MASS EFFECTS IN FACTORIZATION

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Introduction

- 12 GeV Experiments at Jlab
  - Data useful for:
    - TMDs and 3-D structure of the nucleon
    - Transition between hadronic and partonic degrees of freedom
    - Quark-hadron duality
  - Correct interpretation requires proper treatment of factorization:
    - Perturbative QCD can still be used
    - Target hadron masses don't satisfy $m << Q$
      - Target Mass Corrections (TMC)
        - When are they relevant?
        - What can we infer about target structure?

DIS Kinematics

- Breit frame momenta:
  - Target:

$$P^\mu = \left(\frac{Q}{x_N\sqrt{2}}, \frac{x_N M^2}{Q\sqrt{2}}, \mathbf{0}_\mathrm{T} \right)$$

Photon:

$$q^\mu = \left(-\frac{Q}{\sqrt{2}}, \frac{Q}{\sqrt{2}}, \mathbf{0}_\mathrm{T} \right)$$

- where:

$$Q = \sqrt{-q^2} \qquad x_N = -\frac{q^+}{P^+} = \frac{2x_{Bj}}{1+\rho} \qquad \rho^2 = 1 + \frac{4x_{Bj}^2 M^2}{Q^2} \qquad x_{Bj} = \frac{Q^2}{2P\cdot q}$$

DIS Cross Section

- Cross Section:

$$E'\frac{d\sigma}{d^3\mathbf{l}'} = \frac{2\alpha_{\text{em}}^2}{(s-M^2)Q^4}L_{\mu\nu}W^{\mu\nu}$$

- Leptonic Tensor:

$$L^{\mu\nu} = 2(l^\mu l'^\nu + l^\nu l'^\mu - g^{\mu\nu} l\cdot l')$$

- Hadronic Tensor:

$$W^{\mu\nu} = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)F_1(x_N, Q) + \left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)\frac{F_2(x_N, Q)}{P\cdot q}$$

Structure Functions

- Projection Operators: $$F_{1,2}(x_N, Q) = \mathrm{P}^{\mu\nu}_{1,2}(x_N, Q, M)\, W_{\mu\nu}$$

$$P_1^{\mu\nu}(x_N, Q, M) = -\frac{1}{2} g^{\mu\nu} + \frac{2x_{Bj}^2}{\rho^2 Q^2} P^\mu P^\nu$$

$$P_2^{\mu\nu}(x_N, Q, M) = -\frac{x_{Bj}}{\rho^2} g^{\mu\nu} + \frac{12 x_{Bj}^3}{Q^2 \rho^4} P^\mu P^\nu$$

- Transverse and Longitudinal Structure Functions:

$$F_T = 2x_{Bj} F_1 \qquad F_L = \rho^2 F_2 - F_T$$

Target Mass Corrections (TMC)

- Large Number of TMC formalisms:
  - Brady, Accardi, Hobbs, Melnitchouk, PR D84, 074008 (2011)
- Interested in a method that relies on no assumptions about dynamics
  - Aivazis, Olness, Tung, PR D50, 3085 (1994) (AOT)

AOT TMC

- Massless Target Approximation (MTA):

$$x_N \to x_{Bj} \qquad P \to \tilde{P} = \left(\frac{Q}{\sqrt{2} x_{Bj}}, 0, \mathbf{0}_{\mathrm{T}} \right)$$

$$P_1^{\mu\nu}(x_N, Q, M) \to \tilde{P}_1^{\mu\nu}(x_{Bj}, Q, 0) = -\frac{1}{2} g^{\mu\nu} + \frac{2x_{Bj}}{Q^2} \tilde{P}^\mu \tilde{P}^\nu$$

$$P_2(x_N, Q, M)^{\mu\nu} \to \tilde{P}_2^{\mu\nu}(x_{Bj}, Q, 0) = -x_{Bj} g^{\mu\nu} + \frac{12 x_{Bj}^3}{Q^2} \tilde{P}^\mu \tilde{P}^\nu$$

$$W^{\mu\nu} \to \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x_{Bj}, Q) + \left(\tilde{P}^\mu - \frac{\tilde{P} \cdot q}{q^2} q^\mu \right) \left(\tilde{P}^\nu - \frac{\tilde{P} \cdot q}{q^2} q^\nu \right) \frac{F_2(x_{Bj}, Q)}{\tilde{P} \cdot q}$$

AOT TMC

- Collinear factorization without the MTA:
  - Approximate target parton momentum:

$$k \to \hat{k} = \left(\xi P^+, 0, \mathbf{0}_{\mathrm{T}}\right)$$

  - Partonic tensor:

$$\hat{W}^{\mu\nu} \to \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)\hat{F}_1(\hat{x}_{Bj}, Q) + \left(\hat{k}^\mu - \frac{\hat{k}\cdot q}{q^2}q^\mu\right)\left(\hat{k}^\nu - \frac{\hat{k}\cdot q}{q^2}q^\nu\right)\frac{\hat{F}_2(\hat{x}_{Bj}, Q)}{\hat{k}\cdot q}$$

  - where

$$\hat{x}_{Bj} = \frac{Q^2}{2\hat{k}\cdot q} = \frac{Q^2}{2\hat{k}^+\cdot q^-} = \frac{x_N}{\xi}$$

AOT TMC

- AOT Structure functions:

$$F_1^{\mathrm{AOT}}(x_N, Q^2) = \int_{x_N}^{1} \frac{d\xi}{\xi} \hat{F}_1(x_N/\xi, Q^2)\, f(\xi) + O(m^2/Q^2)$$

$$F_2^{\mathrm{AOT}}(x_N, Q^2) = \frac{1+\rho}{2\rho^2} \int_{x_N}^{1} d\xi\ \hat{F}_2(x_N/\xi, Q^2)\, f(\xi) + O(m^2/Q^2)$$

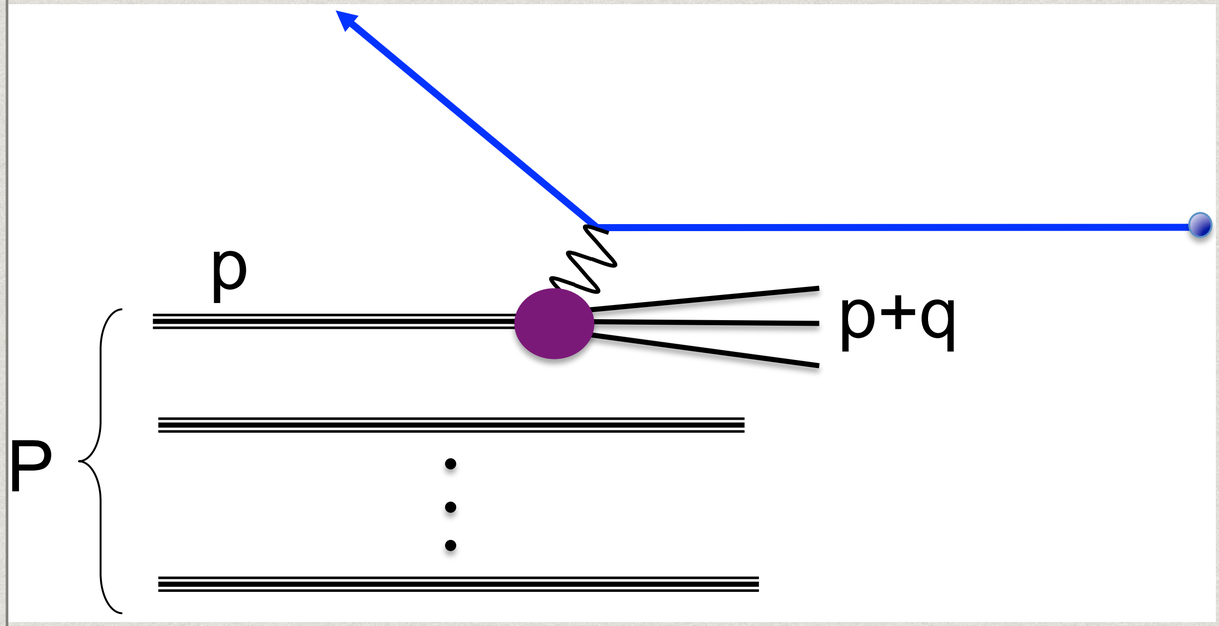
- Applying the MTA:

$$F_1^{\mathrm{MTA}}(x_{Bj}, Q^2) = \int_{x_{Bj}}^{1} \frac{d\xi}{\xi} \hat{F}_1(x_{Bj}/\xi, Q^2)\, f(\xi) + O\left(\max\left[\frac{m^2}{Q^2}, \frac{x_{Bj}^2 M^2}{Q^2}\right]\right)$$

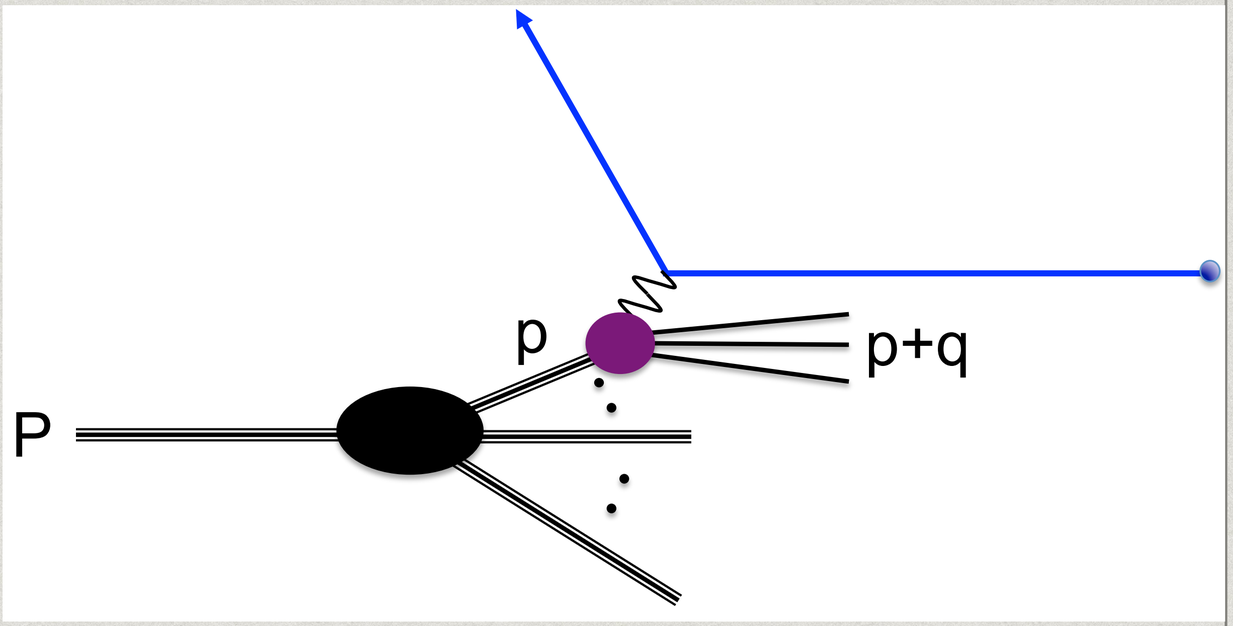
$$F_2^{\mathrm{MTA}}(x_{Bj}, Q^2) = \int_{x_{Bj}}^{1} d\xi\ \hat{F}_2(x_{Bj}/\xi, Q^2)\, f(\xi) + O\left(\max\left[\frac{m^2}{Q^2}, \frac{x_{Bj}^2 M^2}{Q^2}\right]\right)$$

When are target mass effects relevant?

- Target structure depends on scale of partonic interactions relative to subsystem interactions:



Non-Interacting Subsystems



Subsystem Interactions = Partonic Interactions

When are target mass effects relevant?

- Subsystem momentum:

$$p = \left(XP^+, \frac{m_{\mathrm{T}}^2}{2XP^+}, \boldsymbol{p}_{\mathrm{T}} \right)$$

- where

$$m_{\mathrm{T}}^2 = p_{\mathrm{T}}^2 + p^2$$

- Invariant mass of subsystem scattering:

$$m_J^2 = (p+q)^2$$

- Series expansion of momentum fraction:

$$X = x_{Bj} \left[1 + \frac{p_{\mathrm{T}}^2 + m_J^2 - x_{Bj}^2 M^2}{Q^2} - \frac{m_{\mathrm{T}}^2 (p_{\mathrm{T}}^2 + m_J^2) + x_{Bj}^2 M^2 (p_{\mathrm{T}}^2 + m_J^2) - x_{Bj}^4 M^4}{Q^4} + \ldots \right]$$

When are target mass effects relevant?

- p_T, m_T, and m_J:
  - Smallest when subsystems are non-interacting
  - Largest when subsystem and partonic interactions are the same
- Improvement from TMC → Subsystem interactions are at a smaller scale than partonic

Testing in a Simple Field Theory

- AOT methods can be applied to any renormalizable field theory
- Choose a theory for which calculations can be completed without approximation:
  - Calculate exact solution
  - Calculate factorized solutions
  - Compare results

Testing in a Simple Field Theory

- In EM, Melnitchouk, Rogers, Sato, 2017
  - Examined unintegrated low transverse momentum contribution in collinear factorization

$$F_i\left(x_N, Q\right) = \int \frac{d^2\boldsymbol{k}_{\mathrm{T}}}{(2\pi)^2} \mathscr{F}_i(x_N, Q, k_{\mathrm{T}}), \qquad i = 1,2,L, \ldots$$

- For testing TMCs:
  - Transverse momentum integrated collinear factorization

$$F^{(1)}_{i,fact} = \hat{F}^{(0)}_{i,f/f'} \otimes f^{(1)}_{f'/P} + \hat{F}^{(1)}_{i,f/f'} \otimes f^{(0)}_{f'/P} \qquad A \otimes B \equiv \int_{x_{Bj,N}}^{1} \frac{d\xi}{\xi} A(x_{Bj,N}/\xi) B(\xi)$$

The Simple Field Theory

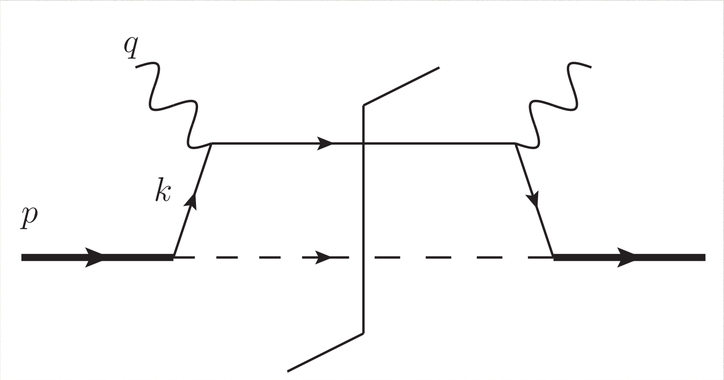
- Interaction Lagrangian Density:

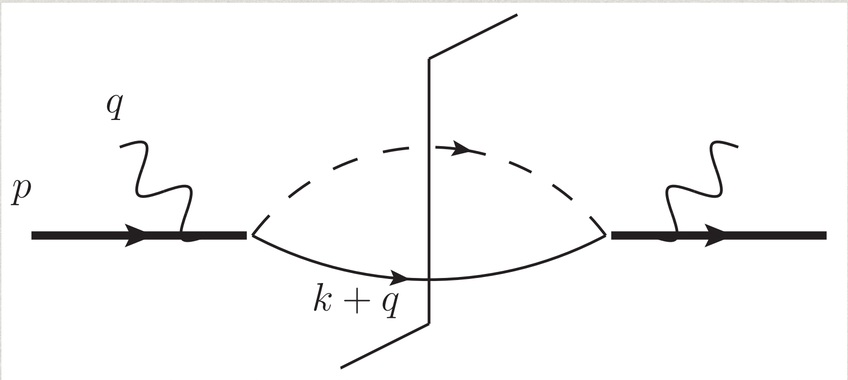
$$\mathscr{L}_{\text{int}} = -\lambda \overline{\Psi}_N \psi_q \phi \; + \; \text{H.c.}$$

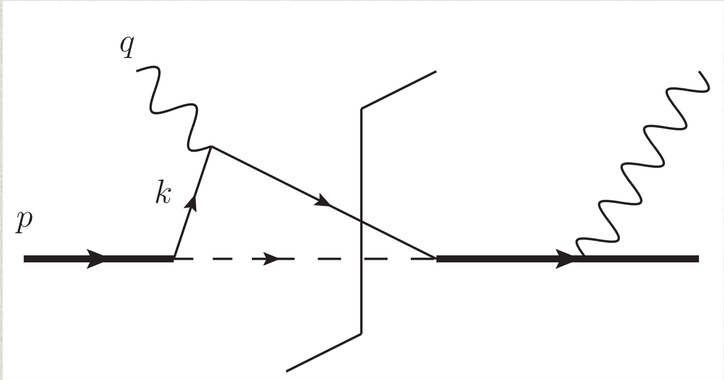
- where:
  - Ψ_N = Spin-1/2 “nucleon” with mass m_p
  - ψ_q = Spin-1/2 “quark” with mass m_q
  - ϕ = Scalar “di-quark” with mass m_s
- Nucleon and quark interact with the photon, but di-quark is a spectator
- Coupling constant:

$$a_\lambda = \frac{\lambda^2}{16\pi^2}$$

Order a_λ Diagrams



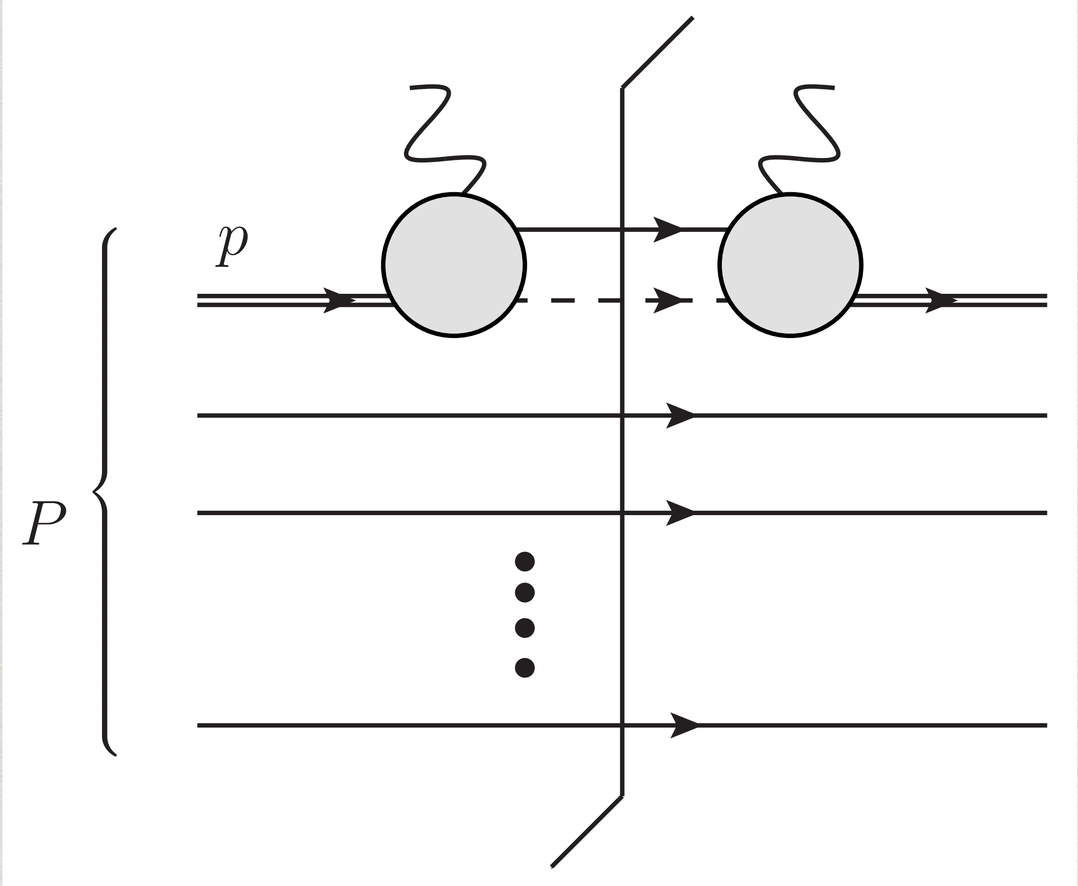




TMCs in the Yukawa Theory

- Need target mass ~ Q
- If $m_p > m_q + m_s$, unstable particle decay
- Larger target of mass M made up of non-interacting subsystems

$$\frac{d\sigma}{d^3\mathbf{l}'} = \int dX \frac{d\sigma^p}{d^3\mathbf{l}'} \delta(X - c)$$



TMCs in the Yukawa Theory

- Exact

$$F_1^P(x_N) = \frac{1}{c} F_1^p(x_N^p) \qquad F_2^P(x_N) = \frac{\left(Q^2 - M^2 x_N^2\right)\left(Q^2 + m_p^2 x_N^2\right)^2}{\left(Q^2 + M^2 x_N^2\right)^2\left(Q^2 - m_p^2 x_N^2\right)} F_2^p(x_N^p)$$

- Massless Target Approximation

$$F_1^P(x_N) = \frac{1}{c} F_{1,fact}^p(x_{Bj}/c) \qquad F_2^P(x_N) = F_{2,fact}^p(x_{Bj}/c)$$

- AOT Target Mass Corrected

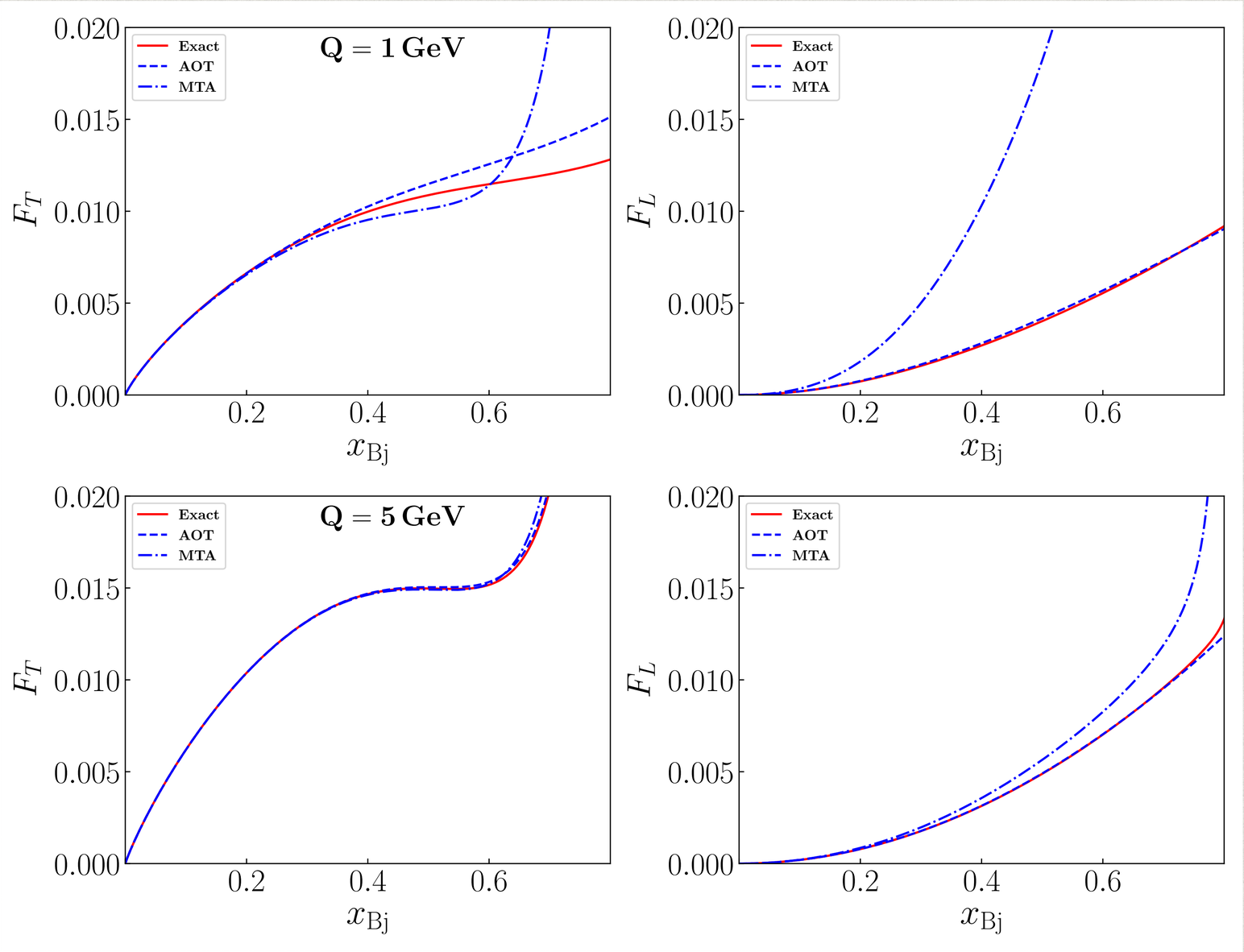
$$F_1^P(x_N) = \frac{1}{c} F_{1,fact}^p(x_N/c) \qquad F_2^P(x_N) = \frac{Q^2\left(Q^2 - M^2 x_N^2\right)}{\left(Q^2 + M^2 x_N^2\right)^2} F_{2,fact}^p(x_N/c)$$

- Where

$$x_N = \frac{2x_{Bj}}{1 + \sqrt{1 + \frac{x_{Bj}^2 M^2}{Q^2}}} \qquad x_{Bj} = \frac{Q^2}{2P \cdot q} \qquad x_N^p = \frac{2x_{Bj}^p}{1 + \sqrt{1 + \frac{x_{Bj}^{p2} m_p^2}{Q^2}}} \qquad x_{Bj}^p = \frac{Q^2}{2p \cdot q} = \frac{x_N/c}{1 - \frac{x_N^2 M^2}{c^2 Q^2}}$$

Results

- c=0.8
- M=0.938 GeV
- m_p=m_q=m_s=0.0938 GeV



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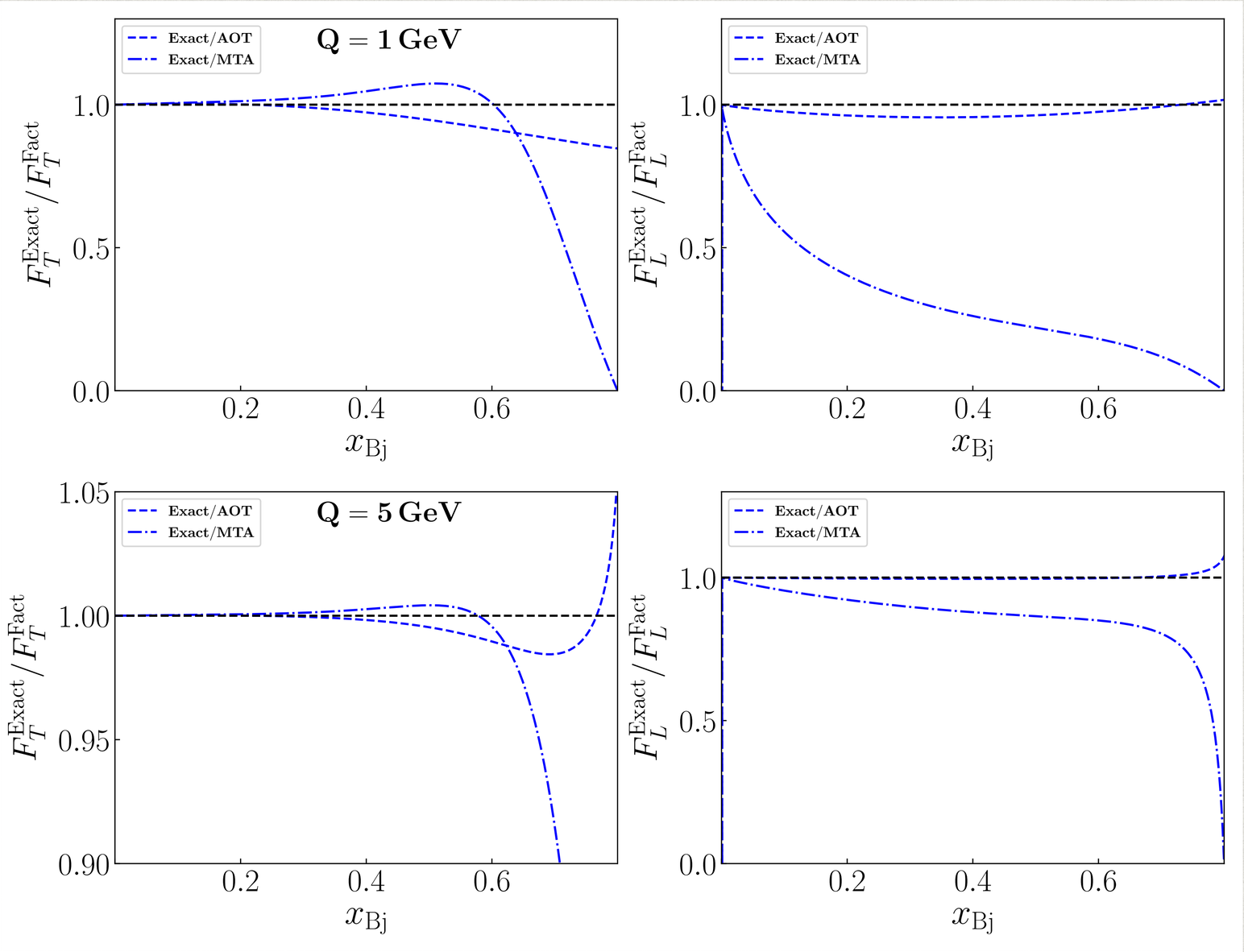

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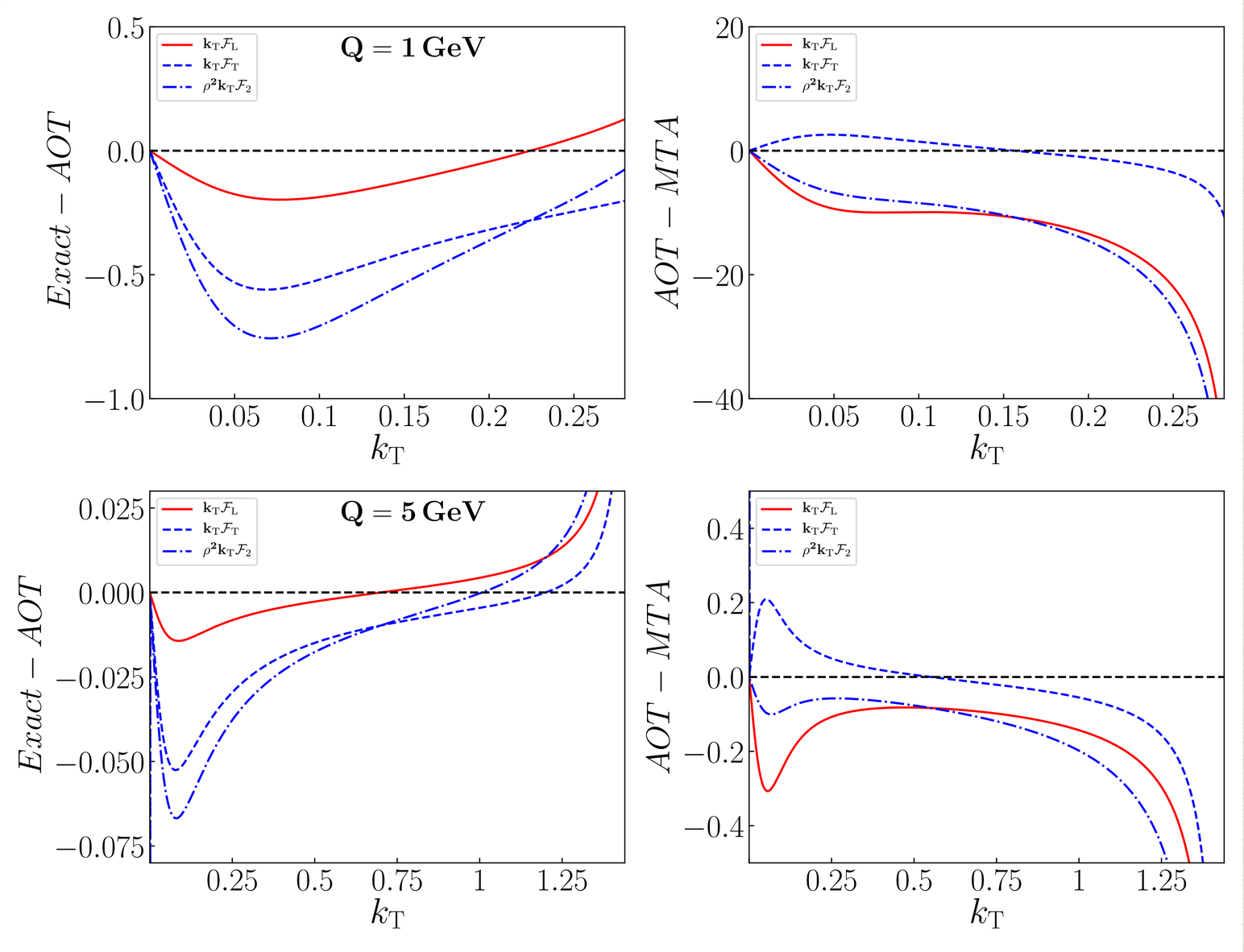

Conclusion

- Target masses are relevant when:
  - x_{Bj} is large
  - $M \sim Q$
  - $x_{Bj} M/Q$ Corrections >> Dynamic Corrections
- Large TMC improvement relative to $F_1(F_T)$ or F_2 data implies distinctive subsystem and partonic interaction scales
- Due to larger TMC effects, F_L data would be best suited for comparing the relative effectiveness of different TMC methodologies