

Hubble-type solutions of hydrodynamics with bulk viscosity

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Introduction

- Ideal fluid (=perfect fluid) hydrodynamics:
 - Well-proven theory, clear basic equations
 - Good at describing bulk development in heavy-ion collisions
 - ⇔ QGP viscosity (experimentally) small
 - Analytic solutions (vs. numerical ones)
 - Non-relativistic: many well known simple fireball solutions
 - Relativistic: historical + (more or less) recent developments
- Viscous hydrodynamics:
 - Non-relativistic: basic equations clear (?)
 - Fireball NR solutions → generalization works
 - Relativistic: basic equations NOT well settled
 - Landau theory vs. Eckart's theory (1940s):
1st order (parabolic PDEs, acausality, instability...)
 - Israel-Stewart theory: hyperbolic PDEs (complicated; more coefficients)
 - Exact relativistic viscous solutions **lacking**
- Goal: study relativistic viscous effect in simplest possible way

Basic equations

- Ideal fluid (=perfect fluid) case (a reminder):
 - $T_{\mu\nu}$ stress-energy-momentum tensor, $\partial_\nu T^{\mu\nu}=0$.
 - $T_{\mu\nu}=(\varepsilon+p)u_\mu u_\nu - pg_{\mu\nu}$, =definition of p , ε .
 \Rightarrow energy equation: $(\varepsilon+p)\partial_\nu u^\nu + u^\nu \partial_\nu \varepsilon=0$
 \Rightarrow Euler equation: $(\varepsilon+p)u^\nu \partial_\nu u^\mu = (g^{\mu\rho} - u^\mu u^\rho)\partial_\rho p$.
 - Need Equation of State (EoS): eg. $\varepsilon=\kappa(T)p$.
 - Simplest choice: $\kappa=\text{const.}$
 - Two cases in what follows:
 $p=nT$ (with conserved charge n , $\partial_\nu(nu^\nu)=0$) vs.
 $p=p_0(T/T_0)^{\kappa+1}$ (no conserved charge, only σ entropy density)
- Viscous hydrodynamics, first order theory:
 - $T_{\mu\nu}=(\varepsilon+p)u_\mu u_\nu - pg_{\mu\nu} + q_\mu u_\nu + q_\nu u_\mu + \pi_{\mu\nu}$
 q_μ : thermal conduction ($q_\mu u^\mu=0$), $\pi_{\mu\nu}$: viscous tensor ($\pi_{\mu\nu}u^\nu=0$).
 - Conservation of particle number (charge): $\partial_\mu N^\mu=0$
 $N^\mu=nu^\mu + j^\mu$ with $u_\mu j^\mu=0$.
 j^μ : ambiguity in definition of u^μ .

Eckart vs. Landau frame

- Eckart frame: $N^\mu = nu^\mu$ (ie: $j^\mu = 0$): choice for definition of u^μ
 - $q_\mu = \lambda(g_{\mu\nu} - u_\mu u_\nu) \cdot (\partial^\nu T - Tu^\rho \partial_\rho u^\nu)$
 - $\pi_{\mu\nu} = \eta[(g_{\mu\rho} - u_\mu u_\rho)\partial^\rho u_\nu + (g_{\nu\rho} - u_\nu u_\rho)\partial^\rho u_\mu - \frac{2}{d}(g_{\mu\nu} - u_\mu u_\nu)\partial^\rho u_\rho] + \zeta(g_{\mu\nu} - u_\mu u_\nu)\partial^\rho u_\rho$
 - Expressions: from increase of entropy
- λ : thermal conductivity, η : shear viscosity, ζ : bulk viscosity, $d=3$.
- Landau frame: $q_\mu = 0$, choice for definition of u^μ
 - $j_\mu = \lambda\left(\frac{nT}{\varepsilon + p}\right)^2 (g_{\mu\nu} - u_\mu u_\nu) \cdot \partial^\nu \frac{\mu}{T}$
 - $\pi_{\mu\nu}$ similar form as for Eckart (definition of u^μ not the same...)
- In what follows: take $\lambda=0$, investigate role of ζ
 - η : famous lower limit $\frac{\eta}{s} \geq \frac{\hbar}{4\pi}$ from AdS/CFT
P. Kovtun, D. T. Son, A. O. Starinets, PRL 94, 111601 (2005)
 - ζ : not straightforward; general consensus $\zeta \ll \eta$.
Eg. in monatomic gases, $\zeta=0$
I. M. Khalatnikov, Sov. Phys. JETP 2, 169 (1956)

Hubble-type solution & generalization

- Well-known (in fact, simplest) 3D solution in perfect fluid case:

$$u^\mu = \frac{x^\mu}{\tau} \quad n = n_0 \left(\frac{\tau_0}{\tau} \right)^d, \quad T = T_0 \left(\frac{\tau_0}{\tau} \right)^{d/\kappa}.$$

See e.g. T. Csörgő, *et al.*, PLB **565**, 107 (2003)

- Ansatz for viscous solution (with $\lambda=0$, continuity works)

$$u^\mu = \frac{x^\mu}{\tau} \quad n = n_0 \left(\frac{\tau_0}{\tau} \right)^d, \quad T \equiv T(\tau), \quad \Rightarrow \quad p \equiv p(\tau)$$

- Choice for ζ bulk viscosity & Eos??? Cases investigated are:

- 1 Case A: $\zeta = \zeta_0$ constant, no conserved n , $p = p_0 (T/T_0)^{\kappa+1}$
- 2 Case B: $\zeta = \zeta_0$ constant, conserved n , $p = nT$
- 3 Case C: $\zeta = \zeta_0 (T/T_0)^\kappa$ (ie. $\zeta \propto s$), no conserved n , $p = p_0 (T/T_0)^{\kappa+1}$
- 4 Case D: $\zeta = \zeta_0 (n/n_0)$ (proxy for $\zeta \propto s$), conserved n , $p = nT$
- 5 Case E: $\zeta = \zeta_0 (T/T_0)^\kappa$ (other proxy for $\zeta \propto s$), conserved n , $p = nT$

Solving the equations. . .

- Some intermediate steps:
 - All terms containing η **shear viscosity cancel**
 \Rightarrow Hubble profile: ideal to study effect of ζ
 - Also λ (heat conductive) terms cancel (if present at all)
 - Euler equation is automatically **satisfied** (because assumed only τ dependence)
 - In simplest case (case A,B) possibility for ellipsoidal generalization with arbitrary $\mathcal{V}(S)$, $S = \frac{r_x^2}{X_0^2 t^2} + \frac{r_y^2}{Y_0^2 t^2} + \frac{r_z^2}{Z_0^2 t^2}$.
- Only energy equation to solve: reduces to 1 ODE for $p(\tau)$

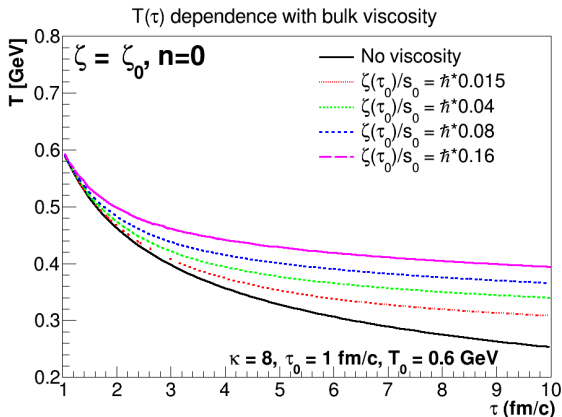
$$\kappa \frac{dp}{d\tau} + \frac{d(\kappa+1)}{\tau} p - \frac{d^2}{\tau^2} \zeta(p, \tau) = 0.$$

Solution: straightforward, in each cases.

Case A

$\zeta = \zeta_0$ constant, no conserved n , $p = p_0 (T/T_0)^{\kappa+1}$

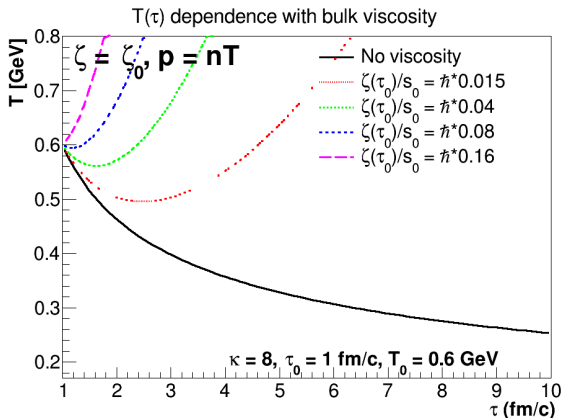
$$p(\tau) = \left[p_0 - \frac{d^2}{(\kappa+1)d - \kappa \tau_0} \zeta_0 \right] \left(\frac{\tau_0}{\tau} \right)^{d \frac{\kappa+1}{\kappa}} + \frac{d^2}{(\kappa+1)d - \kappa \tau} \zeta_0.$$



Case B

$\zeta = \zeta_0$ constant, conserved n , $p = nT$

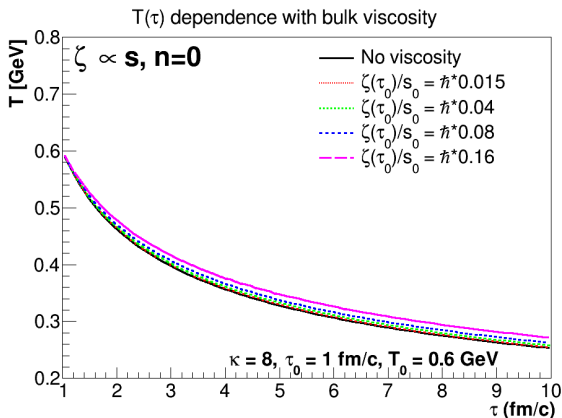
$$p(\tau) = \left[p_0 - \frac{d^2}{(\kappa+1)d - \kappa \tau_0} \zeta_0 \right] \left(\frac{\tau_0}{\tau} \right)^{d \frac{\kappa+1}{\kappa}} + \frac{d^2}{(\kappa+1)d - \kappa \tau} \zeta_0.$$



Case C

$\zeta = \zeta_0 (T/T_0)^\kappa$ (ie. $\zeta \propto s$), no conserved n , $p = p_0 (T/T_0)^{\kappa+1}$

$$p(\tau) = p_0 \left\{ \left(1 + \frac{d^2}{(\kappa+1)(\kappa-d)} \frac{\zeta_0}{p_0 T_0} \right) \left(\frac{T_0}{T} \right)^{\frac{d}{\kappa}} - \frac{d^2}{(\kappa+1)(\kappa-d)} \frac{\zeta_0}{p_0} \frac{1}{T} \right\}^{\kappa+1}$$

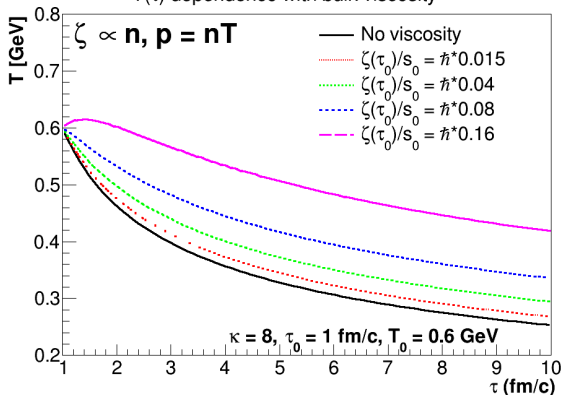


Case D

$\zeta = \zeta_0(n/n_0)$ (proxy for $\zeta \propto s$), conserved n , $p = nT$

$$p(\tau) = \left[p_0 + \frac{d^2}{\kappa - d} \frac{\zeta_0}{\tau_0} \right] \left(\frac{\tau_0}{\tau} \right)^{\frac{\kappa+1}{\kappa} d} - \frac{d^2}{\kappa - d} \frac{\zeta_0}{\tau_0} \frac{\tau_0^{d+1}}{\tau^{d+1}}$$

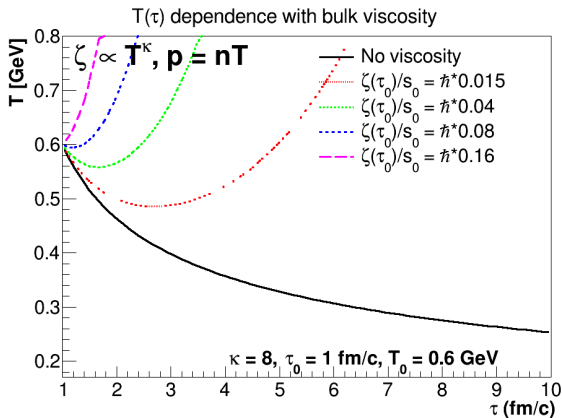
T(τ) dependence with bulk viscosity



Case E

$\zeta = \zeta_0 (T/T_0)^\kappa$ (other proxy for $\zeta \propto s$), conserved n , $p = nT$

$$\frac{p(\tau)}{p_0} = \left\{ \left(1 - \frac{d^2 \zeta_0 / (p_0 \tau_0)}{(d-1)(\kappa^2 + \kappa) + d} \right) \left(\frac{\tau_0}{\tau} \right)^{\frac{d}{\kappa}} + \frac{d^2 \zeta_0 / (p_0 \tau_0)}{(d-1)(\kappa^2 + \kappa) + d} \left(\frac{\tau}{\tau_0} \right)^{d \frac{\kappa}{\kappa+1} - 1} \right\}^{\kappa+1}$$



Summary and outlook

- Hydrodynamical models: perfect fluid vs. viscous fluids
- A (simple) viscous solution presented: Hubble flow, energy development distorted by bulk viscosity
- Illustrations plotted for (somewhat) equivalent values of ζ in different scenarios
- Different assumptions, different quantitative effects
- Further steps
 - Of course many possibilities (beyond simplest Hubble velocity field. . .)

Thank you for your attention!

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