Theoretical Progress at the Frontiers of Small-x Physics

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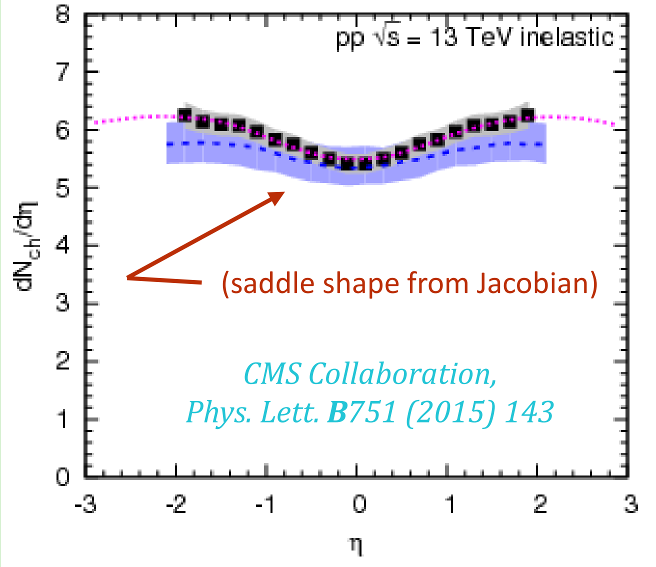
XLIX International Symposium
on Multiparticle Dynamics

Friday, Sept. 13, 2019

3 Meanings of "Small x"

Low-x Kinematics

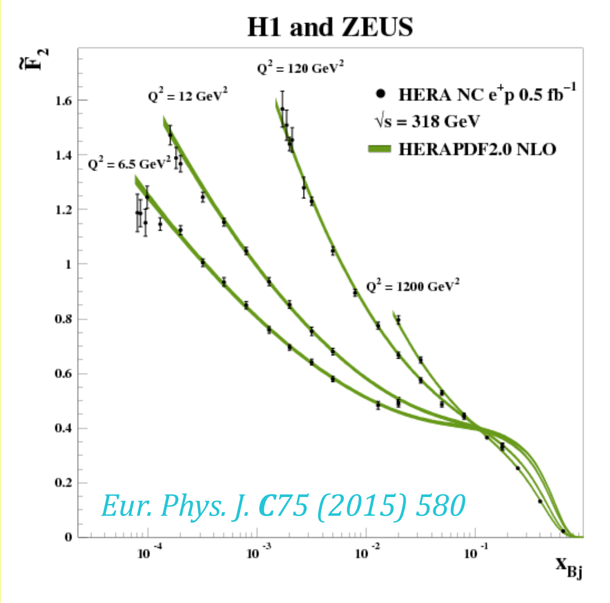


(saddle shape from Jacobian)

CMS Collaboration,
Phys. Lett. ***B****751 (2015) 143*

$x \ll \alpha_s$

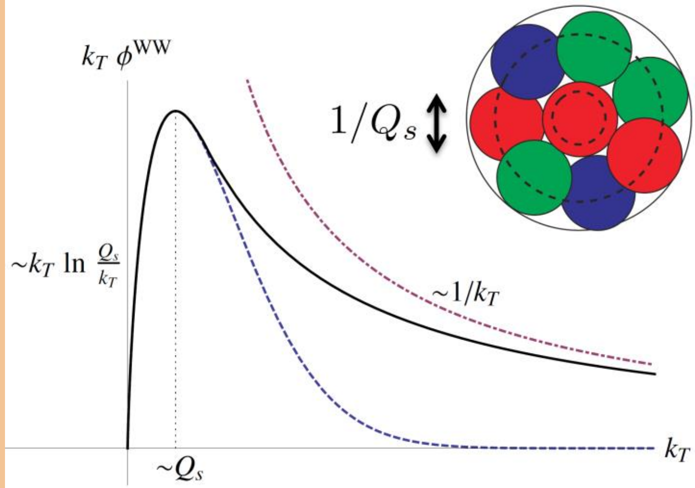
Linear Evolution



Eur. Phys. J. ***C****75 (2015) 580*

$x \ll e^{-1/\alpha_s}$

Nonlinear Evolution



M.D.S*, arXiv: 1407.4047*

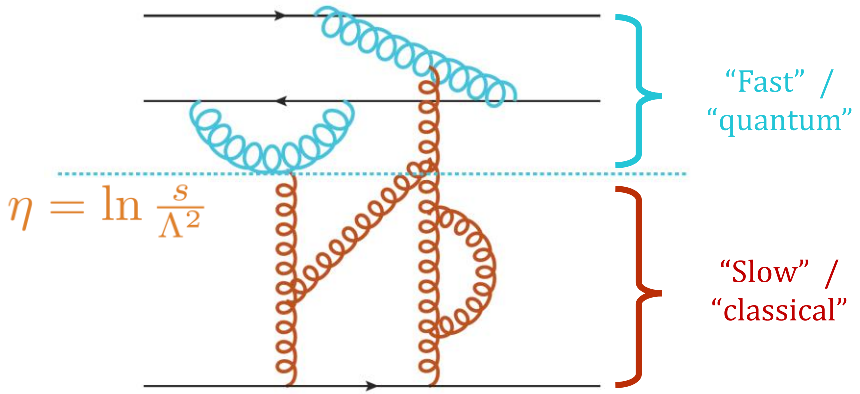
$\Lambda_{QCD} \ll Q_s(x)$

Leading Order is Well-Established

- There are **many different formalisms** on the market
  - All are **equivalent** at LO

Background Field Method

I. Balitsky, A. Tarasov
JHEP ***1510*** *(2015) 017*



Hamiltonian EFT

Y. Kovner, M. Lublinsky, Y. Mulian
JHEP ***1408*** *(2014) 114*

$$
\begin{aligned}
H_{int} \equiv \int dx^- \, d^2\mathbf{x} \Big(& -g f^{abc} A_i^b A_j^c \partial_i A_j^a + \frac{g^2}{4} f^{abc} f^{ade} A_i^b A_j^c A_i^d A_j^e \\
& - g f^{abc} (\partial_i A_i^a) \frac{1}{\partial^+} (A_j^b \partial^+ A_j^c) + \frac{g^2}{2} f^{abc} f^{ade} \frac{1}{\partial^+} (A_i^b \partial^+ A_i^c) \frac{1}{\partial^+} (A_j^d \partial^+ A_j^e) \\
& + 2g^2 f^{abc} \frac{1}{\partial^+} (A_i^b \partial^+ A_i^c) \frac{1}{\partial^+} (\psi_+^\dagger t^a \psi_+) + 2g^2 \frac{1}{\partial^+} (\psi_+^\dagger t^a \psi_+) \frac{1}{\partial^+} (\psi_+^\dagger t^a \psi_+) \\
& - 2g (\partial_i A_i^a) \frac{1}{\partial^+} (\psi_+^\dagger t^a \psi_+) - g \psi_+^\dagger t^a (\sigma_i \partial_i) \frac{1}{\partial^+} (\sigma_j A_j^a \psi_+) - g \psi_+^\dagger t^a \sigma_i A_i^a \frac{1}{\partial^+} (\sigma_j \partial_j \psi_+) \\
& - i g^2 \psi_+^\dagger t^a t^b \sigma_i A_i^a \frac{1}{\partial^+} (\sigma_j A_j^b \psi_+) \Big) .
\end{aligned}
$$

Weight Functional

L. McLerran, R. Venugopalan,
Phys. Rev. ***D****49 (1994) 2233*

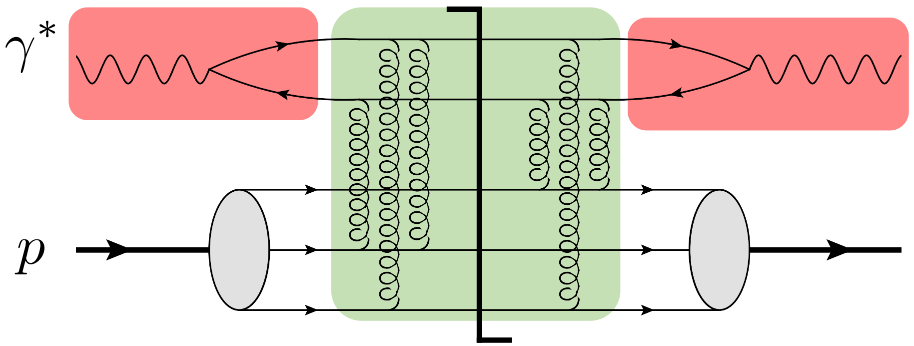
$$
\begin{aligned}
Z = & \int [dA_t dA_+][d\psi^\dagger d\psi][d\rho] \\
& exp\left(iS + ig \int d^4x A_+(x) \delta(x^-) \rho(x) - \frac{1}{2\mu^2} \int d^2x_t \rho^2(0, x_t) \right)
\end{aligned}
$$

Leading Order is Well-Established

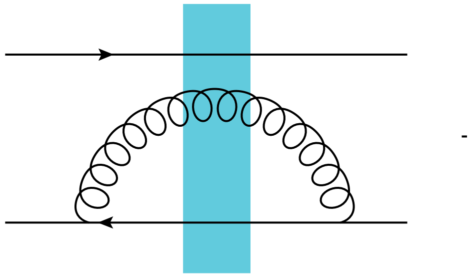
- There are **many different formalisms** on the market
  - All are **equivalent** at LO

- Physical picture is clear
  - Hadronic **wave functions** ("Impact Factor")
  - **Shockwave** interactions (Wilson lines)
  - Gluon **cascade** (small-x evolution)

Wave Functions



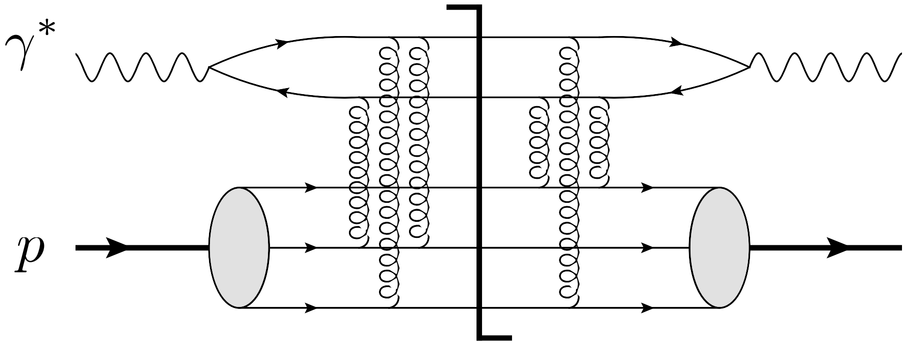
Wilson Lines

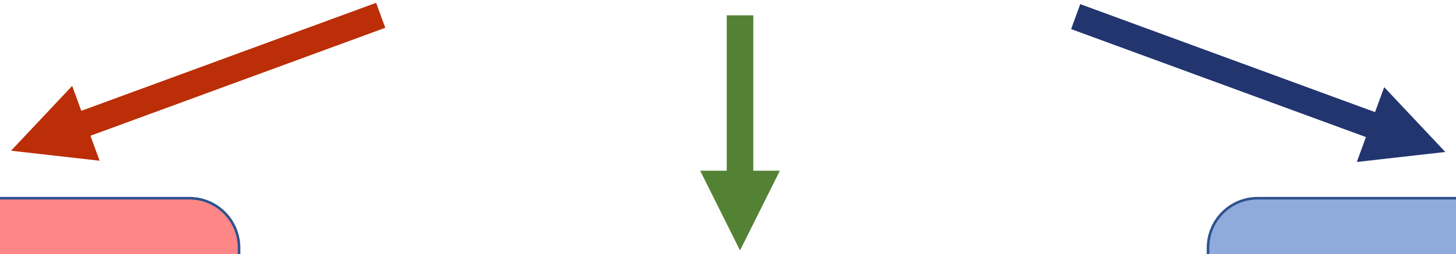


Small-x Evolution

$$\frac{d\mathcal{O}}{dY} = -\,H_{JIMWLK}\,\mathcal{O}$$

Active Frontiers of Small-x Theory





NLO

$\mathcal{O}(\alpha_s)$

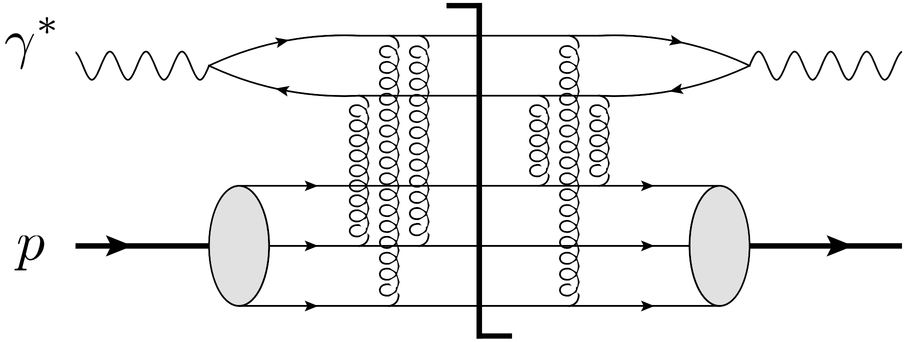
Sub-eikonal

$\mathcal{O}(\frac{1}{s})$

Correlations

$v_n\{4\}, v_n\{6\}, \cdots$

The NLO Frontier



NLO

$$\mathcal{O}(\alpha_s) \to \mathcal{O}\left(\frac{1}{\ln 1/x}\right)$$

Y. Kovchegov, H. Weigert, Nucl. Phys. ***A784*** *(2007) 188*

- **Running coupling corrections**
  - rcBK evolution

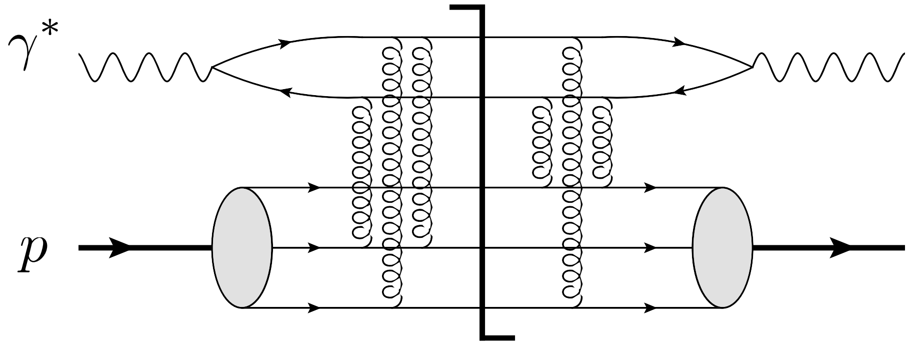
I. Balitsky, G. Chirilli, Phys. Rev. ***D83*** *(2011) 031502*

- **NLO splittings in wave functions**
  - Beyond leading-log gluon cascade

Y. Hatta et al., Nucl. Phys. ***A760*** *(2005) 172*

- **NLO shockwave corrections**
  - QCD odderon (antisymmetric, C-odd)

The Sub-eikonal Frontier



C. Shen, B. Schenke, Nucl. Phys. **A982** (2019) 411

- **Finite-Energy Kinematics**
  - ➢ Non-instantaneous shockwave

C. Shen, B. Schenke, Nucl. Phys. **A982** (2019) 411

- **Background Field Corrections**
  - ➢ New gauge fields

Sub-eikonal

$\mathcal{O}(\frac{1}{s})$

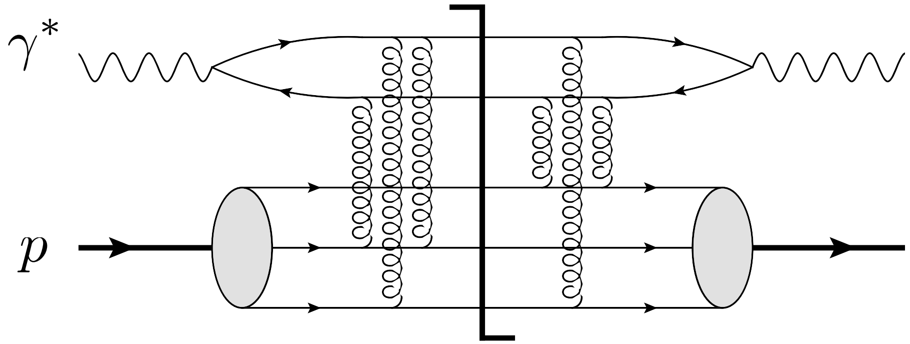
M. Gyulassy, P. Levai, I. Vitev, Nucl. Phys. **B594** (2001) 371

- **In-Medium Branching**
  - ➢ LPM effect (radiative jet quenching)

T. Altinoluk et al., JHEP **1407** (2014) 068

- **Quark Exchange**
  - ➢ Spin, flavor transfer

The Multiparticle Frontier



Y. Kovchegov, V. Skokov, Phys. Rev. ***D97*** *(2018) 094021*

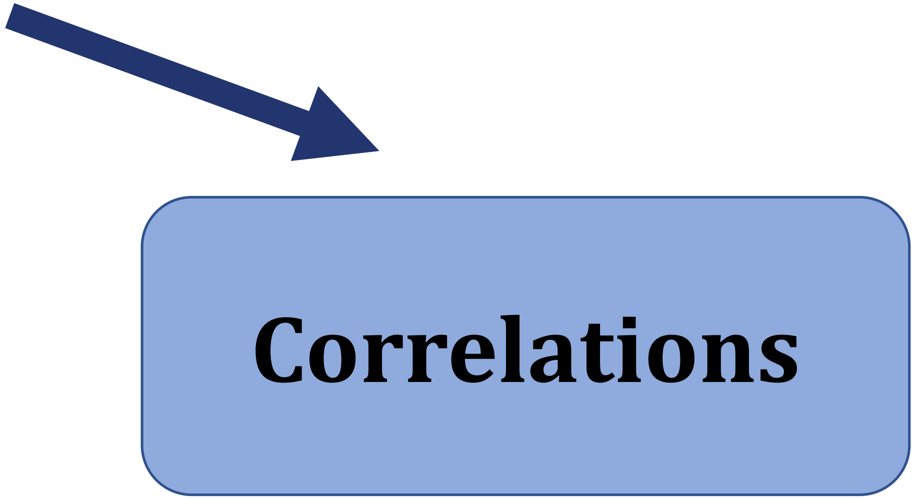
- **Two-gluon correlations**
  - ➢ Breaking of back-to-back symmetry at NLO

T. Altinoluk et al., Eur. Phys. J. ***C78*** *(2018) 702*

- **Multi-gluon correlations**
  - ➢ Challenging even at LO (“glasma graphs”)

M. Martinez, ***M.D.S.****, D. Wertepny, JHEP* ***1902*** *(2019) 024*

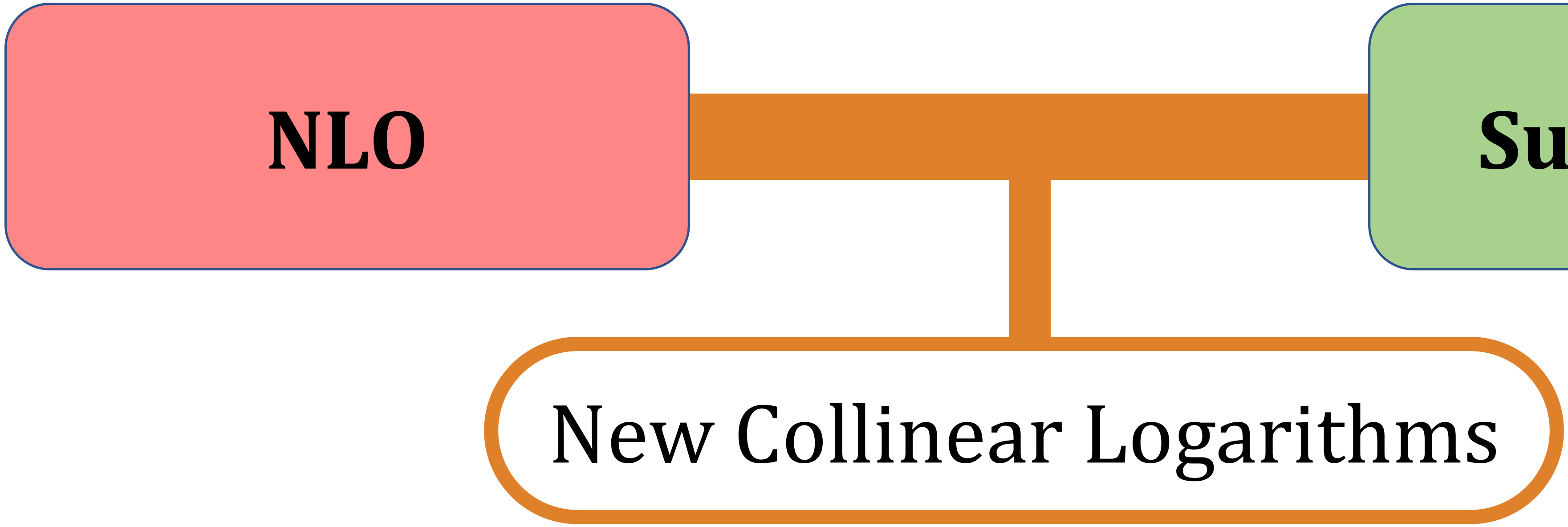
- **Quark correlations**
  - ➢ Arise at NLO but carry new quantum numbers



Correlations

$$v_n\{4\}, v_n\{6\}, \cdots$$

A Common Thread

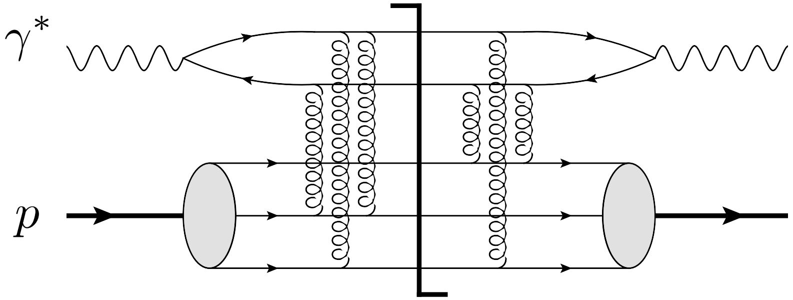


NLO

Sub-eikonal

New Collinear Logarithms

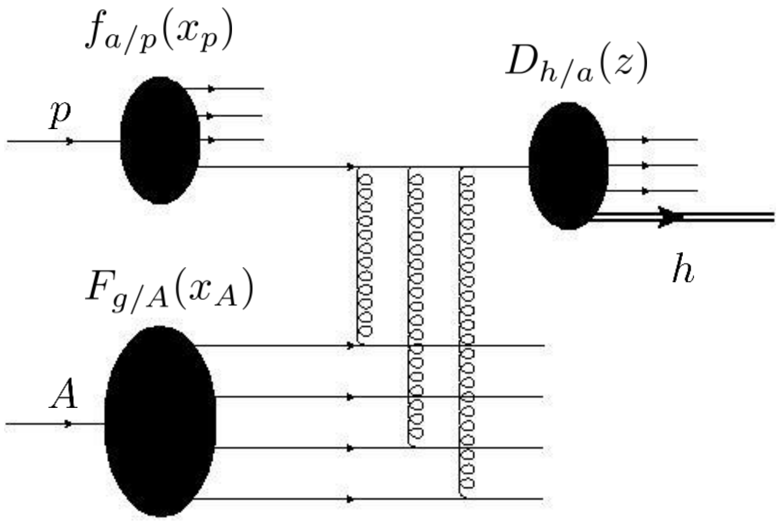
- **Significantly modify the standard small-x paradigm**



Collinear + Small-x "Hybrid Factorization"

$$p + A \rightarrow h + X$$

- **Forward production in "Dilute / Dense" collisions**
  - "Dilute": collinear large-x PDFs
  - "Dense": dipole amplitudes (~ small-x gluon TMDs)
  - Collinear fragmentation functions if needed



- **Relies on separation of transverse momentum scales**
  - Hadron recoils against intrinsic Qs
  - Neglect $O(\Lambda_{QCD})$ transverse momenta

G. Chirilli, B.-W. Xiao, F. Yuan, Phys. Rev. ***D86*** *(2012) 054005*

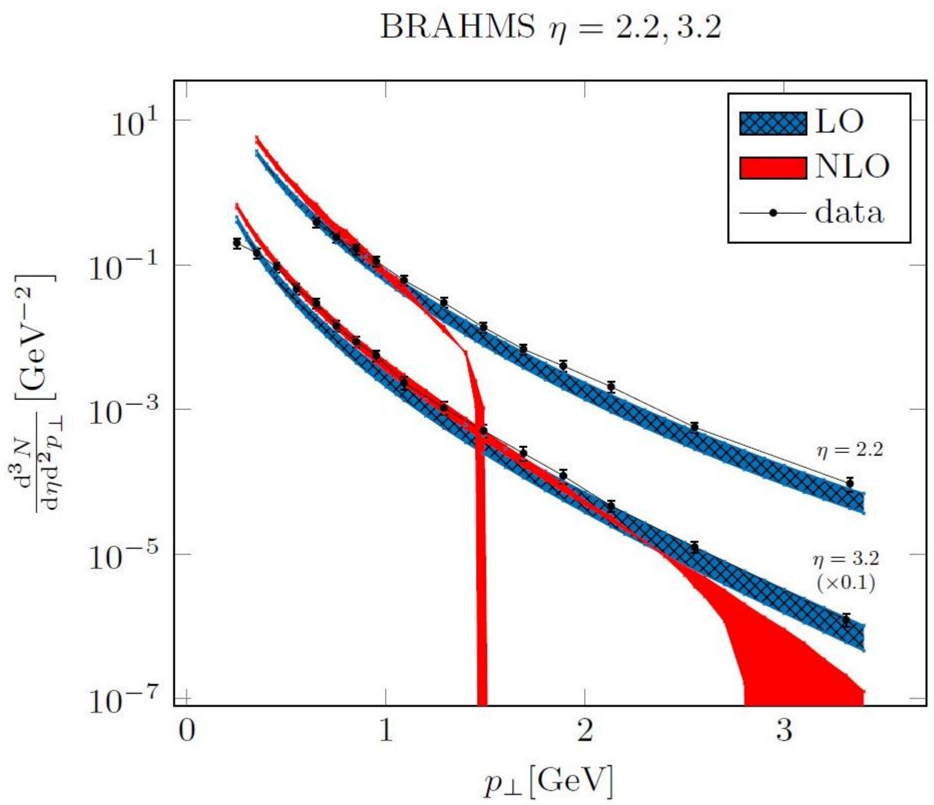
$$p_T \sim Q_s \gg \Lambda_{QCD}$$

- **Proven up to NLO**
  - Divergences can be absorbed into DGLAP + JIMWLK

NLO Gone Wrong: The Negativity Problem

A. Stasto, B.-W. Xiao, D. Zaslavsky, Phys. Rev. Lett. ***112*** *(2014) 012302*

- **Despite its theoretical appeal, something goes wrong in the numerics of NLO**
  - ➢ At low pT, the corrections are well behaved
  - ➢ But at higher pT, a negative correction blows up
  - ➢ Drives the cross-section negative

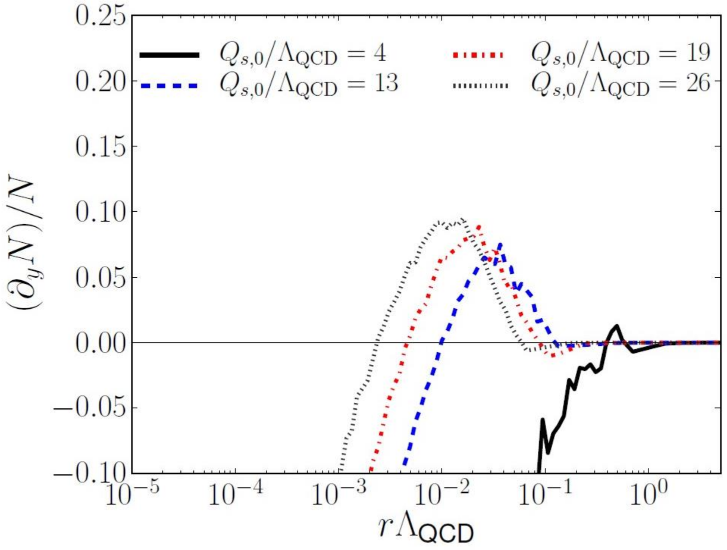


- **What has gone wrong?**
  - ➢ Is there a fundamental problem at NLO?
  - ➢ Or is there physics that has been missed?

The Negativity Problem in Small-x Evolution

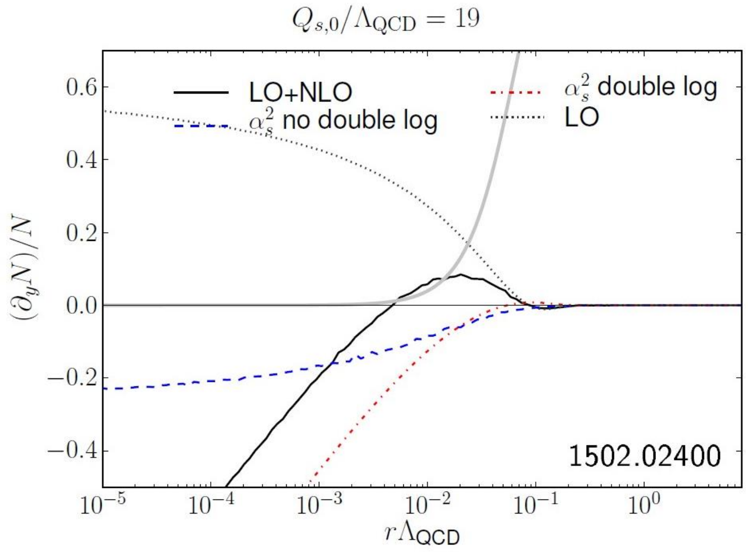
T. Lappi, H. Mantysaari, Phys. Rev. ***D91*** *(2015) 074016*

- **The same negativity problem also arises in small-x evolution at NLO**
    - ➢ The “evolution speed” becomes negative due to large NLO corrections



- **The problematic term is one which generates an additional large logarithm in the gluon cascade**

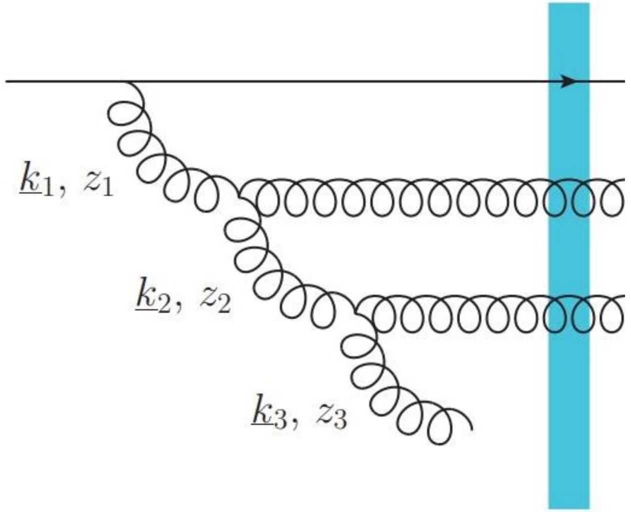
$$\frac{\partial S_{xy}}{\partial Y} \supset -\frac{\bar{\alpha}_s^2}{4\pi} \int d^2 z \frac{(x-y)_\perp^2}{(x-z)_\perp^2 \, (z-y)_\perp^2} (S_{xz} S_{zy} - S_{xy}) \ln \frac{(x-z)_\perp^2}{(x-y)_\perp^2} \ln \frac{(y-z)_\perp^2}{(x-y)_\perp^2}$$



Lifetime Ordering: New Constraints Needed at NLO

B. Ducloue et al., JHEP. ***1904*** *(2019) 081*

- **In both cases, the physical origin of the instability is the need to impose lifetime ordering at NLO**
  - ➢ Imposes conditions which link the longitudinal and transverse integrals



- **At LO, lifetime ordering is unimportant**
  - ➢ Integrals are well-behaved and obtain leading log without it
  - ➢ Transverse integral can be unrestricted

$$\Delta x_1^+ \gg \Delta x_2^+ \gg \Delta x_3^+ \gg \cdots$$

$$\frac{k_{1T}^2}{z_1} \ll \frac{k_{2T}^2}{z_2} \ll \frac{k_{3T}^2}{z_3} \ll \cdots$$

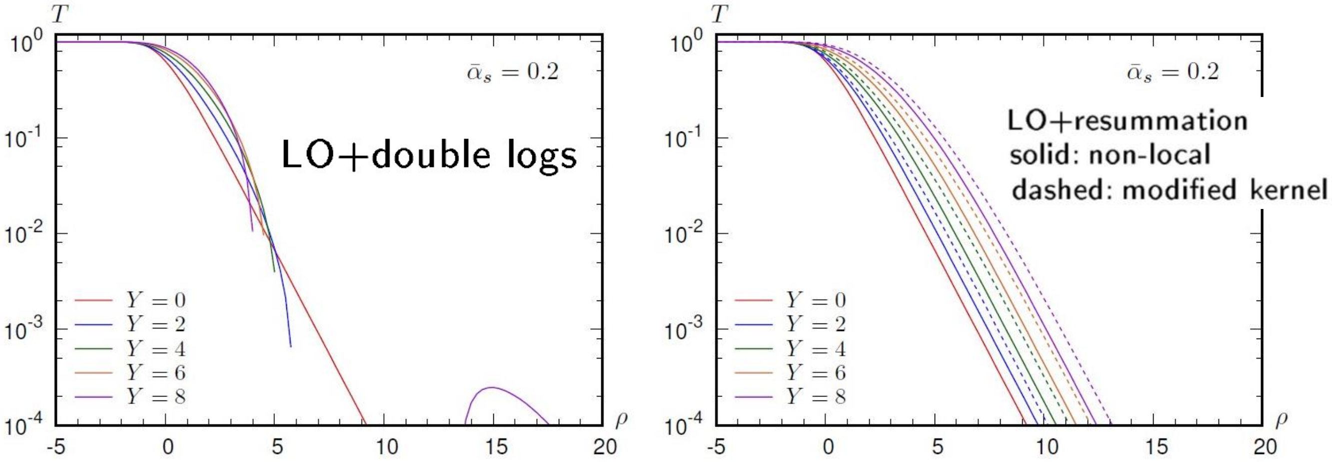
- **At NLO, corrections from collinear splittings which violate lifetime ordering can spoil the convergence**

$$x_{1T}^2 z_1 \gg x_{2T}^2 z_2 \gg x_{3T}^2 z_3 \gg \cdots$$

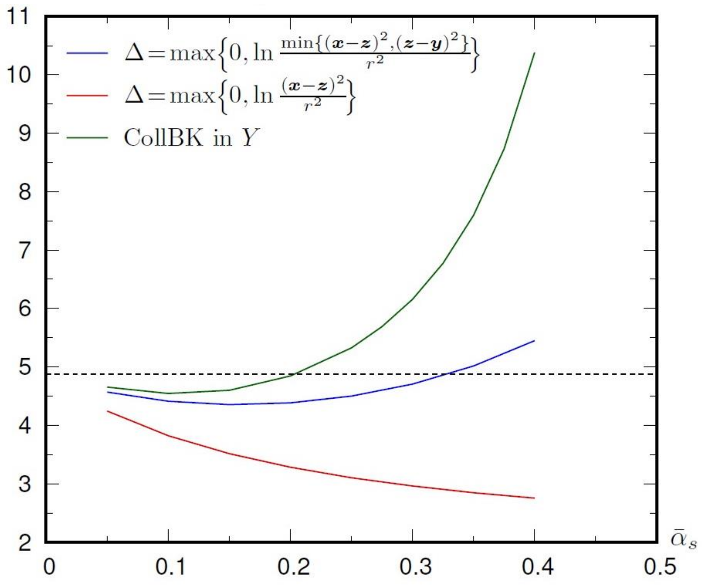
Collinear Resummation Schemes at NLO

E. Iancu et al., Phys. Lett. ***B744*** *(2015) 293*

- **There are multiple resummation schemes for the collinear logs**
  - ➢ All equivalent with NLO accuracy
- **Resummation solves the negativity problem**
  - ➢ Introduces scheme dependence
- **Clearly resummation is necessary to make physical NLO predictions**
  - The “best” approach is debatable



$$\frac{\bar\lambda_s}{\bar\alpha_s} = \frac{1}{\bar\alpha_s}\frac{\mathrm{d}\ln Q_s^2}{\mathrm{d}\eta}\bigg|_{\eta\to\infty}$$

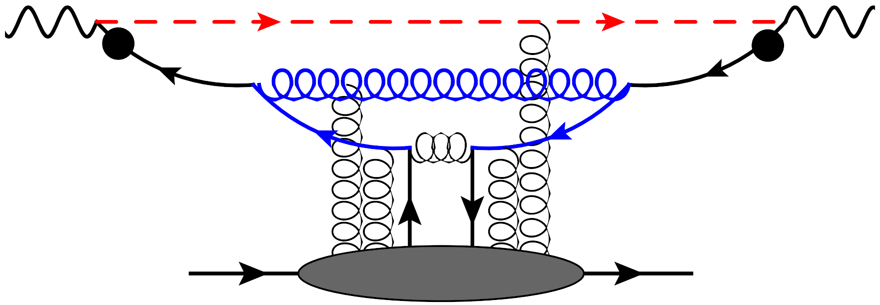


Collinear Logs in the Sub-eikonal Sector

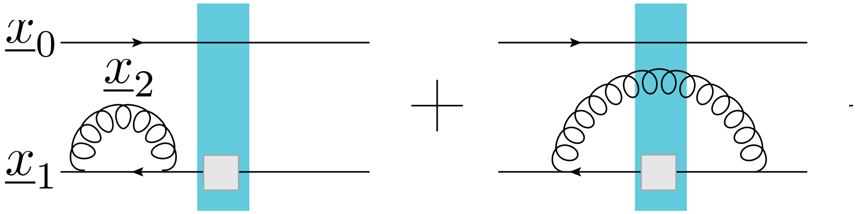
R. Kirschner, L. Lipatov, Sov. Phys. JETP ***56*** *(1982) 266*

- **Spin asymmetries at small x are one class of sub-eikonal corrections**
  - ➢ Small-x tails of helicity PDFs

$$A_{LL} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}$$



- **Partons must transfer spin through the shockwave**
  - ➢ Sensitive to short-distance structure around the polarized parton
  - ➢ Generates a collinear logarithm



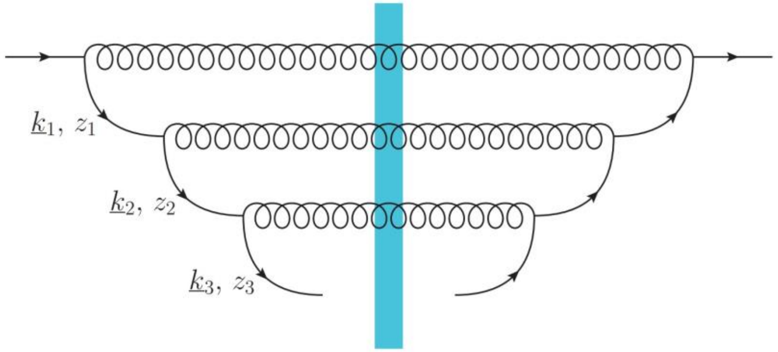
- **Small-x evolution of spin is double logarithmic**
  - Resummation parameter demands systematic treatment of collinear logs

$$\alpha_s \int \frac{dz}{z} \int \frac{dx_{21}^2}{x_{21}^2} \sim \alpha_s \ln^2 \tfrac{1}{x}$$

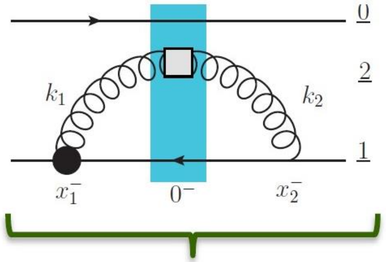
Same Problems Have Same Solutions

Y. Kovchegov, D. Pitonyak, ***M.D.S.****,, JHEP* ***1601*** *(2016) 072*

- **Deriving the double-logarithmic evolution equations requires enforcing lifetime ordering**
  - ➢ Effect enters at LO for this observable

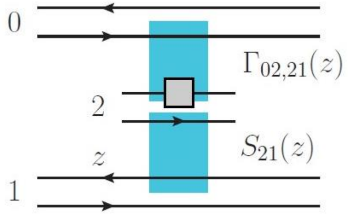


- **Like the NLO case, lifetime ordering complicates the longitudinal ordering in k^+**
  - ➢ Even in the large-Nc limit, not all dipoles are independent
  - ➢ Introduces messy auxiliary functions



Constrained by lifetime of x_{21}

- **The resummation parameter $\alpha_s \, ln^2 \, {}^{1}/_{x}$ provides systematic control over how to treat the collinear logs**
  - ➢ No scheme dependence arising from different choices of how to treat the logs



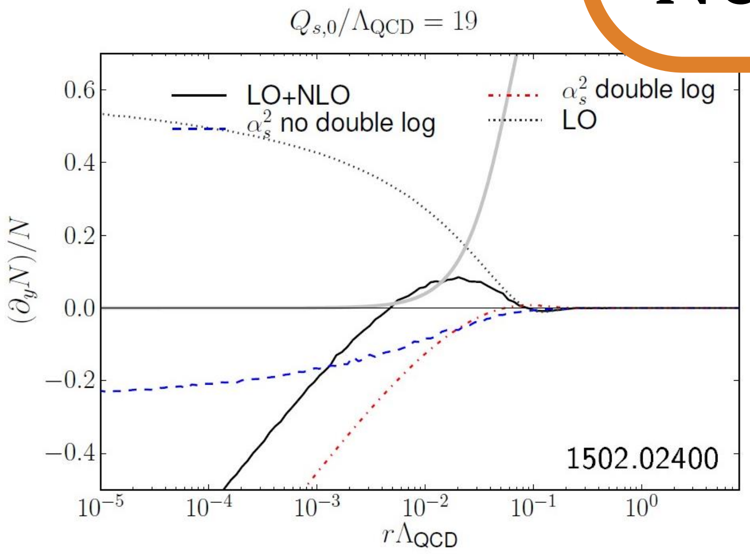
Evolution of x_{02} dipole is constrained by x_{21}

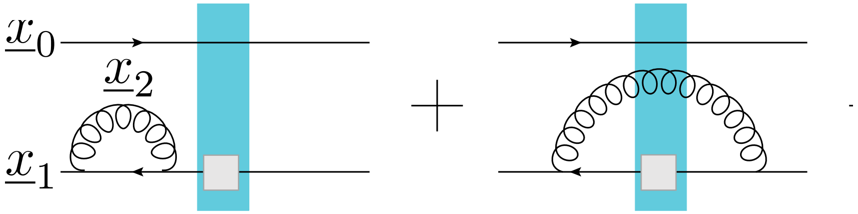
A Common Thread… And a Deeper Meaning?

NLO

Sub-eikonal

New Collinear Logarithms





$$\alpha_s \int \frac{dz}{z} \int \frac{dx_{21}^2}{x_{21}^2} \sim \alpha_s \ln^2 \frac{1}{x}$$

Frontiers of Small x at a Glimpse

- **The NLO Frontier**
  - Significant progress has been made. Evolution equations and solutions mostly under control. Emphasis is shifting to calculation of full processes at NLO.
  - New physical effects appear at NLO which were absent at LO. Is this worrisome, or is it understood?

- **The Sub-eikonal Frontier**
  - Many disparate types of corrections which challenge the entire small-x framework
  - Easy to miss some contributions (quark exchange) because they violate intuition
  - Can all $O(^1/_s)$ corrections be systematically included?

- **The Multiparticle Frontier**
  - Two-gluon correlations understood analytically and numerically
  - Emphasis extending to many-gluon correlations and quark correlations

A Closing Perspective

- **There are many small-x frameworks on the market, all consistent at LO**
  - Are they still consistent at NLO, or do differences emerge?
  - Which frameworks are better than others?

- **What is the regime of validity of the small-x paradigm?**
  - Is it systematically improvable to higher accuracies?
  - Or does something fundamental break down when higher-order or sub-eikonal corrections are included?

- **Extending the boundary of "civilized small-x theory" is essential**
  - Discovering saturation will require teasing a small signal a large background
  - Need robust, precise global analyses which consistently combines many processes
  - Discovering or falsifying gluon saturation is a critical test of the UV-completeness of QCD