Relativistic hydro solution with application to pseudo-rapidity distribution at RHIC and LHC

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Motivation

The next phase … will focus on detailed investigations of the QGP, "both to quantify its properties and to understand precisely how they emerge from the fundamental properties of QCD"

-- The frontiers of nuclear science, a long range plan

- What is the initial temperature and thermal evolution of the produced matter?

- What is the viscosity of the produced matter? ... http://www.bnl.gov/physics/rhiciiscience/

Preliminary work

- The exact solutions and results of the perfect fluid. （CNC, CKCJ)

  Csörgő, Nagy, Csanád (CNC) arXiv: 0605070, 0710.0327, 0805.1562,
  Csanád, et. arXiv:1609.07176. Z. F. Jiang, et. arXiv: 1711.10740, 1806.05750.
  Csörgő, et. arXiv: 1805.01427, 1806.11309, 1810.00154.

Outline

The next phase … will focus on detailed investigations of the QGP, "both to quantify its properties and to understand precisely how they emerge from the fundamental properties of QCD"

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Outline

1. A perturbative solution of viscous hydrodynamics.

   Z. F. Jiang, et. arXiv:1808.10287.

2. Final state spectrum compared with the RHIC and the LHC data.

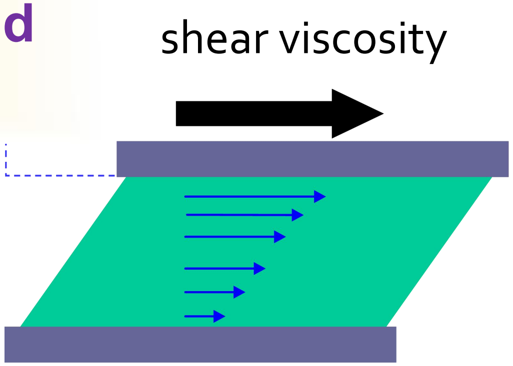
3. Summary and outlook.

Relativistic accelerated viscous hydrodynamic

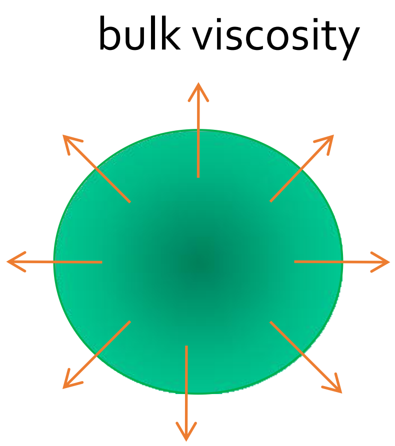
Longitudinal acceleration effect makes the fluid cooling faster.
The viscosity will creating heat and makes the fluid cooling slower.

$$T^{\mu\nu} = eu^{\mu}u^{\nu} - (p + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$u^{\mu} = (\cosh\Omega, 0, 0, \sinh\Omega) \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu}u^{\nu}$$



Shear viscosity tensor: $\pi^{\mu\nu}$ **Bulk viscosity:** Π.



Shear tensor:

$$\sigma^{\mu\nu} \equiv \partial^{\langle\mu}u^{\nu\rangle} \equiv (\frac{1}{2}(\Delta^{\mu}_{\alpha}\Delta^{\nu}_{\beta} + \Delta^{\mu}_{\beta}\Delta^{\nu}_{\alpha}) - \frac{1}{d}\Delta^{\mu\nu}\Delta_{\alpha\beta})\partial^{\alpha}u^{\beta}.$$

The fundamental equations of the viscous fluid:

$$e = \kappa p, \ \partial_{\mu}T^{\mu\nu} = 0.$$

Equations of viscous hydrodynamic

The second law of thermodynamics: $\partial_\mu S^\mu \geq 0$

$$\tau_\pi \Delta^{\alpha\mu} \Delta^{\beta\nu} \dot{\pi}_{\alpha\beta} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{1}{2}\pi^{\mu\nu} \frac{\eta T}{\tau_\pi} \partial_\lambda \left(\frac{\tau_\pi}{\eta T} u^\lambda \right)$$

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta(\partial \cdot u) - \frac{1}{2}\Pi \frac{\zeta T}{\tau_\Pi} \partial_\lambda \left(\frac{\tau_\Pi}{\zeta T} u^\lambda \right)$$

Israel-Stewart equations.

viscous hydro: near-equilibrium system

The Navier-Stokes approximation,

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} \qquad \Pi = -\zeta\left(\partial_\rho u^\rho\right)$$

The shear viscosity and bulk viscosity,

Strongly coupled AdS/CFT prediction: $\eta / s \geq 1/4\pi \approx 0.08$ D.T. Son, et,al. 05

Via lattice calculation: $\zeta / s \leq 0.015\,(\text{for } 3T_c)$ H.B. Meyer, et,al. 07 10.3717

Accelerating viscous hydrodynamic equation

Based on the conservation law, the energy equation and Euler equations are :

$$\begin{cases} D = u^{\mu}\partial_{\mu}, \quad \theta = \nabla_{\mu}u^{\mu} \\ \nabla^{\alpha} = \Delta^{\mu\alpha}\partial_{\mu}, \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^{\mu}u^{\nu} \end{cases}$$

$$\begin{cases} De = -(e+P+\Pi)\theta + \sigma_{\mu\nu}\pi^{\mu\nu} \\ (e+P+\Pi)Du^{\alpha} = \nabla^{\alpha}(P+\Pi) - \Delta_{\nu}^{\alpha}u_{\mu}D\pi^{\mu\nu} - \Delta_{\nu}^{\alpha}\nabla_{\mu}\pi^{\mu\nu} \end{cases}$$

Accelerating viscous hydrodynamic equation

In Rindler coordinate, the energy equation and Euler equation reduce to:

$$\begin{cases} \tau\frac{\partial T}{\partial\tau}+\tanh(\Omega-\eta_s)\frac{\partial T}{\partial\eta_s}+\frac{\Omega'}{\kappa}T=\frac{\Pi_d}{\kappa}\frac{\Omega'^2}{\kappa}\cosh(\Omega-\eta_s), \\ \tanh(\Omega-\eta_s)\left[\tau\frac{\partial T}{\partial\tau}+T\Omega'\right]+\frac{\partial T}{\partial\eta_s}=\frac{\Pi_d}{\kappa}[2\Omega'(\Omega'-1)+\Omega''\coth(\Omega-\eta_s)]\sinh(\Omega-\eta_s) \end{cases}$$

$$\begin{cases} \Omega'=\frac{\partial\Omega}{\partial\eta_s} \\ \Pi_d=\left(\frac{\zeta}{s}+\frac{2\eta}{s}(1-\frac{1}{d})\right) \end{cases}$$

Bjorken approximation:

$$\Pi_d=0,\ \Omega(\eta_s)=\eta_s$$

J.D.Bjorken, Phys.Rev. D27 (1983) 140-151.

Without accelerating:

$$\Pi_d\neq 0\ \ \Omega(\eta_s)=\eta_s$$

A. Muronga, Phys. Rev. C69(2004), 034904.

A perturbative case,

$$\Omega=\lambda\eta_s=(1+\varepsilon)\eta_s,\qquad |\varepsilon|<<1.$$

$$\Pi_d\neq 0$$

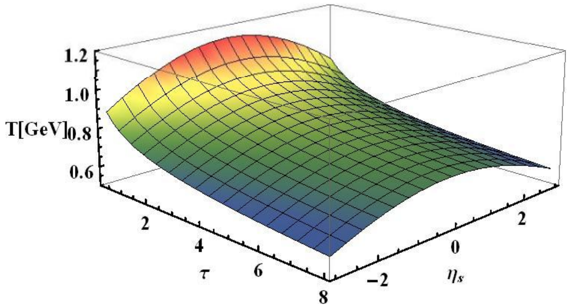
M. Csanád,ET. 2017. Universe. 3. 1-9.
Z. F. Jiang, et. arXiv: 1711.10740

Up to $\mathcal{O}(\epsilon)$,

$$\begin{cases} \tau\frac{\partial T}{\partial\tau}+\frac{\epsilon+1}{\kappa}T=\frac{\Pi_d}{\kappa}\frac{2\epsilon+1}{\tau}+\mathcal{O}(\epsilon^2). \\ T_1(\eta_s)\left(1-\frac{1}{\kappa}\right)\epsilon\eta_s+\frac{\epsilon\Pi_d}{(\kappa-1)\tau_0}\left(1-\frac{1}{\kappa}\right)\eta_s+\frac{\partial T_1(\eta_s)}{\partial\eta_s}+\mathcal{O}(\epsilon^2)=0 \end{cases}$$

Solutions form hydrodynamic equations

The temperature profile:



$$T(\tau,\eta_s) = T_0\left(\frac{\tau_0}{\tau}\right)^{\frac{1+\epsilon}{\kappa}} \times \left[\exp[-\frac{1}{2}\epsilon(1-\frac{1}{\kappa})\eta_s^2] + \frac{R_0^{-1}}{\kappa-1}\left(2\epsilon+\exp[-\frac{1}{2}\epsilon(1-\frac{1}{\kappa})\eta_s^2]-(2\epsilon+1)\left(\frac{\tau_0}{\tau}\right)^{\frac{\kappa-\epsilon-1}{\kappa}}\right)\right]$$

Contribution from ideal terms.

Contribution from viscous effect

$$R_0^{-1} = \frac{\Pi_d}{T_0\tau_0}$$

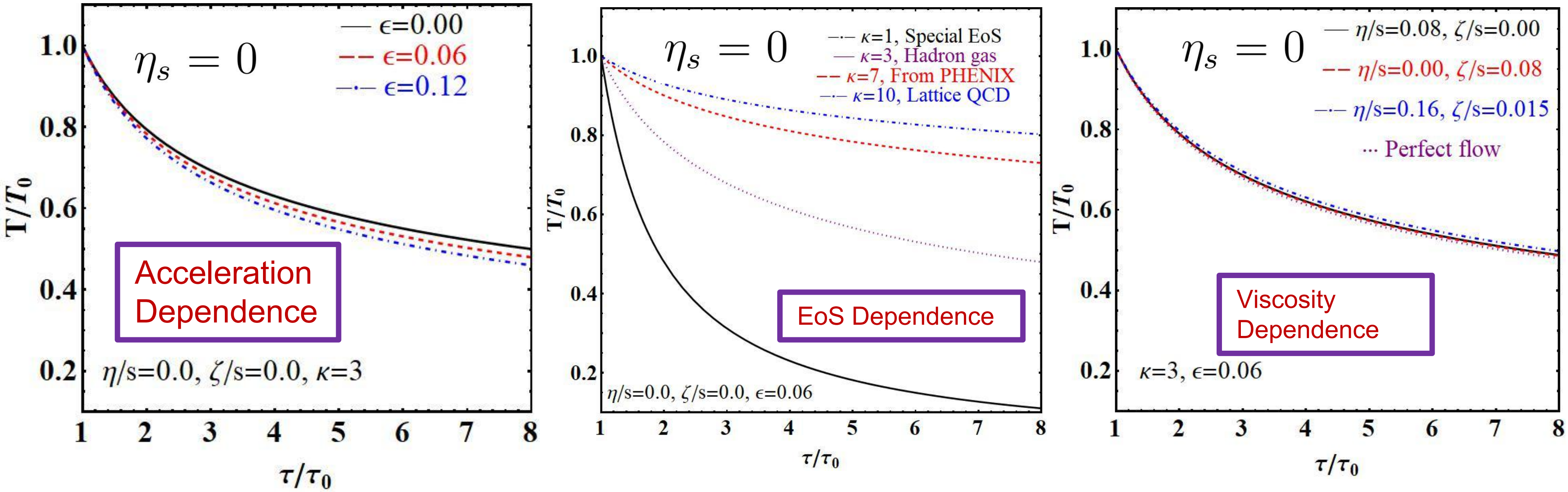
Reynolds number
[A. Muronga, arXiv: 0309055]

- A non-zero Reynolds numbers R_0^{-1} makes cooling rate smaller,
- A non-vanishing acceleration ϵ makes the cooling rate is larger.

Open question: setting viscosity as the perturbative term.

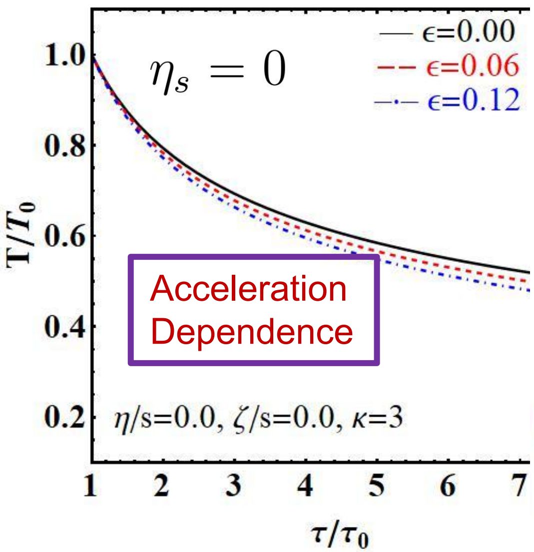
(Z.F. Jiang, C.B. Yang, Chi Ding, Xiang-Yu Wu. arXiv: 1808.10287)

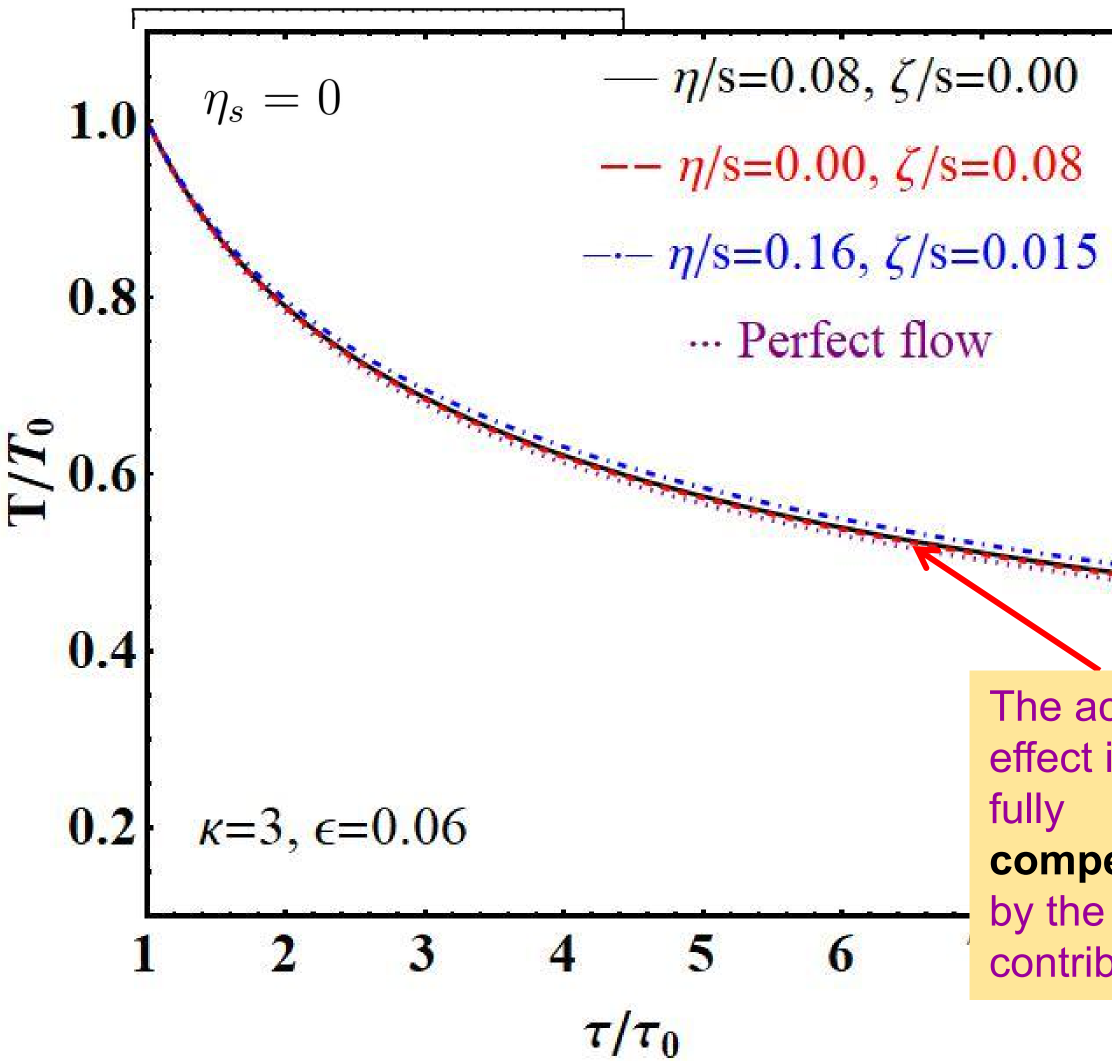
Temperature evolution



- Acceleration effect comes from the pressure gradient, makes the cooling ratio **larger** than non-acceleration flow.
  [M. Nagy, T. Csörgő, M. Csanád: arXiv:0709.3677v1]
- EoS is an important modified factor.
  κ=1 a very special case, CNC solution.
  κ=7 comes from [PHENIX, arXiv:nucl-ex/0608033v1] .
- Viscosity effect make the cooling rate samller. [H. Song, S. Bass, U. Heinz. et, PRL2011]

Temperature evolution





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The acceleration effect is almost fully **compensated** by the viscous contribution.

The final state spectrum

Freeze-out hypersurface:

$$p_\mu d\Sigma^\mu = m_T \tau_f \cosh^{\frac{2-\Omega'}{\Omega'-1}}((\Omega'-1)\eta_s)\cosh(\Omega - y) r dr d\phi d\eta_s$$

[M. I. Nagy, T. Csörgő, M. Csanád: arXiv:0709.3677v1]

The transverse momentum distribution (academic study):

$$\frac{d^2N}{2\pi p_T dp_T dy} = \frac{\pi R_0^2}{(2\pi)^3}\int_{-\infty}^{+\infty} m_T \cosh((\epsilon+1)\eta_s - y)\exp\left[-\frac{m_T}{T(\tau,\eta_s)}\cosh((\epsilon+1)\eta_s - y)\right]$$
$$\times\left(\tau_f \cosh^{\frac{1-\epsilon}{\epsilon}}(\epsilon\eta_s) + \frac{1+\epsilon}{T^3(\tau,\eta_s)}\left[\frac{1}{3}\frac{\eta}{s}(p_T^2 - 2m_T^2\sinh^2((\epsilon+1)\eta_s - y))\right.\right.$$
$$\left.\left. - \frac{1}{5}\frac{\zeta}{s}(p_T^2 + m_T^2\sinh^2((\epsilon+1)\eta_s - y))\right]\right) d\eta_s$$

-Temperature solution,

-viscosity, acceleration parameter, mass, space-time rapidity...

D. Teaney, 2003. P. R. C 68, 034913, a special case when there is no acceleration effect (ε=0).

(Pseudo-) Rapidity distribution

Rapidity distribution

Contribution from perfect fluid

$$\frac{dN}{dy} = \frac{\pi R_0^2}{(2\pi)^3}\int_0^{+\infty}\left\{\cosh^{\frac{1-\epsilon}{\epsilon}}(\epsilon\eta_s)\frac{4\tau_f T^3(\tau,\eta_s)}{\cosh^2((\epsilon+1)\eta_s-y)} + \frac{48(1+\epsilon)T^2(\tau,\eta_s)}{\cosh^4((\epsilon+1)\eta_s-y)}\right.$$
$$\left.\times\left[\frac{1}{3}\frac{\eta}{s}(1-2\sinh^2((\epsilon+1)\eta_s-y)) - \frac{1}{5}\frac{\zeta}{s}\cosh^2((\epsilon+1)\eta_s-y)\right]\right\}d\eta_s$$

Rapidity distribution (academic study),
- the integral value $error \propto m^3$, this is a good approximation for the particle that mass m is little.

Pseudo-rapidity distribution

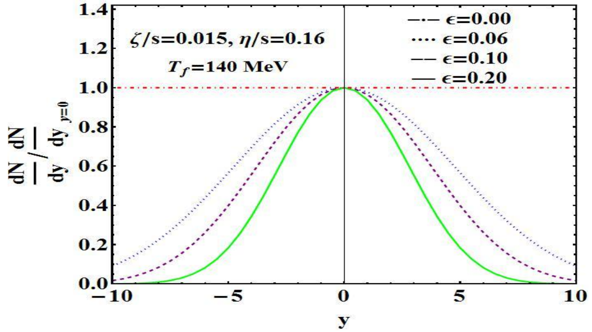
$$\frac{dN}{d\eta} = \frac{\pi R_0^2}{(2\pi)^3}\int_{-\infty}^{+\infty}d\eta_s\int_0^{+\infty}dp_T\sqrt{1-\frac{m^2}{m_T^2\cosh^2 y}}\,m_T p_T\cosh((\epsilon+1)\eta_s-y)\exp\left[-\frac{m_T}{T(\tau,\eta_s)}\cosh((\epsilon+1)\eta_s-y)\right]$$
$$\times\left(\tau_f\cosh^{\frac{1-\epsilon}{\epsilon}}(\epsilon\eta_s) + \frac{1+\epsilon}{T^3(\tau,\eta_s)}\left[\frac{1}{3}\frac{\eta}{s}(p_T^2-2m_T^2\sinh^2((\epsilon+1)\eta_s-y)) - \frac{1}{5}\frac{\zeta}{s}(p_T^2+m_T^2\sinh^2((\epsilon+1)\eta_s-y))\right]\right)$$

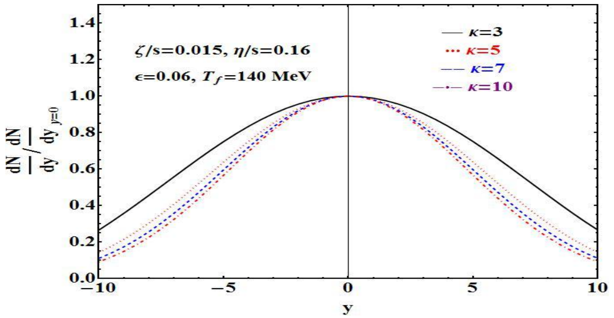
Contribution from perfect fluid

Contribution from viscous effect

Particle's distribution (academic study)

Numerical results (Rapidity distribution):





Acceleration parameter extracted from RHIC and the LHC data:

$\sqrt{s_{NN}}$ /[GeV]		$\frac{dN}{d\eta}\Big\vert_{\eta=\eta_0}$	ϵ	χ^2/NDF
130	Au+Au	563.9±59.5	0.076±0.003	9.41/53
200	Au+Au	642.6±61.0	0.062±0.002	12.23/53
200	Cu+Cu	179.5±17.5	0.060±0.003	2.41/53
2760	Pb+Pb	1615±39.0	0.035±0.003	5.50/41
5020	Pb+Pb	1929±47.0	0.032±0.002	33.0/27
5440	Xe+Xe	1167±26.0[41]	0.030±0.003	−/−

$$\epsilon = A\left(\frac{\sqrt{s_{NN}}}{\sqrt{s_0}}\right)^{-B}$$

$$\sqrt{s_0} = 1\ \text{GeV}$$

$$A = 0.045 \text{ and } B = 0.23$$

Z.F. Jiang, C.B. Yang, Chi Ding, Xiang-Yu Wu. arXiv: 1808.10287. Accepted by Chin. Phys. C

Particle's distribution
- at finial state, the dn/dy and $dn/d\eta$ are effected sensitively by the acceleration parameter and EoS.
- The model’s prediction for XeXe@5440 GeV works well!
- A simple description of acceleration parameters is obtained.

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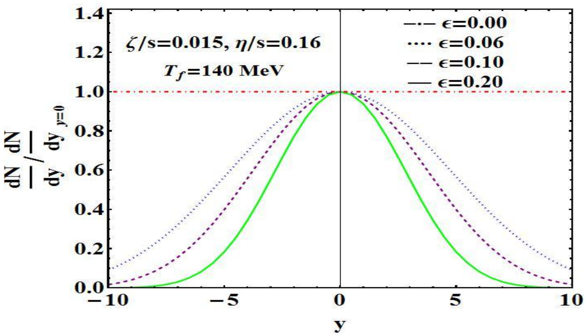
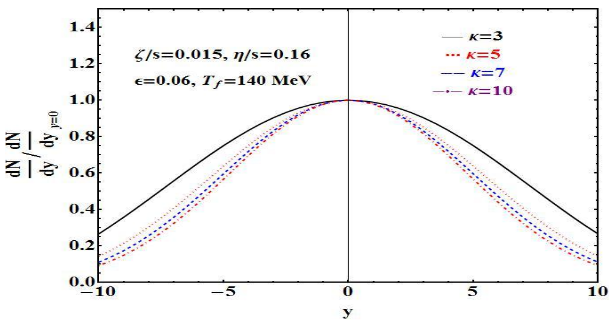
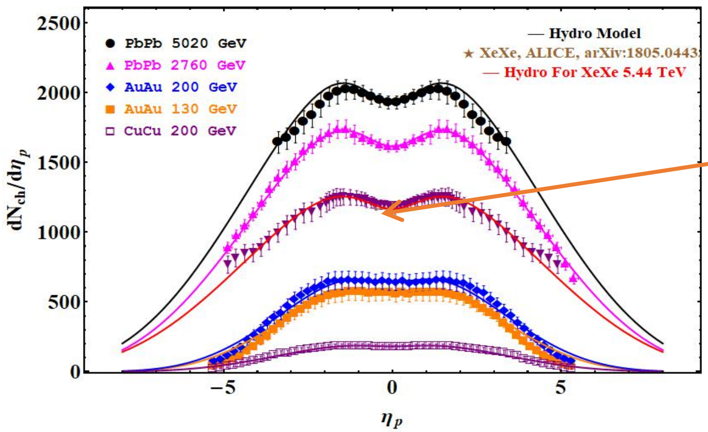




Numerical results (pseudo-rapidity distribution):



Red line: prediction for XeXe @ 5440 GeV. Data: ALICE@QM2018, 1805.04432.

Z.F. Jiang, C.B. Yang, Chi Ding, Xiang-Yu Wu. arXiv: 1808.10287. Accepted by Chin. Phys. C

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A brief summary and outlook

Summary:

1. A perturbative solution with viscous correction are obtained.
2. The final state spectrum are obtained and described well the RHIC and the LHC data.

Outlook:

1. 2rd I-S problem, magenetic hydrodynamics, CLVisc 3+1D code;
2. Fast parton disturbance evolution on accelerating medium background...

Thank you for your attention

[*arXiv: 1609.07176, 1711.10740, 1805.01427, 1806.05750, 1808.10287…*]