Aspects of supersymmetric plasma dynamics

Jakub Jankowski

Faculty of Physics, University of Warsaw

I. Aniceto, B. Meiring, J.J. and M. Spaliński, JHEP **1902**, 073 (2019)
C. Ecker, J.J., and M. Spaliński to appear ...

Outline

- Fast hydrodynamisation on the level of energy-momentum tensor caused by exponentially damped, *transient* modes
- Energy-momentum tensor admits a *transseries* expansion incorporating hydrodynamical and transient degrees of freedom
- Universal aspects of thermalisation on the level of non-local probes. Absence of hydrodynamisation

SYM vs. QCD plasma

- $\mathcal{N} = 4$ SYM = gluons + 6 scalars + 4 fermions ($m = 0$)
- Similarities

  → deconfined phase

  → strongly coupled

  → no SUSY at finite T

  → at weak coupling similar to pQCD plasma

  A. Czajka, S. Mrówczyński, Phys. Rev. D **86**, 025017 (2012)
- Differences

  → no running coupling

  → no confinement-deconfinement phase transition

  → exactly conformal EoS
- Perspective

  first principles derivation of hydrodynamic gradient expansion in a strongly coupled gauge theory

Bjorken flow

- Boost invariant metric $ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$
- Energy momentum tensor is diagonal

$$T_{\mu\nu} = \mathrm{diag}\{\epsilon(\tau), P_L(\tau), P_T(\tau), P_T(\tau)\}$$

- Conditions: $\nabla_\mu T^{\mu\nu} = 0$ and $T^\mu_\mu = 0$ imply

$$P_L = -\epsilon - \tau\dot{\epsilon}\ , \qquad\qquad P_T = \epsilon + \frac{1}{2}\tau\dot{\epsilon}$$

- Evolution of the system is captured by a single function $\epsilon(\tau)$
- Strict for an infinite energy collision of infinitely large nuclei

J. D. Bjorken, Phys. Rev. D **27**, 140 (1983)

Bjorken flow

- Energy density *defines* local effective temperature

$$\epsilon(\tau) = \frac{3}{8} N_c^2 \pi^2 T(\tau)^4$$

- Dimensionless time variable measured in units of relaxation time $\tau_\pi \sim 1/T(\tau)$, i.e., $w = \tau T(\tau)$
- At late times we have

$$T(\tau) = \frac{\Lambda}{(\Lambda\tau)^{1/3}} \left(1 - \frac{1}{6\pi^2} \frac{1}{(\Lambda\tau)^{2/3}} + \frac{\log 2 - 1}{36\pi^2} \frac{1}{(\Lambda\tau)^{4/3}} + \dots \right)$$

and energy scale Λ is the only trace of initial conditions

J. D. Bjorken, Phys. Rev. D **27**, 140 (1983)

W. Florkowski,*et al.* Rept. Prog. Phys. **81**, no. 4, 046001 (2018)

Gravity dual of Bjorken flow

- Using gauge/gravity duality one can compute the non-perturbative QFT energy-momentum tensor from the dual geometry
- For the simplest case the $d = 5$ geometry is determined by

$$R_{ab} + 4g_{ab} = 0$$

- Can be embedded into $d = 10$ SUGRA
- Detailed studies of dynamics of SYM plasma

M. P. Heller, R. A. Janik, P. Witaszczyk, Phys. Rev. Lett. **108**, 201602 (2012)

J. J, G. Plewa and M. Spaliński, JHEP **1412**, 105 (2014)

R. A. Janik, Lect. Notes Phys. **828**, 147 (2011)

Pressure anisotropy and universality

- Pressure anisotropy is defined to be

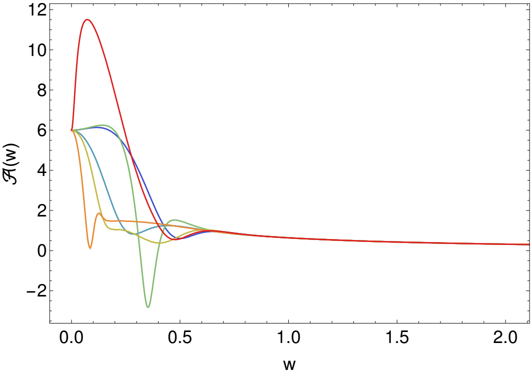
$$\mathcal{A}(w) = \frac{p_L - p_T}{p}$$

where $p = \epsilon/3$ is equilibrium pressure

- $T(\tau) \sim \frac{\Lambda}{(\Lambda\tau)^{4/3}} + \cdots$ implies that $\mathcal{A}(w)$ becomes independent of initial conditions at late time
- Since $\mathcal{A} = 0$ for equilibrium it is a measure of the distance from equilibrium, and is computable from AdS/CFT
- Universal approach to equilibrium: initial state information is dissipated exponentially at early times
- Can one find something similar for non-local probes?

Thermalization $\neq$ Hydrodynamization

$$\mathcal{A}(w_0) = \frac{p_L - p_T}{p} \sim 1.3 \text{ at } w_0 \sim 0.7$$



J. J, G. Plewa and M. Spaliński, JHEP **1412**, 105 (2014)

High-order gradient expansion

- AdS/CFT allows to compute *all* coefficients of

$$\varepsilon_{\text{hydro}}(\tau) \sim \frac{\Lambda}{(\Lambda\tau)^{4/3}} \sum_{n=0}^{\infty} \varepsilon_n^{(0)} (\Lambda\tau)^{-2n/3}$$

- Coefficients $\varepsilon_n^{(0)} \sim \Gamma(n+\beta) A_1^{-n-\beta}$ for $n \gg 1$
- A_1 is determined by the dual black hole quasinormal mode frequency
- $\mathrm{Im}\, A_1 \sim \tau_0^{-1}$ where τ_0 is the equilibration time, $\tau_0 \sim 0.5 - 1\ \mathrm{fm/c}$ at RHIC and LHC

M. P. Heller *et al.* Phys. Rev. Lett. **110**, no. 21, 211602 (2013)

I. Aniceto, B. Meiring, J. J. and M. Spaliński, JHEP **1902**, 073 (2019)

Transient modes

- Exponentially damped modes

$$\varepsilon(\tau) \sim \varepsilon_{\text{hydro}}(\tau) + \frac{\Lambda\sigma_1}{(\Lambda\tau)^{4/3}} \sum_{n=0}^{\infty} \varepsilon_n^{(1)} (\Lambda\tau)^{-2n/3} e^{-A_1(\Lambda\tau)^{2/3}} + \cdots$$

- Series $\varepsilon_n^{(1)} \sim \Gamma(n+\beta_2) A_2^{-n-\beta_2}$ for $n \gg 1$
- All information is stored in the $\varepsilon_n^{(0)}$ coefficients: resurgence property
- σ_1 is a transseries parameter that encodes initial state
- First, strong evidence for resurgence in strongly coupled QFT

M. P. Heller *et al.* Phys. Rev. Lett. **110**, no. 21, 211602 (2013)

I. Aniceto, B. Meiring, J. J. and M. Spaliński, JHEP **1902**, 073 (2019)

Making sens of divergent series

- The Borel transform is defined

$$\mathcal{B}\left[\varepsilon_{\mathrm{hydro}}\right](\xi) = \xi^{\beta-\frac{1}{2}} \sum_{n=0}^{\infty} \frac{\varepsilon_n^{(0)}}{\Gamma(n+\beta+\frac{1}{2})} \xi^n$$

- Has finite radius of convergence
- First singularities appear at $\xi = A_1$ and $\xi = \bar{A}_1$
- Singularity encodes the coefficients $\epsilon_n^{(1)}$
- Analyze numerically by Borel-Padé approximant

I. Aniceto, G. Basar and R. Schiappa, arXiv:1802.10441 [hep-th]

I. Aniceto, B. Meiring, J. J. and M. Spaliński, JHEP **1902**, 073 (2019)

Numerical analysis and Borel-Pade approximant

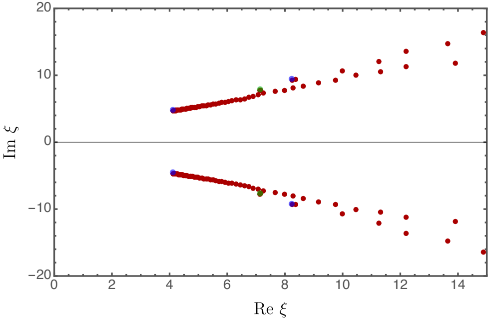
- Provided limited number of coefficients one uses Borel-Pade approximant to analytically continue the Borel transform

$$\mathrm{BP}_N\left[\Phi\right](s) = \frac{P_N(s)}{Q_N(s)}$$

  where $P_N(s)$ and $Q_N(s)$ are polynomials

- Poles of the approximant resemble the singularity structure of the exact Borel transform

Borel plane



Poles of the Borel-Padé approximant $\mathrm{BP}_{189}\left[\epsilon_{\mathrm{hydro}}\right]$, in the complex $\xi-$plane $\xi = A_1, \overline{A_1}, 2A_1, 2\overline{A_1}$ $\quad \xi = A_2, \overline{A_2}$

I. Aniceto, B. Meiring, J. J. and M. Spaliński, JHEP **1902**, 073 (2019)

Large order relations

- For the hydrodynamic series at the leading singularity

$$\varepsilon_n^{(0)} \sim -\frac{S_{0\to 1}}{2\pi \mathrm{i}} \frac{\Gamma(n+\beta)}{A_1^{n+\beta}} \left(\varepsilon_0^{(1)} + \frac{A_1 \varepsilon_1^{(1)}}{n+\beta-1} + \right.$$

$$\left. + \frac{A_1^2 \varepsilon_2^{(1)}}{(n+\beta-1)(n+\beta-2)} + \cdots \right) + \mathrm{c.c.} + \cdots$$

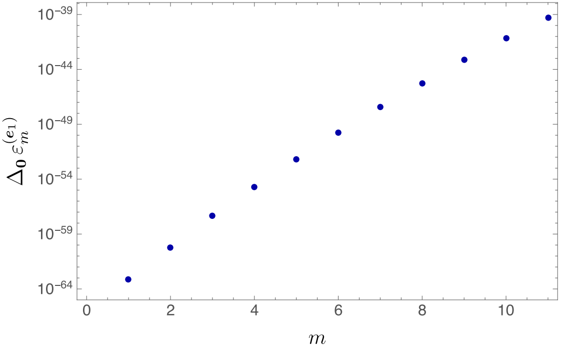
where $S_{0\to 1}$ is the Stokes constant $(\beta = \beta_{\mathbf{0}} - \beta_1)$

- Large order relations contain contributions from *all* sectors and couplings between them

- Stokes constant is determined numerically

$$S_{0\to 1} = 0.01113\cdots - \mathrm{i}0.03050\cdots$$

I. Aniceto, G. Basar and R. Schiappa, arXiv:1802.10441 [hep-th]



$$\Delta_{\boldsymbol{n}} \varepsilon_k^{(\boldsymbol{m})} \equiv \frac{\varepsilon_k^{(\boldsymbol{m})} \left.\right|_{\boldsymbol{n}-\text{predicted}} - \varepsilon_k^{(\boldsymbol{m})} \left.\right|_{\text{numerical}}}{\varepsilon_k^{(\boldsymbol{m})} \left.\right|_{\text{numerical}}}, \quad k \geq 1,$$

I. Aniceto, B. Meiring, J. J. and M. Spaliński, JHEP **1902**, 073 (2019)

Entanglement entropy and holography

- The entanglement entropy of a certain subsystem R is defined by
$$S = -\mathrm{tr}_R \left(\rho_R \log \rho_R \right)$$
where $\rho_R = \mathrm{tr}_{\bar{R}} \rho$ is the partial trace over the complement $\bar{R}$

- In holography the entanglement entropy of a region R in the field theory is equal to the area A of an extremal bulk surface homologous to R
$$S = \frac{A}{4G_5}$$

S. Ryu and T. Takayanagi, Phys. Rev. Lett. **96**, 181602 (2006)

S. Ryu and T. Takayanagi, JHEP **0608**, 045 (2006)

Entanglement Entropy and Bjorken flow

- We choose as an entangling region

$$R = \{\tau = \tau_0, -\ell_{||}/2 \leq x_{||} \leq \ell_{||}/2, -\ell_{\perp}/2 < \vec{x}_{\perp} < \ell_{\perp}/2\}$$

as an entangling region with $\ell_{\perp} \to \infty$

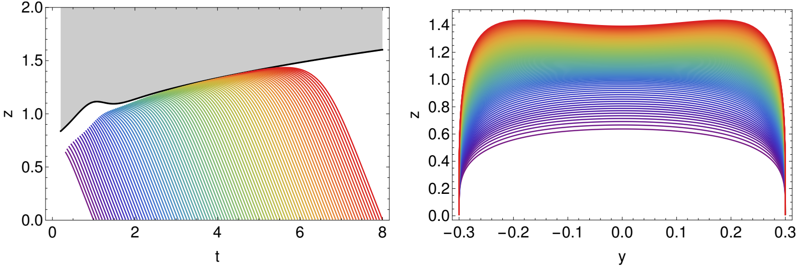
- We can either fix $\ell_{||}$ or ℓ_y using the relation

$$\ell_y = 2\sinh^{-1}\left(\ell_{||}/(2\tau)\right)$$

- S is obtained numerically for the whole time range

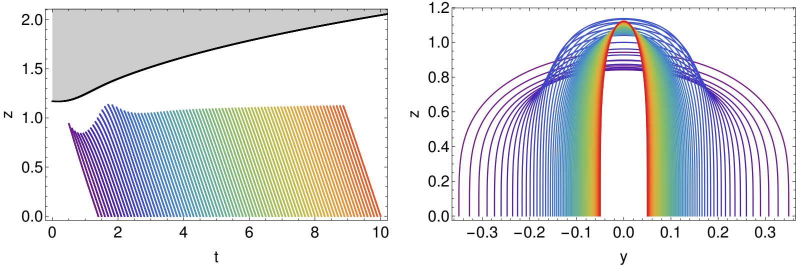
C. Ecker, arXiv:1809.05529 [hep-th]

Extremal surfaces for fixed rapidity



Typical example for RT surfaces with fixed rapidity separation $\ell_y = 0.6$. At later values of the proper time these surfaces come arbitrarily close to the horizon (black line) without ever crossing it

Extramal surfaces for fixed spatial separation



Family of RT-surfaces with fixed spatial separation $\ell_{||} = 1$ such as used to compute the time evolution of the entanglement entropy. To keep the spatial separation $\ell_{||}$ fixed we change the rapidity separation ℓ_y

Late time behaviour for Entanglement Entropy

- We expect S to have the late time expansion

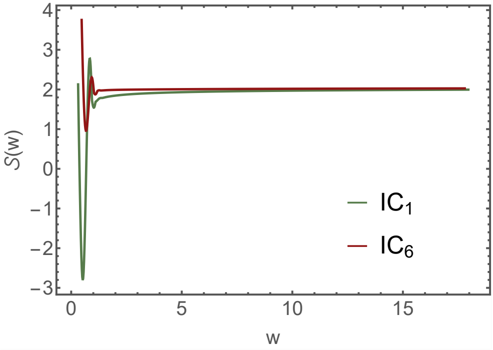
$$S(\tau) = S^{(0)} + \frac{N^2 \Lambda^4 s^{(4)} \ell_{\parallel}^2 \ell_{\perp}^2}{(\Lambda\tau)^{\frac{4}{3}}} + \dots$$

where $S^{(0)} = -\frac{2\sqrt{\pi}\Gamma(\frac{2}{3})^3}{\Gamma(\frac{1}{6})^3} \left(\frac{\ell_\perp}{\ell_\parallel}\right)^2$ is the vacuum entanglement entropy, and $s^{(4)} \approx 7.34209$

- This suggest a *universal* quantity

$$\mathcal{S}(w) \equiv \frac{S - S^{(0)}}{\ell_{\parallel}^2 \ell_{\perp}^2 \mathcal{E}} \sim \mathcal{S}_\infty \left(1 + O(1/w)\right)$$

with $\mathcal{S}_\infty = \frac{8 s^{(4)}}{3\pi^2} \approx 1.98376$ found numerically



$$\mathcal{S}(w) \equiv \frac{S - S^{(0)}}{\ell_{\parallel}^2 \ell_{\perp}^2 \mathcal{E}} \sim \mathcal{S}_{\infty} \left(1 + O(1/w)\right)$$

for fixed boundary separation $\ell_{\parallel} = 0.5$ and six initial states

Summary

- Hydrodynamisation of energy-momentum tensor is understood in terms of exponentially damped modes within resurgent, transeries expansion
- Universal aspects of thermalization on the level of non-local probes, but no hydrodynamisation effect
- Non-local probes thermalize much slower than one-point functions