluon radiation, aka, TMDs

$$\frac{d\sigma}{dy_1 dy_2 dP_\perp^2 d^2k_\perp} \propto H(P_\perp^2)\int d^2x_\perp d^2y_\perp e^{ik_\perp\cdot(x_\perp - y_\perp)}\widetilde{W}_{x_A}(x_\perp, y_\perp)$$

Sudakov

$$\Rightarrow \quad H(P_\perp^2)\int d^2x_\perp d^2y_\perp e^{ik_\perp\cdot R_\perp} \boxed{e^{-S_{sud}(P_\perp,R_\perp)}}\widetilde{W}_{x_A}(x_\perp, y_\perp)$$

Mueller-Xiao-Yuan 2013

TMD at small-x: Sudakov and BFKL (BK)

- Start with the factorized TMDs, with full operator definitions
- Calculate the high order corrections in dipole formalism
  - With proper subtraction
- Solve the TMD evolution with BK-evolved dipole (quadrupole) amplitude

Subtracted TMD at small-x

$$f_g^{(sub.)}(x, r_\perp, \mu_F, \zeta_c) = f_g^{unsub.}(x, r_\perp)\sqrt{\frac{S^{\bar{n},v}(r_\perp)}{S^{n,\bar{n}}(r_\perp)S^{n,v}(r_\perp)}}$$

WW-gluon
Dipole gluon

Subtract the endpoint
Singularity (Collins 2011)

$$\zeta_c^2 = x^2(2v \cdot P)^2/v^2$$

- TMD evolution follows Collins 2011
  - with resummation, doesn’t depend on scheme
  - Beta_0 term missing though
- Small-x evolution follows the relevant BK-evolution, respectively
  - Dipole: BK
  - WW: DMMX

Final results

$$xG^{(1)}(x, k_\perp, \zeta_c = \mu_F = Q)$$

Hard scale entering TMD
Factorization, e.g., Higgs

$$-\frac{2}{\alpha_S}\int\frac{d^2x_\perp d^2y_\perp}{(2\pi)^4}e^{ik_\perp\cdot r_\perp}\mathcal{H}^{WW}(\alpha_s(Q))e^{-\mathcal{S}_{sud}(Q^2,r_\perp^2)}$$
$$\times\mathcal{F}^{WW}_{Y=\ln 1/x}(x_\perp, y_\perp)\ ,$$

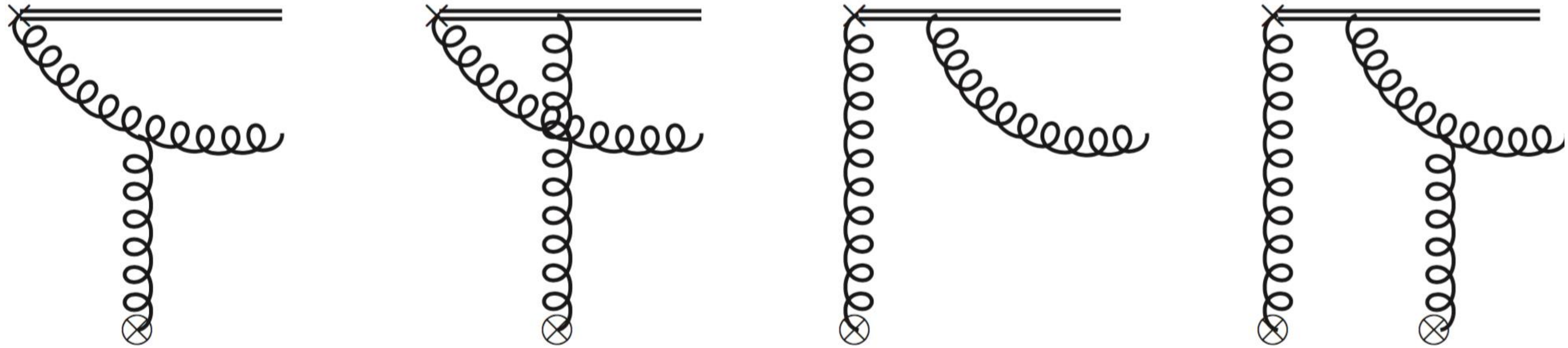
Pert. corrections

Small-x evolution

Sudakov resum.

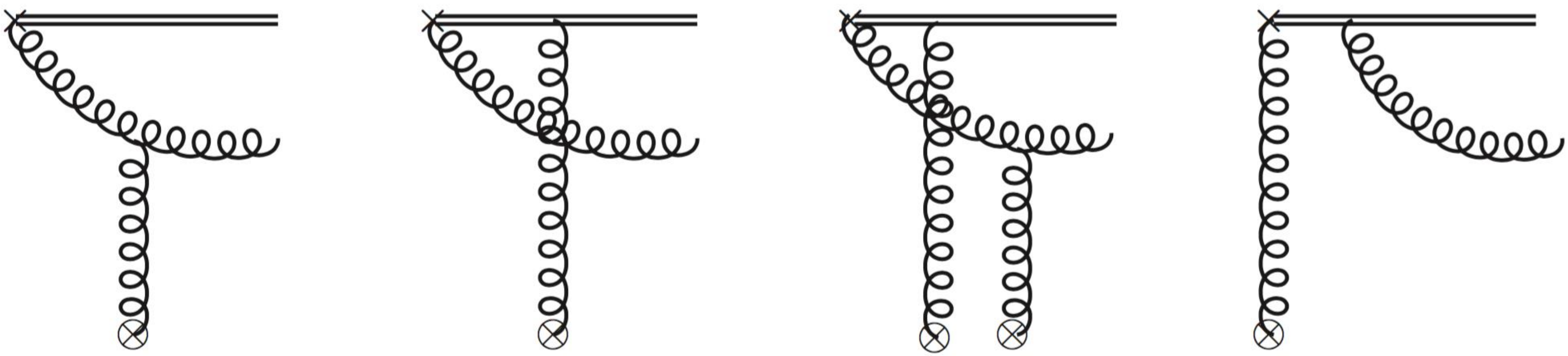
One-loop examples

Gauge link goes to $-\infty$



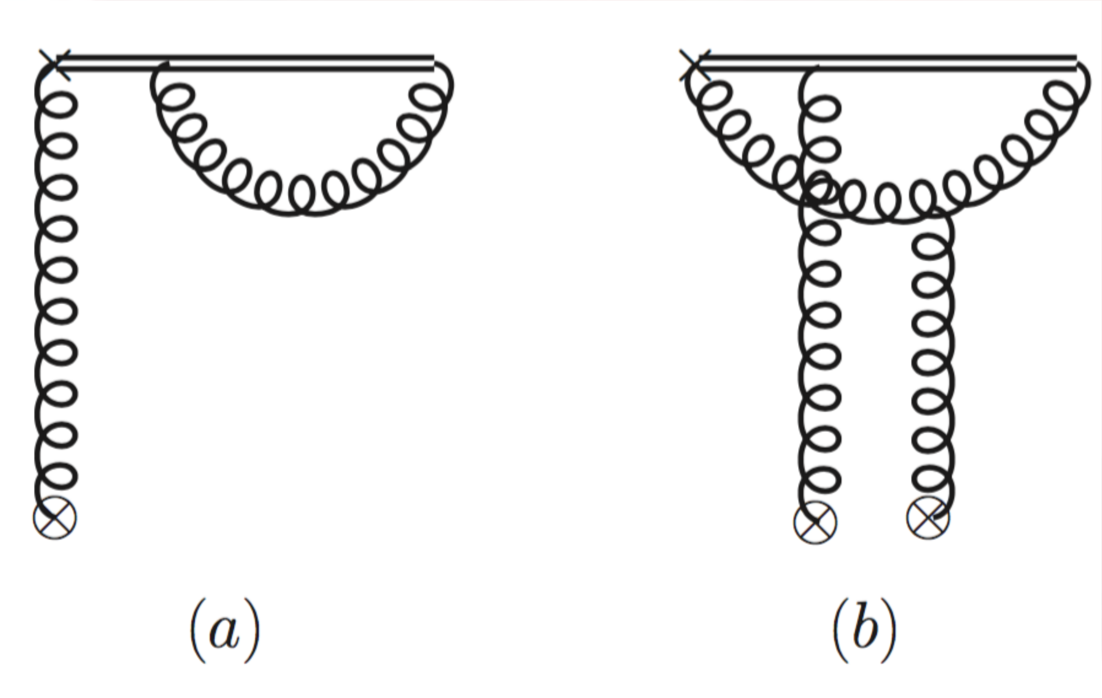
(a) (b) (c) (d)

Gauge link goes to $+\infty$



(a) (b) (c) (d)

Virtual is the same



One-loop result

- WW-gluon (universal)

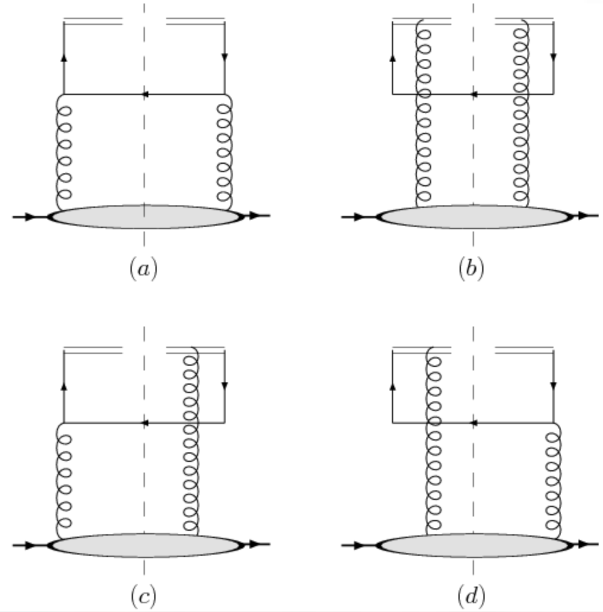
$$xG^{(WW)}_{(-\infty)}(x,r_\perp)|^{(1)} =$$

Sudakov double logs

$$\frac{\alpha_s}{2\pi}C_A\left\{\left(\frac{-2}{\alpha_s}\right)\mathcal{F}^{(WW)}(r_\perp)\left[\frac{1}{2}\left(\ln\frac{\zeta_c^2}{\mu^2}\right)^2-\frac{1}{2}\left(\ln\frac{\zeta_c^2 r_\perp^2}{c_0^2}\right)^2\right]\right.$$
$$\left.+\ln\left(\frac{1}{x}\right)\left(\frac{-2}{\alpha_s}\right)\int\mathbf{K}_{\rm DMMX}\otimes\mathcal{F}^{(WW)}(x_g,r_\perp)\right\}\ ,$$

Small-x logs (BK-type of evolution)

TMD quark at small-x



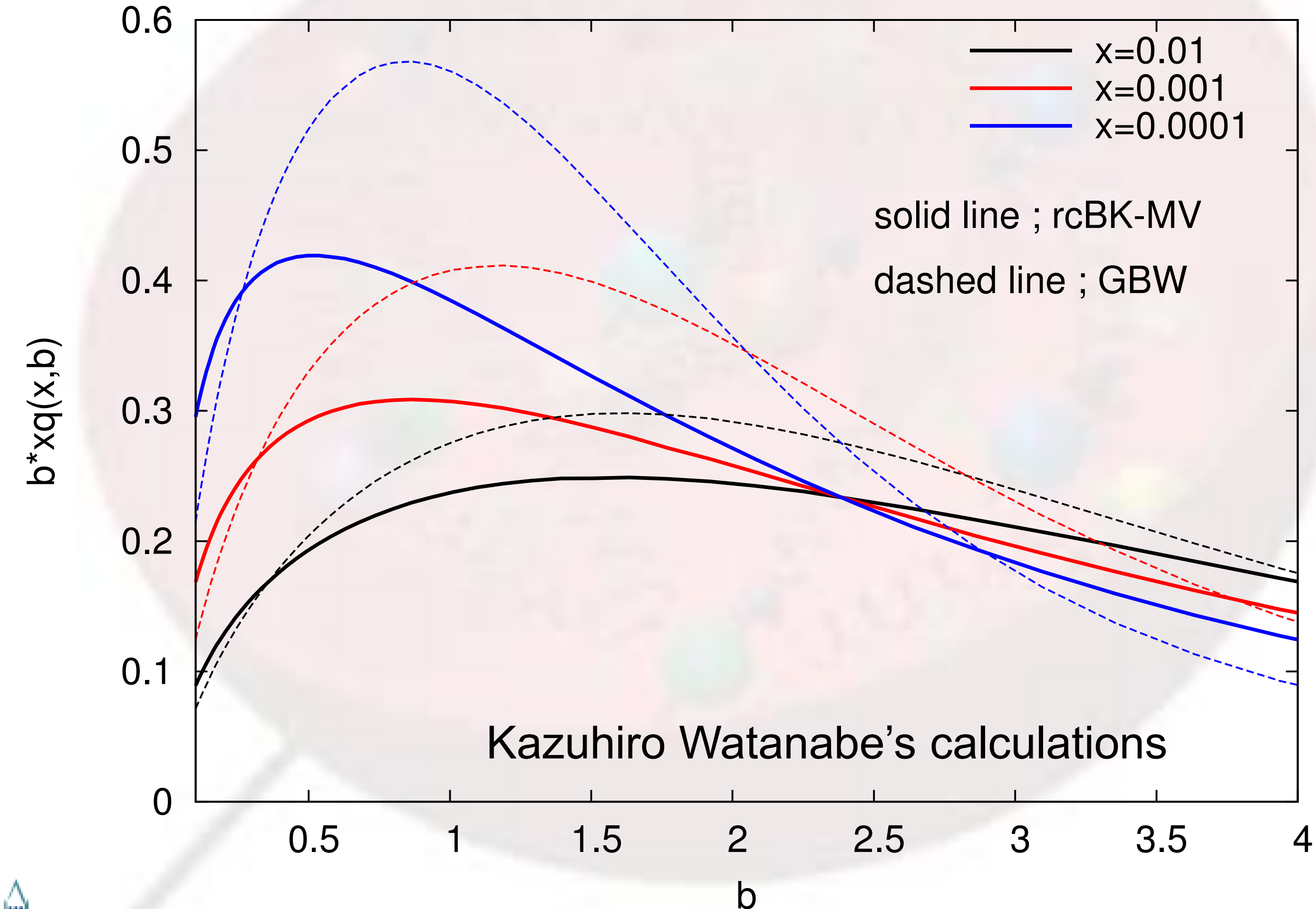
McLerran-Venugopalan 98

$$q(x,k_\perp) = \frac{N_c}{8\pi^4}\int\frac{dx'}{x'^2}\int d^2b d^2q_\perp F(q_\perp,x')A(q_\perp,k_\perp)$$

- Can be calculated from the dipole amplitude, and can be applied to DIS and Drell-Yan processes
  - Universality

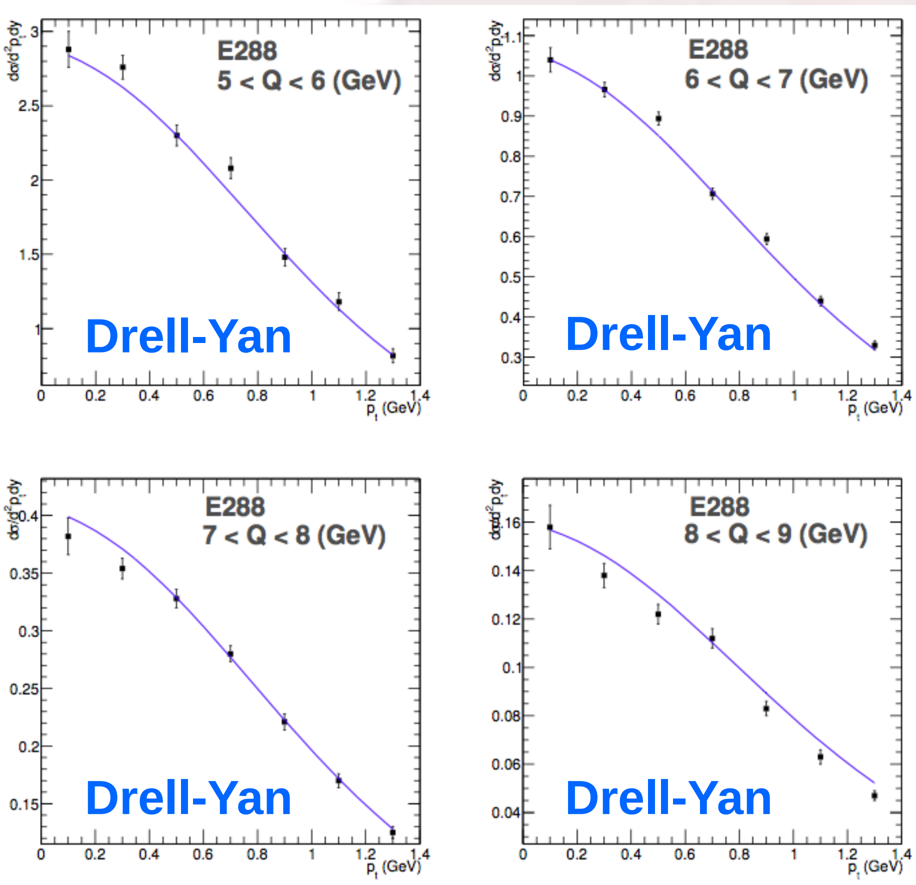
Marquet-Xiao-Yuan 2009

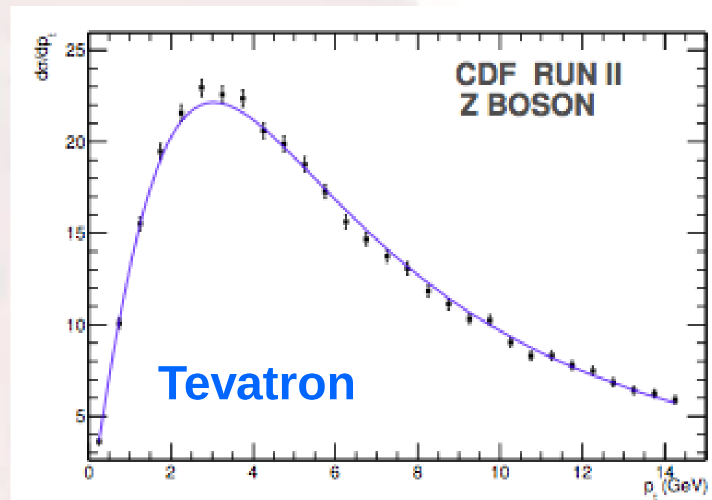
TMD quark at small-x

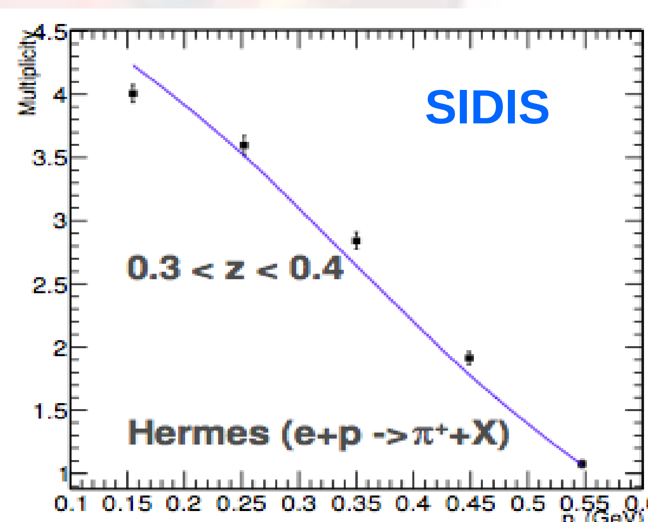


What we know the TMD quarks (not small-x)

Sun-Issacson-Yuan-Yuan, 2014

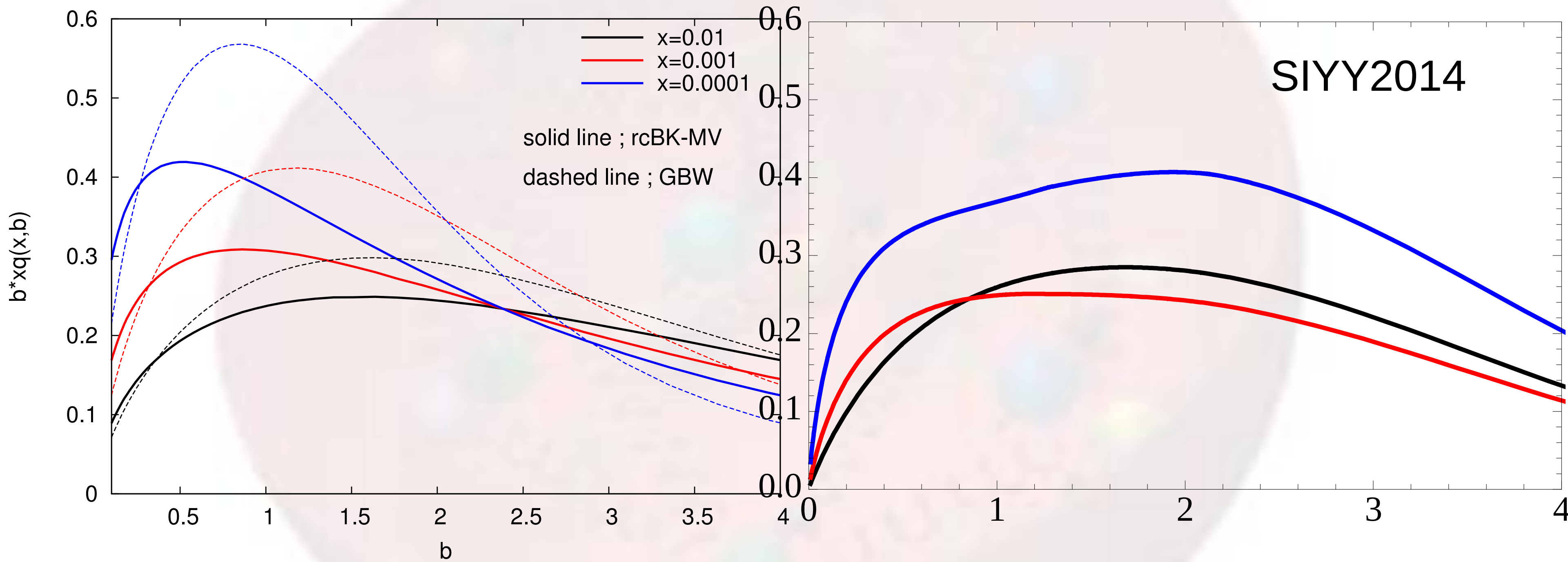






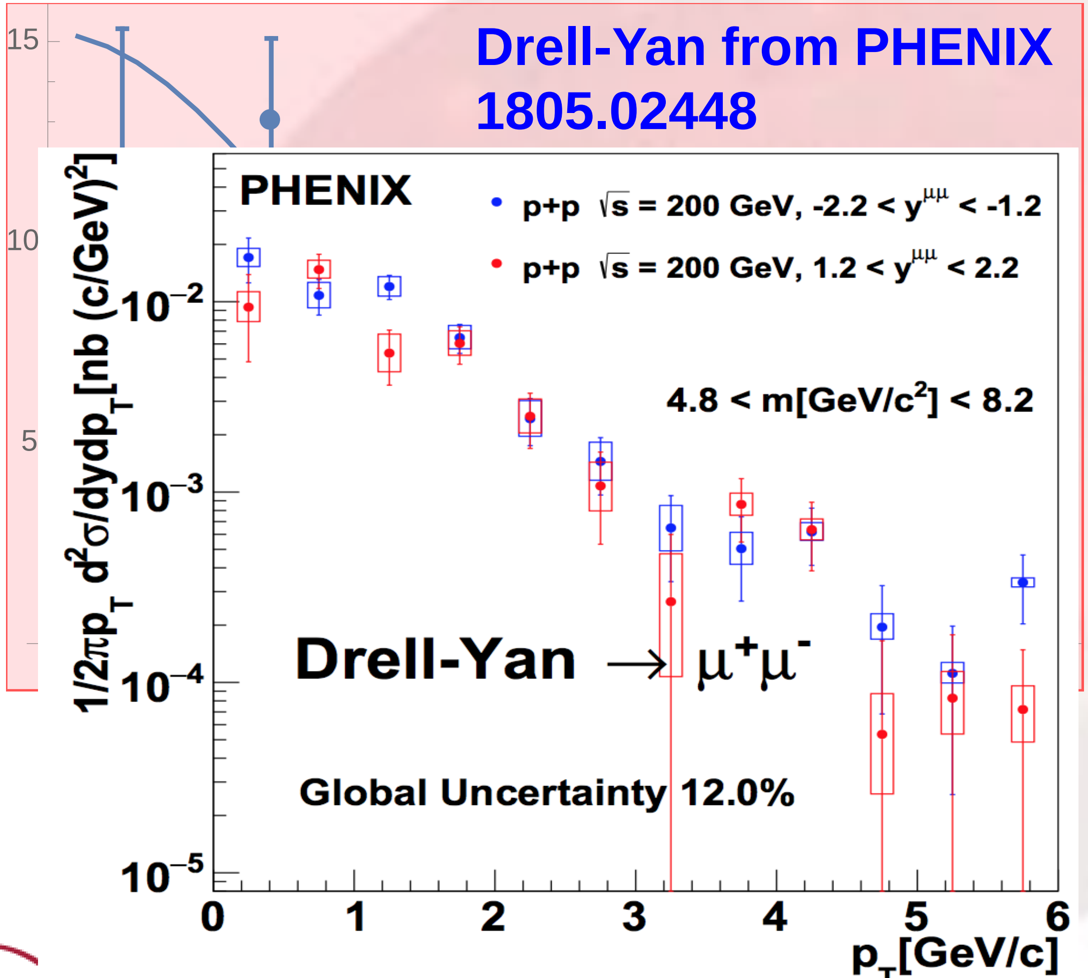
See also, BLNY 2002

TMD quark at small-x: CGC vs Collinear



- Realistic comparison will shed light on the TMD quarks at small-x (work in progress)

We need more data at small-x



LHCb, pp and pA:
Drell-Yan and Upsilon

EIC:
SIDIS and di-hadron

Looking Forward: EIC

- Precise, detailed, mapping of quark distribution at small-x
  - TMD fragmentation functions will be well understood too
- Electron-nucleus (*eA*) collisions provide information on the nuclear modification of quark distribution at small-x
  - BK evolution shall become more evident

Short Summary

- Theory developments in recent years provide solid ground to study TMDs at small-x
- Looking forward to new data from RHIC/LHC, and of course, EIC

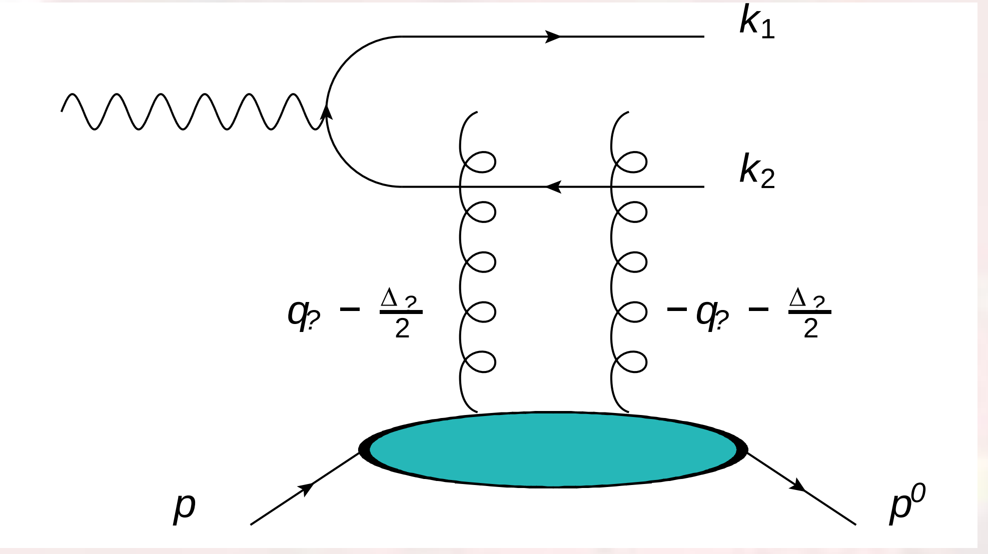
Strategy forward

- Establish the case for the TMDs at small-x
  - Common language between hadron physics and small-x physics communities
- Extend to the GPDs/DVCS at small-x
  - Tons of data when EIC is on-line!!
- Extend to Wigner distributions at small-x
  - Nucleon/nucleus tomography, finally!

Probing 3D Tomography of Protons at Small-x at EIC

Diffractive back-to-back dijet productions at EIC:

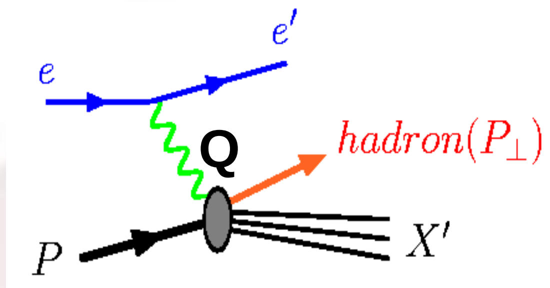
Hatta-Xiao-Yuan,1601.01585



- In the Breit frame, by measuring the recoil of final state proton, one can access Δ_T. By measuring jets momenta, one can approximately access q_T.
- The diffractive dijet cross section is proportional to the square of the Wigner distribution.

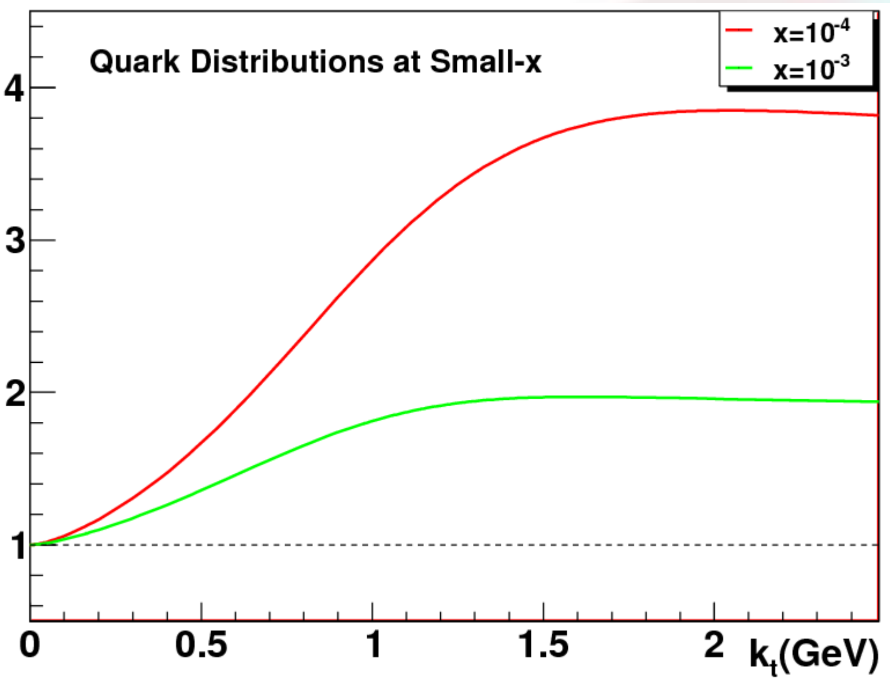
Back-up

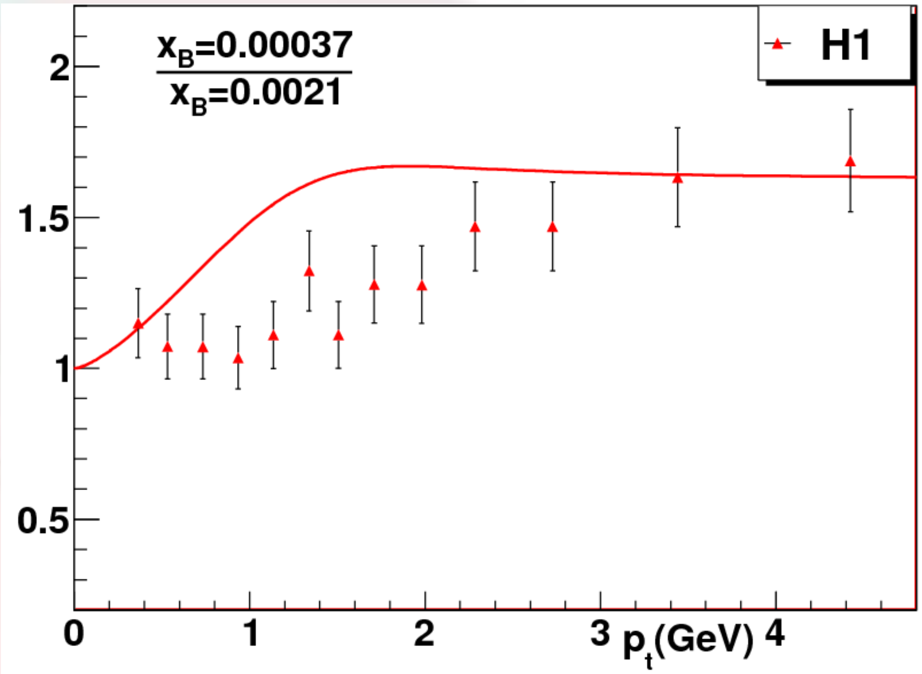
SIDIS at small-x



- What are the relevant scales
  - Q, virtuality of the photon
  - Pt, transverse momentum of hadron
  - Qs, saturation scale
- We are interested in the region of Q>>Qs, Pt
  - TMD factorization makes sense

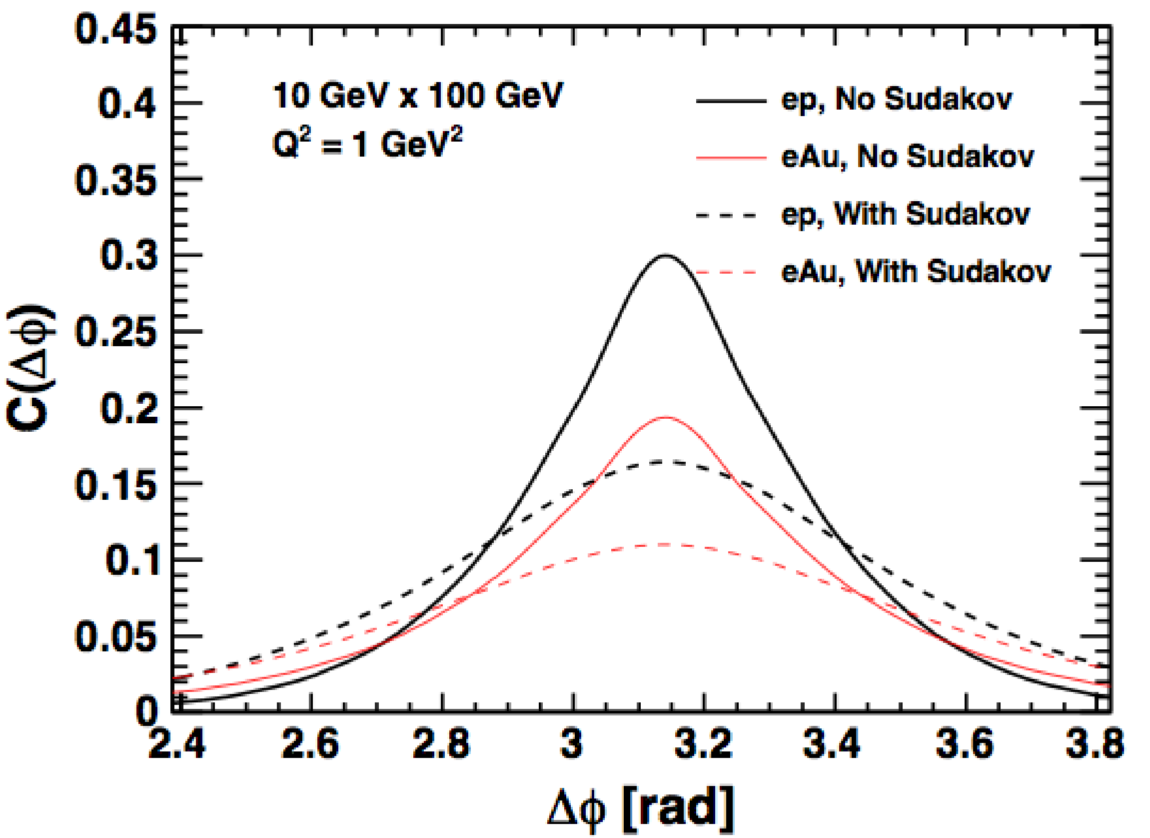
Implication from HERA



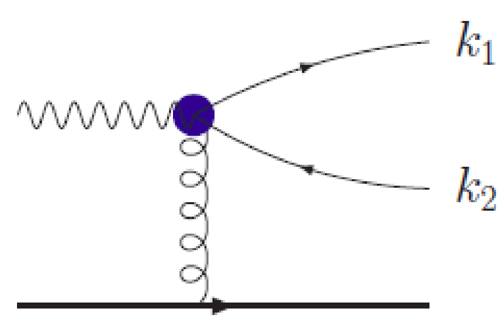


Ratio relative to that at 10^{-2}

Similarly: Gluon TMDs

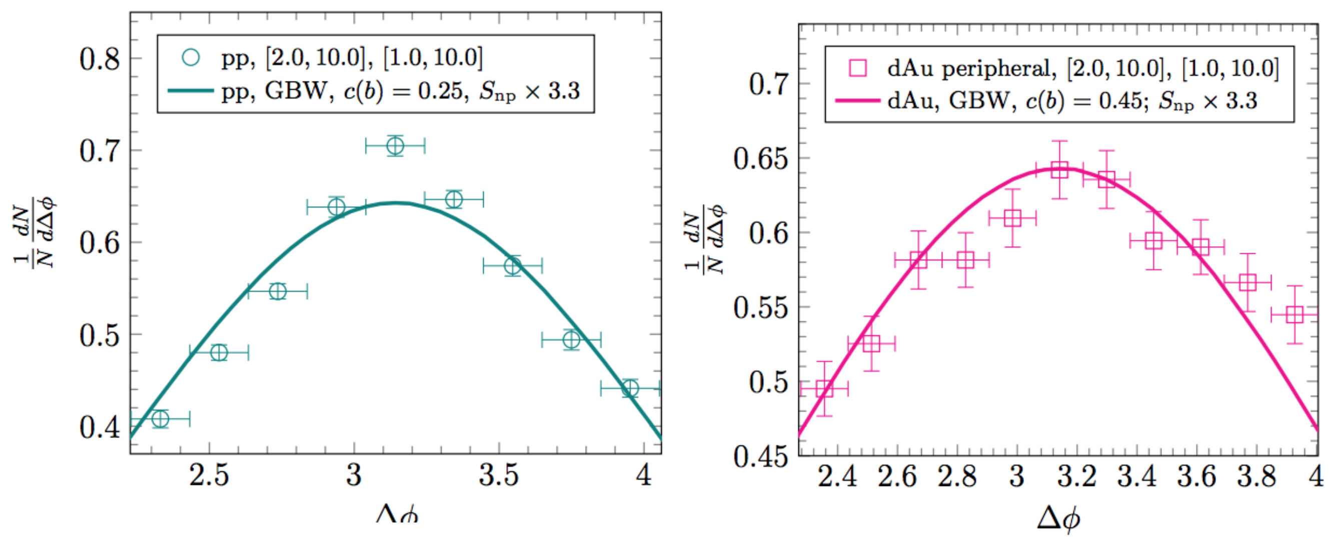


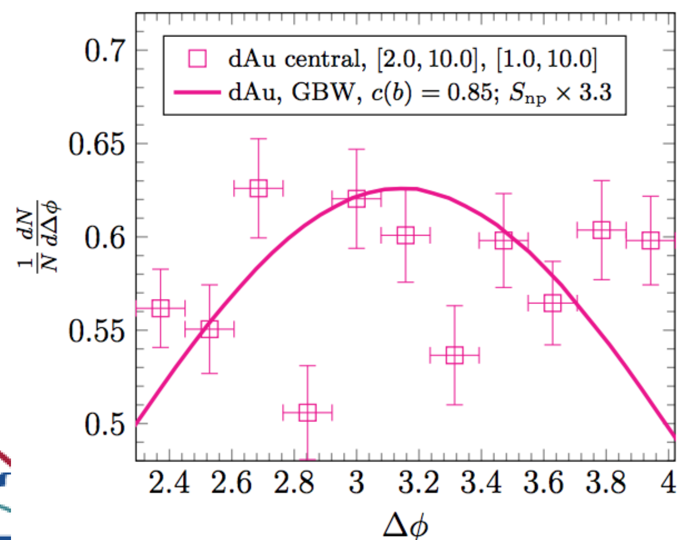
Di-hadron azimuthal Correlations at the Electron-ion Collider



Zheng, Aschenauer,Lee,Xiao, Phys.Rev. D89 (2014) 074037

Compare to RHIC Data





Saturation and Sudakov resummation in a single formula to describe both pp and dAu,

Stasto-Wei-Xiao-Yuan, 1805.0571

TMD gluon from quarkonium productions

Sun,C.-P.Yuan,F.Yuan, PRD 2013
earlier work: Bergers-Qiu-Wang, 2004

- Low pt distribution

$$\frac{d\sigma}{d^2P_\perp dy}|_{P_\perp \ll M} = \frac{1}{(2\pi)^2}\int d^2b\, e^{i\vec{P}_\perp\cdot\vec{b}} W(b,M,x_1,x_2)$$

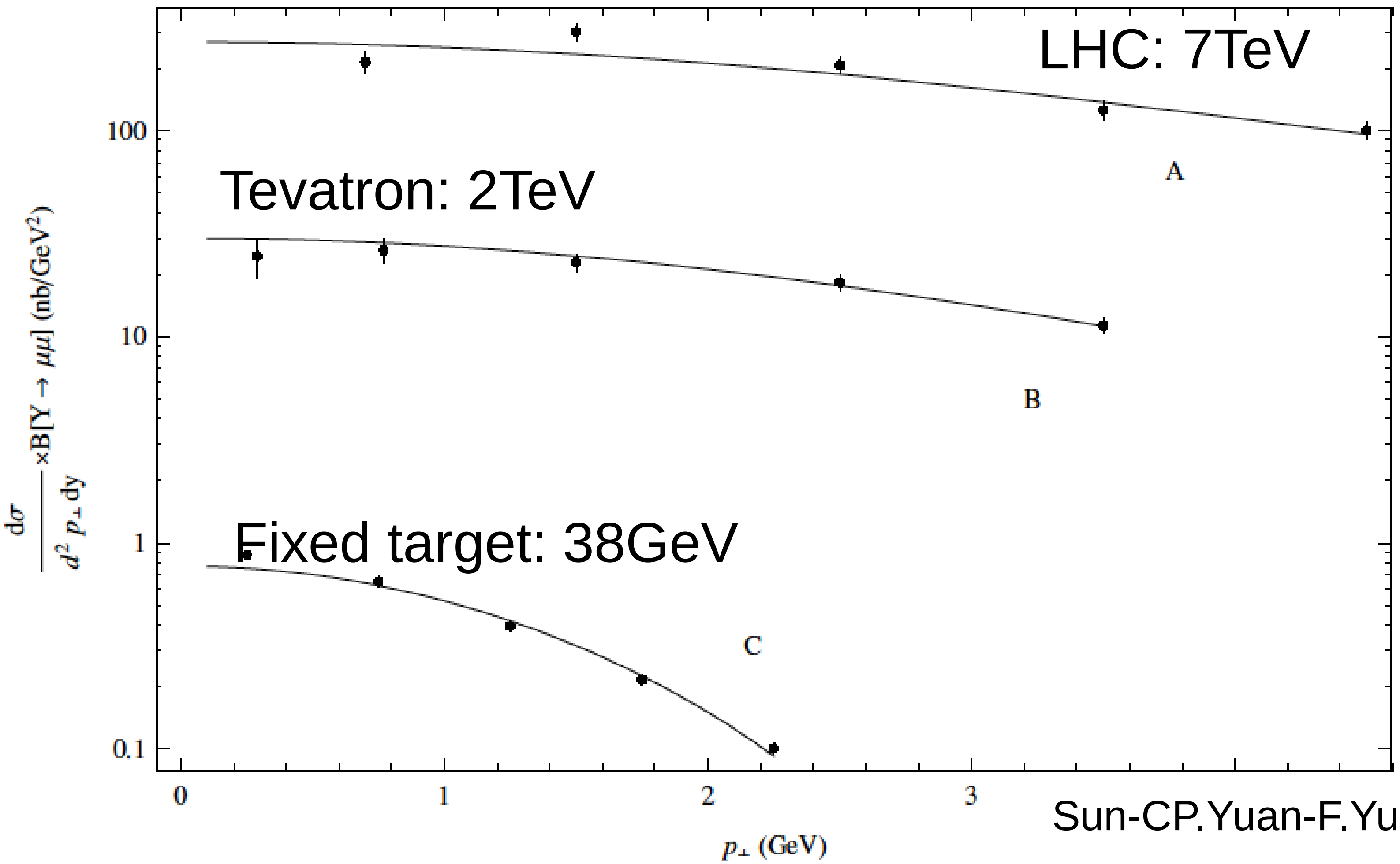
$$W(b,M^2) = e^{-S_{Sud}(M^2,b,C_1,C_2)} W(b,C_1,C_2)$$

- Non-perturbative input

$$W(b) = W(b_*)W^{NP}(b)$$

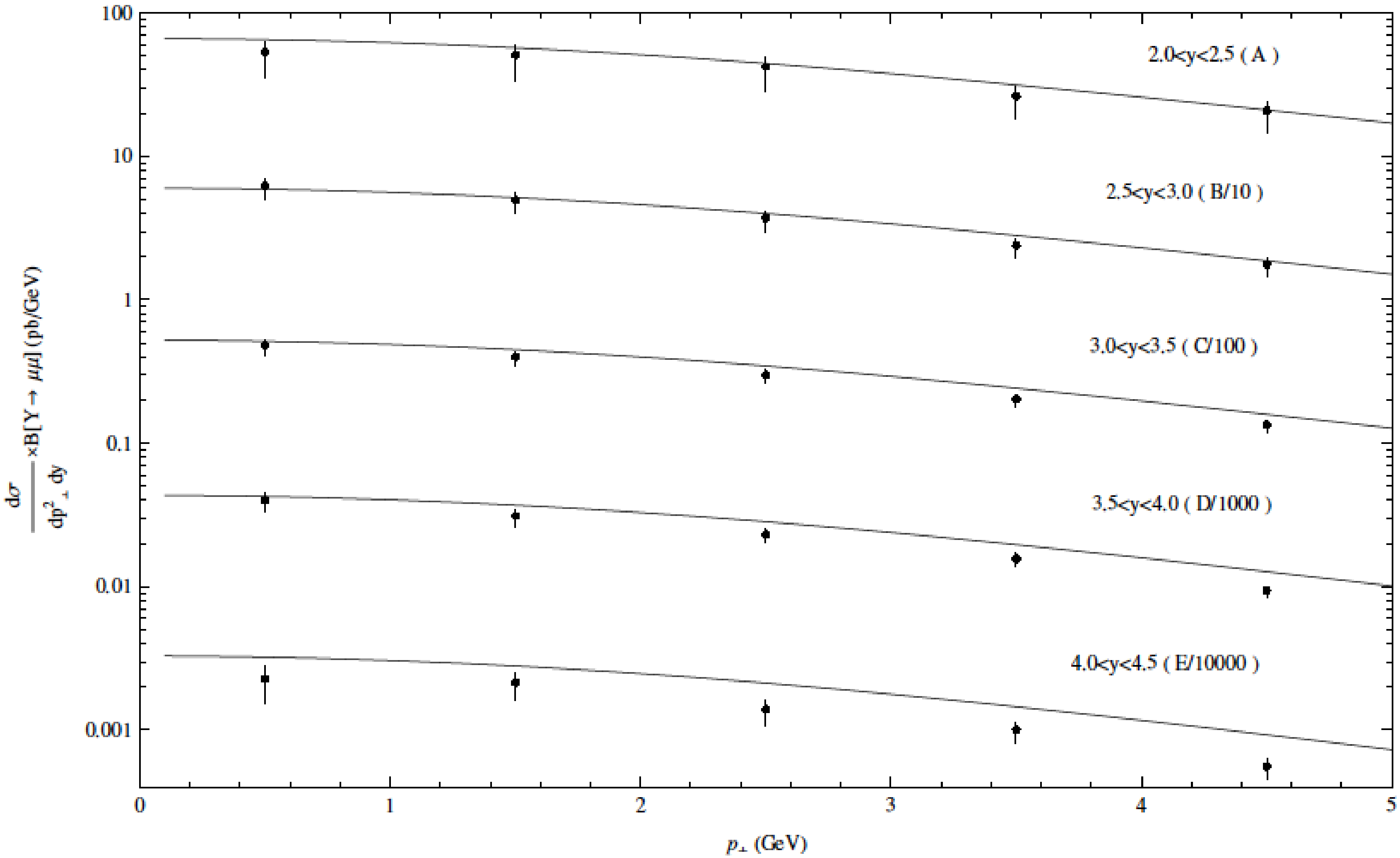
$$W^{NP}(b) = \exp\left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \ln(100 x_1 x_2)\right] b^2$$

Quarkonium production as a probe to the gluon TMDs (Υ)



Sun-CP.Yuan-F.Yuan, 2013

More data from LHC

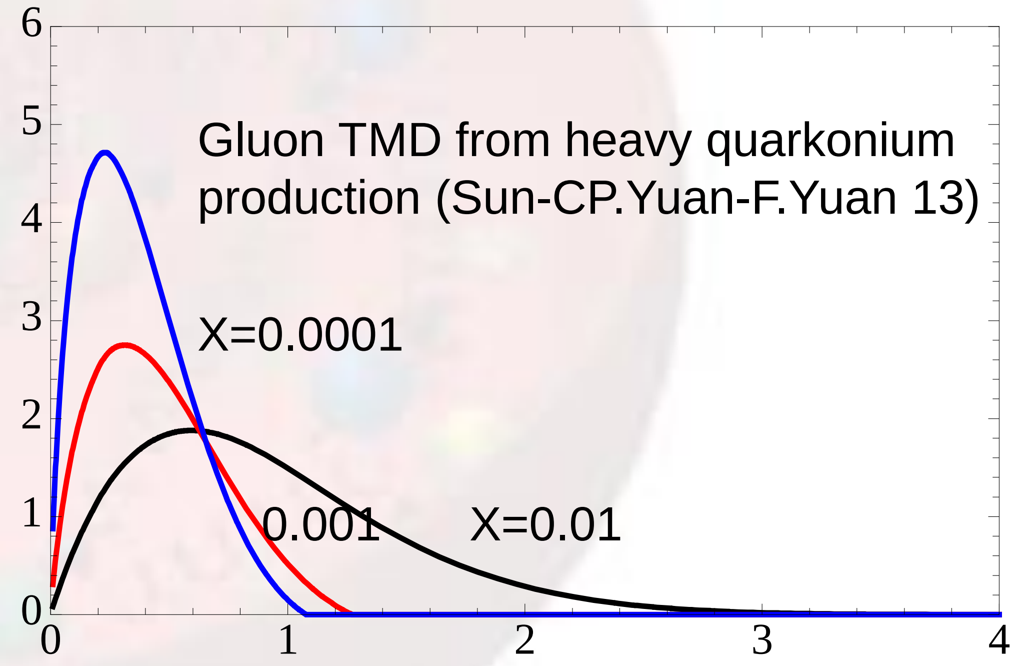


Sun-CP.Yuan-F.Yuan, 2013

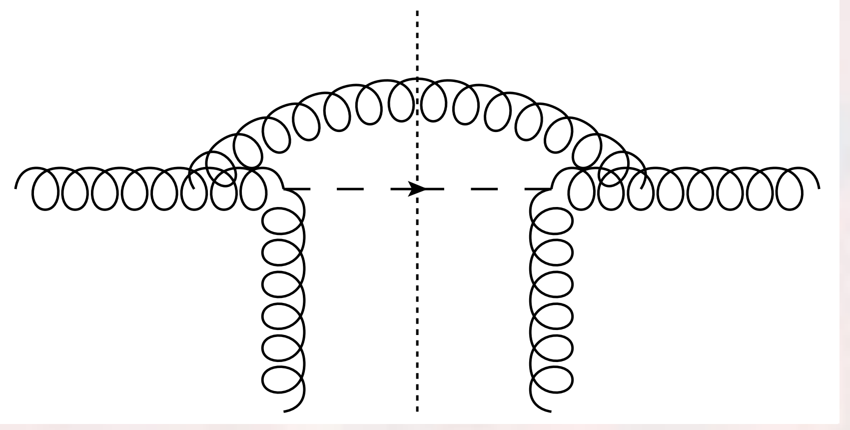
Gluon TMD:
CGC vs Collinear

Model-dependent way
??
Qiu-Sun-Xiao-Yuan 14
Ma-Venugopalan 14/15
Ducloué-Lappi-Mäntysaari 15

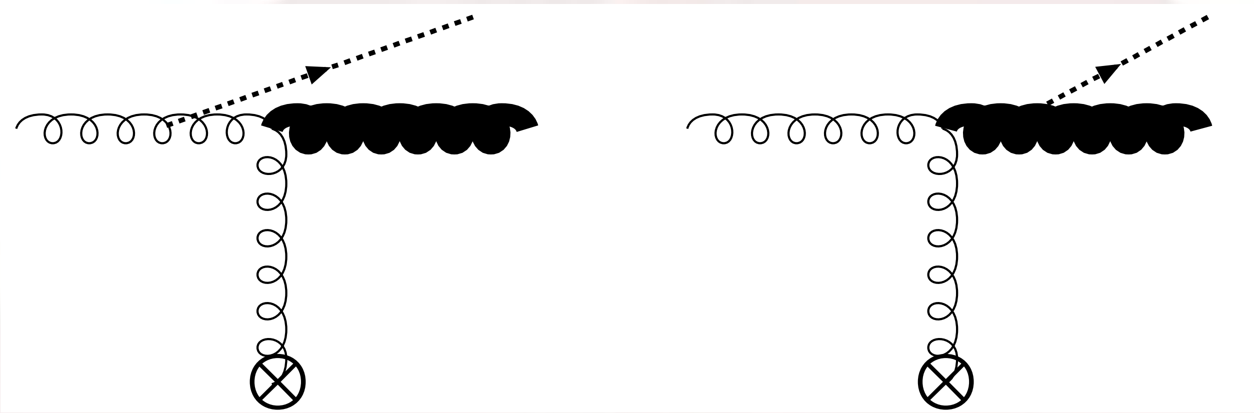
(work in progress)



Examples



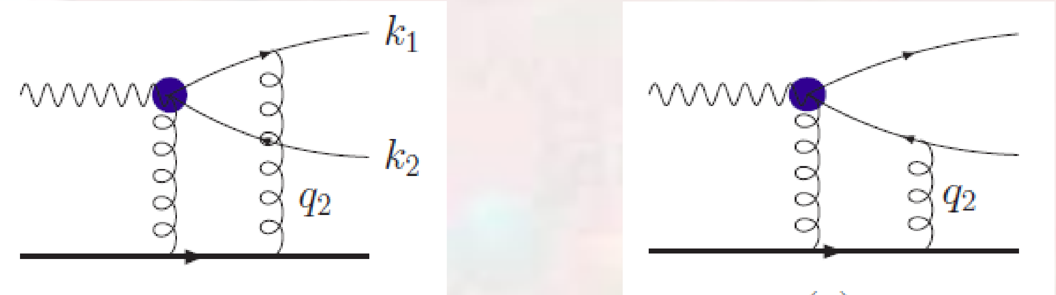
- Contributes to
  - Collinear gluon from the proton
  - Collinear gluon from nucleus
  - Soft gluon to Sudakov double logs



- Only contributes to small-x collinear gluon

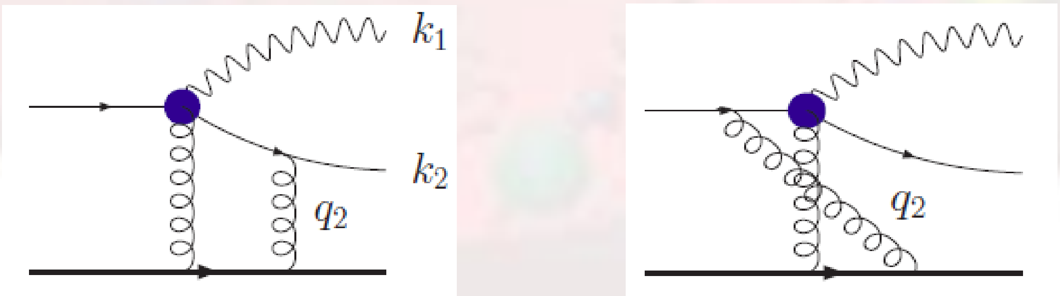
Gluon TMDs

- DIS dijet probes the WW gluon



(b) (c)

- Photon-jet in pA probes dipole gluon



(e) (f)

Dominguez-Marquet-Xiao-Yuan 2011