New results from L3 using Levy-expansions

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$e^+e^- \longrightarrow$ hadrons

- a clean environment for studying hadronization
- everything is jets – no spectators
- at $\sqrt{s} = M_Z$ almost all events are

  2-jet $e^+e^- \longrightarrow q\bar{q}$ or 3-jet $e^+e^- \longrightarrow q\bar{q}g$

- event hadronization axis is the $q\bar{q}$ direction
  estimate by the thrust axis, *i*.e., axis $\vec{n}_T$ for which
  $T = \frac{\sum |\vec{p}_i \cdot \vec{n}_T|}{\sum |\vec{p}_i|}$ is maximal
- 3-jet events are planar.
  Estimate event plane by thrust, major axes.
  Major is analogous to thrust, but in plane perpendicular to $\vec{n}_T$.
- Require $\vec{n}_T$ within central tracking chamber
  $\Longrightarrow 4\pi$ acceptance

BEC Introduction

$$R_2 = \frac{\rho_2(p_1, p_2)}{\rho_1(p_1)\rho_1(p_2)} \Rightarrow \frac{\rho_2(Q)}{\rho_0(Q)} \qquad\qquad Q^2 = (p_1 - p_2)^2$$

Assuming particles produced incoherently
with spatial source density $S(x)$,

$$R_2(Q) = 1 + \lambda|\widetilde{S}(Q)|^2$$

where $\widetilde{S}(Q) = \int \mathrm{d}x\, e^{iQx} S(x)$ – Fourier transform of $S(x)$

$\lambda = 1$ — $\lambda < 1$ if production not completely incoherent
and other effects reducing BEC

Assuming $S(x)$ is a spherical Gaussian with radius $r \Longrightarrow$

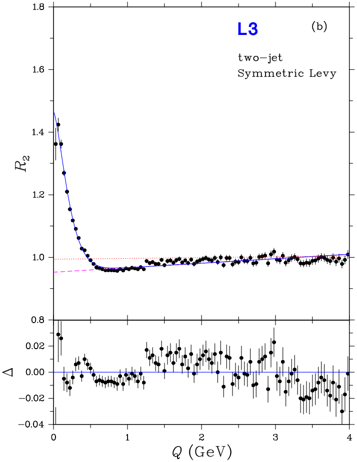
$$R_2(Q) = 1 + \lambda\, \mathrm{e}^{-(Qr)^2}$$

BEC – ‘Classic’ Parametrizations

$$R_2 = \frac{\rho(p_1,p_2)}{\rho_0(p_1,p_2)} = \gamma \cdot [1 + \lambda G] \cdot (1 + \epsilon Q)$$

- Gaussian
  $G = \exp\left(-(rQ)^2\right)$
- Edgeworth expansion
  $G = \exp\left(-(rQ)^2\right) \cdot \left[1 + \frac{\kappa}{3!} H_3(rQ)\right]$
  Gaussian if $\kappa = 0$ $\kappa = 0.71 \pm 0.06$
- symmetric Lévy
  $G = \exp\left(-|rQ|^{\alpha}\right)$, $0 < \alpha \le 2$
  α is index of stability
  Gaussian if $\alpha = 2$ $\alpha = 1.34 \pm 0.04$

Cannot accomodate the anticorrelation seen as a dip in R_2 below unity in the region $0.6 < Q < 1.5\,\mathrm{GeV}$



	Gauss	Edgeworth	sym. Lévy
CL:	10^{-15}	10^{-5}	10^{-8}

L3, EPJC 71 (2011) 1648

The τ-model

T.Csörgő, W.Kittel, W.J.Metzger, T.Novák, Phys.Lett.**B663**(2008)214
T.Csörgő, J.Zimányi, Nucl.Phys.**A517**(1990)588

- Assume avg. production point is related to momentum:
$$\overline{x}^\mu(p^\mu) = a\tau p^\mu$$
where for 2-jet events, $a = 1/m_\mathrm{t}$
$$\tau = \sqrt{\bar{t}^2 - \bar{r}_z^2} \text{ is the “longitudinal” proper time}$$
$$\text{and } m_\mathrm{t} = \sqrt{E^2 - p_z^2} \text{ is the “transverse” mass}$$
- Let $\delta_\Delta(x^\mu - \overline{x}^\mu)$ be dist. of production points about their mean, and $H(\tau)$ the dist. of τ. Then the emission function is
$$S(x,p) = \int_0^\infty \mathrm{d}\tau H(\tau)\delta_\Delta(x - a\tau p)\rho_1(p)$$
- In the plane-wave approx. F.B.Yano, S.E.Koonin, Phys.Lett.**B78**(1978)556.
$$\rho_2(p_1,p_2) = \int \mathrm{d}^4x_1 \mathrm{d}^4x_2 S(x_1,p_1)S(x_2,p_2)\left(1 + \cos\left([p_1 - p_2][x_1 - x_2]\right)\right)$$
- Assume $\delta_\Delta(x^\mu - \overline{x}^\mu)$ is very narrow — a δ-function. Then
$$R_2(p_1,p_2) = 1 + \lambda\,\mathrm{Re}\widetilde{H}\left(\frac{a_1Q^2}{2}\right)\widetilde{H}\left(\frac{a_2Q^2}{2}\right), \qquad \widetilde{H}(\omega) = \int \mathrm{d}\tau H(\tau)\exp(i\omega\tau)$$

BEC in the τ-model

- Assume a Lévy distribution for $H(\tau)$
  Since no particle production before the interaction, $H(\tau)$ is one-sided.
  Characteristic function is
  $$\widetilde{H}(\omega) = \exp\left[-\tfrac{1}{2}\left(\Delta\tau|\omega|\right)^{\alpha}\left(1 - i\,\mathrm{sign}(\omega)\tan\left(\tfrac{\alpha\pi}{2}\right)\right) + i\,\omega\tau_0\right] \;, \quad \alpha \neq 1$$
  where
  - α is the index of stability, $0 < \alpha \leq 2$;
  - τ_0 is the proper time of the onset of particle production;
  - $\Delta\tau$ is a measure of the width of the distribution.
- Then, R_2 depends on Q, a_1, a_2

$$\begin{aligned} R_2(Q, a_1, a_2) \quad = \quad & \gamma\left\{1 + \lambda\cos\left[\frac{\tau_0 Q^2(a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right)\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right]\right. \\ & \left.\cdot\exp\left[-\left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha}\frac{a_1^{\alpha} + a_2^{\alpha}}{2}\right]\right\}\cdot(1 + \epsilon Q) \end{aligned}$$

BEC in the τ-model

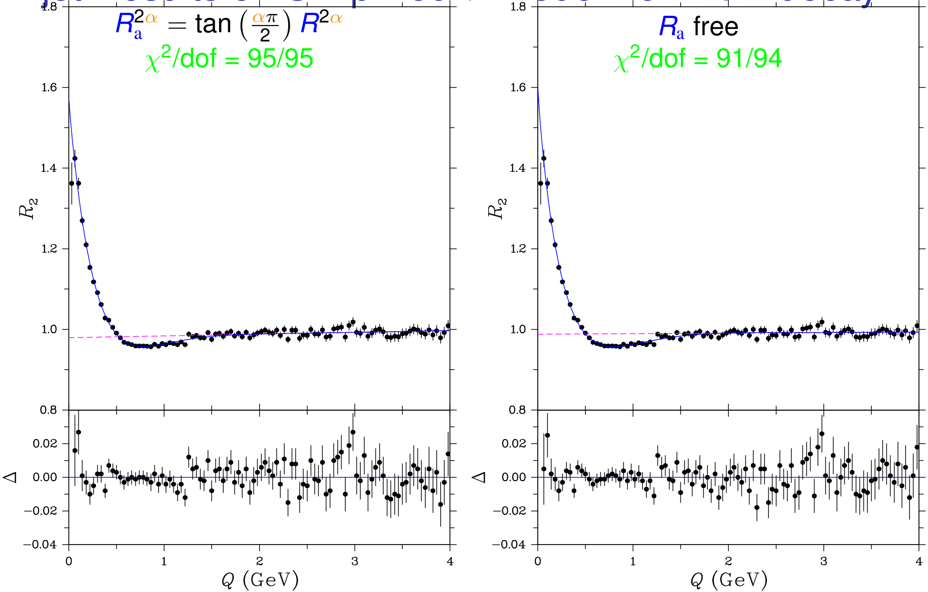
$$R_2(Q, a_1, a_2) = \gamma \left\{ 1 + \lambda \cos\left[\frac{\tau_0 Q^2 (a_1 + a_2)}{2} + \tan\left(\frac{\alpha\pi}{2}\right) \left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2} \right] \cdot \exp\left[- \left(\frac{\Delta\tau Q^2}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2} \right] \right\} \cdot (1 + \epsilon Q)$$

Simplification:

- effective radius, R, defined by $R^{2\alpha} = \left(\frac{\Delta\tau}{2}\right)^{\alpha} \frac{a_1^\alpha + a_2^\alpha}{2}$
- Assume particle production begins immediately, $\tau_0 = 0$
- Then
$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha} \right) \exp\left(-(RQ)^{2\alpha} \right) \right] \cdot (1 + \epsilon Q)$$
  where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

  Compare to sym. Lévy parametrization:
$$R_2(Q) = \gamma \left[1 + \lambda \qquad \exp\left(-|rQ|^{\alpha} \right) \right] (1 + \epsilon Q)$$
- R describes the BEC peak
- R_a describes the anticorrelation dip
- τ-model: both anticorrelation and BEC are related to ‘width’ $\Delta\tau$ of $H(\tau)$

2-jet Results on Simplified τ-model from L3 Z decay



$R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$ agrees well with data

L3, EPJC 71 (2011) 1648

τ-model vs. sym. Lévy

- Simplified τ-model:

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_\mathrm{a} Q)^{2\alpha} \right) \exp\left(-(RQ)^{2\alpha} \right) \right] \cdot (1 + \epsilon Q)$$

where $R_\mathrm{a}^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

- R describes the BEC peak
- R_a describes the anticorrelation dip
- τ-model: Both anticorrelation and BEC are related to 'width' $\Delta\tau$ of $H(\tau)$ i.e. to the temporal distribution of production

- Symmetric Lévy parametrization:

$$R_2(Q) = \gamma \left[1 + \lambda \qquad \exp\left(-|rQ|^{\alpha} \right) \right] (1 + \epsilon Q)$$

- r describes the BEC peak
- the anticorrelation dip is NOT described
- BEC is related to the spatial distribution of the production points

But suppose we did not have the τ-model (or don't believe it):
What to do then?

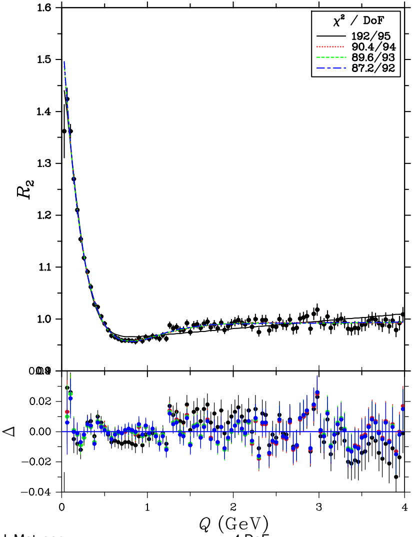
Lévy polynomials

Expand about the Symmetric Lévy distribution using Lévy Polynomials, l_i

Then the Symmetric Lévy parametrization becomes De Kock, Eggers, Csörgő, PoS WPCF 2011 (2011) 033

$$R_2(Q) = \gamma\left[1 + \lambda \exp\left(-|rQ|^{\alpha}\right)\left(1 + \sum c_i l_i\right)\right] \cdot (1 + \epsilon Q)$$

Csörgő, Pasechnik, Ster, arXiv.1807.02897

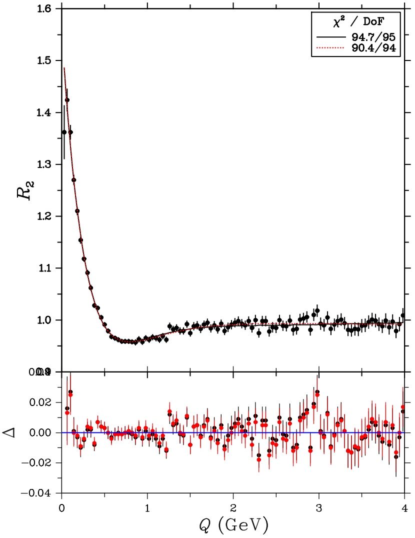


l_i are orthonormal

Fits to succeeding orders provide improved χ^2:

- Order 0: very bad χ^2
- Order 1: good χ^2
- Orders 2-3 give: only marginal further improvement

Lévy polynomials *vs.* τ-model



- χ^2 of order-1 Sym. Lévy polynomial fit
  is a bit better than τ-model
- but not much difference in fits
  difference is mainly for
  $Q > 1.5\,\mathrm{GeV}$

Lévy polynomials *vs.* τ-model

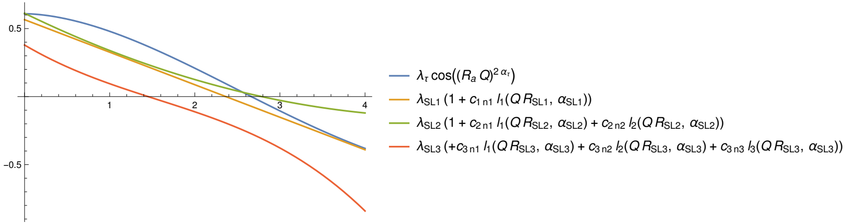
- Simplified τ-model:

$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha}\right) \exp\left(-(RQ)^{2\alpha}\right)\right] \cdot (1 + \epsilon Q)$$

where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$

- Symmetric Lévy polynomial parametrization:

$$R_2(Q) = \gamma \left[1 + \lambda \quad \left(1 + \sum c_i l_i\right) \quad \exp\left(-|rQ|^{\alpha}\right)\right] \cdot (1 + \epsilon Q)$$



- τ-model describes dip by the cosine term
- Sym. Lévy by Lévy polynomial(s)

Lévy polynomials *vs.* τ-model

- Simplified τ-model:
$$R_2(Q) = \gamma \left[1 + \lambda \cos\left((R_a Q)^{2\alpha}\right) \exp\left(-(RQ)^{2\alpha}\right)\right] \cdot (1 + \epsilon Q)$$
where $R_a^{2\alpha} = \tan\left(\frac{\alpha\pi}{2}\right) R^{2\alpha}$
- Symmetric Lévy polynomial parametrization:
$$R_2(Q) = \gamma \left[1 + \lambda \quad (1 + \sum c_i l_i) \quad \exp\left(-|rQ|^{\alpha}\right)\right] \cdot (1 + \epsilon Q)$$

R_a	$2\alpha = 0.88 \pm 0.02$	$\lambda = 0.61 \pm 0.03$	$R = 0.78 \pm 0.04$ fm
SL order 1	$\alpha = 1.07 \pm 0.06$	$\lambda = 0.16 \pm 0.03$	$r = 0.54 \pm 0.03$ fm
SL order 2	$\alpha = 1.01 \pm 0.10$	$\lambda = 0.23 \pm 0.03$	$r = 0.43 \pm 0.04$ fm
SL order 3	$\alpha = 1.36 \pm 0.25$	$\lambda = 0.22 \pm 0.03$	$r = 0.54 \pm 0.05$ fm

Values of parameters differs between τ-model and Sym. Lévy
and between orders of Sym. Lévy

Does expansion improve the τ-model?

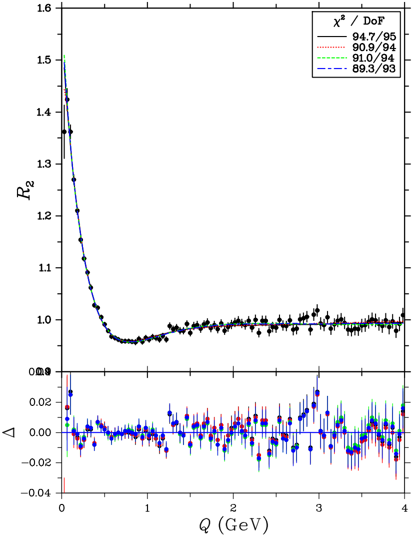
Lacking (so far) an orthogonal polynomial expansion for the asymmetric Lévy distribution $H(\tau)$ of the τ-model, we use a derivative expansion:

$$R_2(Q) = \gamma\left[1+\lambda\left\{\cos\left((R_\mathrm{a}Q)^{2\alpha}\right)\exp\left(-(RQ)^{2\alpha}\right) + \sum c_n \frac{\mathrm{d}^n}{\mathrm{d}Q^n}\cos\left((R_\mathrm{a}Q)^{2\alpha}\right)\exp\left(-(RQ)^{2\alpha}\right)\right\}\right]\cdot(1+\epsilon Q)$$

	order 0	order 1	order 0, R_a free	order 1, R_a free
α	0.44 ± 0.01	0.43 ± 0.01	0.41 ± 0.02	0.40 ± 0.03
R (fm)	0.78 ± 0.04	0.84 ± 0.05	0.79 ± 0.04	0.83 ± 0.07
R_a (fm)	–	–	0.69 ± 0.04	0.60 ± 0.06
λ	0.61 ± 0.03	0.67 ± 0.05	0.63 ± 0.03	1 at limit
γ	0.979 ± 0.002	0.979 ± 0.002	0.988 ± 0.005	0.992 ± 0.006
ϵ	0.005 ± 0.001	0.005 ± 0.001	0.001 ± 0.002	0.000 ± 0.002
c_1	–	0.0008 ± 0.0005	–	0.072 ± 0.015
χ^2/DoF	$94.7/95$	$90.9/94$	$91.0/94$	$89.3/93$
CL	49%	57%	57%	59%

- Orders 0-1 $\sim 1\sigma$ difference
- Order 1 has somewhat better χ^2, as does order 0, R_a free

τ-model expansion



order	χ^2/DoF	CL
order 0	94.7/95	49%
order 1	90.9/94	57%
order 0, R_a free	91.0/94	57%
order 1, R_a free	89.3/93	59%

- Difference of two χ^2 is also a χ^2
- Small CL$(\chi_1^2 - \chi_2^2, \mathrm{DoF}_1 - \mathrm{DoF}_2)$ is reason to reject Hypothesis 1
- CL(94.7 – 90.9, 1 dof) = 5.1%
  Not small enough to reject order 0
- Other χ^2 differences are smaller; so CL larger
- expansion not needed
  R_a free does not give significant improvement

Conclusions

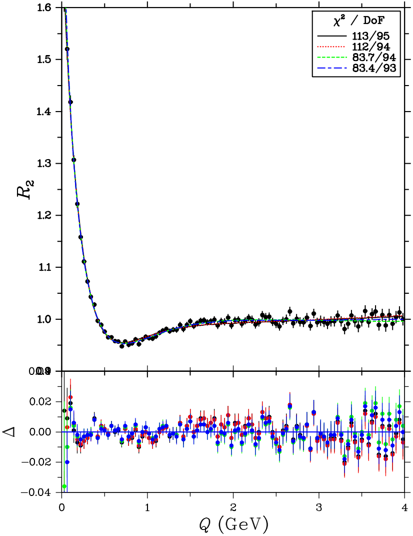
- Expansions provide a test of whether the assumed function is (approximately) correct
  and if only approximately, help to locate the differences
- for 2-jet events
  - for τ-model expansion is not needed;
    assumption that $H(\tau)$ is an asymmetric Lévy distribution is OK
  - for symmetric Lévy order-1 expansion is required;
    modification of the symmetric Lévy required is similar to that of the τ-model

τ-model– 3-jet events

- at $\sqrt{s} = M_Z$ almost all events are
  2-jet $\quad e^+e^- \longrightarrow q\bar{q}$ or 3-jet $\quad e^+e^- \longrightarrow q\bar{q}g$

- for 2-jet events hadronizaton is basically 1+1 dimension,
  which lead in the τ-model to the dependence on
  τ, the longitudinal proper time
  m_t, the transverse mass
- for 3-jet events this is more complicated
  So, we might expect the τ-model to work less well

τ-model expansion – 3-jet events



order	χ^2/DoF	CL
order 0	113.2/95	10%
order 1	112.4/94	9%
order 0, R_a free	83.7/94	77%
order 1, R_a free	83.4/93	75%

- CL(113.2 – 112.4, 1 dof) = 37%
  CL(83.7 – 83.4, 1 dof) = 58%
  Order 1 gives no significant improvement
  expansion not needed
- However,
  CL(113.2 – 83.7, 1 dof)$= 6 \cdot 10^{-8}$
- R_a free does give significant improvement

Conclusions – 3-jet events

- τ-model expansion not needed
  $\Longrightarrow H(\tau) =$ asymmetric Lévy distribution is OK
- significant improvement is obtained letting R_a free
  i.e., by lessening the connection of simplified τ-model
  between the BEC peak and antisymmetric dip
  presumably due to the more complicated structure of the event

BACKUP

$\tau_0 > 0$?

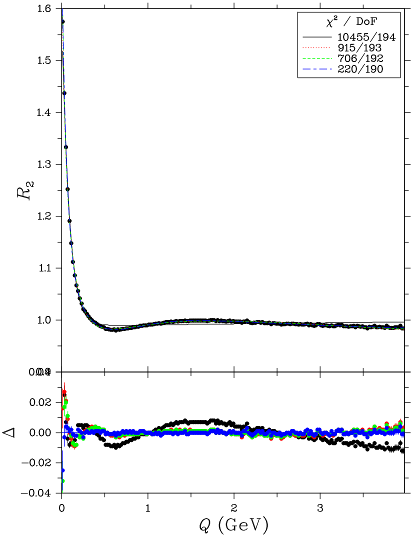
jets	R_a	τ_0	χ^2/DoF	CL
2	constr.	0	94.7/95	49%
2	free	0	91.0/94	57%
2	constr.	free	91.4/94	56%
		$\tau_0 = 0.068 \pm 0.033$ fm		2σ
2	free	free	91.0/93	54%
		$\tau_0 = 0.004 \pm 0.111$ fm		0σ
3	constr.	0	113.2/95	10%
3	free	0	83.7/94	77%
3	constr.	free	84.0/94	76%
		$\tau_0 = 0.196 \pm 0.026$ fm		7.5σ
3	free	free	80.8/93	81%
3		$\tau_0 = 0.116 \pm 0.052$ fm		2.2σ
3		CL(83.7 − 80.8, 1) = 9%		

Conclusion 2-jet: R_a free and/or $\tau_0 > 0$ do not give significant improvement
Both free same as R_a free with $\tau_0 = 0$

Conclusion 3-jet:

- R_a free or $\tau_0 > 0$ *do* give significant improvement
- R_a free and $\tau_0 > 0$ give insignificant further improvement
- Difficult physically to understand how to have $\tau_0^{\text{3-jet}} > \tau_0^{\text{2-jet}}$

Lévy polynomials in pp



CMS sees anticorrelation in pp at LHC

PRC97,064912(2018)

ATLAS also (unpublished) in PhD thesis
R. Astaloš http://hdl.handle.net/2066/143448

Using data from a figure in this thesis:

- Sym. Lévy: χ^2/DOF = 10455/194 – does not fit
- χ^2 of τ-model (R_a free) (Order 0) is much better 915/193
- χ^2 of τ-model (R_a free) (Order 1) is better 706/192
- Sym. Lévy polynomial (Order 4) is better 220/190