Fluctuations and clustering of multiplicities

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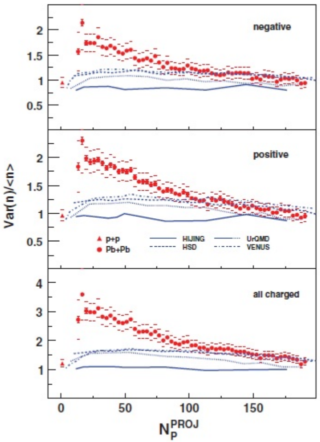
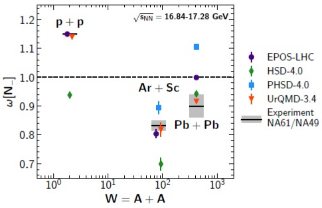
Outline

- In this talk we present predictions of the dynamical clusters model (DCM) for **multiplicity fluctuations of charged particles** produced in collisions of relativistic ions.
- The DCM predictions are compared to the recent measurements of the charged particles multiplicity fluctuations done by **NA49** [Phys. Rev. C 75, 064904] and **NA61/SHINE** [PoS(EPS-HEP2017)167; J. Phys. G 45, 115104] experiments.

Motivation





Figures taken from [Phys. Rev. C 75, 064904] and [J. Phys. G 45, 115104].

Introduction

- Collision of relativistic ions leads to a production of hot quark-gluon plasma, which cools and at $T = 155 \pm 10$ MeV transits to a hadron gas of that temperature.

- The hot quark-gluon system during the transition is effectively quenched by the cold physical vacuum. The so-called **self-organized criticality** is the appropriate mechanism leading to **universal scale-free behaviour** [arXiv:1901.10407v1].

Introduction

- Self-organized criticality (SOC) is a property of non-equilibrium dynamical systems that have a **critical point as an attractor**. The macroscopic properties of such systems are characterized by the the spatial and/or temporal **scale-invariance of the phase transition critical point**. Unlike equilibrium systems which require the tuning of parameters to get a critical behavior, non-equilibrium SOC systems tune itself during evolution in the direction of criticality [Phys. Rev. Lett. 59, 381].
- A remarkable feature of active matter is the **propensity to self-organize**. One striking instance of this ability to generate spatial structures is the **cluster phase**, where cluster broadly distributed in size constantly move and evolve through particle exchange.

Introduction/Model description

- Thus, we can define cluster as a quasi-neutral gas of charged and neutral particles which exhibits collective behavior. The characteristic space scale of this shielding is the Debye length (or radius):

$$\lambda_D^2 = \frac{kT}{4\pi e^2 n},$$

where n is the density of charged pions.

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- In the Debye sphere of the volume:

$$V = \frac{4}{3}\pi\lambda_D^3$$

  we have $N \simeq 143$ charged pions, what corresponds at $\sqrt{s_{NN}} = 17$ GeV to the number of projectile participants $N_p \simeq 18$.

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- Within this model there is also possible to obtain **multiplicity fluctuations** since the status of the hadronizing sources is known.
- In the canonical ensemble (CE) with exact conservation of charges, scaled variance of the multiplicity distribution of any particle does not converge to the corresponding grand canonical (GCE) value even in the thermodynamic limit, unlike the mean.

Introduction/Model description

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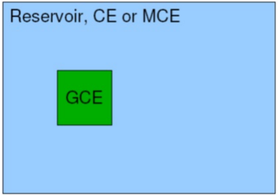
- We split a CE with a large volume into cluster, which is a Grand Canonical Ensemble with the rest of the system being a reservoir.



GCE is defined as a small subsystem of a large reservoir.

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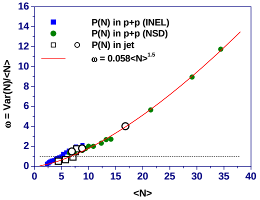
- Variance of multiplicity distribution in a cluster, $Var(N)$, depends on the mean multiplicity $\langle N \rangle$ of the system, $Var(N) = \langle N \rangle + \langle N \rangle^2 / k$.



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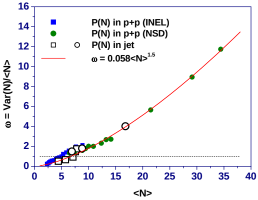

- The rough formula $\omega = 0.058\langle N \rangle^{1.5}$ asserts Taylor's law, $Var(N) = a \cdot \langle N \rangle^b$ with exponent $b > 2$. Such behaviour corresponds a geometrical random walk (as opposed to the ordinary additive random walk) if multiplicity density at each step grows on average (super-critical model) [Proc. R. Soc. B 280, 20122955].

Experimental data

- The NA49 experiment located at CERN SPS analyzed fluctuations of the number of charged particles produced in Pb+Pb collisions at $\sqrt{s_{NN}} = 17.3$ GeV [Phys. Rev. C 75, 064904].

- The NA49 experiment registered multiplicity distributions of particles produced in the forward hemisphere in restricted rapidity interval $1.1 < y_{c.m} < 2.6$. Such restriction corresponds to fraction of about 17% of accepted charged particles.

- The magnitude of multiplicity fluctuations was determined by value of scaled variance of multiplicity distribution, $\omega(N)$, defined as:

$$\omega(N) = \frac{Var(N)}{\langle N \rangle},$$

where $Var(N)$ is the variance of the distribution and $\langle N \rangle$ is the average multiplicity.

Experimental data

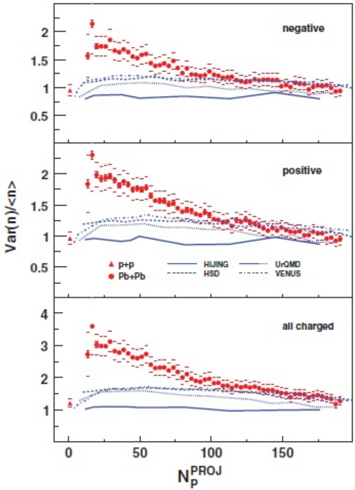

Figure taken from [Phys. Rev. C 75, 064904].

Model results

To describe the NA49 data on centrality dependence of multiplicity fluctuations the following dynamical clusterization method was used:

- Each projectile nucleon participating in collision “produces” particles independently,

$$\langle N \rangle = N_p \cdot \langle N_{ch} \rangle,$$

  where $\langle N \rangle$ is the average multiplicity produced in Pb+Pb collisions at particular centrality, N_p is the number of nucleons from projectile nucleus participating in collision and $\langle N_{ch} \rangle$ is the average multiplicity produced in proton-proton interactions.

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- Having calculated $\langle N \rangle$, multiplicity in a given event of collision is calculated according to NBD distribution **with the shape parameter** k **dependent on** $\langle N \rangle$.

- This is up to certain value of $N_p = N_p^{max}$, for which clusters of secondary particles may be formed. The value of N_p^{max} is sampled from Gamma distribution with $\langle N_p^{max} \rangle = 18$ and $Var\left(N_p^{max}\right) = 9 \cdot \langle N_p^{max} \rangle$.

Model results

- The rest of colliding projectile nucleons, $m = N_p - N_p^{max}$ do not contribute their produced particles to the cluster. The produced by them particles are emitted according to Binomial distribution:

$$P_{BD}(N, n, p) = \binom{n}{N} p^N (1-p)^{n-N}$$

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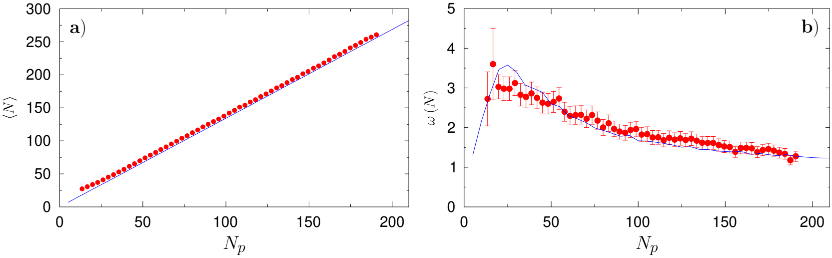
- Clusters of particles are formed with a certain probability, $p_c = 0.25$.

- If cluster of particles is not formed then all colliding nucleons emit their particles according to Binomial distribution.

Model results



Average number of all charged particles (panel a)) and scaled variance of all charged multiplicity distribution (panel b)) of particles produced in Pb+Pb collisions plotted as a function of number of nucleons from projectile nucleus which participate in the collision. Circles – NA49 data [Phys. Rev. C 75, 064904].

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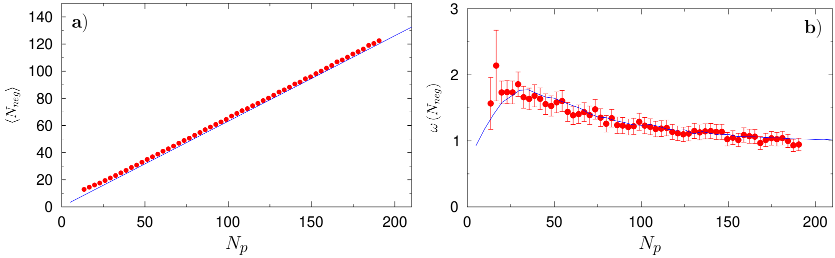

Average number of negatively charged particles (panel a)) and scaled variance of negatively charged multiplicity distribution (panel b)) of particles produced in Pb+Pb collisions plotted as a function of number of nucleons from projectile nucleus which participate in the collision. Circles – NA49 data [Phys. Rev. C 75, 064904].

The other possibility - LogNormal instead of NBD in the cluster

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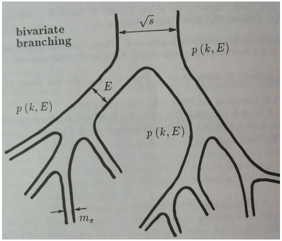
- Each of the created quarks may again emit a gluon and so on.



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- The bivariate branching process has a multiplicative nature, i.e. the number of particles in a given generation is proportional to the number of particles in the previous one.

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- Taking logarithms of both sides and expanding $\ln(1+\epsilon_i)$ near 1 we see that right hand side of the above equation is a sum of a random components:

$$\ln(N/N_0) = \sum_i \epsilon_i.$$

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- Such a sum should be distributed normally due to the Central Limit Theorem. Hence, also a left hand side, i.e. a logarithm of multiplicity should be distributed normally. In other words, multiplicity should be distributed lognormally:

$$P(N) = \frac{1}{\sqrt{2\pi}\sigma} \cdot \frac{1}{N} \exp\left(-\frac{[\ln N - \mu]^2}{2\sigma^2}\right).$$

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- Formally we get the LogNormal distribution in the limit of a large number of steps. Thus LogNormal distribution is continuous. In the case of use of LogNormal distribution for the multiplicities we should apply:

$$P_N = \int_N^{N+1} P(N)\, dN.$$

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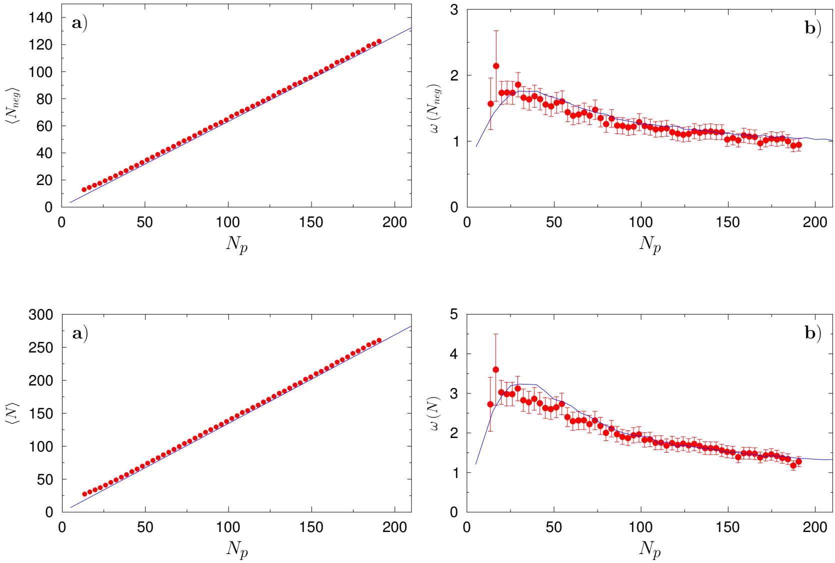

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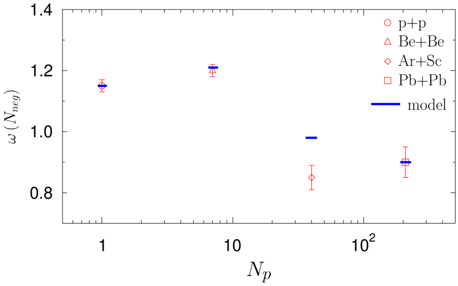
- Using similar considerations we have obtained the values for scaled variance of negatively charged particles multiplicity distribution produced in p+p and the most central (1%) Be+Be, Ar+Sc and Pb+Pb collisions, and emitted to forward hemisphere, $y_\pi > 0$ [PoS(EPS-HEP2017)167; J. Phys. G 45, 115104].



Conclusions

- We discussed the recently measured event-by-event multiplicity fluctuations in relativistic heavy-ion collisions.
- It is shown that the observed non-monotonic behaviour of the scaled variance of multiplicity distribution as a function of collision centrality (such effect is not observed in a widely used string-hadronic models of nuclear collisions) can be fully explained by the correlations between produced particles promoting cluster formation.
- The ability to generate spatial structures (cluster phase) sign the propensity to self-organize of hadronic matter. Multiplicity clustering provide new insights on non-monotonic behaviour of multiplicity fluctuations.
- This talk is based on the manuscript: **arXiv:1904.01366**.

Additional slides

Imprints of acceptance

- Let us assume that $g(M)$ presents a real distribution which describe multiplicity distribution in the full phase space. Scaled variance ω is given by parameters of such distribution. For example:

$$\omega = \begin{cases} 1 + \langle M \rangle / k & \text{for NBD} \\ 1 & \text{for Poisson} \\ 1 - \langle M \rangle / k & \text{for BD} \end{cases} \tag{1}$$

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- However, in the experiment we measure the multiplicity only within some window in rapidity Δy. Roughly, for fixed acceptance α we have

$$\omega = \alpha \cdot \omega_{\alpha=1}, \tag{2}$$

and scaled variance decrease monotonically with decreasing acceptance.

Imprints of acceptance

- Of course, the above discussed procedure is not correct. Let us assume that the detection process is a Bernoulli process described by the BD with the generating function

$$F(z) = 1 - \alpha + \alpha \cdot z, \tag{3}$$

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- The number of the registered particles is
$$N = \sum_{i=1}^{M} n_i, \tag{4}$$
where n_i follows the BD with the generating function $F(z)$ and M comes from $g(M)$ with the generating function $G(z)$. The measured multiplicity distribution $P(N)$ is therefore given by the generating function
$$H(z) = G[F(z)] \tag{5}$$
and finally we have
$$P(N) = \frac{1}{N!} \left. \frac{d^N H(z)}{dz^N} \right|_{z=0} \tag{6}$$

Imprints of acceptance

- Note that such procedure applied to NBD, Poisson distribution (PD) or BD gives again the same distributions but with modified parameters. The scaled variance is given by:

$$\omega = \begin{cases} 1 + \alpha\langle M\rangle/k = 1 + \langle N\rangle/k & \text{for NBD} \\ 1 & \text{for PD} \\ 1 - \alpha\langle M\rangle/k = 1 - \langle N\rangle/k & \text{for BD} \end{cases} \quad (7)$$

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- For all mentioned in Eq. 7 distributions $\omega \to 1$ when $\alpha \to 0$. In the case of small acceptance, the observed $P(N)$ tends to PD.