The Current Status of Studies of TMDs

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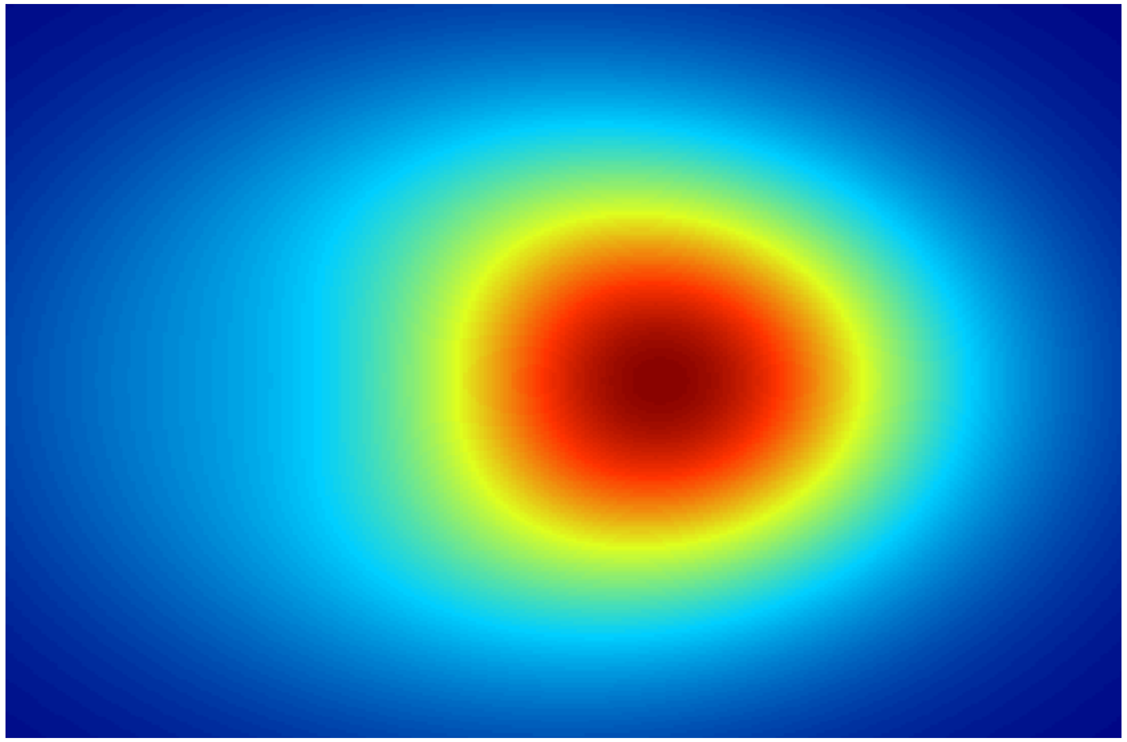


The polarized proton in momentum space as "seen" by the virtual photon

Factorization theorems help us to relate functions that describe the hadron structure and the experimental observables

Factorization is a *controllable approximation* and the goal of theorists and phenomenologists is to test and improve the region of applicability of factorization and/or construct new factorization theorems

Hadron structure is the ultimate goal of measurements and phenomenology

The main goal of phenomenology now is to have a well defined methodology that allows to study hadron structure

Why Spin?

Xiangdong Ji at DIS08

Spin is a fundamental quantum degree of freedom

Spin plays a critical role in determining the basic structure of fundamental interactions

Test of a theory is not complete without a full test of spin-dependent decays and scattering

Spin provides a unique opportunity to probe the inner structure of a composite system (such as the proton)

Quark TMDs

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Collinear functions

8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each depends on Bjorken-x, transverse momentum, the scale

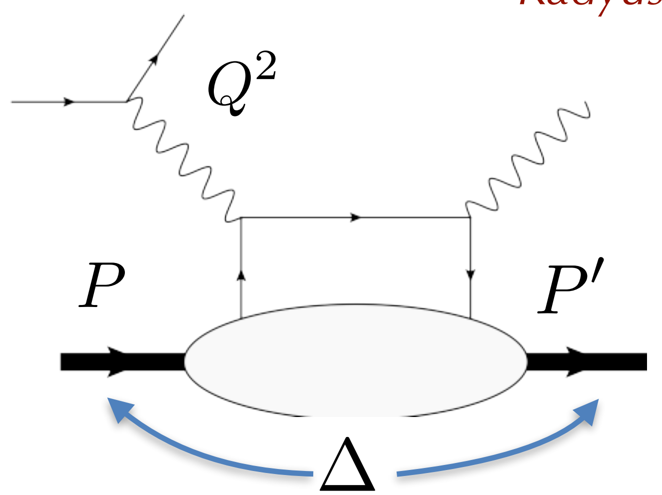
Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

GPDs   TMDs

DVCS

Ji (1997)
Radyushkin (1997)



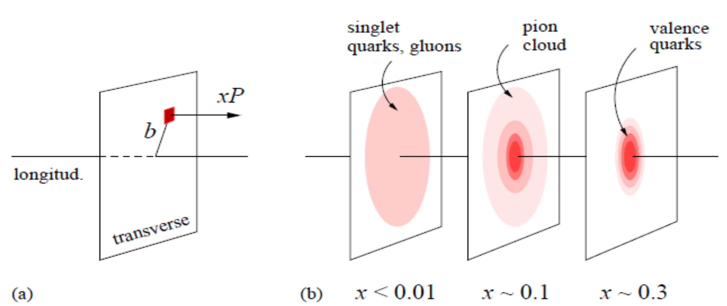
Q^2 ensures hard scale, pointlike interaction

$\Delta = P' - P$ momentum transfer can be varied independently

Connection to 3D structure   Burkardt (2000) Burkardt (2003)

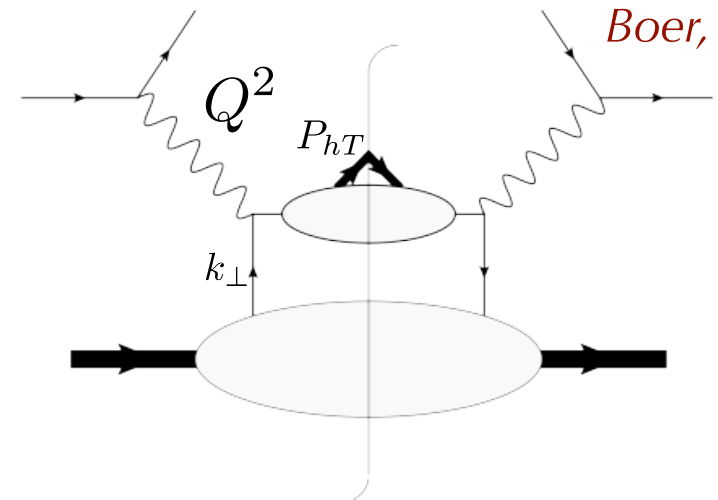
$$\rho(x,\vec{r}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{r}_\perp} H_q(x,\xi=0,t=-\vec{\Delta}_\perp^2)$$

Drell-Yan frame $\Delta^+ = 0$ Weiss (2009)



SIDIS

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)



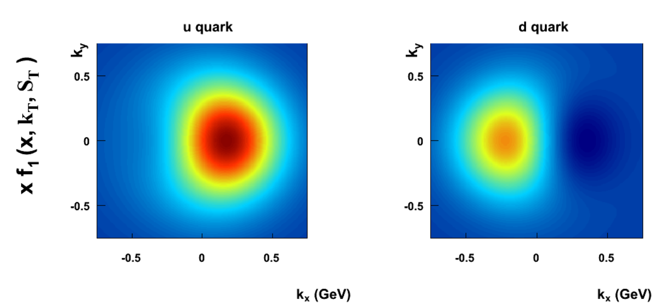
Q^2 ensures hard scale, pointlike interaction

P_{hT} final hadron transverse momentum can be varied independently

Connection to 3D structure   Ji, Ma, Yuan (2004) Collins (2011)

$$\tilde{f}(x,\vec{b}) = \int d^2k_\perp e^{-i\vec{b}\cdot\vec{k}_\perp} f(x,\vec{k}_\perp)$$

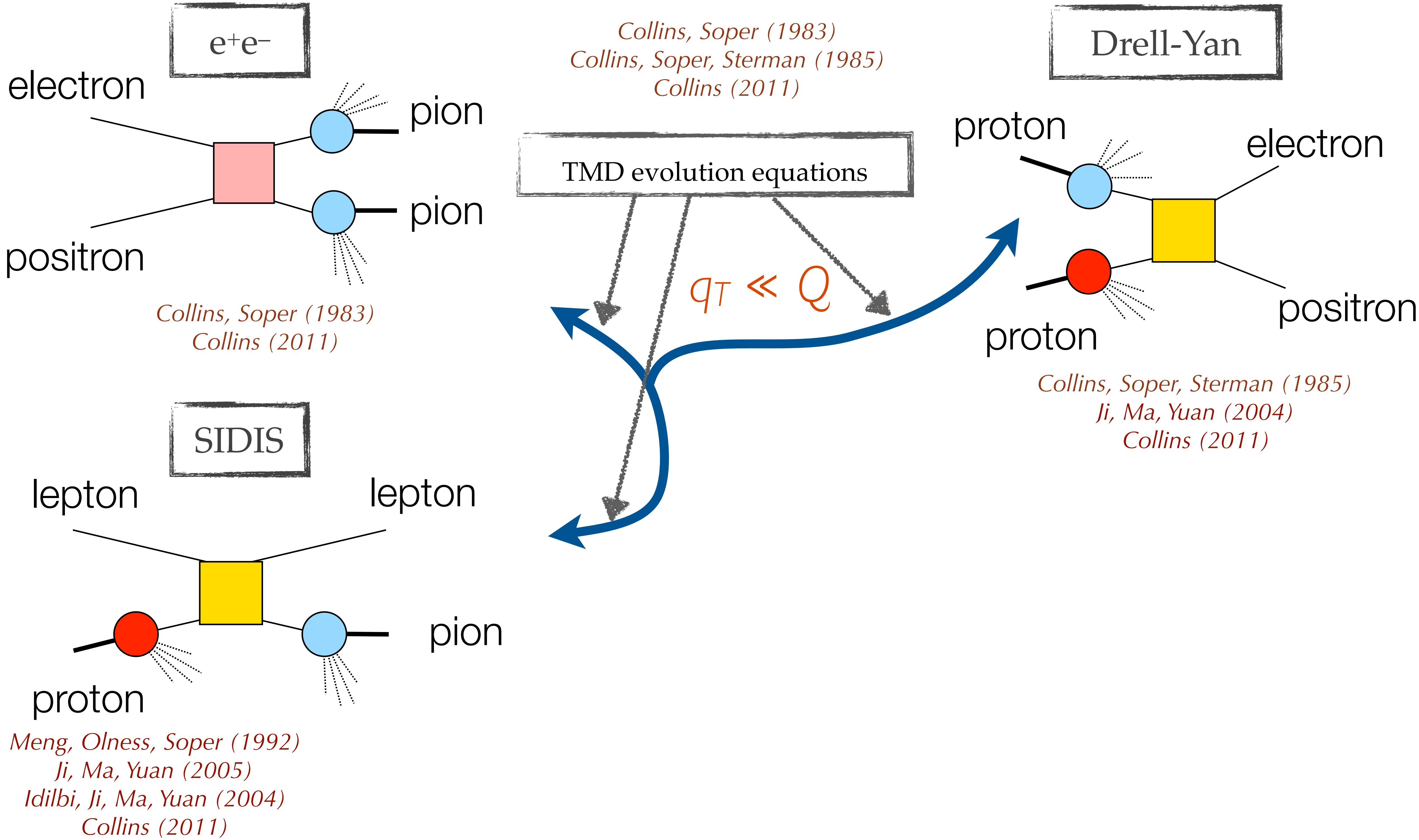
$\vec{b}$ is the transverse separation of parton fields in configuration space



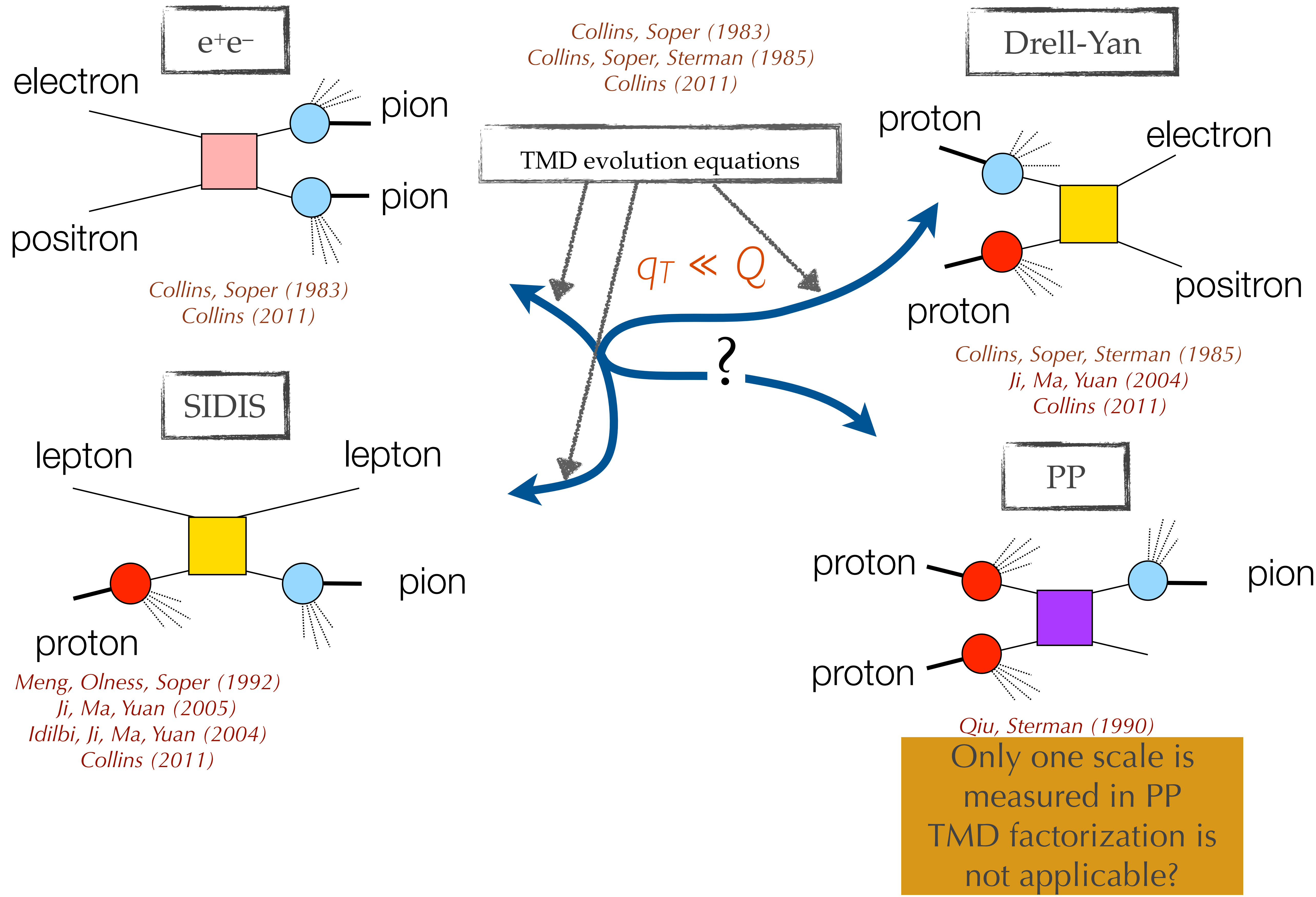
AP (2012)

Why TMDs, factorization, and evolution

TMD factorization



TMD factorization



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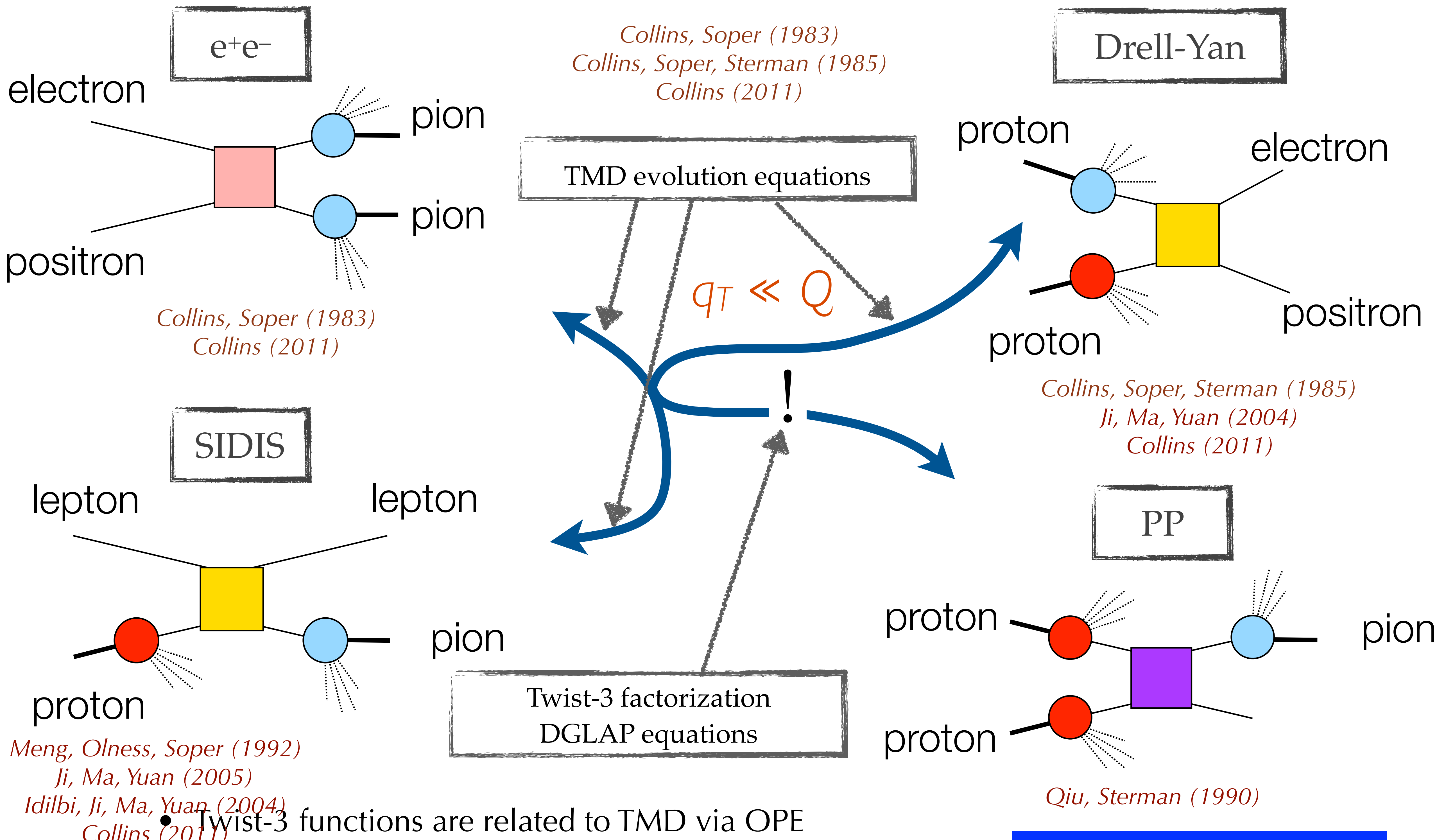
TMD factorization



- Twist-3 functions are related to TMD via OPE
- TMD and twist-3 factorizations are related in high QT region
- Global analysis of TMDs and twist-3 is possible:

All four processes can be used.

- Data are from HERMES, COMPASS, JLab,

BaBar, Belle, RHIC, LHC, Fermilab

Global fit is needed.
Work in progress

TMD factorization vs CSS

- Collins-Soper-Sterman formalism (1985) provides a method to calculate a differential in q_T cross section taking into account logarithmic corrections up to all orders in a_s.

- TMD formalism (Collins 2011) and CSS results can be matched

$$
\begin{aligned}
\frac{d\sigma}{dQ^2 dy dq_T^2} = {} & \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} H^{\mathrm{DY}}_{j\bar{j}}(Q, \mu_Q, a_s(\mu_Q)) \int \frac{d^2 \boldsymbol{b}_T}{(2\pi)^2} e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T} \\
& \times e^{-g_{j/A}(x_A, b_T; b_{\max})} \int_{x_A}^{1} \frac{d\xi_A}{\xi_A} f_{j_A/A}(\xi_A; \mu_{b_*}) \tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*; \mu^2_{b_*}, \mu_{b_*}, a_s(\mu_{b_*})\right) \\
& \times e^{-g_{\bar{j}/B}(x_B, b_T; b_{\max})} \int_{x_B}^{1} \frac{d\xi_B}{\xi_B} f_{j_B/B}(\xi_B; \mu_{b_*}) \tilde{C}^{\mathrm{PDF}}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*; \mu^2_{b_*}, \mu_{b_*}, a_s(\mu_{b_*})\right) \\
& \times \exp\left\{ -g_K(b_T; b_{\max}) \ln\frac{Q^2}{Q_0^2} + \tilde{K}(b_*; \mu_{b_*}) \ln\frac{Q^2}{\mu^2_{b_*}} + \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma_j(a_s(\mu')) - \ln\frac{Q^2}{(\mu')^2} \gamma_K(a_s(\mu')) \right] \right\} \\
& + \text{suppressed corrections.}
\end{aligned}
$$

Collins, Rogers Phys.Rev. D96 (2017), 054011

- The main advantage of TMD formalism with respect to CSS - well defined individual TMD (universal) functions - is a possibility of a global fit of experimental data from various processes .

- Universal or quasi-universal TMD functions encode the hadron structure we are interested in.

- Perturbative quantities are known up to several orders in log resummation in CSS and can be readily used in TMD formalism

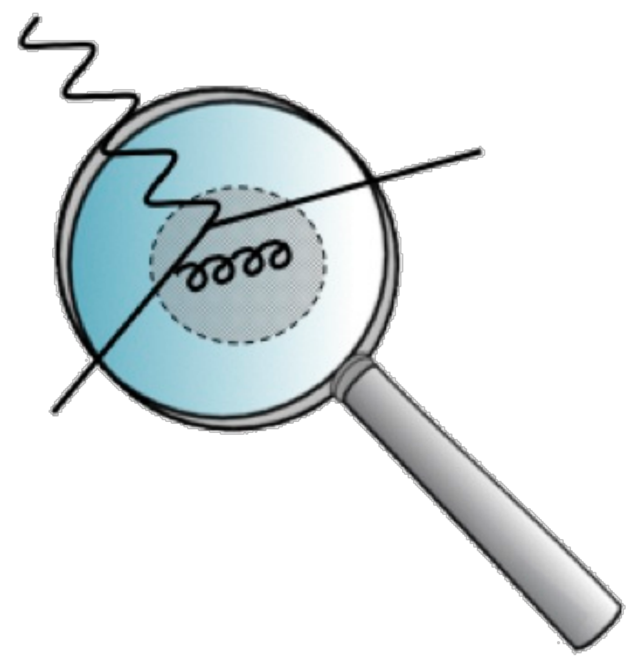
- As TMD formalism deals with regulated TMD functions, some ambiguity in hard factor that is present in CSS is resolved *Prokudin, Sun, Yuan, Phys.Lett. B750 (2015) 533-538*

Just like collinear PDFs, TMDs also depend on the scale of the probe = evolution

Collinear PDFs

$$F(x, Q)$$

- ✓ DGLAP evolution
- ✓ Resum $\left[\alpha_s \ln(Q^2/\mu^2)\right]^n$
- ✓ Kernel: purely **perturbative**



TMDs

$$F(x, k_\perp; Q)$$

- ✓ Collins-Soper/rapidity evolution equation
- ✓ Resum $\left[\alpha_s \ln^2(Q^2/k_\perp^2)\right]^n$
- ✓ Kernel: can be **non-perturbative** when $k_\perp \sim \Lambda_{\rm QCD}$

$$F(x, Q_i)$$
$$\downarrow$$
$$R^{\rm coll}(x, Q_i, Q_f)$$
$$\downarrow$$
$$F(x, Q_f)$$

$$F(x, k_\perp, Q_i)$$
$$\downarrow$$
$$R^{\rm TMD}(x, k_\perp, Q_i, Q_f)$$
$$\downarrow$$
$$F(x, k_\perp, Q_f)$$

TMD evolution and non-perturbative component

- Fourier transform back to the momentum space, one needs the whole b region (large b): need some non-perturbative extrapolation
  - Many different methods/proposals to model this non-perturbative part

$$F(x, k_\perp; Q) = \frac{1}{(2\pi)^2}\int d^2b e^{ik_\perp \cdot b} F(x, b; Q) = \frac{1}{2\pi}\int_0^\infty db\, b J_0(k_\perp b) F(x, b; Q)$$

Collins, Soper, Sterman 85, ResBos, Qiu, Zhang 99, Echevarria, Idilbi, Kang, Vitev, 14, Aidala, Field, Gamberg, Rogers, 14, Sun, Yuan 14, D'Alesio, Echevarria, Melis, Scimemi, 14, Rogers, Collins, 15, Vladimirov, Scimemi 17…

- Eventually evolved TMDs in b-space

$$F(x, b; Q) \approx C \otimes F(x, c/b^*) \times \exp\left\{-\int_{c/b^*}^{Q} \frac{d\mu}{\mu}\left(A \ln\frac{Q^2}{\mu^2} + B\right)\right\} \times \exp\left(-S_{\text{non-pert}}(b, Q)\right)$$

longitudinal/collinear part

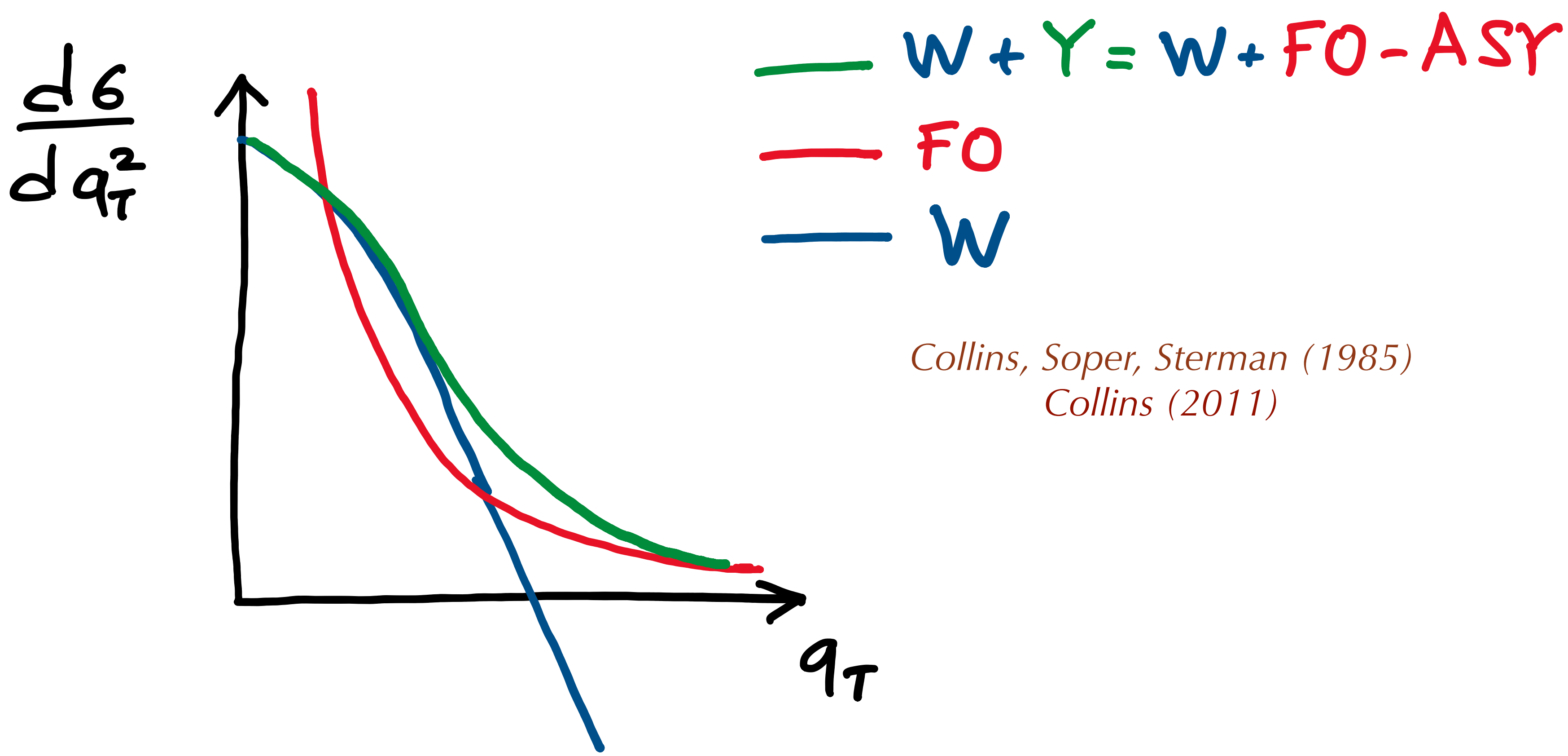
transverse part

✓ Non-perturbative: fitted from data
✓ The key ingredient – ln(Q) piece is spin-independent

Since the polarized scattering data is still limited kinematics, we can use unpolarized data to constrain/ extract key ingredients for the non-perturbative part

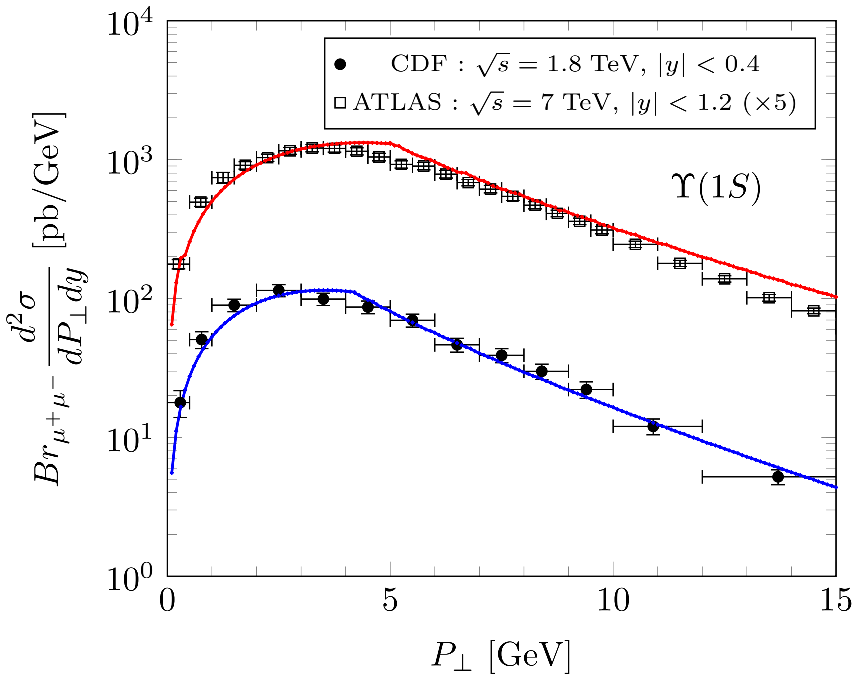
TMD factorization

- TMD factorization organizes a differential in q_T cross section as a convolution of TMD functions (W term) in the region of applicability of TMD factorization $q_T \ll Q$
- CSS formalism provides a W+Y method to make the cross section accurate in a wide region of q_T by adding a Y term, which is a difference of a Fixed Order calculation in collinear approximation and its asymptotic expansion $q_T \to 0$
- At some large $q_T \sim Q$ calculation is switched to a Fixed Order



Collins, Soper, Sterman (1985)
Collins (2011)

Success of TMD factorization predictive power



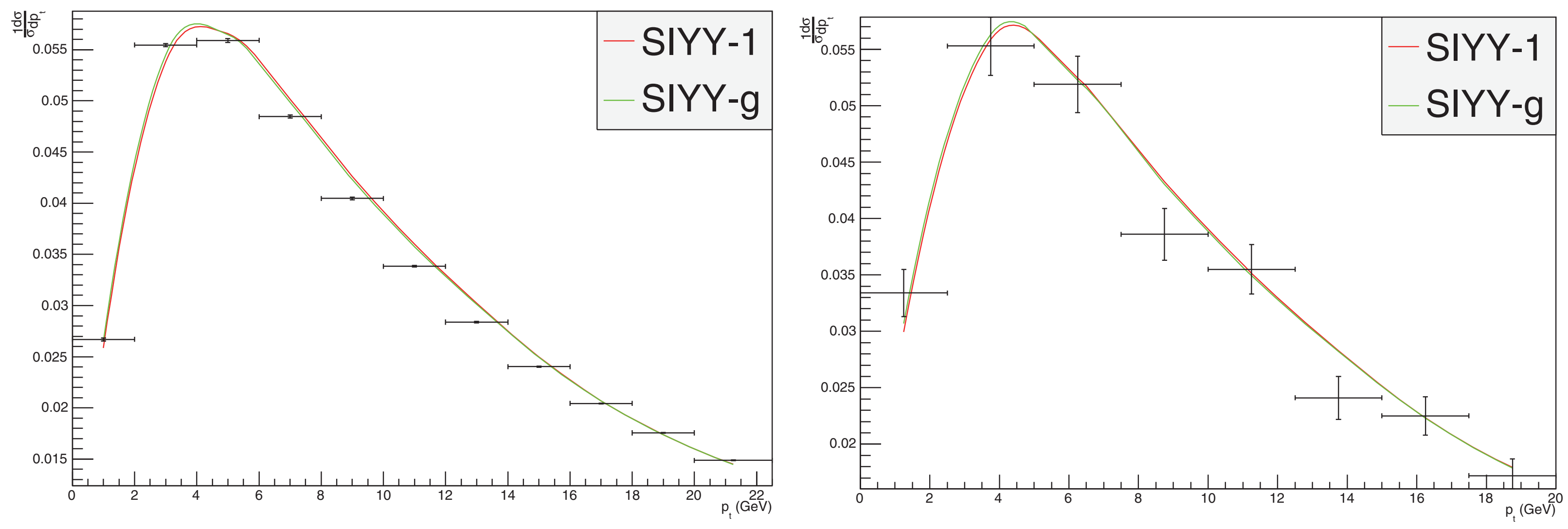


Fig. 8. Compare the resummation prediction for Z boson production at the LHC.[49–51] The data in left one is from the ATLAS collaboration, the right one is for CMS collaboration. These data are not included in our fit.

Upsilon production at the LHC

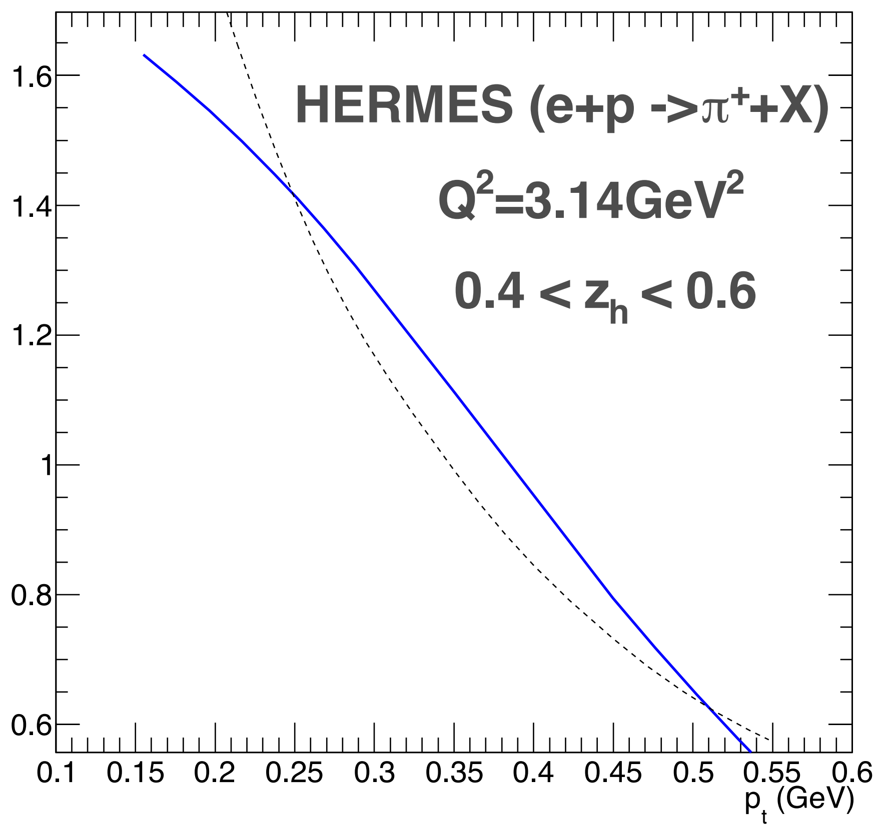
Qiu, Watanabe arXiv:1710.06928

Z boson production at the LHC

Sun, Isaacson, Yuan, Yuan arXiv:1406.3073

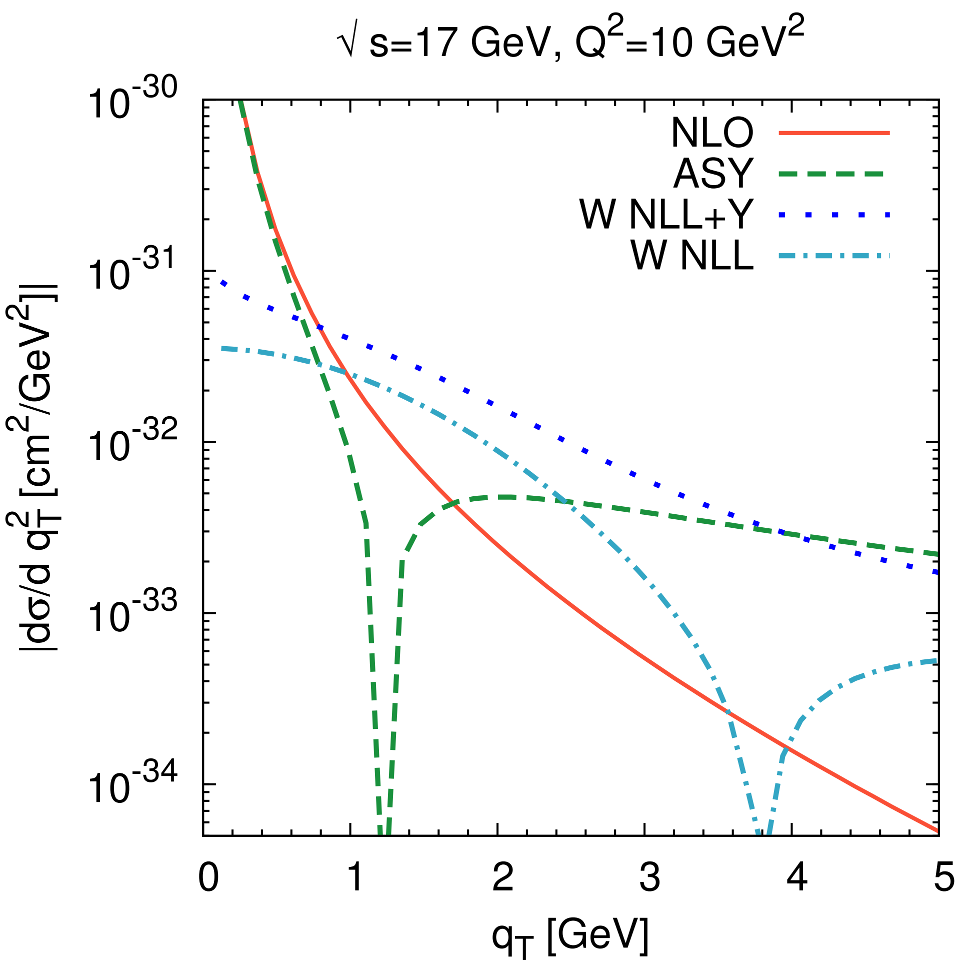
- TMD factorization (with an appropriate matching to collinear results) aims at an accurate description (and prediction) of a differential in q_T cross section in a wide range of q_T
- LHC results at 7 and 13 TeV are accurately predicted from fits of lower energies

"Problems" of TMD factorization at low Q



W (solid line) and Y terms (dashed line)

Sun, Isaacson, Yuan, Yuan arXiv:1406.3073



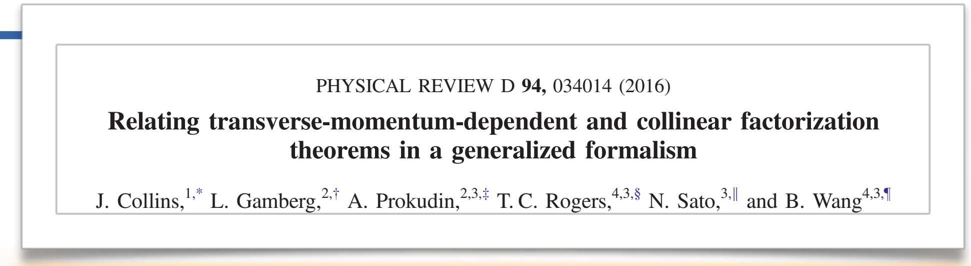
Boglione, Gonzalez, Melis, AP arXiv:1412.1383

- At low Q the Y term becomes unreasonably large (larger than the W term) in the region of the maximal validity of TMD factorization (cross section should be given by W with a small error)
- W term changes sign at a different qT compared to ASY, making matching problematic
- The reason: Y=FO-ASY has constant terms that do not depend on q_T and may be large compared to W if cross section itself is small

PHYSICAL REVIEW D **94,** 034014 (2016)

Relating transverse-momentum-dependent and collinear factorization theorems in a generalized formalism

J. Collins,[1,*] L. Gamberg,[2,†] A. Prokudin,[2,3,‡] T. C. Rogers,[4,3,§] N. Sato,[3,‖] and B. Wang[4,3,¶]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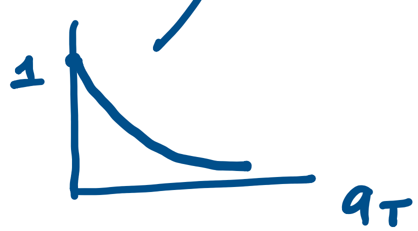
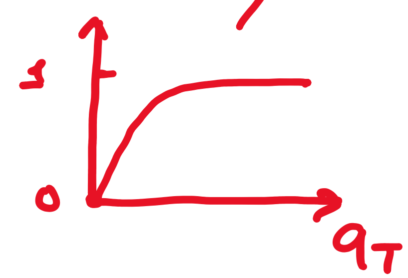
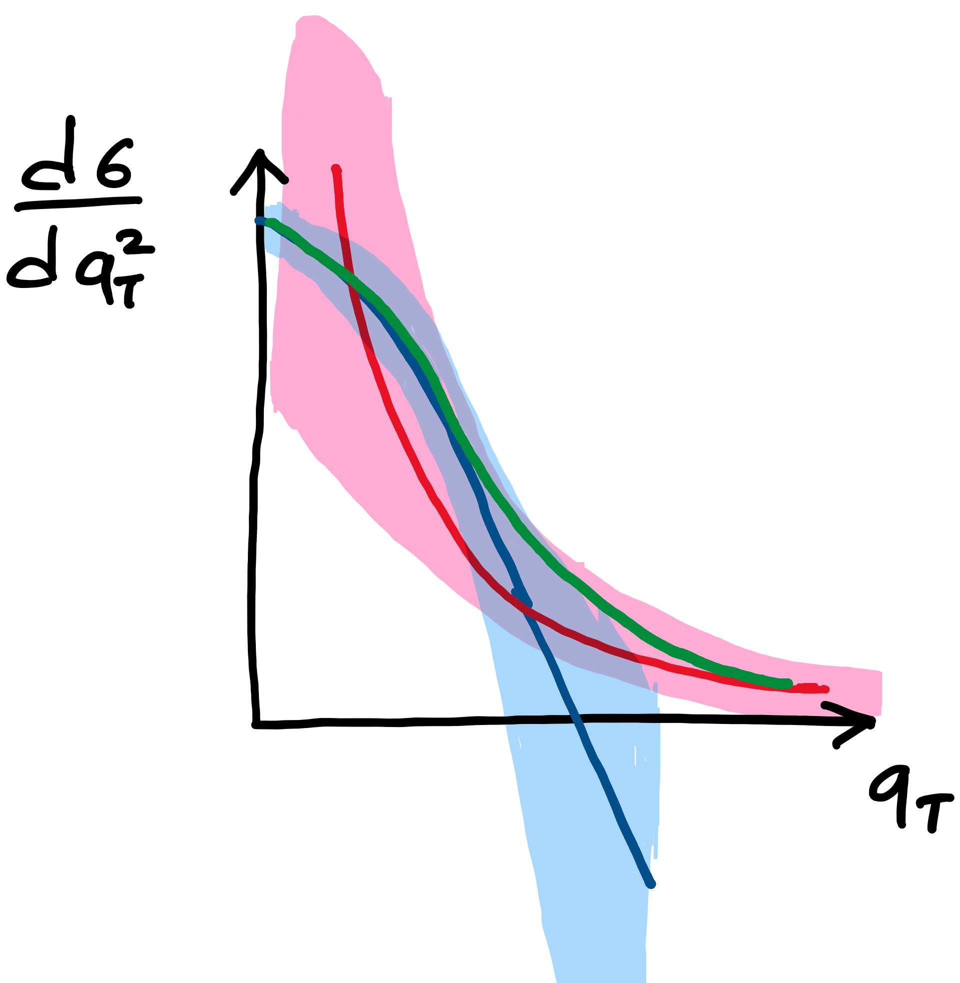
- It is all about the theoretical errors: modify W and Y=FO-ASY preserving the overall precision

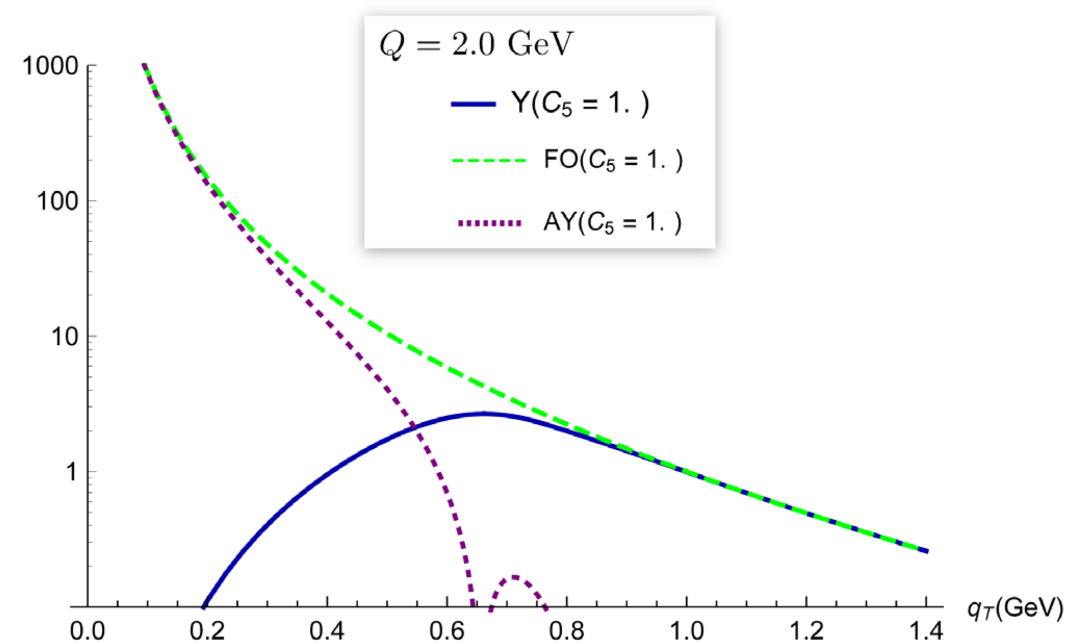
$$W + FO\text{-}ASY \rightarrow W \cdot \Xi(q_T) + (FO\text{-}ASY) \cdot X(q_T)$$





Similar to "profile" function used in
Abbate et al PRD 83 (2011)
Hoang et al PRD 91 (2015)





Collins, Gamberg, AP, Rogers, Sato, Wang arXiv:1605.00671

PHYSICAL REVIEW D 94, 034014 (2016)

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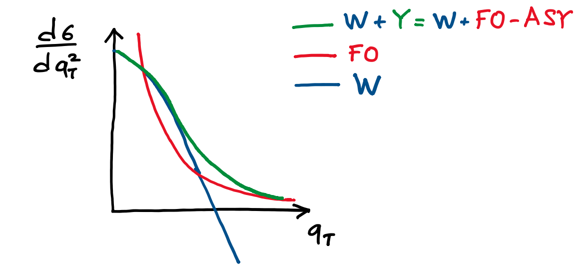
- Tame the Sudakov form factor in the small b_T by introducing b_{min}, $b_c(b_T=0) = b_{min}$

$$W(q_T) = \int \frac{d^2 b_T}{(2\pi)^2} \tilde{W}(b_T)$$



$$\int d^2 q_T (W+Y) \overset{CSS}{=\!=} \int \cancel{d^2 q_T W} \to \emptyset + \int d^2 q_T Y \to \sigma_{collinear}$$

$$\int d^2 q_T (W+Y) \overset{iCSS}{=\!=} \underbrace{\int d^2 q_T W}_{\text{non zero}} + \int d^2 q_T Y \to \sigma_{collinear}$$

$$\int d^2 q_T W(q_T) = \lim_{b_T \to 0} \tilde{W}(b_T)$$

- The integration over q_T means limit in b_T, and we restore the “naive” interpretation of integrals of TMDs. See talk by Leonard Gamberg.

- Corrections from Y term are important even in iCSS, especially for higher q_T

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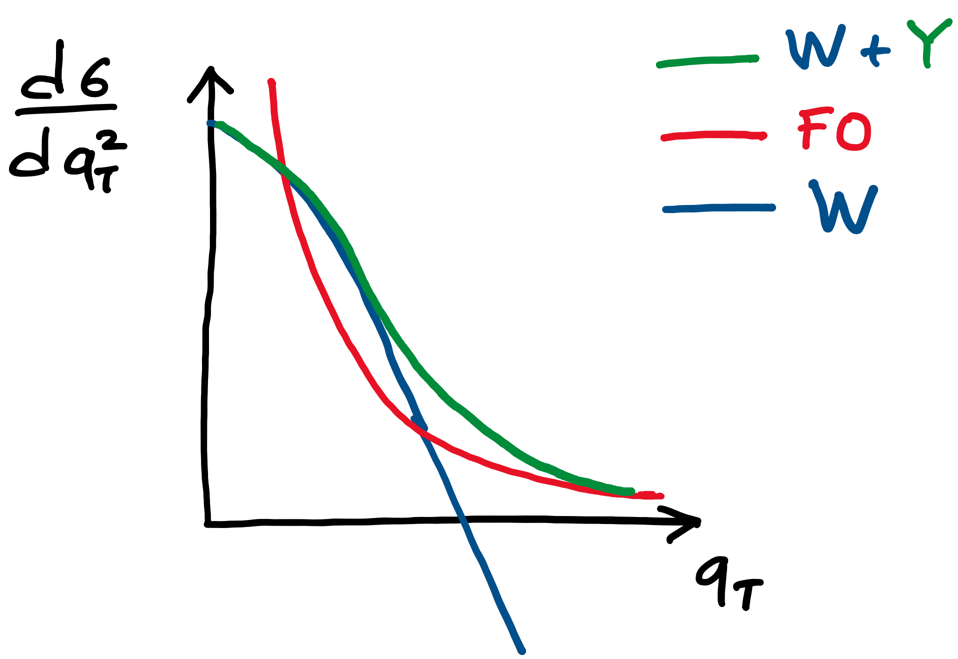
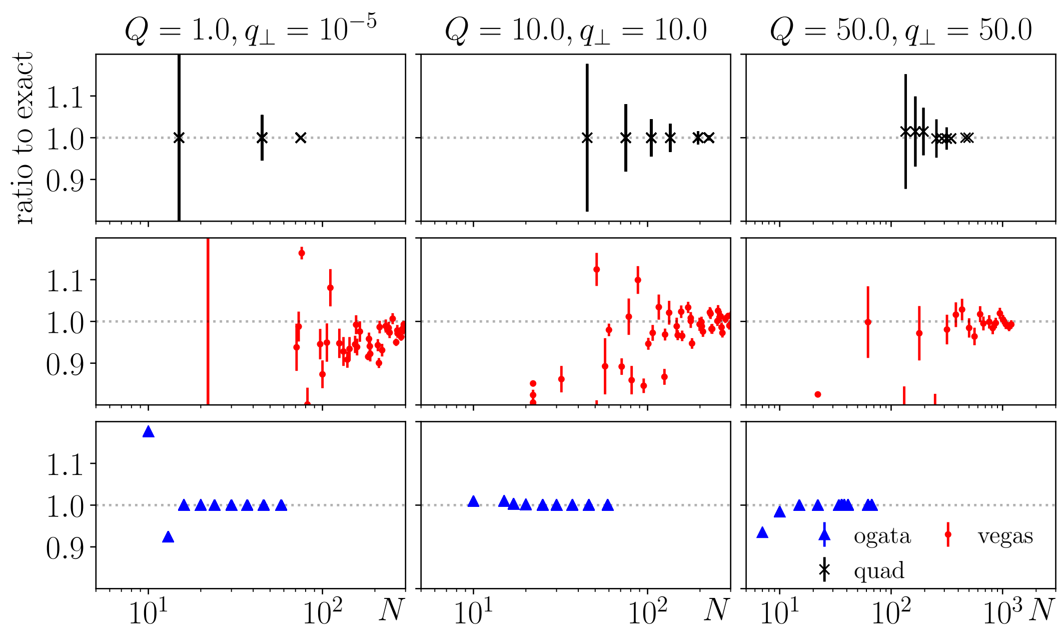
Numerical tools we need

- Numerical precision and the speed of FT and the **is important**, especially in high q_T, where matching and switching happens





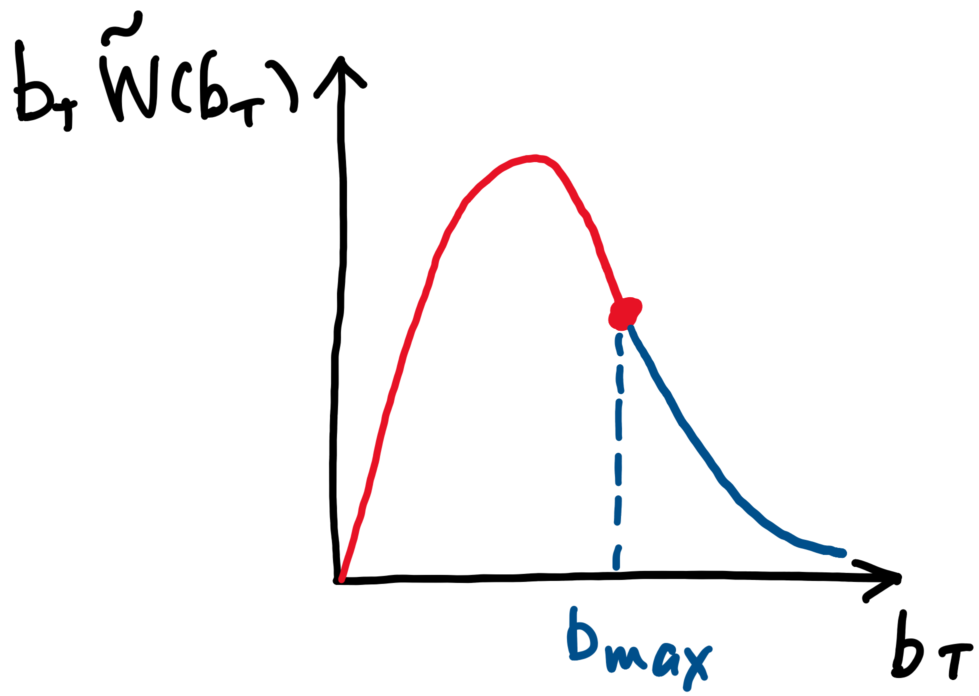
N~time

Kang, AP, Sato, Terry in preparation 2018

- Ogata integration method allows to boost the speed and improve precision of the numerical FT
- Especially important for future fits (including polarized data)
- To be released as a Python package

Alternative implementations to consider

- Switching from non perturbative to perturbative regimes in b_T space directly



PHYSICAL REVIEW D **63** 114011

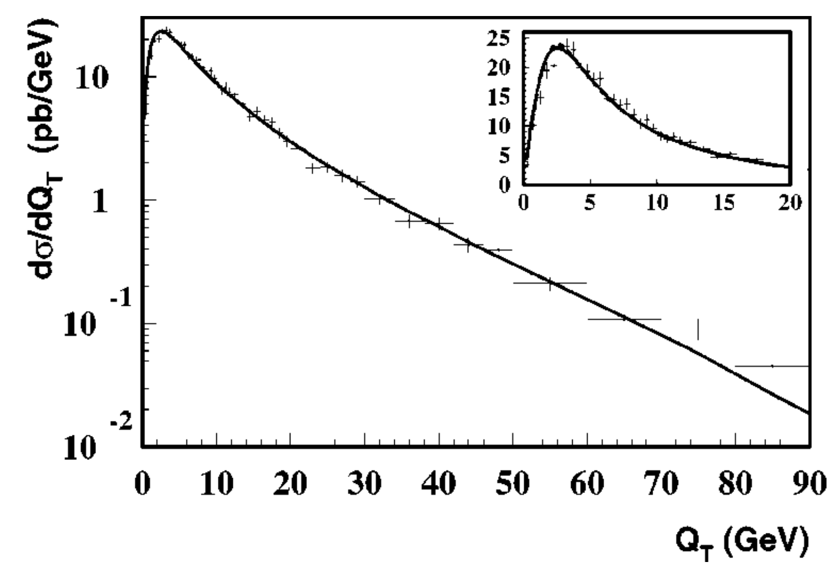
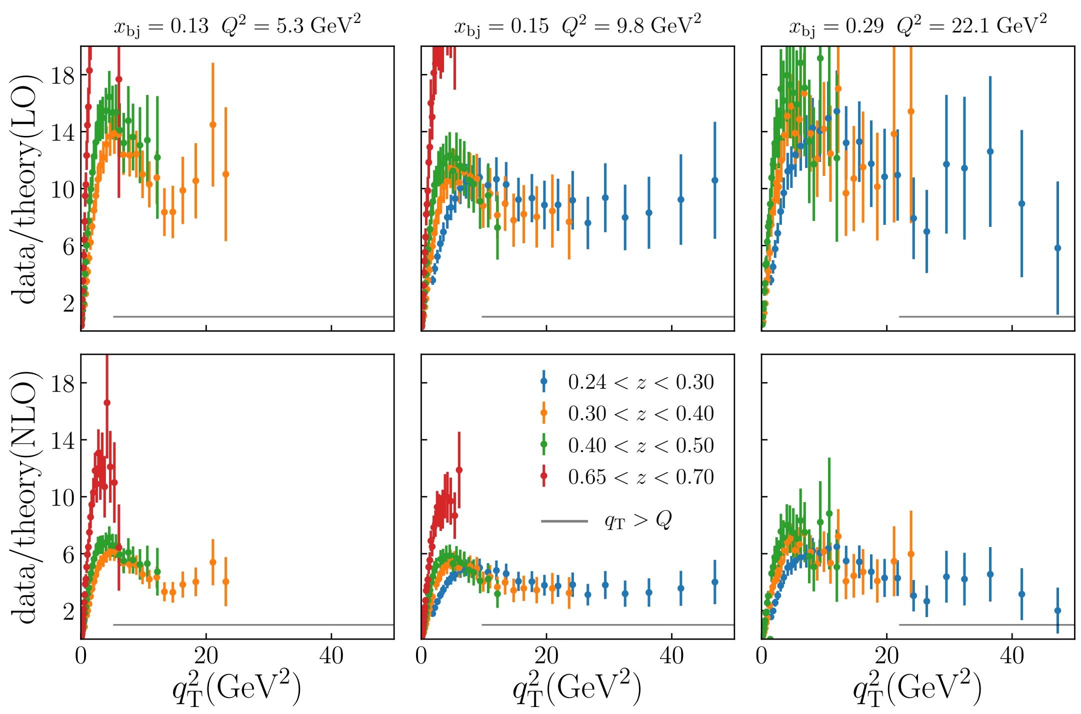


FIG. 7. Comparison between the b-space resummed Q_T distribution and CDF data [29]. The inset shows the Q_T<20 GeV region.

Qiu, Zhang 2001

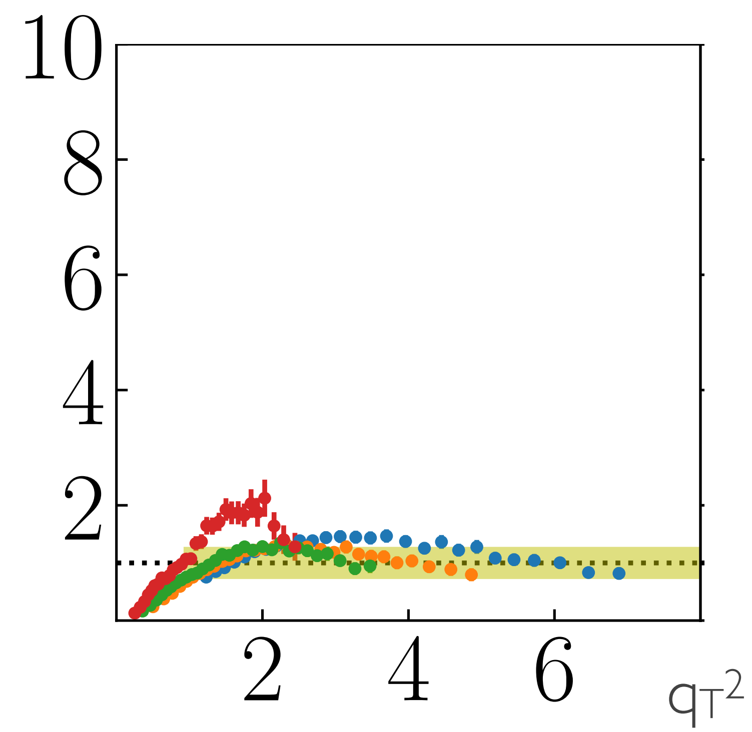
- Resummation in q_T space directly?

"Problems" of factorization at low Q and large qT



Gonzalez, Rogers, Sato, Wang arXiv:1808.04396

Possible solution? - refit collinear PDFs and FFs



Sato, presentation at INT-18-3 program

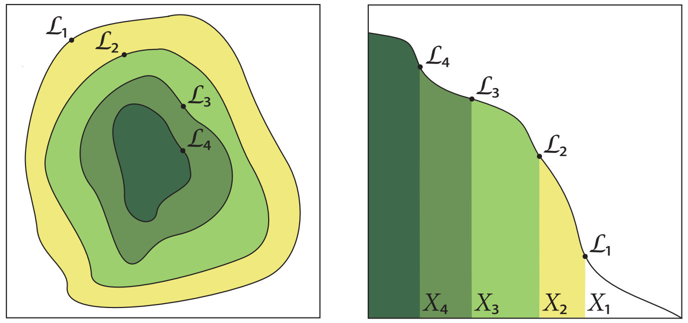
- SIDIS at low Q is challenging in both low and high q_T regions
- Possibly we need to refit PDFs and FFs
- Higher twist contributions might be important *Liu, Qiu in preparation 2018*
- Most probably combination of both

Fits of DY and SIDIS data

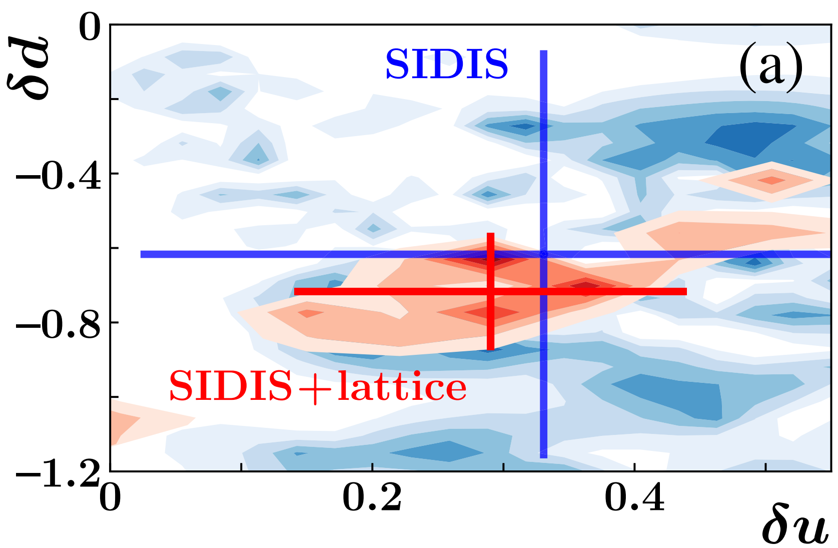
	Framework	W+Y	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	W+Y	✘	✘	✔	✔	98
QZ 2001 hep-ph/0506225	NLO-NLL	W+Y	✘	✘	✔	✔	28 (?)
RESBOS resbos@msu	NLO-NNLL	W+Y	✘	✘	✔	✔	>100 (?)
Pavia 2013 arXiv:1309.3507	No evolution	W	✔	✘	✘	✘	1538
Torino 2014 arXiv:1312.6261	No evolution	W	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	W	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	LO-NLL	W	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
SIYY 2014 arXiv:1406.3073	NLO-NLL	W+Y	✘	✔	✔	✔	140+SIDIS
Pavia 2017 arXiv:1703.10157	LO-NLL	W	✔	✔	✔	✔	8059
SV 2017 arXiv:1706.01473	NNLO-NNLL	W	✘	✘	✔	✔	309

- Perturbative precision improves with time
- Y term is not yet commonly used in fits
- TMD Collaboration should perform a fit of all available data

Recent important developments



*Sato et al., P.R. D**94** (16) 114004*



Lin, Melnitchouk, AP, Sato, PRL 120 (18) n.15

- New JAM and JAM3D methodology (Jefferson Lab Angular Momentum Collaboration)
- Packages in Python are ready for use in parton model approximation

  https://github.com/JeffersonLab/jam3d

- First extraction of tensor charge from SIDIS data including constraints from lattice QCD on g_T
- Regions of fragmentation in SIDIS

  Boglione, Collins, Gamberg, Gonzalez, Rogers, Sato arXiv:1611.10329

- First implementation of **all** structure functions in SIDIS in WW approximation

  Bastami et al, arXiv:1807.10606
  https://github.com/prokudin/WW-SIDIS

- …

- TMD related studies have been extremely active in the past few years, lots of progress have been made
- We look forward to the future experimental results from COMPASS, RHIC, Jefferson Lab, LHC, Fermilab, future Electron Ion Collider
- Many TMD related groups are created throughout the world:

Italy, Netherlands, Belgium, Germany, Japan, China, Russia, and the USA

- We have all tools for a reliable fit of DY+SIDIS data, that would allow to compare various methods (schemes) of implementation of TMD evolution.
- We have all tools for a reliable description of polarized data and predictions, for instance W asymmetry at RHIC due to Sivers effect, tensor charge extraction, predictions and impact studies for the future EIC, etc
- Robust methodology will be crucial and essential for future endeavors in studies of the structure of the nucleon and beyond