When, and how much, is $U_A(1)$ restored ?

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(1) The issue of the (effective) restoration of the $U_A(1)$ symmetry

• In QCD, $U_A(1)$ and $SU_A(3)$ chiral symmetry are explicitly broken only slightly by the current (Lagrangian) masses of u and d quarks, & not too badly by s-quark mass m_s, so that **chiral limit(s)** make sense [with 3 (or 2) $m_q \to 0$].

• Even approximate chiral $SU_A(3)$ symmetry = absent due to **DChSB**, marked by appearance of $\langle \bar{q}q \rangle$ condensates and $Ps = \pi^{0,\pm}, K^{0,\pm}, \bar{K}^0, \eta$ as very light (almost-)Goldstone bosons ... but it should be restored as a crossover around $T_{\rm Ch} \sim 155$ MeV (for $\mu \sim 0$), $\langle \bar{q}q \rangle(T) \to 0$, as lattice calculations now agree.

• Even in the chiral limit, $m_q \to 0$, the $U_A(1)$ symmetry is broken explicitly on the quantum level by nonabelian ("gluon") axial anomaly:

$$\partial_\alpha \bar{\psi}(x)\gamma^\alpha\gamma_5 \frac{\lambda^0}{2}\psi(x) \propto F^a(x) \cdot \widetilde{F}^a(x) \equiv \epsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu}(x) F^a_{\rho\sigma}(x) \neq 0 \,,$$

which holds at any E and T $\Rightarrow$? Would only $T \to \infty$ restore $U_A(1)$?!?!

NO, since DChSB and $U_A(1)$ anomaly are tied through quark bi-linear operators such as $\langle \bar{q}q \rangle$ and QCD topological susceptibility χ $\Rightarrow$ Expect an effective restoration signaled by $U_A(1)$-violating quantities (e.g., large $M_{\eta'}$, difference π-$a_0(980)$, ...) vanishing or diminishing over the chiral symmetry crossover ... **BUT** ...

(2) ... still debatable what happens with $U_A(1)$ symmetry restoration!

- Presently, no consensus within lattice community whether $U_A(1)$ is badly broken or effectively restored at the chiral crossover critical temper. $T = T_{\rm Ch}$ [Sharma for HotQCD collaboration, e-Print: arXiv:1801.08500]

- Already older works found sizable $U_A(1)$ breaking above $T_{\rm Ch}$ [Bernard+al, PRL78 (1997)598, Chanrasekharan+al, PRL82(1999)2463, Ohno+al, PoS LATTICE 2011(2011)210 arXiv:1111.1939] ... and, this is confirmed by some recent works: notably by HotQCD collab. [Bazavov+al,PRD86(2012)9094503] and by Karsch & collaborators [Buchoff+al,PRD89(2014) 054514, Sharma+al, NPA956(2016)793, Dick+al,PRD91(2015)095504] as high as $T = 1.5\ T_{\rm Ch}$.

- BUT, some recent works claim that $U_A(1)$ breaking above $T_{\rm Ch}$ is overestimated in the continuum limit (blaming lattice artifacts near ChLim). Some then conclude that $U_A(1)$ anomaly is consistent with zero above $T_{\rm Ch}$, including also Graz group Rohrhofer+al, Phys.Rev.D96(2017)094501 arXiv 1707.01881, but most vocal were researchers around JLQCD collaboration [Cossu+al, PRD93(2016)034507 arXiv:1510.07395, PRD87&88 (2013)114514&019901 .... These disappearances of $U_A(1)$ anomaly seem to be associated with the chiral limit - see, *e.g.*, Tomiya+al, PRD96(2017)034509.

- Then our model approach to η-η' may show that these two kinds of results can be reconciled, since it is consistent with both - depending whether one uses the chiral or "massive" $q\bar{q}$ condensates: arXiv 1809.00379 by Horvatić, Kekez & D.K.

(3) Quantum-level breaking of $U_A(1)$ causes anomalously high $\eta' \approx \eta_0$ mass

QCD chiral behavior (reproduced by, e.g., DS approach) of the non-anomalous parts of masses of light $q\bar{q}'$ pseudoscalars (i.e., all parts except ΔM_{η_0}) : $M^2_{q\bar{q}'} = \mathrm{const}\,(m_q + m_{q'}), \ (q, q' = u, d, s)$.

$\Rightarrow$ non-anomalous parts of the masses cancel in Witten-Veneziano rel. (WVR):

$$M_{\eta'}{}^2 + M_\eta{}^2 - 2\,M_K{}^2 = \chi_{\mathrm{YM}}\,\frac{2N_f}{f_\pi^2} = \mathrm{anomalous\ \ mass}^2 \equiv M_{U_A(1)}{}^2 \approx \Delta M_{\eta_0}{}^2\,,$$

$$\chi = \int d^4x\,\langle 0|Q(x)Q(0)|0\rangle\,, \qquad Q(x) = \frac{g^2}{64\pi^2}\epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}(x)F^a_{\rho\sigma}(x)$$

QCD topological susceptibility χ = a direct measure of $U_A(1)$ breaking, so $U_A(1)$ restoration is indicated by vanishing or asymptotic reduction of χ and quantities related to it, like $M_{U_A(1)} \approx \Delta M_{\eta_0} \approx \Delta M_{\eta'}$.

- $Q(x)$ = topological charge density operator
- In WVR, χ is pure-gauge, YM one, $\chi_{\mathrm{YM}} \leftrightarrow \chi_{quench}$, obtained reliably by lattice - harder for χ of light-flavorQCD, but can use DiVecchia-Veneziano

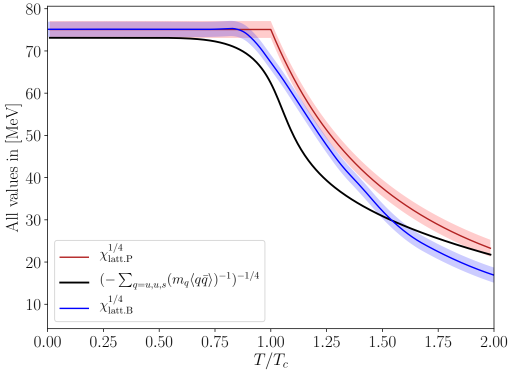
relation:
$$\chi = -\frac{\langle\bar{q}q\rangle_0}{\sum\limits_{q=u,d,s}\frac{1}{m_q}} + \mathcal{C}(\mathrm{unknown\ corrections, higher}\,\mathcal{O}\,\mathrm{in\,small}\,m_q)$$

.

(4) T-dependence of QCD topological susceptibility χ = encouraging

- $\chi(T)$ from lattice: Petreczky&al. PLB(2016) and Borsany&al. Nature(2016)
- $\chi(T)$ from our usual DS model: rank-2 separable, phenomenologically successful at $T = 0$, no additional fitting for $T > 0$, $\Rightarrow$ condensates $\langle \bar{q}q \rangle(T)$ $(q = u, d, s)$ realistically away from the chiral limit $\Rightarrow$

$$\chi = \frac{-1}{\frac{1}{m_u \langle \bar{u}u \rangle} + \frac{1}{m_d \langle \bar{d}d \rangle} + \frac{1}{m_s \langle \bar{s}s \rangle}} + \mathcal{C}_m \qquad (1)$$



(5) Shore's generalization of WV = example of the interplay between the breaking of $U_A(1)$ and chiral symmetries

$$(f^0_{\eta'})^2 M^2_{\eta'} + (f^0_{\eta})^2 M^2_{\eta} = \frac{1}{3}\left(f^2_\pi M^2_\pi + 2 f^2_K M^2_K\right) + 6A \qquad (2)$$

$$f^0_{\eta'} f^8_{\eta'} M^2_{\eta'} + f^0_{\eta} f^8_{\eta} M^2_{\eta} = \frac{2\sqrt{2}}{3}\left(f^2_\pi M^2_\pi - f^2_K M^2_K\right) \qquad (3)$$

$$(f^8_{\eta'})^2 M^2_{\eta'} + (f^8_{\eta})^2 M^2_{\eta} = -\frac{1}{3}\left(f^2_\pi M^2_\pi - 4 f^2_K M^2_K\right) \qquad (4)$$

The role of $\chi_{\rm YM}$ taken over by the full QCD topological charge parameter A ,

$$A = \frac{\chi}{1 + \chi\left(\frac{1}{\langle \bar{u}u\rangle\, m_u} + \frac{1}{\langle \bar{d}d\rangle\, m_d} + \frac{1}{\langle \bar{s}s\rangle\, m_s}\right)} \qquad (5)$$

A behaves with T as a full QCD quantity, **but**, at $T = 0$, $A = \chi_{\rm YM} + \mathcal{O}(\frac{1}{N_c})$

Note (1)+(3) $\Rightarrow (f^0_{\eta'})^2 M^2_{\eta'} + (f^0_{\eta})^2 M^2_{\eta} + (f^8_{\eta})^2 M^2_{\eta} + (f^8_{\eta'})^2 M^2_{\eta'} - 2 f^2_K M^2_K = 6A$

- Then, large N_c limit and 'off-diagonal' $f^0_\eta, f^8_{\eta'} \to 0$, as well as $f^0_{\eta'}, f^8_\eta, f_K \to f_\pi$, recovers the standard WV.

(6) Approximate all 3 light condensates by $\langle \bar{q}q \rangle_0$, the chiral-limit one!

This reduces the full QCD topological charge A, Eq. (5), to the remarkable Leutwyler-Smilga relation (LS), which is still valid for both large and small values of m_q:

$$\chi_{\rm YM} = \frac{\chi}{1 + \frac{\chi}{\langle \bar{q}q \rangle_0} \sum_{q=u,d,s} \frac{1}{m_q}} \equiv \widetilde{\chi} \to \widetilde{\chi}(T) \approx A(T)$$

where for the light quarks $$\chi = - \frac{1}{\sum_{q=u,d,s} \frac{1}{m_q \, \langle \bar{q}q \rangle_0}} + \mathcal{C}(m)$$

.

- $\mathcal{C}(m)$ = small corrections of higher orders in small m_q, ... but $\mathcal{C}(m)$ should not be neglected, since $\mathcal{C}(m) = 0$ would imply that $\chi_{\rm YM} = \infty$.
- LS relation fixes the value of the correction at $T = 0$:

$$\frac{1}{\mathcal{C}(m)} = \sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q}q \rangle_0} - \chi_{\rm YM}(0) \left(\sum_{q=u,d,s} \frac{1}{m_q \langle \bar{q}q \rangle_0} \right)^2 .$$

(7) Anomaly, NS-S mass matrix, mixing of η-η', & WVR connection

- $SU(3)$ flavor-broken ($X = f_\pi / f_{s\bar{s}} < 1$) nonstrange ($NS$) – strange ($S$) basis

$$|\eta_{NS}\rangle = \frac{1}{\sqrt{2}}(|u\bar{u}\rangle + |d\bar{d}\rangle) = \frac{1}{\sqrt{3}}|\eta_8\rangle + \sqrt{\frac{2}{3}}|\eta_0\rangle \ ,$$

$$|\eta_S\rangle = |s\bar{s}\rangle = -\sqrt{\frac{2}{3}}|\eta_8\rangle + \frac{1}{\sqrt{3}}|\eta_0\rangle \ .$$

- the η–η' matrix in this basis (but isosymmetric, $M_{u\bar{u}} = M_{d\bar{d}} = M_\pi$) is

$$\hat{M}^2 = \begin{pmatrix} M^2_{\eta_{NS}} & M^2_{\eta_S \eta_{NS}} \\ M^2_{\eta_{NS}\eta_S} & M^2_{\eta_S} \end{pmatrix} = \begin{pmatrix} M^2_{u\bar{u}} + 2\beta & \sqrt{2}\beta X \\ \sqrt{2}\beta X & M^2_{s\bar{s}} + \beta X^2 \end{pmatrix} \xrightarrow{\phi} \begin{pmatrix} M^2_\eta & 0 \\ 0 & M^2_{\eta'} \end{pmatrix}$$

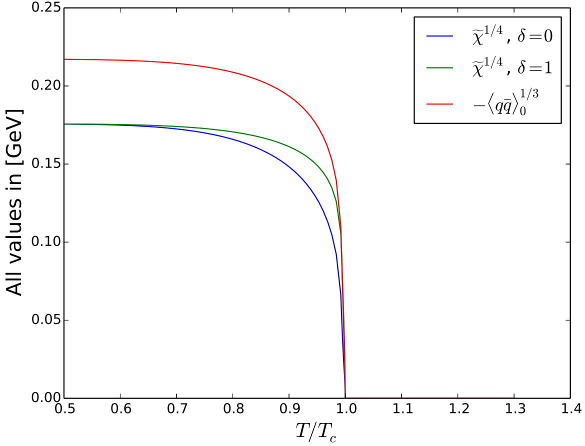
- NS–S mixing relations (related to η_8-η_0 ones by $\theta = \phi - \arctan\sqrt{2}$)

$$|\eta\rangle = \cos\phi|\eta_{NS}\rangle - \sin\phi|\eta_S\rangle \ , \quad |\eta'\rangle = \sin\phi|\eta_{NS}\rangle + \cos\phi|\eta_S\rangle \ .$$

- almost-Goldstones obey $M^2_{q\bar{q}'} = \mathrm{const}\,(m_q + m_{q'}) \ \Rightarrow$ express fictitious (at low T) RLA pseudoscalar $s\bar{s}$ mass: $M^2_{s\bar{s}} = 2M^2_K - M^2_\pi$.
  β is related to anomalous mass, since the matrix trace shows

$$\mathrm{Tr}[\textbf{anom. part of } \hat{M}^2] \equiv M^2_{U_A(1)} = \beta\,(2+X^2) = M^2_\eta + M^2_{\eta'} - 2M^2_K = \frac{2N_f}{f^2_\pi}\chi_{\rm YM}$$

(8) Chiral condensate $\langle q\bar{q}\rangle_0(T)$ and resulting $\widetilde{\chi}(T)$

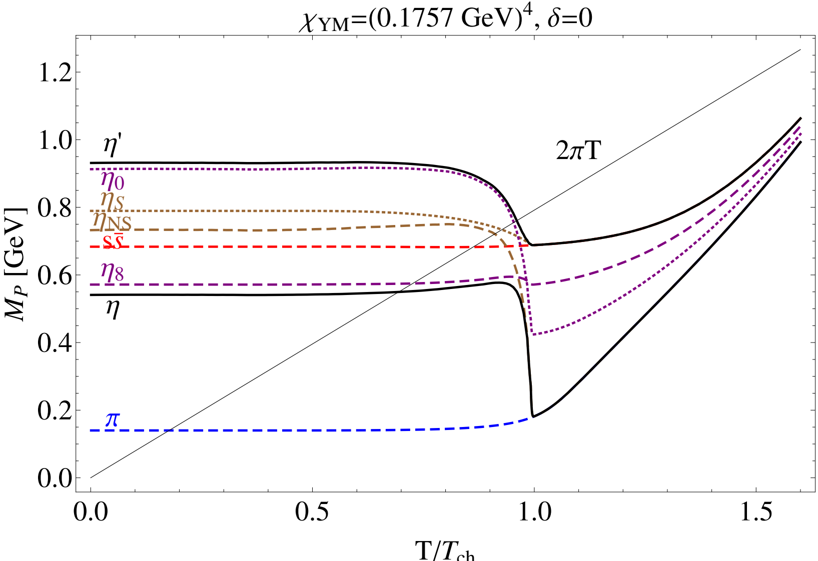


This sharp chiral transition enforces at $T = T_c \equiv T_{\rm Ch}$ the abrupt transition to the NS-S asymptotic regime of the vanishing $U_A(1)$ anomaly influence: $M_{\eta'}(T) \to M_{s\bar{s}}(T)$, and $M_\eta(T) \to M_{\rm NS}(T) \to M_\pi(T)$, and $\phi(T) \to 0$.

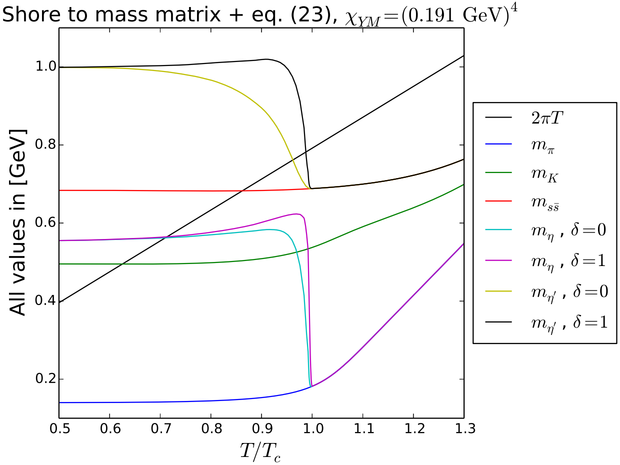
(9) Prediction good for η', but for η not supported by any experiment

[Benić, Horvatić, Kekez and Klabučar, Phys. Rev. D **84** (2011) 016006.]:

Anomalous contribution from WVR:

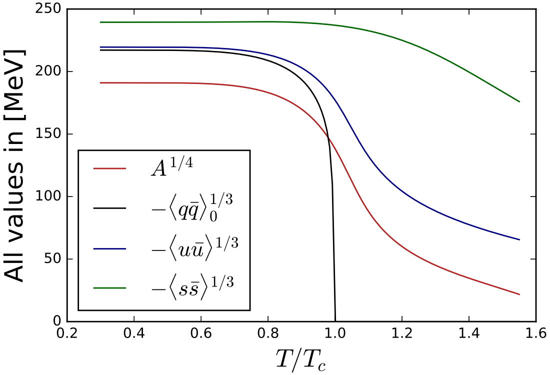


(10) WV → Shore, variations of model, or parameters, do not matter much



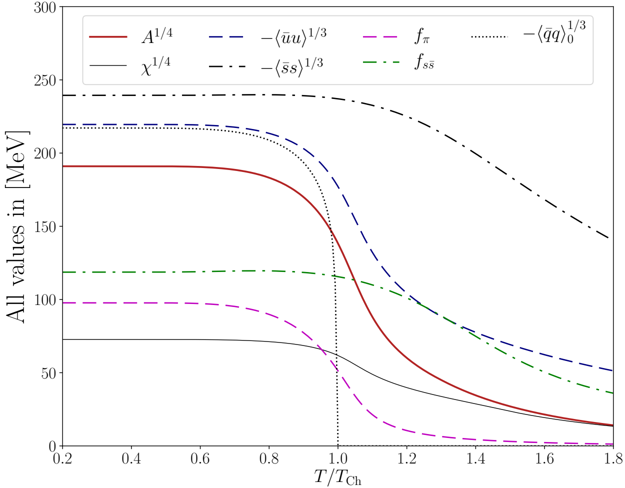
... In Shore's generalization with $A \approx \widetilde{\chi}$, mass drop prediction still good for η' (where Csörgő and collaborators had found this in RHIC data), but again an even larger mass drop for η, which is not supported by any experiment.

(11) A solution: $U_A(1)$ breaking from realistic condensates



Instead of the fast-falling chiral-limit condensate $\langle \bar{q}q \rangle_0$, try $\langle \bar{q}q \rangle$ condensates with realistic explicit chiral symmetry breaking: replace $m_q \langle \bar{q}q \rangle_0 \rightarrow m_q \langle \bar{q}q \rangle$, $(q = u, d, s)$ in χ, like in the original A.

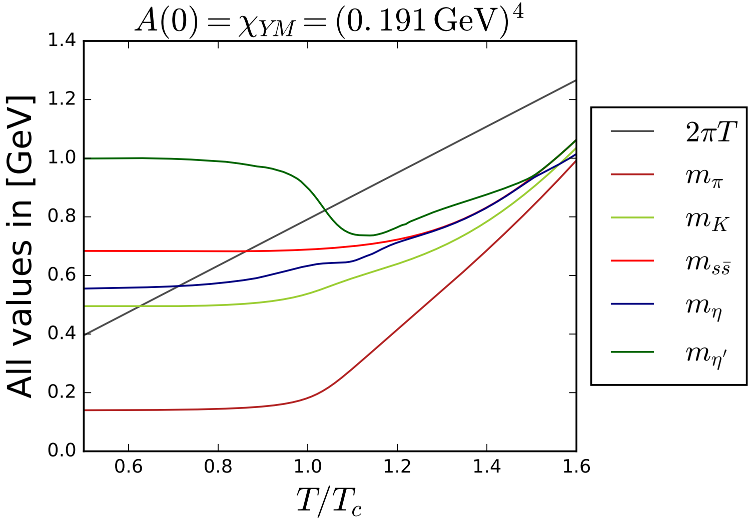
(12) Compare T-dependence of $\langle \bar{q}q \rangle$ & decay const's f_P with A & χ



FKS scheme on Shore $\Rightarrow$ how f_P influence elements of the η-η' mass matrix:

$$X = \frac{f_\pi}{f_{s\bar{s}}}, \qquad M^2_{\rm NS} = M^2_\pi + \frac{4A}{f^2_\pi}, \qquad M^2_{\rm NSS} = \frac{2\sqrt{2}A}{f_\pi f_{s\bar{s}}}, \qquad M^2_{\rm S} = M^2_{s\bar{s}} + \frac{2A}{f^2_{s\bar{s}}}$$

(13) $\Rightarrow$ T dependence of light pseudoscalars including η and η'

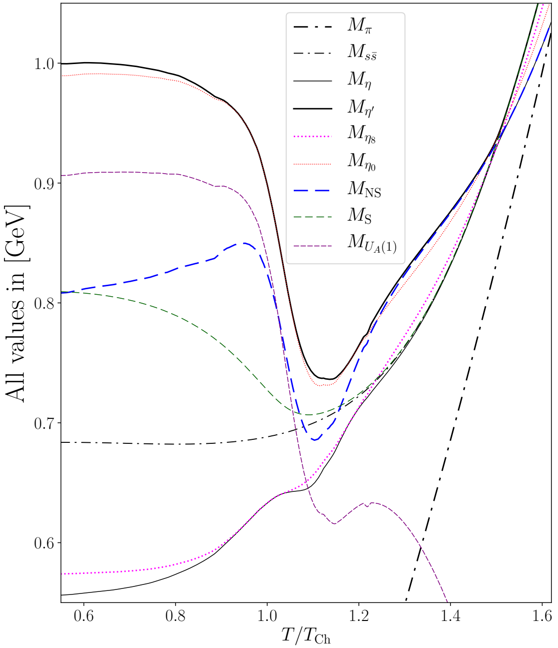


Simplest Ansatz $\mathcal{C}(T) = \mathcal{C}(0)$ fails for $T > 1.6\,T_c$, as $\chi(T)$ and $A(T) < 0$. $\Rightarrow$
Try different $\mathcal{C}(T)$, but first look in detail zoomed η-η' complex.

(14)Zoomed η-η' complex

- $M_{\eta'}(T)$ is not changed much as condensates are changed from chiral to massive: $M_{\eta'}(T)$ falls again around T_{Ch} by 300 to 200 MeV, corresponding to melting of $\sim \frac{1}{3} M_{U_A(1)}$.
  - But η does not exhibit any mass drop at all, now. It stays predominantly η_8 till anticrossing at $\sim 1.5\ T_{\mathrm{Ch}}$.

Similarly $\eta' \sim \eta_0$ long after T_{Ch}, and only after this anti-X with η, η' tends to a pure $s\bar{s}$, while η tends to a pure η_{NS} degenerate with the pion.

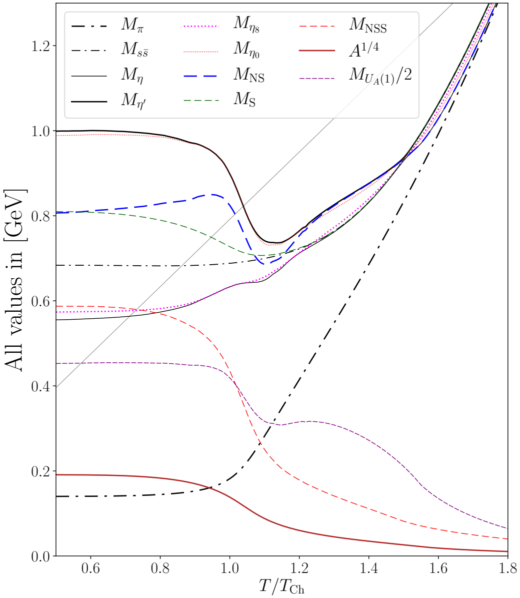


(15) T-dependence of $M_P(T)$ up to $T = 1.8\,T_{\rm Ch}$

- $\mathcal{C}(T) \neq$ *const*, adjusted to enable reaching arbitrary high T's, results otherwise very similar to previous case with $\mathcal{C}(T) = \mathcal{C}(0)$.
  - Other limitations of rank-2 separable model make it hard to find solutions beyond $T \sim 1.8\,T_c$.

But it is enough to exhibit cleanly the asymptotic regime beyond anticrossing at $1.5\,T_{\rm Ch}$.

Along with A, the more direct influence on anomalous masses is given by $M_{\rm NS\,S}$ and $(\frac{1}{2})M_{U_A(1)}$.



(16) Summary

- Our approach ties the $U_A(1)$ SB to the DChSB very closely $\Rightarrow$ the restoration of the chiral symmetry must lead to the restoration of the $U_A(1)$ symmetry at least partially, on the level of the η' & η masses.
- **Full $U_A(1)$ restoration occurs together with the chiral restoration at $T = T_{Ch}$ only for the chiral condensate $\langle q\bar{q}\rangle_0$.**
- Condensates with **explicit** ChSB fall with T much more slowly than $\langle q\bar{q}\rangle_0$. **Then, η does not exhibit any mass drop at all,** while the **significant drop of the η' mass signals only a partial restoration of $U_A(1)$ symmetry, consuming only about $\frac{1}{3} M_{U_A(1)}$.**
- For realistic explicit chiral breaking, there is an intermediate region between the chiral restoration at $T = T_{Ch}$ and the η-η' anticrossing at $T = 1.5\, T_{Ch}$ which marks the effective $U_A(1)$ restoration.

  There, the anomalous contributions to the masses become sufficiently weak, and η-η' complex enters the NS-S asymptotic regime of the vanishing $U_A(1)$ anomaly influence:

  $M_{\eta'}(T) \to M_{s\bar{s}}(T)$, & $M_\eta(T) \to M_{\mathrm{NS}}(T) \to M_\pi(T)$, & $\phi(T) \to 0$.