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# Theoretical perspectives on TMDs

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Supported by DOE Topical TMD collaboration

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$$f(x, k_T^2)$$

Detailed information on 3-dim momentum structure of nucleons

Various TMD effects

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SIDIS & DY

Transverse Single-Spin asymmetry

imaginary part needed

→ T-odd TMDs (Sivers function)

"Sign change"

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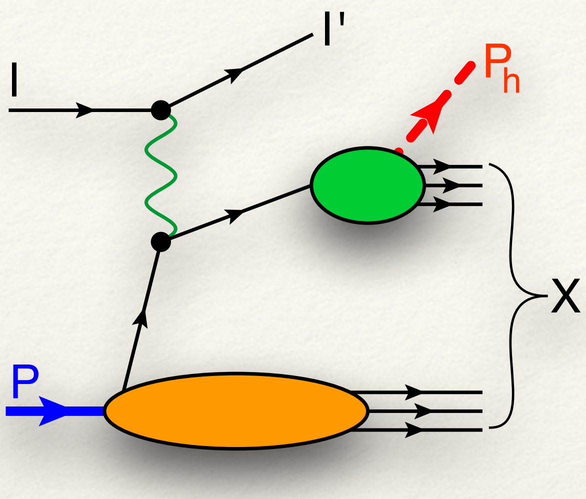
## Linear gluon polarization

Proton collisions at LHC

T-even TMD for linearly polarized gluons

→ azimuthal modulations of unpolarized cross section

# TMD factorization: SIDIS $eN^{\uparrow} \rightarrow e\pi X$




$$\frac{d\sigma^{\text{SIDIS}}}{d^2 P_{h\perp}}$$

SIDIS Spin Structure Functions:

[Bacchetta, Diehl, Goeke, Metz, Mulders, MS, JHEP (2007)]

$$\frac{d\sigma^{\text{SIDIS}}}{d^2 P_{h\perp}} \propto \left[F_{UU,T} + S_T \sin(\phi_h - \phi_s)\; F_{UT}^{\text{Siv}} + 16 \text{ S.F.}\right]$$

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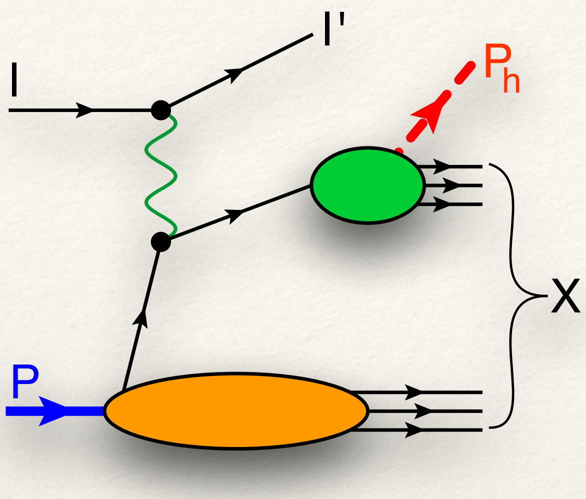


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$P_{hT}$ small → sensitive to partonic transverse momentum $k_T$

→ Transverse Momentum Dependent (TMD) distributions

$$f(x, k_T^2)$$

$$F \propto |H|^2 \int d^2 k_T \int d^2 p_T\, \delta^{(2)}(k_T - p_T - P_{h\perp}/z)\, f(x, k_T^2)\, D(z, p_T^2) \equiv |H|^2\, [f \otimes D]$$

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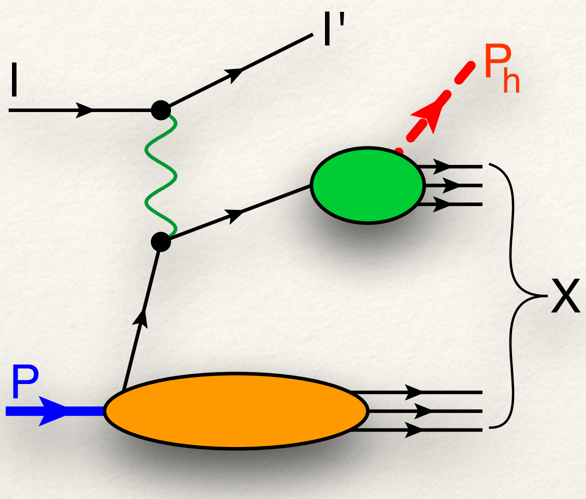


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## Sivers effect (HERMES, COMPASS, JLab & EIC)

$$A_{UT}^{\rm Siv} = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} = \frac{f_{1T}^{\perp} \otimes D_1}{f_1 \otimes D_1}$$

Sivers function

# (Naive) definition of the Sivers function

$$f_1^q(x, k_T^2) \propto \int \frac{d\lambda\, d^2 z_T}{(2\pi)^3}\, \mathrm{e}^{i\lambda x + i k_T \cdot z_T} \langle P | \bar{q}(0)\, \not{n} \mathcal{W}\, q(\lambda n + z_T)\, | P \rangle$$

Unpolarized TMD

$$(k_T \times S_T)\, f_{1T}^{\perp, q}(x, k_T^2) \propto \int \frac{d\lambda\, d^2 z_T}{(2\pi)^3}\, \mathrm{e}^{i\lambda x + i k_T \cdot z_T} \langle P, S_T | \bar{q}(0)\, \not{n} \mathcal{W}\, q(\lambda n + z_T)\, | P, S_T \rangle$$

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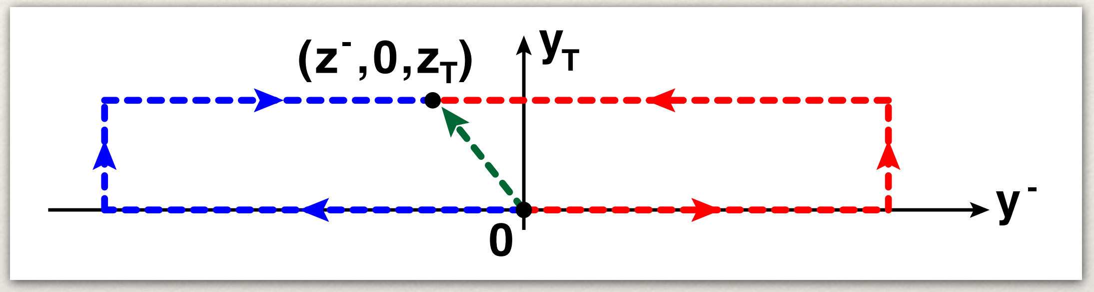
$$\mathcal{W}[a; b] = \mathcal{P}\mathrm{e}^{-ig \int_a^b ds \cdot A(s)}$$



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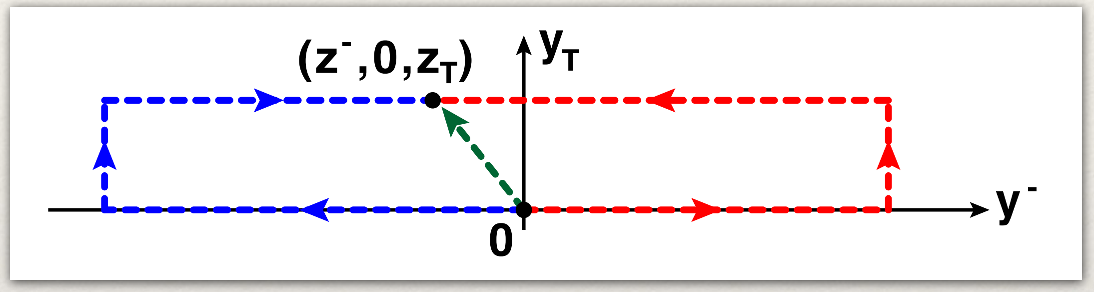
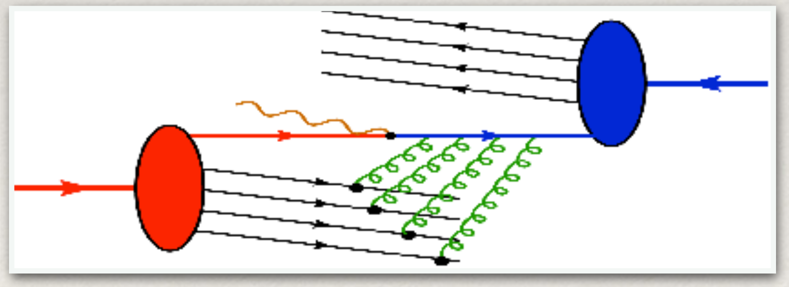
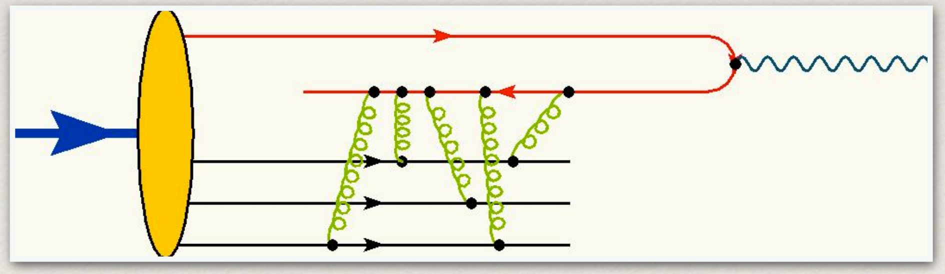



Initial State Interactions: Drell-Yan



Final State Interactions: SIDIS



$\mathcal{PT}$ - Transformation on the quark correlator → ISI ⇔ FSI

→ sign switch of Sivers function "time-reversal (T)-odd" $f_{1T}^{\perp}\Big|_{DIS} = -f_{1T}^{\perp}\Big|_{DY}$

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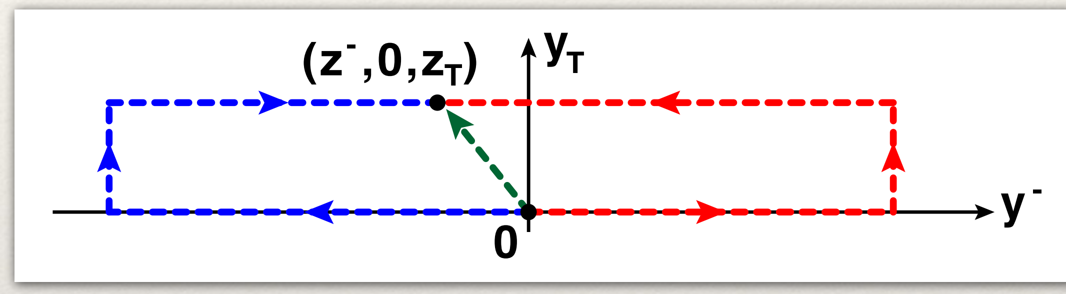


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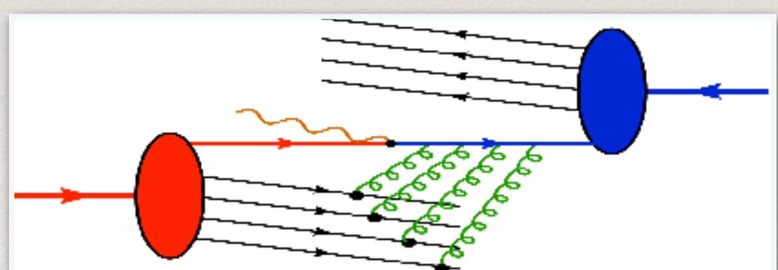

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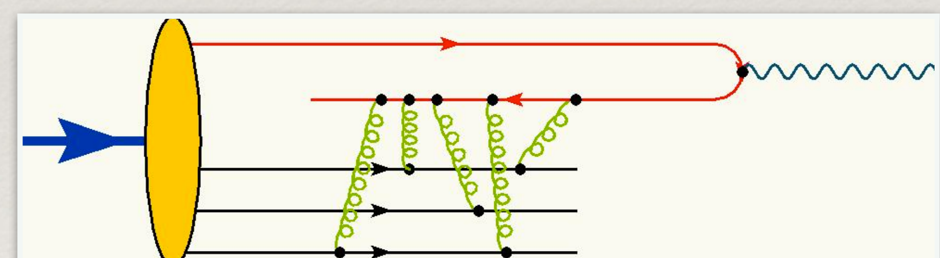

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→ sign switch of Sivers function “time-reversal (T)-odd” $f_{1T}^{\perp}\big|_{DIS} = -f_{1T}^{\perp}\big|_{DY}$

→ important theoretical prediction to test TMD factorization, experimental leads (DY) that point in this direction RHIC, COMPASS & SpinQuest

# Linearly Polarized Gluons

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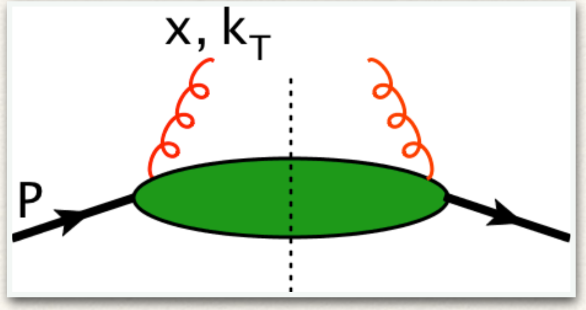


## TMD gluonic matrix element

$$\Gamma^{\alpha\beta;\,[\mathcal{W},\mathcal{W}']}(x, k_T) = \int \frac{d\lambda d^2 z_T}{(2\pi)^3}\, \mathrm{e}^{ix\lambda + ik_T\cdot z_T}\, \langle P, S|\, F^{n\alpha}(0)\, \mathcal{W}\, F^{n\beta}(\lambda n + z_T)\, \mathcal{W}'\, |P, S\rangle$$

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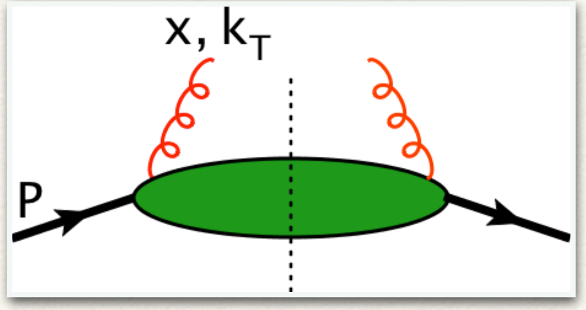


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past-pointing WL → Initial State Interactions: pp → color singlet + X (DY-type)

$$\Gamma^{\alpha\beta;[-,-]}$$

future-pointing WLs → Final State Interactions: ep → jets + X (SIDIS-type)

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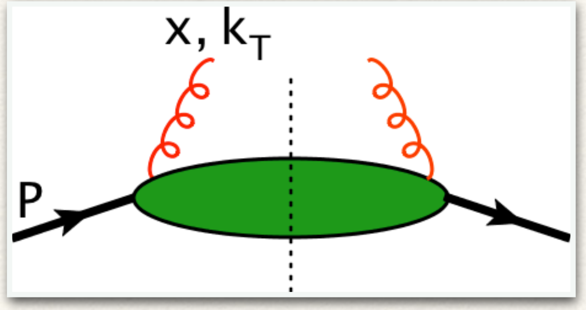


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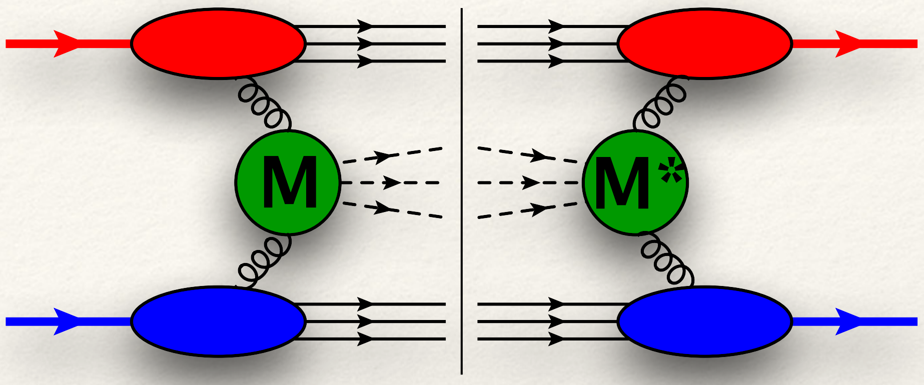
## Parameterization (unpolarized nucleon)

unpolarized & linearly polarized gluons :
helicity flip TMDs ➙ azimuthal modulations

$$\Gamma^{\alpha\beta}(x,k_T) = \frac{1}{2x}\left[ -g_T^{\alpha\beta}\; f_1^g(x,k_T^2) + \frac{k_T^\alpha k_T^\beta - \frac{1}{2}k_T^2 g_T^{\alpha\beta}}{M^2}\; h_1^{\perp g}(x,k_T^2)\right]$$

# Gluon TMDs from pp - collisions

[Lansberg, Pisano, M.S., NPB 2017]




General TMD expression for gluon fusion:

$$\frac{d\sigma}{d^4q\ldots}(q_T \ll Q) \propto \mathcal{C}[\Gamma^{\alpha\alpha'}(x_a, k_{aT})\,\Gamma^{\beta\beta'}(x_b, k_{bT})]\;(\mathcal{M}_{\alpha\beta}\,(\mathcal{M}_{\alpha'\beta'})^*)$$

$$\mathcal{C}[w\,f\,g] = \int d^2k_{aT}\int d^2k_{bT}\,\delta^{(2)}(k_{aT}+k_{bT}-q_T)\,w(k_{aT},k_{bT})\,f(x_a,k_{aT})\,g(x_b,k_{bT})$$

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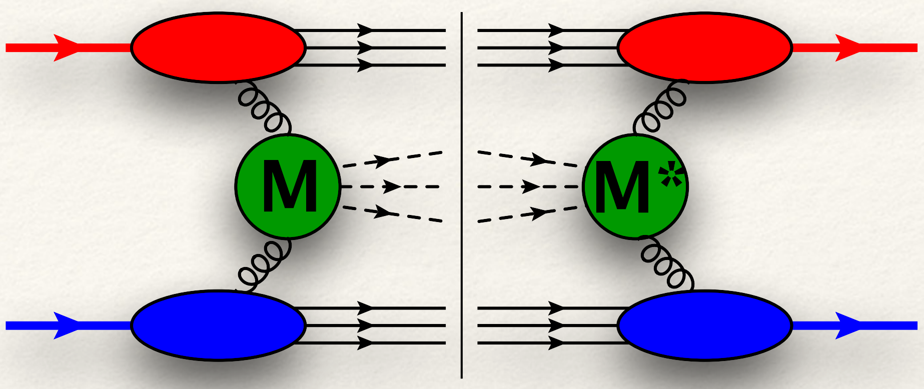


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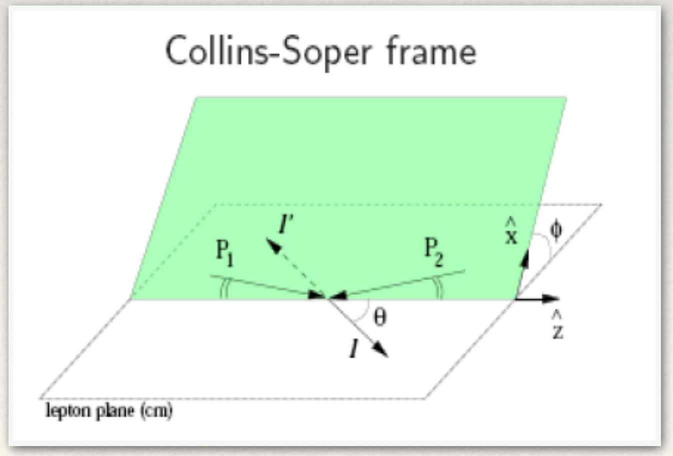
## 2-particle (pair) final states in pp - collisions

$$\frac{d\sigma^{gg}}{d^4q\; d\Omega}\Big|_{q_T \ll Q} = \hat{F}_1\; [f_1^g \otimes f_1^g] + \hat{F}_2\; [h_1^{\perp g} \otimes h_1^{\perp g}] + \cos(2\phi)\hat{F}_3\; [h_1^{\perp g} \otimes f_1^g + f_1^g \otimes h_1^{\perp g}] + \cos(4\phi)\hat{F}_4\; [h_1^{\perp g} \otimes h_1^{\perp g}]$$




Evaluate cross section in
c.m. frame of the produced pair

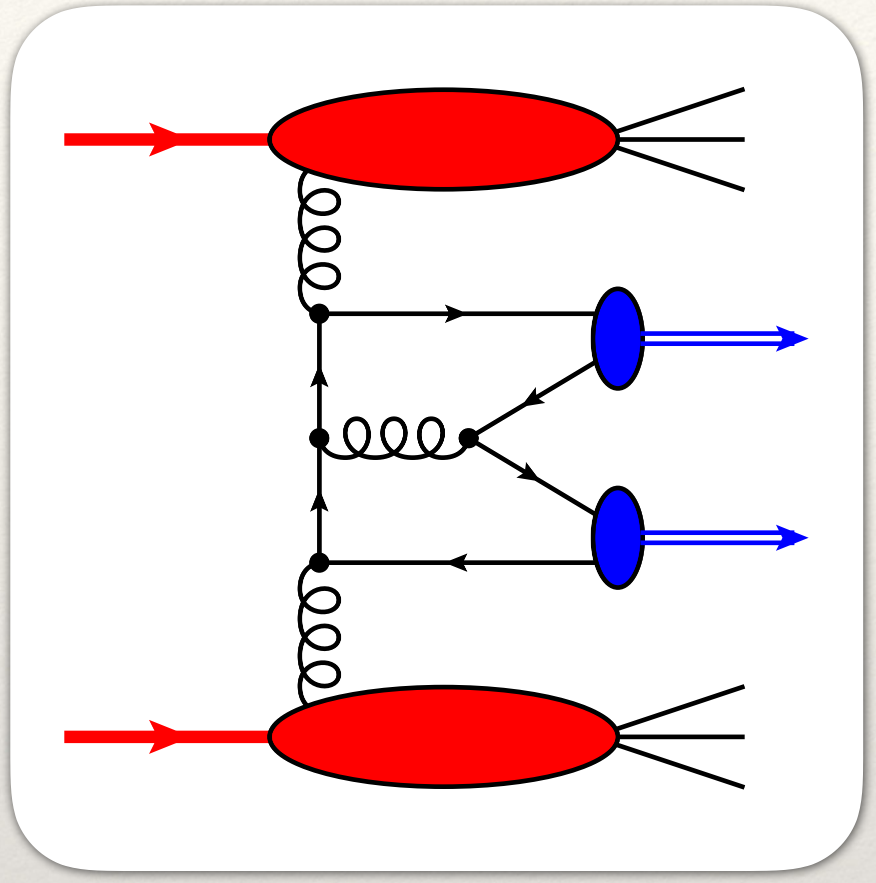
Collins - Soper angles θ , ϕ

Gluon TMDs do not appear in Drell-Yan

# Double $J/\psi(\Upsilon)$- production

[Lansberg, Pisano, Scarpa, M.S., PLB (2018)]

TMD - formalism: $J/\psi$ pair back-to-back; Color singlet



Here: Sample diagram (of about O(40))

Full analytical LO amplitude

$$g + g \to J/\psi + J/\psi$$

($J/\psi$ in color singlet configuration)

Presented in Appendix of
[Qiao, Sun, Sun; 0903.0954]

Helicity formalism: contract with polarization vectors
→ perturbative factors $F_1$, $F_2$, $F_3$, $F_4$

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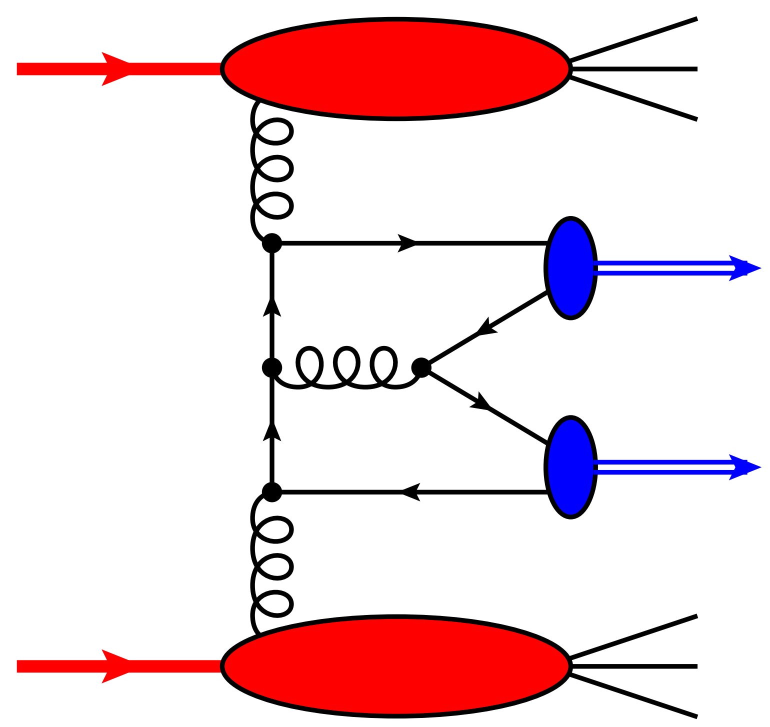

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$$F_i(Q, \cos^2\theta) \propto \frac{\sum_{k=0}^{N_i} c_k(\alpha)\left(\cos^2\theta\right)^k}{\left(1-(1-\alpha^2)\cos^2\theta\right)^4}$$

Q: invariant mass of Quarkonium pair

$$\alpha = \frac{2M_{J/\psi}}{Q}$$

$\alpha = 1$ threshold limit

$\alpha = 0$ high-energy limit

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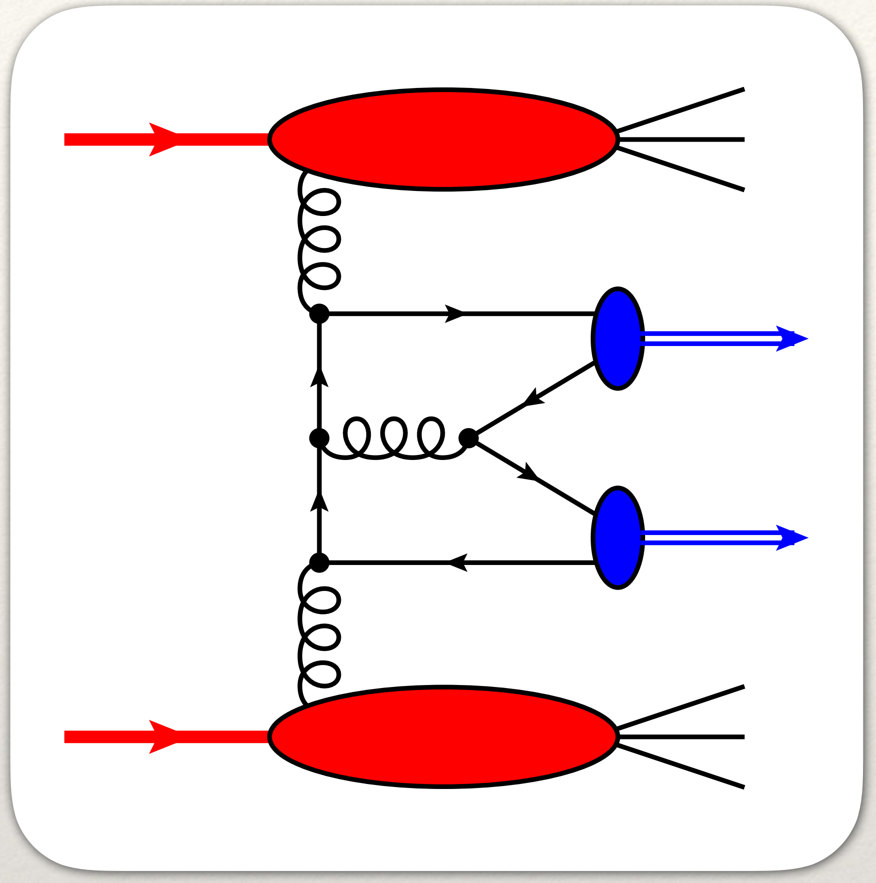

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Numerically: $F_2(Q, \cos^2\theta) \ll F_1(Q, \cos^2\theta)$ $F_4(\theta = \frac{\pi}{2}, Q \gg 2M_{J/\psi}) \to F_1$ Large!

# Azimuthal modulation: no data available → predictions

## Suggested observables: weighted cross section ratios

$$\frac{\int d\phi \cos(2\phi) \left[\frac{d\sigma\, d\phi}{dq_T}\right]_{\rm bins}}{\int_0^{Q/2} dq_T \left[\frac{d\sigma}{dq_T}\right]_{\rm bins}} \propto \frac{\mathcal{C}[w_3\, h_1^{\perp g}\, f_1^g] + \mathcal{C}[w_3\, f_1^g\, h_1^{\perp g}]}{\int_0^{Q/2} dq_T\, \mathcal{C}[f_1^g\, f_1^g]}$$

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Input for unpolarized & linearly pol. gluon TMD: Gaussian models

Model 1

$$h_1^{\perp g}(x, k_T^2) = \frac{M^2}{\langle k_T^2\rangle} \frac{g(x)}{\pi \langle k_T^2\rangle} \exp\left(1 - \frac{3\, k_T^2}{2\langle k_T^2\rangle}\right)$$

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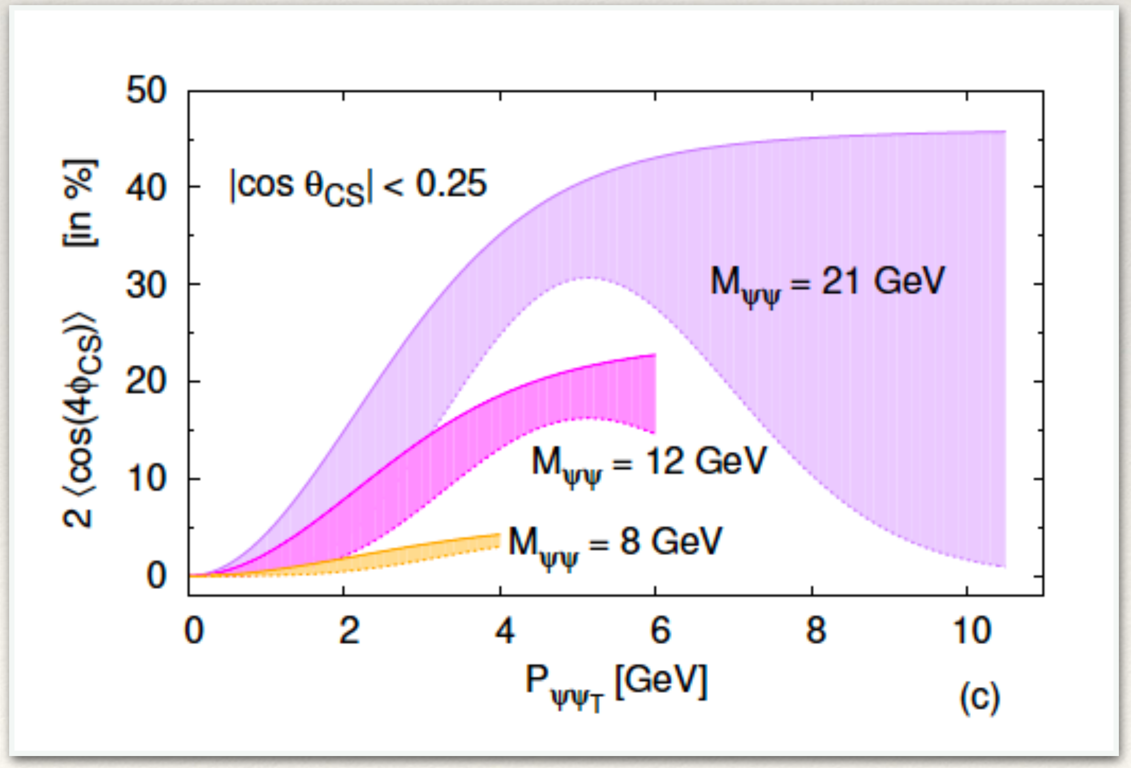


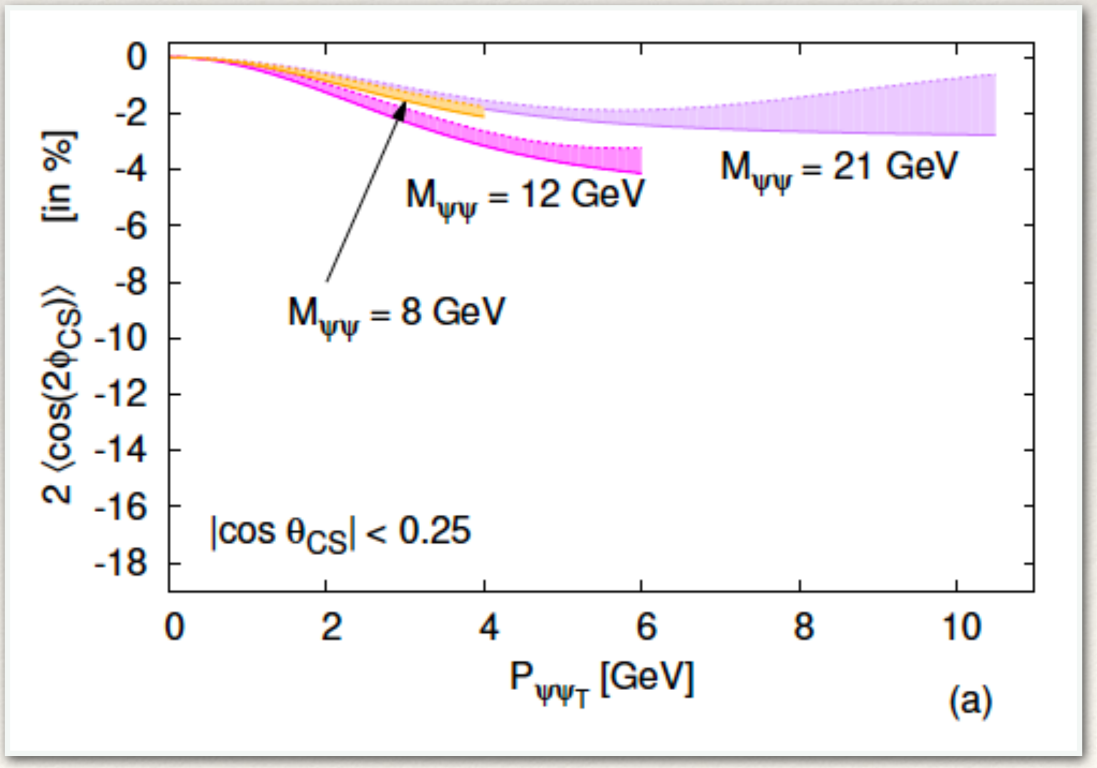


- Large effects for cos(4ϕ) for Q larger than threshold: 10% - 20%, cos(2ϕ) few percent

# Azimuthal modulations including TMD evolution

[Scarpa, Boer, Echevarria, Lansberg, Pisano, M.S., in preparation]

Evolved unpolarized and linearly polarized TMDs

$$f_1^g(x, b_T; \mu; \xi) \propto \left[\int dz\, C_1^{q,g}(z)\, f_1^{q,g}(z)\right] \mathrm{e}^{(S_{\mathrm{pert}}+S_{\mathrm{non-pert}})}$$

LO: $f_1^g(x) + \mathcal{O}(\alpha_s)$

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LO: $\frac{\alpha_s}{\pi} \int_x^1 d\xi\, C_{A,F} \left(\frac{\xi - x}{\xi\, x}\right) f_1^{g,q}(\xi) + \mathcal{O}(\alpha_s^2)$ $\alpha_s$ suppression!

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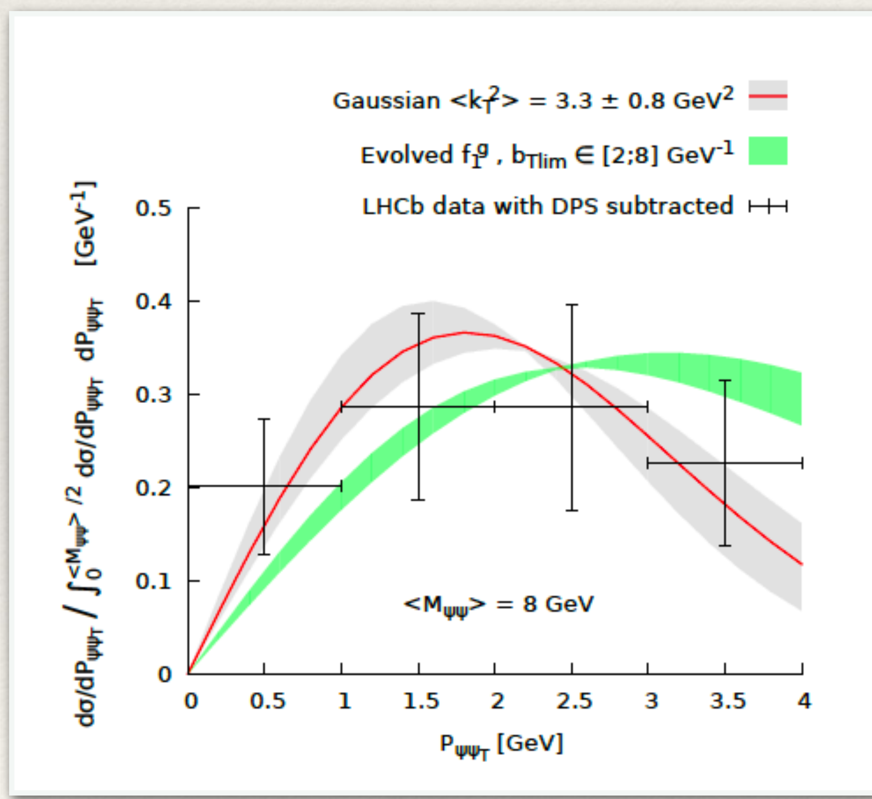
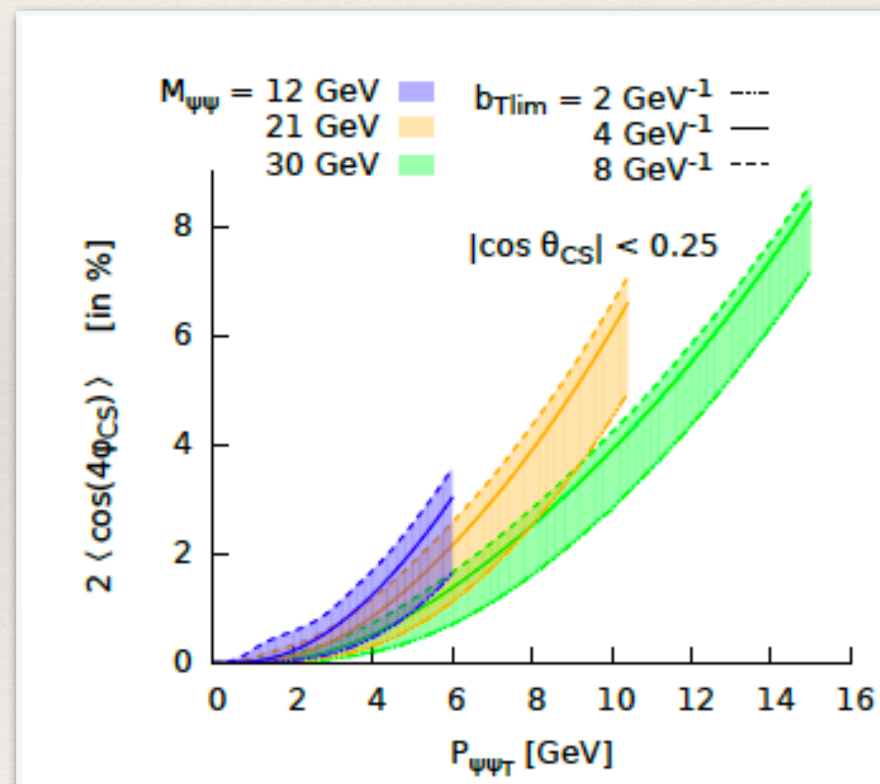
TMD evolution effects azimuthal asymmetries

TM distribution




cos(4ϕ) modulation




- TMD evolution diminished effects for cos(4ϕ) for Q larger than threshold: 4% - 8%
- May still be feasible at LHC → high-luminosity upgrade at LHC

# Summary

- Sivers effect in SIDIS and DY: sign change about to be verified/falsified
- Evolved Sivers function through Quark-Gluon correlations
- Gluon TMDs ➙ new aspects on the 3D gluonic structure of the nucleon ➙ linear gluon polarization
- Promising final state in pp: $J/\psi$ pairs at the LHC, particularly large: $\cos(4\phi)$ azimuthal modulation
- Data is coming: first extraction of unpolarized gluon TMD
- Evolution shrinks azimuthal modulation by a factor of about 2.
- Important for EIC: An idea what to expect for gluon TMDs

# Back-up slides

# Proper TMD definition & Soft function

[Collins; Ji, Yuan; Aybat, Rogers; Echevarria, Idilbi, Scimemi; recent works by Bacchetta et al, Scimemi, Vladimirov et al]

Inclusion of Soft Function $\Longrightarrow$

$$S(b_T) = \frac{1}{N_c} \mathrm{Tr}_c \langle 0|\, \mathcal{W}_n^\dagger(-b_T/2)\, \mathcal{W}_{\bar{n}}(-b_T/2)\, \mathcal{W}_{\bar{n}}^\dagger(b_T/2)\, \mathcal{W}_n(b_T/2)\, |0\rangle$$

$$f(x, b_T) \to \frac{f(x, b_T)}{\sqrt{S(b_T)}}$$

Modification needed in order to have:

1) Renormalizable Matrix Element
2) Finite matching coefficients at small $b_T$

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$$f_1^q(x, \vec{b}_T^2; \mu; \xi) = \sum_{q'} \left( \tilde{C}_{qq'} \otimes q(x) \right) \Big|_{\mu \propto 1/b_*} \mathrm{e}^{S_{\mathrm{pert}}(b_*)} \Big|_{\mu \propto 1/b_*} \mathrm{e}^{g_q(x, b_T) + \frac{1}{2} g_K(b_T) \ln \frac{\xi}{\xi_0}}$$

TMD at large $k_T$, matching coeff NNLO

perturbative Sudakov factor, $N^3LO$

non-perturbative input

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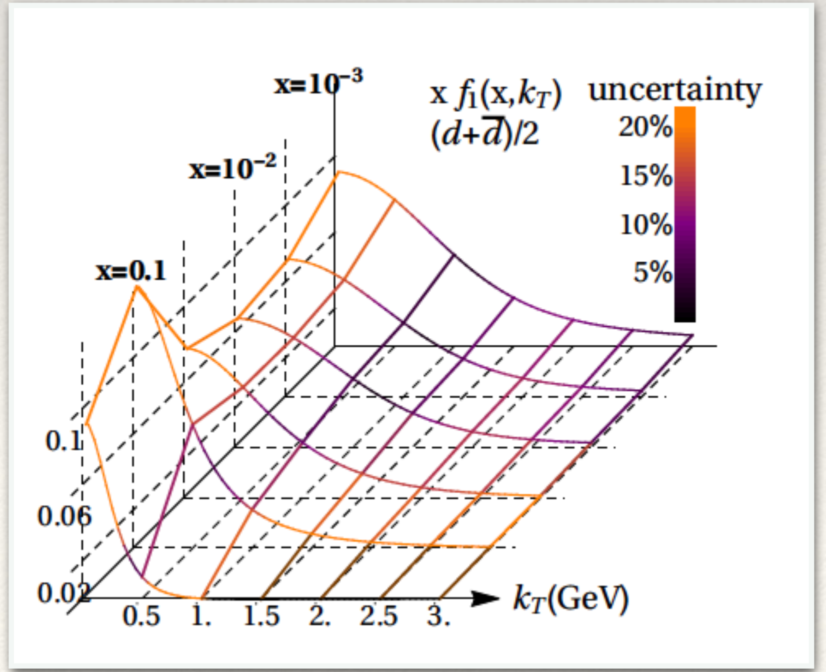


Recent DY fit (Bertone, Scimemi, Vladimirov, JHEP 2019)

Several other fits available:

Pavia group (Bacchetta et al, 2013, 2017)
Torino group (Anselmino et al, 2014)
Cagliari group (D'Alesio et al, 2015)
Echevarria et al (EIKV), 2014
Sun et al (SIYY), 2014

# Evolved Sivers function

[Aybat, Collins, Qiu, Rogers (2011); Scimemi, Tarasov, Vladimirov(2019)]

$$f_{1T}^{\perp,q}(x, b_T, ; \mu; \xi) \propto \left[ \int dz\, dz'\, C_{1T}^{\perp}(z, z')\, F_{FT}^{q}(z, z') \right] e^{(S_{\text{pert}} + S_{\text{non-pert}})}$$

matching coeff NLO,

at LO: $\pi\ F_{FT}(x,x) = f_{1T}^{\perp(1)}(x)$

quark-gluon-quark correlation function!

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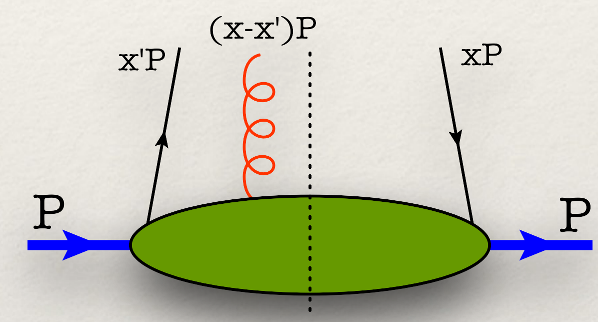
quark-gluon-quark correlation function!

$$2M\, i\epsilon^{Pn\rho S} F_{FT}^{q}(x, x') = \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x'} e^{i\mu(x-x')} \langle P, S_T |\, \bar{q}(0)\, \not{n}\, igF^{n\rho}(\mu n)\, q(\lambda n)\, | P, S_T \rangle$$




→ Quark-Gluon-Quark correlation functions
drive x-dependence of TMDs like Sivers function etc.

→ 'integrated' $F_{FT}(x,x')$: average transverse color Lorentz force on struck quark
[Burkardt, PRD88, 114502], [Aslan, Burkardt, M.S., 1904.03494]

$$F^{n\rho} = [\vec{E} + \vec{n} \times \vec{B}]^{\rho} \propto \int dx \int dx'\, F_{FT}(x, x') \propto \int dx\, x^2\, g_T(x)$$

→ see poster by W. Albaltan (next session)

## $q_T$ - spectrum: Data on J/ψ - pairs available from LHC

LHCb data (2017) at 13 TeV slightly above threshold, Q = 8 GeV

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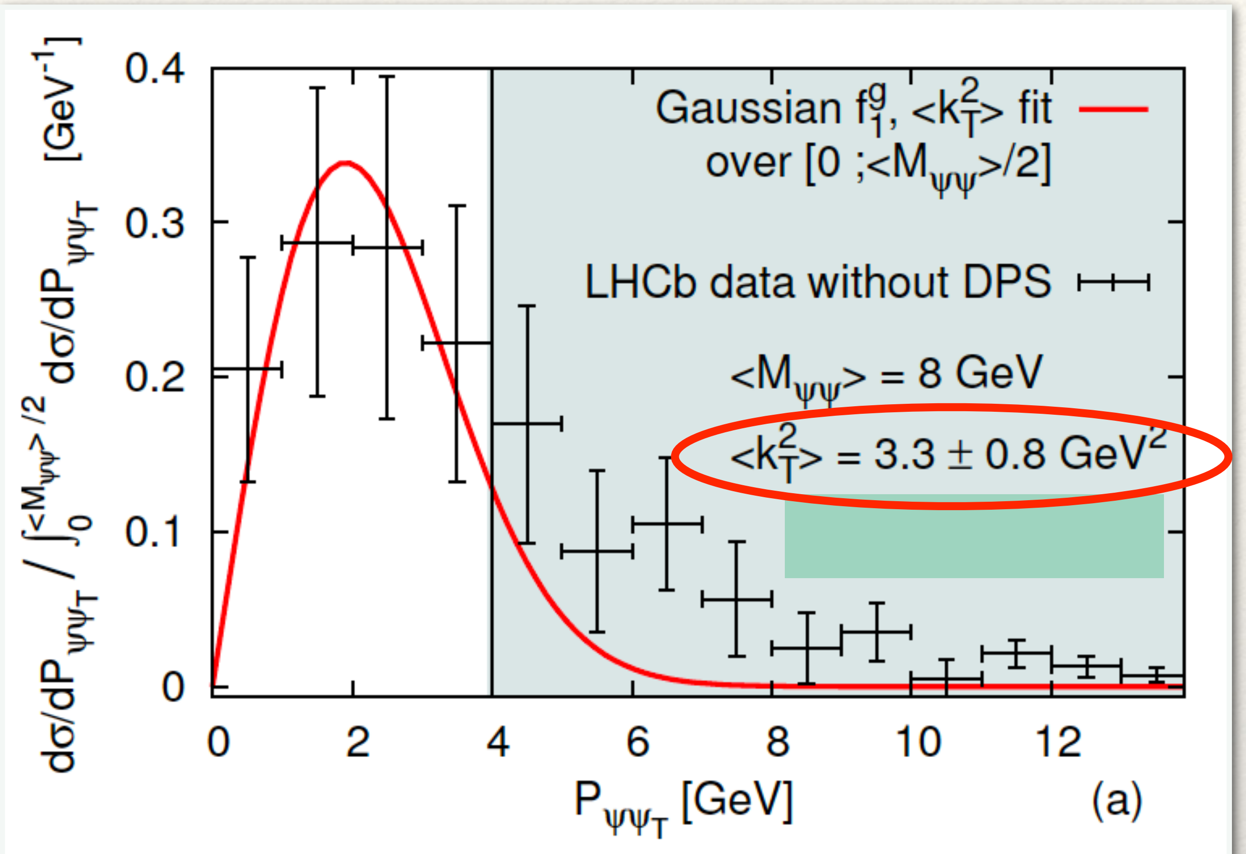


## Fit observable

$$\frac{\left[\frac{d\sigma}{dq_T}\right]_{\text{bins}}}{\int_0^{Q/2} dq_T \left[\frac{d\sigma}{dq_T}\right]_{\text{bins}}} \propto \frac{\mathcal{C}[f_1^g\, f_1^g]}{\int_0^{Q/2} dq_T\, \mathcal{C}[f_1^g\, f_1^g]}$$

## First glimpse on unpol. gluon TMD:

Simple Gaussian ansatz

$$f_1^g(x, k_T^2; Q = 8\,\text{GeV}) = \frac{g(x)}{\pi\langle k_T^2\rangle} \exp\left(-k_T^2/\langle k_T^2\rangle\right)$$

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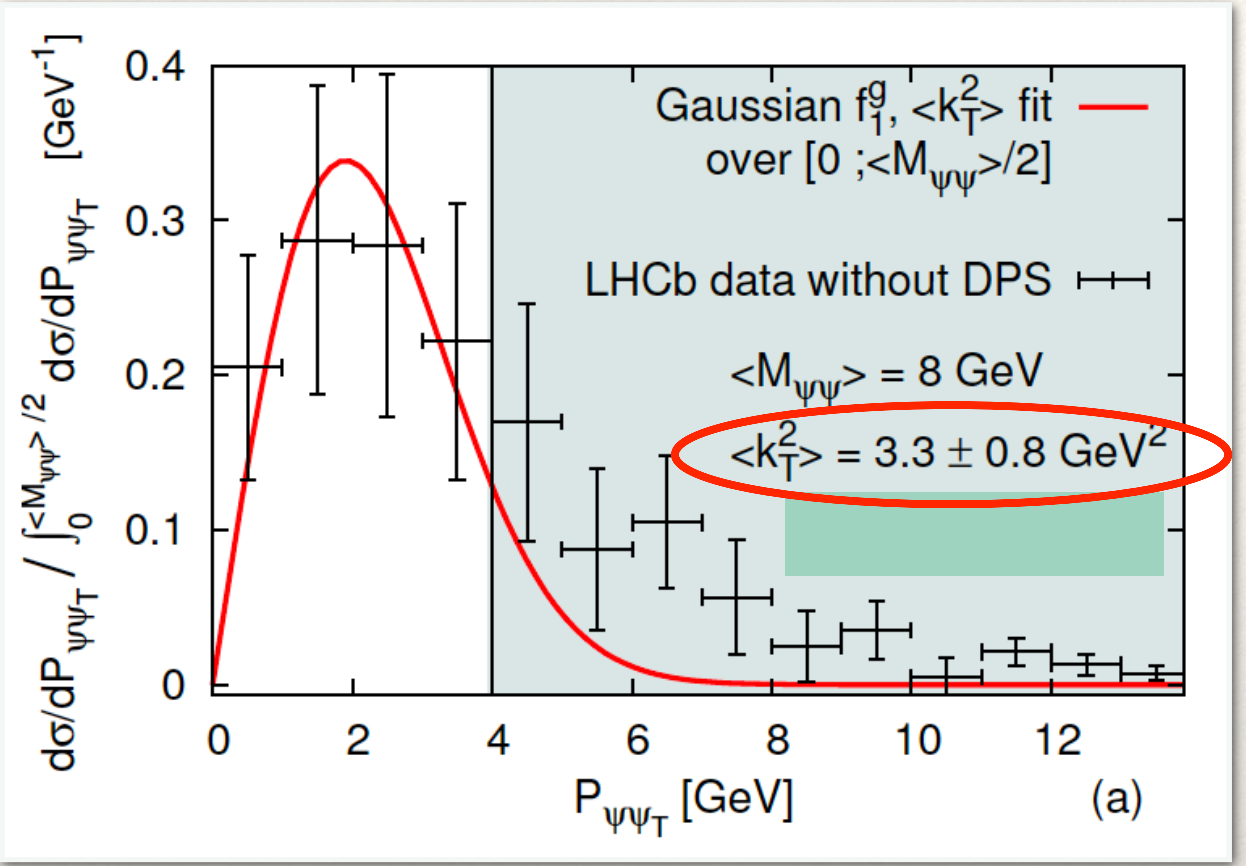


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- One-parameter fit for $<k_T^2>$ , $\chi^2$ = 1.08: *effective*, but not *intrinsic* width
- LHCb data corrected for double-parton scattering
- Expectation: Color Singlet Mode dominant

[Lansberg, Shao, PLB 2015; Ko, Yu, Lee, JHEP 2011; Li, Xu, Liu, Zhang, JHEP 1023]