Soft-gluon corrections for top-pair and single-top processes

Nikolaos Kidonakis

- **Higher-order soft-gluon corrections**

- **Top-pair production**

- **Single-top production**

- **Top with electroweak bosons in new physics**

KENNESAW STATE UNIVERSITY
COLLEGE OF SCIENCE AND MATHEMATICS
Department of Physics

**Top Quark Physics
at the Precision Frontier**

NSF

Higher-order corrections

QCD corrections are very significant for top-quark production processes

**Soft-gluon corrections are important
and they approximate exact results very well**

**We calculate/resum these soft corrections at NNLL accuracy
for the double-differential cross section**

Finite-order expansions - no prescription needed

**Approximate NNLO (aNNLO) and N^{3}LO (aN^{3}LO) predictions
for cross sections and differential distributions**

Soft-gluon corrections

partonic processes
$$f_1(p_1) + f_2(p_2) \to t(p_t) + X$$

define

$s = (p_1 + p_2)^2$, $t = (p_1 - p_t)^2$, $u = (p_2 - p_t)^2$
and $s_4 = s + t + u - \sum m^2$

At partonic threshold $s_4 \to 0$

Soft corrections $\left[\dfrac{\ln^k(s_4/m_t^2)}{s_4}\right]_+$

For the order α_s^n **corrections** $k \le 2n - 1$

Resum these soft corrections for the double-differential cross section

At NNLL accuracy we need two-loop soft anomalous dimensions

Soft-gluon Resummation

moments of the partonic cross section with moment variable N:
$\hat{\sigma}(N) = \int (ds_4/s) \, e^{-Ns_4/s} \hat{\sigma}(s_4)$

factorized expression for the cross section in $4 - \epsilon$ dimensions

$$\sigma^{f_1 f_2 \to tX}(N, \epsilon) = H_{IL}^{f_1 f_2 \to tX}\left(\alpha_s(\mu_R)\right) S_{LI}^{f_1 f_2 \to tX}\left(\frac{m_t}{N\mu_F}, \alpha_s(\mu_R)\right)$$

$$\times \prod J_{\text{in}}\left(N, \mu_F, \epsilon\right) \prod J_{\text{out}}\left(N, \mu_F, \epsilon\right)$$
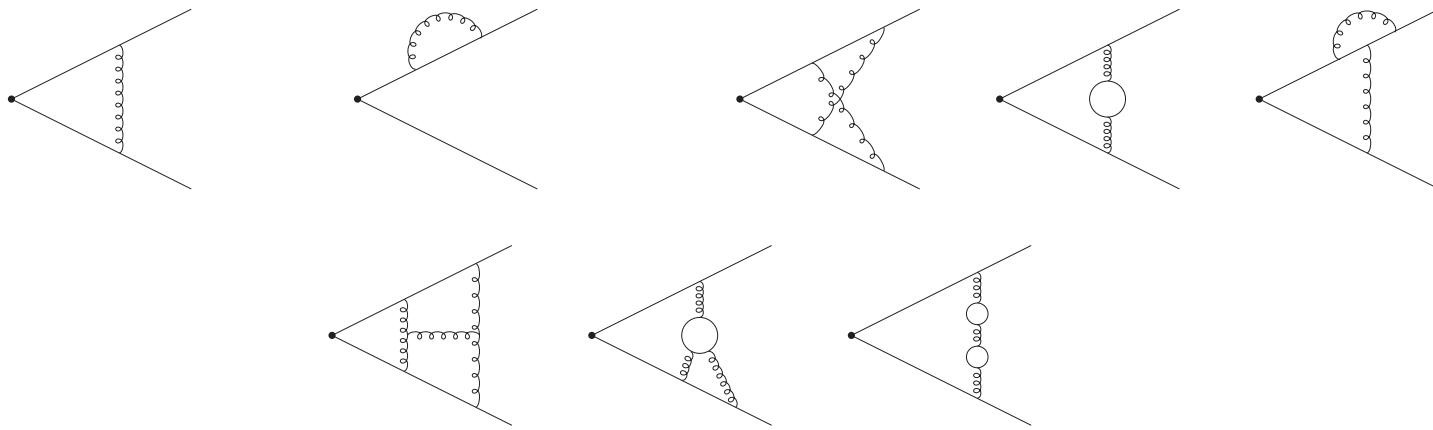
$H_{IL}^{f_1 f_2 \to tX}$ **is hard function and** $S_{LI}^{f_1 f_2 \to tX}$ **is soft function**

S_{LI} **satisfies the renormalization group equation**

$$\left(\mu\frac{\partial}{\partial\mu} + \beta(g_s)\frac{\partial}{\partial g_s}\right) S_{LI} = -(\Gamma_S^\dagger)_{LK} S_{KI} - S_{LK}(\Gamma_S)_{KI}$$

Soft anomalous dimension Γ_S controls the evolution of the soft function

which gives the exponentiation of logarithms of N

Cusp anomalous dimension

A basic ingredient of soft anomalous dimensions

cusp angle $\theta = \cosh^{-1}(p_i \cdot p_j / \sqrt{p_i^2 p_j^2})$ **and** $\Gamma_{\text{cusp}} = \sum_{n=1}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^n \Gamma_{\text{cusp}}^{(n)}$

One loop

$$\Gamma_{\text{cusp}}^{(1)} = C_F(\theta \coth \theta - 1)$$

In terms of $\beta = \tanh(\theta/2) = \sqrt{1 - \frac{4m^2}{s}}$ **we have** $\theta = \ln\left[\frac{(1+\beta)}{(1-\beta)}\right]$ **and**

$$\Gamma_{\text{cusp}}^{(1)} = C_F\left[-\frac{(1+\beta^2)}{2\beta} \ln \frac{(1-\beta)}{(1+\beta)} - 1\right] = -C_F\left(L_\beta + 1\right)$$

Two loops

$$\Gamma_{\text{cusp}}^{(2)} = \frac{K}{2}\Gamma_{\text{cusp}}^{(1)} + \frac{1}{2}C_F C_A \left\{ 1 + \zeta_2 + \theta^2 - \coth\theta \left[\zeta_2\theta + \theta^2 + \frac{\theta^3}{3} + \text{Li}_2\left(1 - e^{-2\theta}\right) \right] \right.$$

$$\left. + \coth^2\theta \left[-\zeta_3 + \zeta_2\theta + \frac{\theta^3}{3} + \theta\,\text{Li}_2\left(e^{-2\theta}\right) + \text{Li}_3\left(e^{-2\theta}\right) \right] \right\}$$

where $K = C_A(67/18 - \zeta_2) - 5n_f/9$

Three loops (very long expression for $C^{(3)}$)

$$\Gamma_{\text{cusp}}^{(3)} = C^{(3)} + K'^{(3)}\Gamma_{\text{cusp}}^{(1)} + K\left[\Gamma_{\text{cusp}}^{(2)} - \frac{K}{2}\Gamma_{\text{cusp}}^{(1)}\right]$$

For $n_f = 5$

$$\Gamma_{\text{cusp}}^{(3)\,\text{approx}}(\beta) = 0.092\,\beta^2 + 2.803\,\Gamma_{\text{cusp}}^{(1)}(\beta)$$

Cusp anomalous dimension

Case where one line is massive and one is massless: simpler expressions
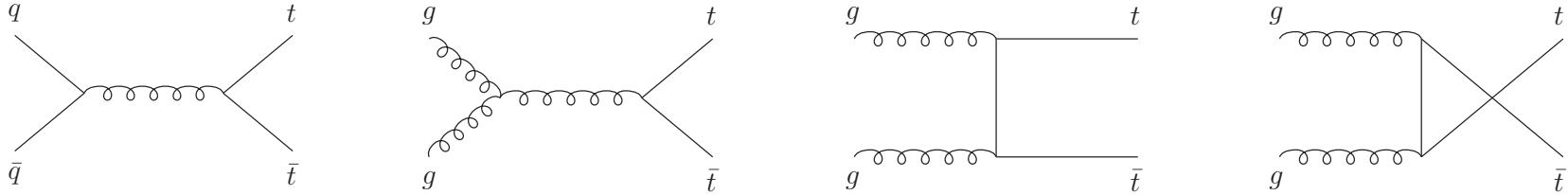
If eikonal line i represents a massive quark and eikonal line j a massless quark, then

$$\Gamma_c^{(1)} = C_F \left[\ln\left(\frac{2 p_i \cdot p_j}{m_i \sqrt{s}} \right) - \frac{1}{2} \right]$$

$$\Gamma_c^{(2)} = C_F \frac{K}{2} \left[\ln\left(\frac{2 p_i \cdot p_j}{m_i \sqrt{s}} \right) - \frac{1}{2} \right] + \frac{1}{4} C_F C_A (1 - \zeta_3)$$

$$\Gamma_c^{(3)} = C_F K'^{(3)} \left[\ln\left(\frac{2 p_i \cdot p_j}{m_i \sqrt{s}} \right) - \frac{1}{2} \right] + C_F C_A \frac{K}{4} (1 - \zeta_3)$$
$$+ C_F C_A^2 \left[-\frac{1}{4} + \frac{3}{8} \zeta_2 - \frac{\zeta_3}{8} - \frac{3}{8} \zeta_2 \zeta_3 + \frac{9}{16} \zeta_5 \right]$$

Top-antitop pair production

At one loop for $q\bar{q} \to t\bar{t}$

$$\Gamma_{11}^{q\bar{q}\,(1)} \quad = \quad \Gamma_{\text{cusp}}^{(1)}\,, \quad \Gamma_{12}^{q\bar{q}\,(1)} = \frac{C_F}{C_A} \ln\left(\frac{t_1}{u_1}\right)\,, \quad \Gamma_{21}^{q\bar{q}\,(1)} = 2\ln\left(\frac{t_1}{u_1}\right)$$

$$\Gamma_{22}^{q\bar{q}\,(1)} \quad = \quad \left(1 - \frac{C_A}{2C_F}\right)\Gamma_{\text{cusp}}^{(1)} + 4C_F \ln\left(\frac{t_1}{u_1}\right) - \frac{C_A}{2}\left[1 + \ln\left(\frac{s m_t^2 t_1^2}{u_1^4}\right)\right]$$
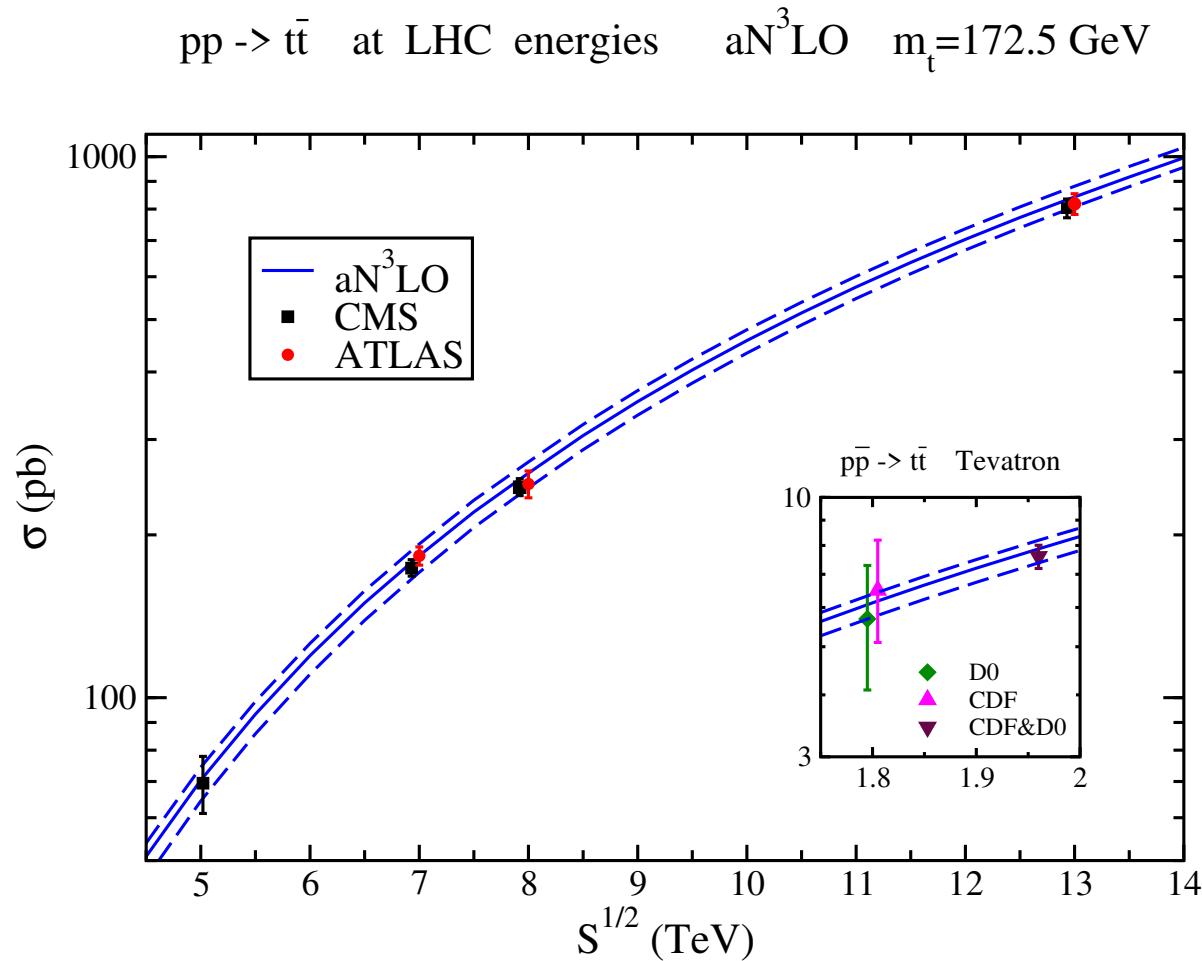
At two loops for $q\bar{q} \to t\bar{t}$

$$\Gamma_{11}^{q\bar{q}\,(2)} \quad = \quad \Gamma_{\text{cusp}}^{(2)}\,, \quad \Gamma_{12}^{q\bar{q}\,(2)} = \left(\frac{K}{2} - \frac{C_A}{2}N_{2l}\right)\Gamma_{12}^{q\bar{q}\,(1)}\,, \quad \Gamma_{21}^{q\bar{q}\,(2)} = \left(\frac{K}{2} + \frac{C_A}{2}N_{2l}\right)\Gamma_{21}^{q\bar{q}\,(1)}$$
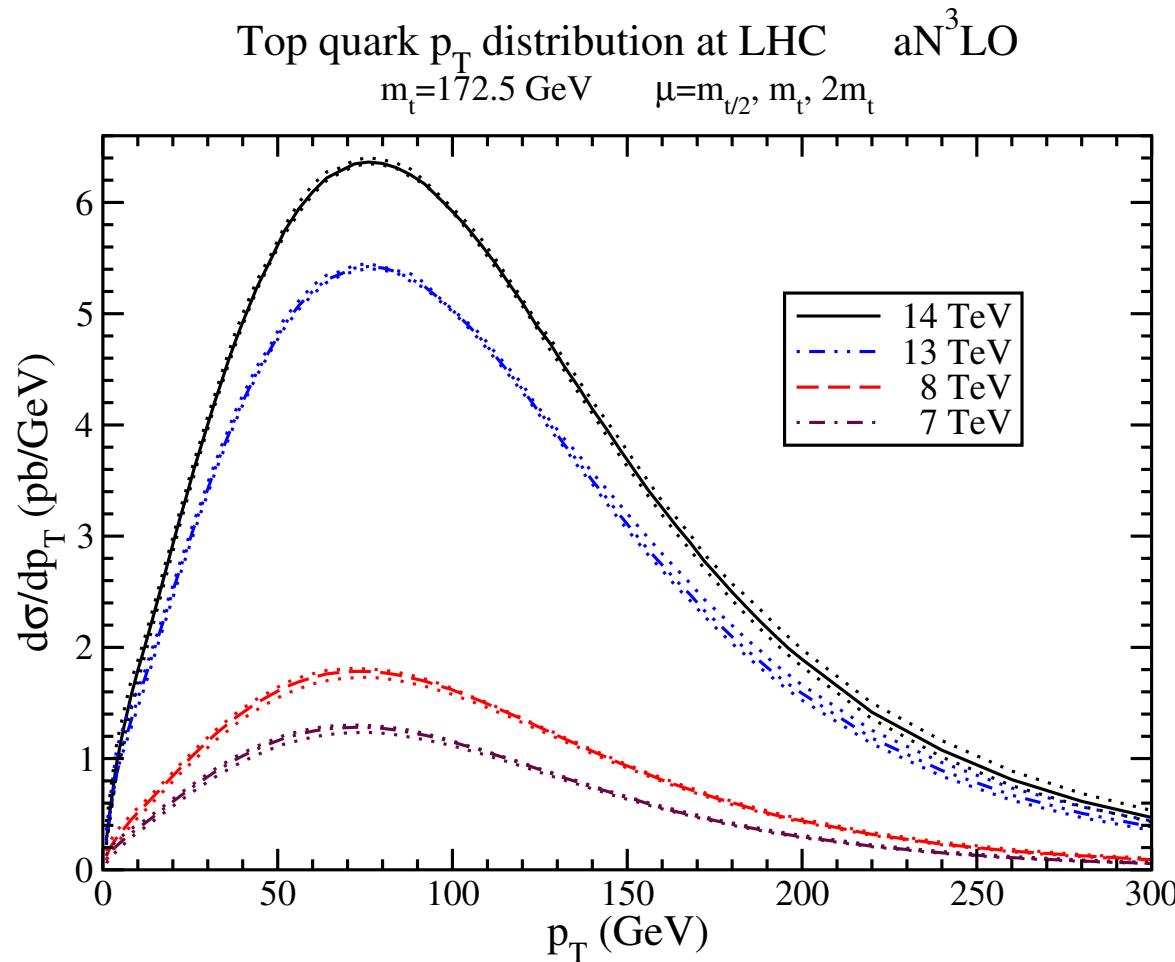
$$\Gamma_{22}^{q\bar{q}\,(2)} \quad = \quad \frac{K}{2}\Gamma_{22}^{q\bar{q}\,(1)} + \left(1 - \frac{C_A}{2C_F}\right)\left(\Gamma_{\text{cusp}}^{(2)} - \frac{K}{2}\Gamma_{\text{cusp}}^{(1)}\right)$$

3×3 matrix for $gg \to t\bar{t}$

Top-antitop pair production at aN³LO with NNLL accuracy
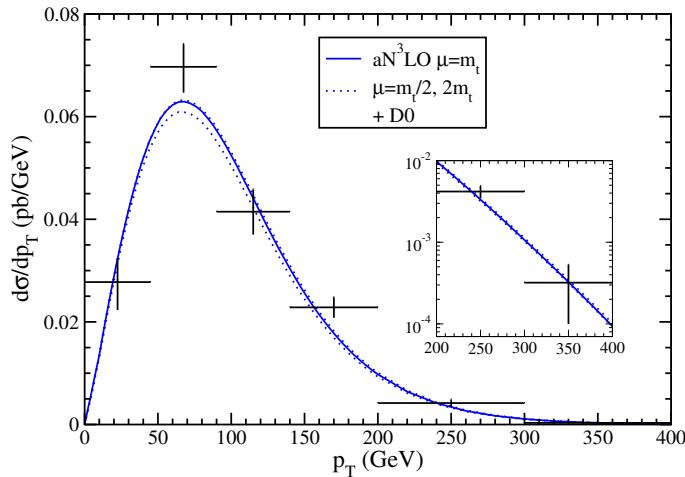
$pp \to t\bar{t}$ at LHC energies aN³LO m_t=172.5 GeV

Top p_T distributions in $t\bar{t}$ production

Top quark p_T distribution at LHC   aN^{3}LO
m_t=172.5 GeV $\mu = m_{t/2}, m_t, 2m_t$

14 TeV
13 TeV
8 TeV
7 TeV

dσ/dp_T (pb/GeV)

p_T (GeV)
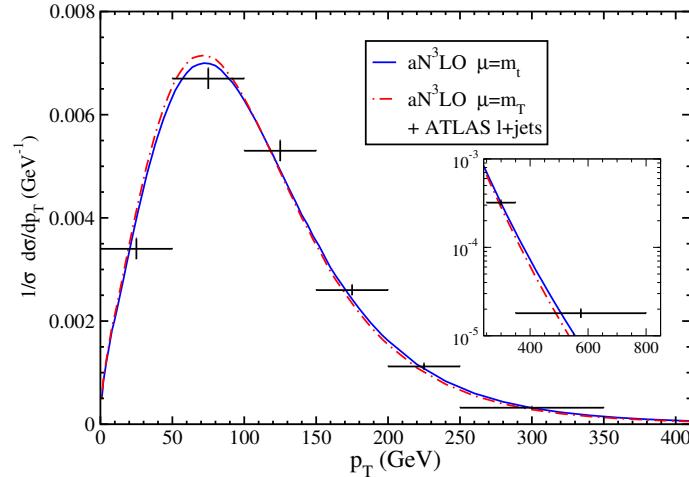
Top p_T distributions in $t\bar{t}$ production
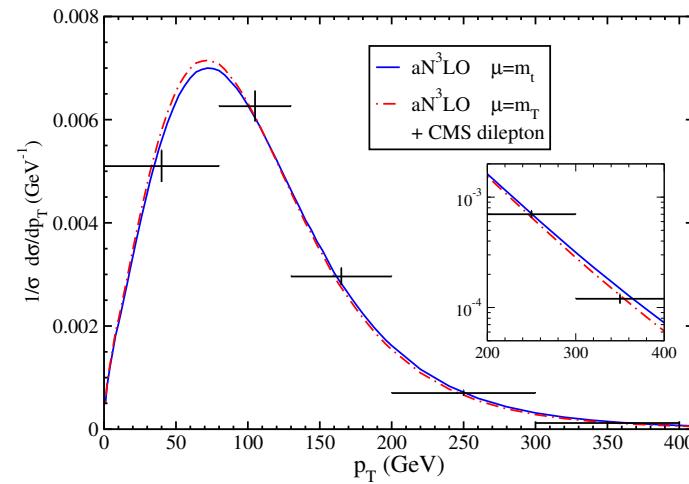
Top quark p_T distribution Tevatron $S^{1/2}$=1.96 TeV m_t=172.5 GeV
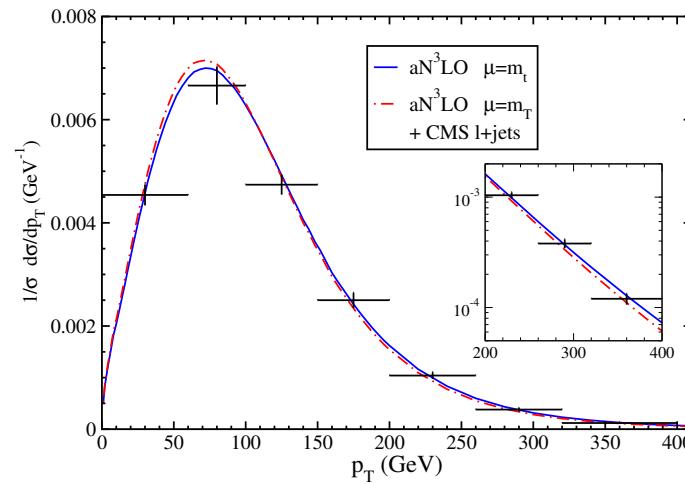
Normalized top p_T distribution at the LHC $S^{1/2}$=7 TeV
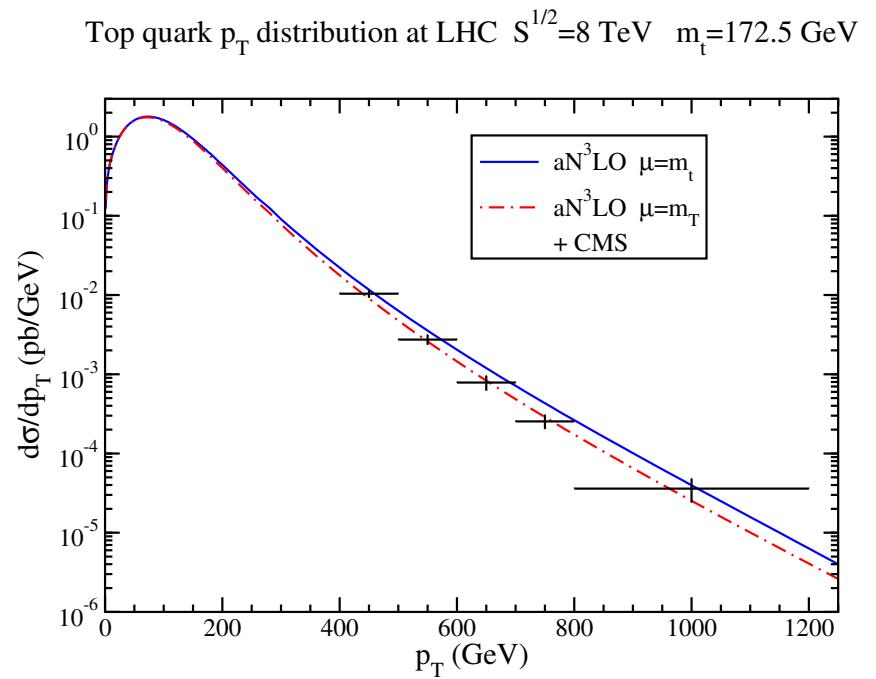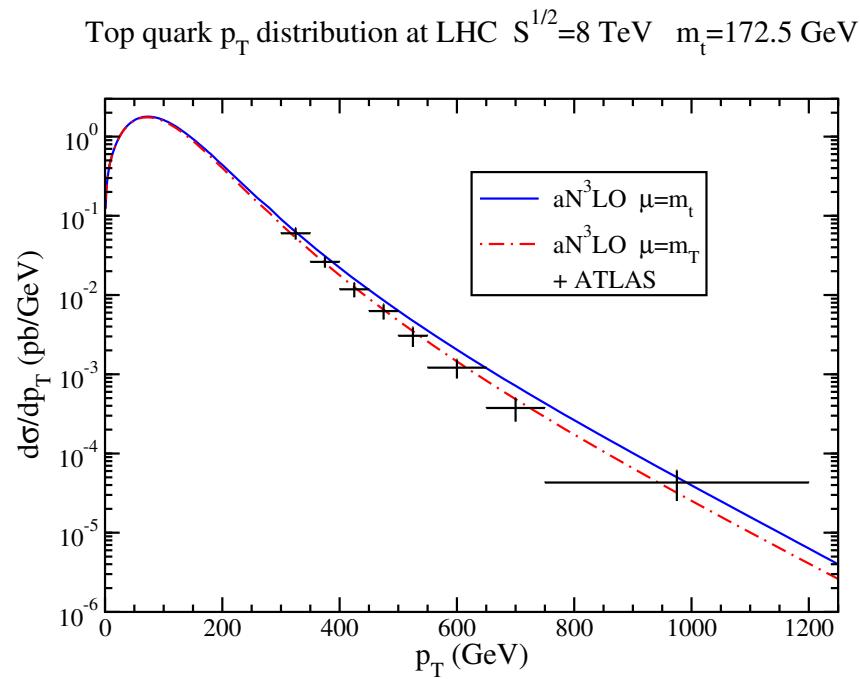
Normalized top p_T distribution at the LHC $S^{1/2}$=7 TeV
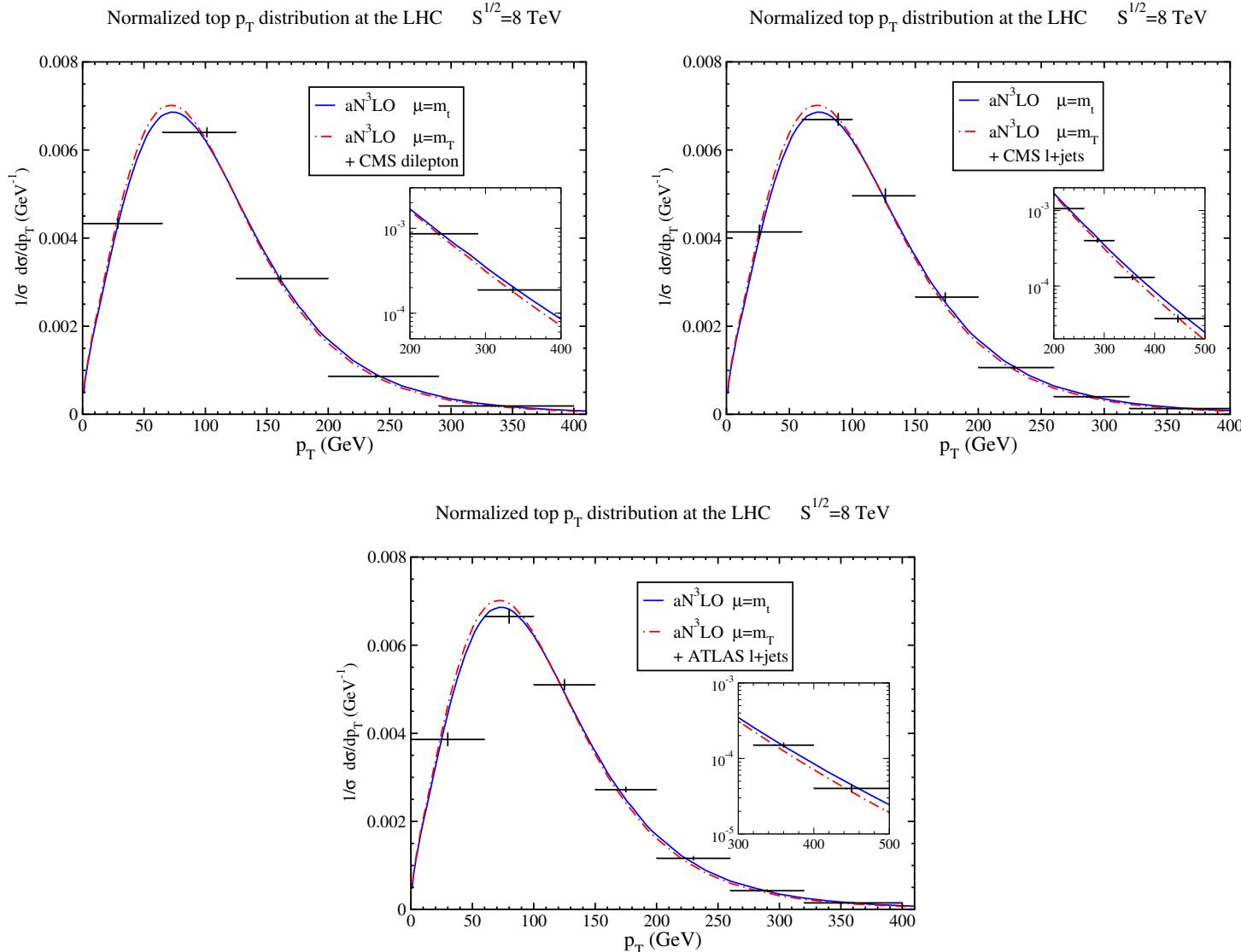
Normalized top p_T distribution at the LHC $S^{1/2}$=7 TeV

Top p_T distributions in $t\bar{t}$ production

Top quark p_T distribution at LHC $S^{1/2}$=8 TeV m_t=172.5 GeV

Top quark p_T distribution at LHC $S^{1/2}$=8 TeV m_t=172.5 GeV

Top p_T distributions in $t\bar{t}$ production

Normalized top p_T distribution at the LHC $S^{1/2}$=8 TeV

Normalized top p_T distribution at the LHC $S^{1/2}$=8 TeV

Normalized top p_T distribution at the LHC $S^{1/2}$=8 TeV

Top p_T distributions in $t\bar{t}$ production

Top quark p_T distribution at 13 TeV LHC   aN^{3}LO
m_t=172.5 GeV

$\mu=m_t$
$\mu=m_t/2, 2m_t$
$\mu=m_T$
+ CMS

dσ/dp_T (pb/GeV)

p_T (GeV)

Normalized top p_T distribution at the LHC   S$^{1/2}$=13 TeV

aN^{3}LO $\mu=m_t$
aN^{3}LO $\mu=m_T$
+CMS dilepton

1/σ dσ/dp_T (GeV^{-1})

p_T (GeV)

Top rapidity distributions in $t\bar{t}$ production

Top rapidity distribution at LHC   aN^{3}LO
m_t=172.5 GeV μ=m_t/2, m_t, $2m_t$

dσ/dY (pb)

- 14 TeV
- 13 TeV
- 8 TeV
- 7 TeV

Y
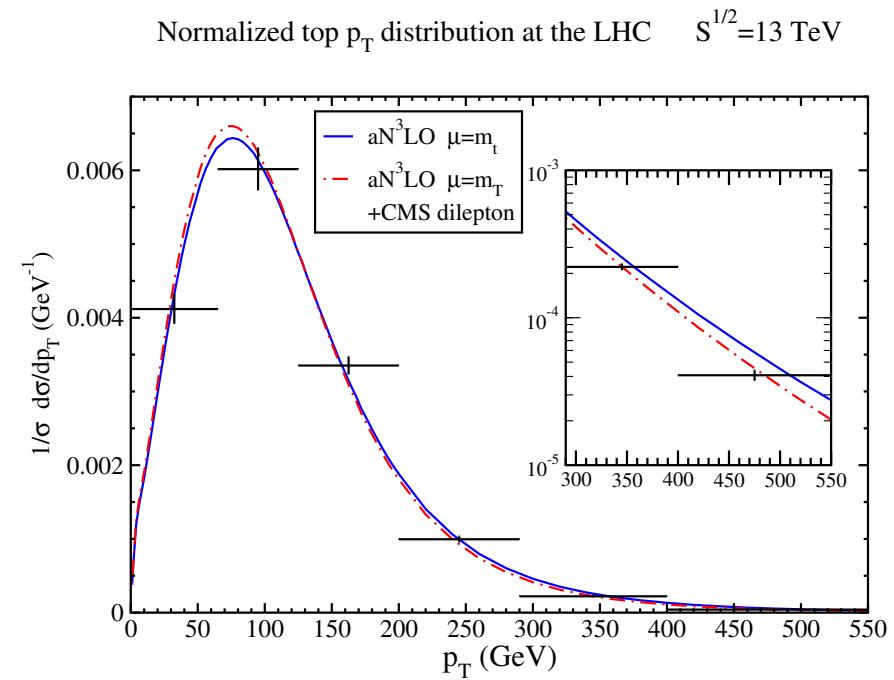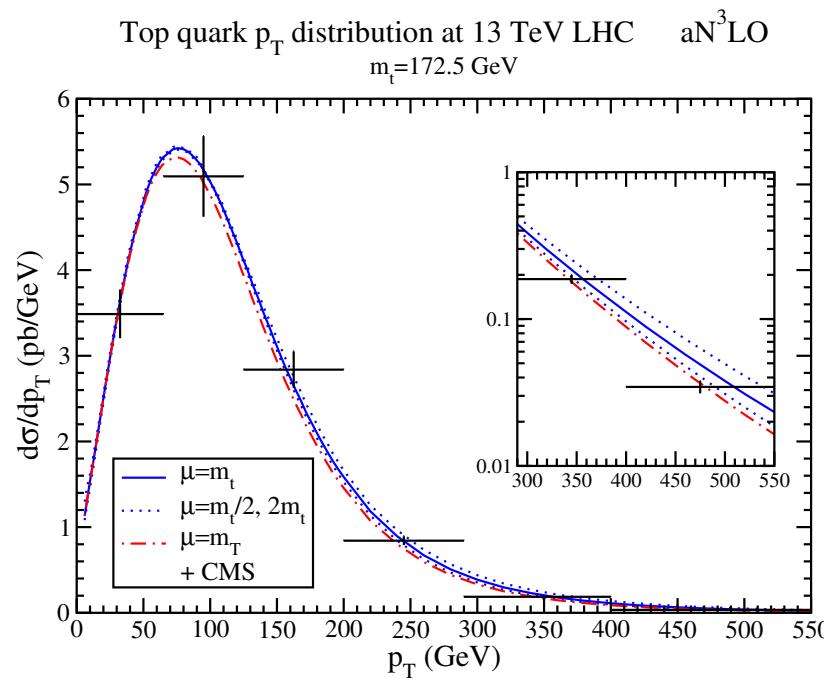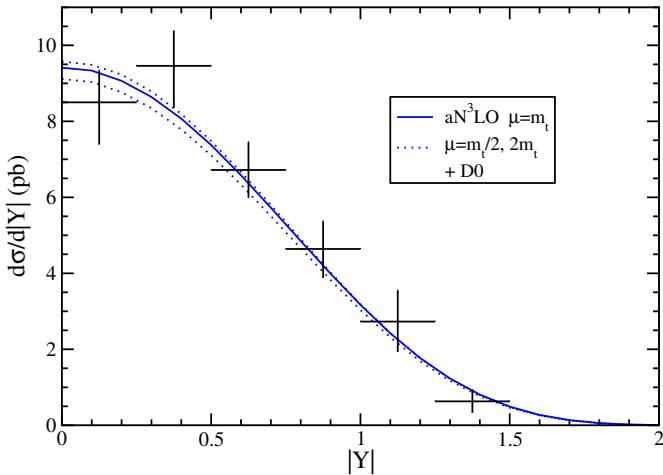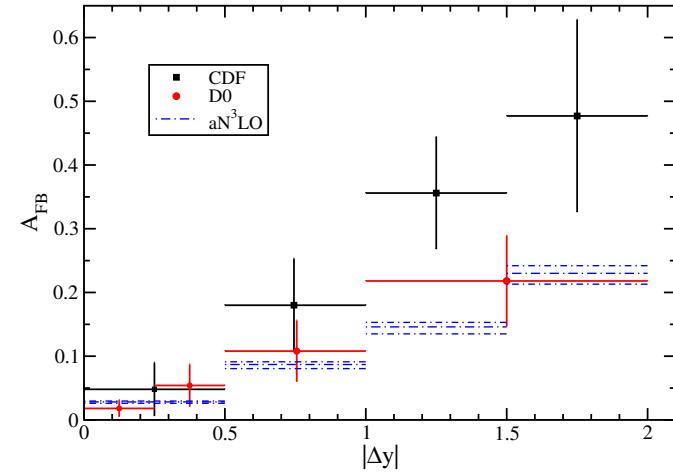
Top rapidity distributions in $t\bar{t}$ production
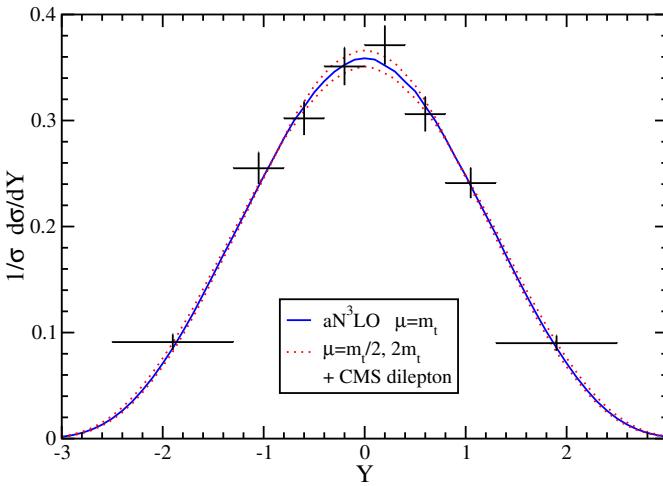
Top quark rapidity at Tevatron $S^{1/2}$=1.96 TeV m_t=172.5 GeV
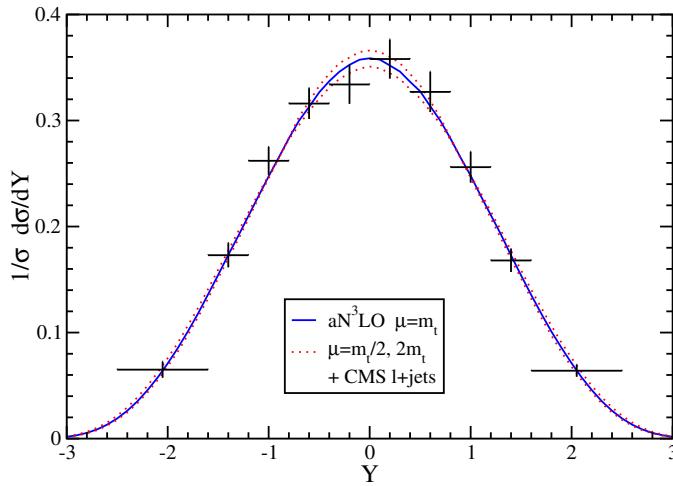
A_{FB} at Tevatron $S^{1/2}$=1.96 TeV

Normalized top rapidity distribution at LHC $S^{1/2}$=7 TeV
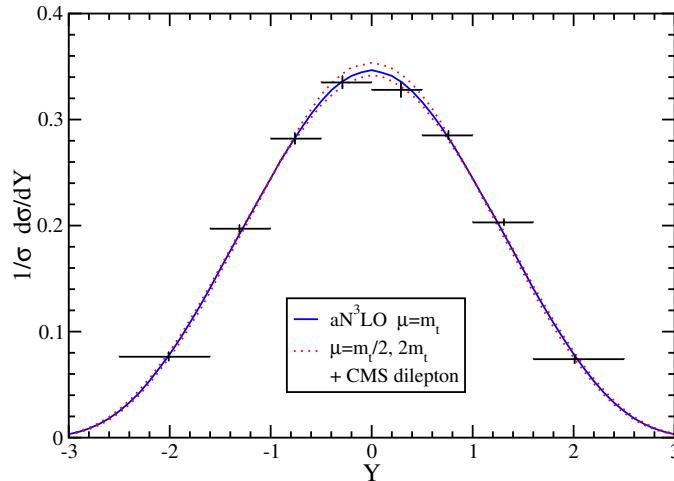
Normalized top rapidity distribution at LHC $S^{1/2}$=7 TeV
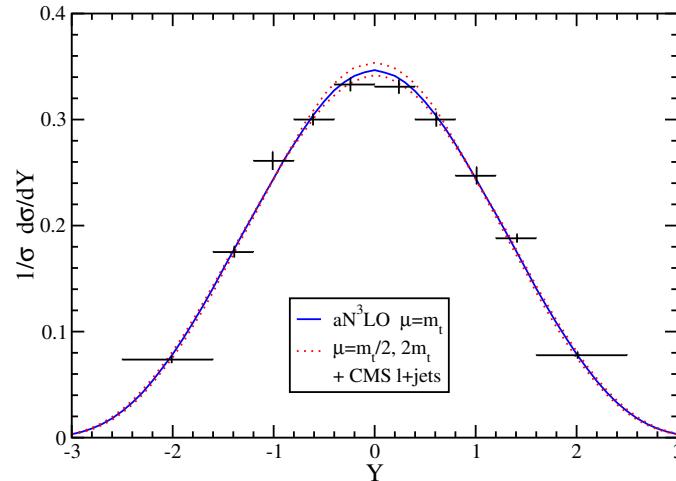
Top rapidity distributions in $t\bar{t}$ production
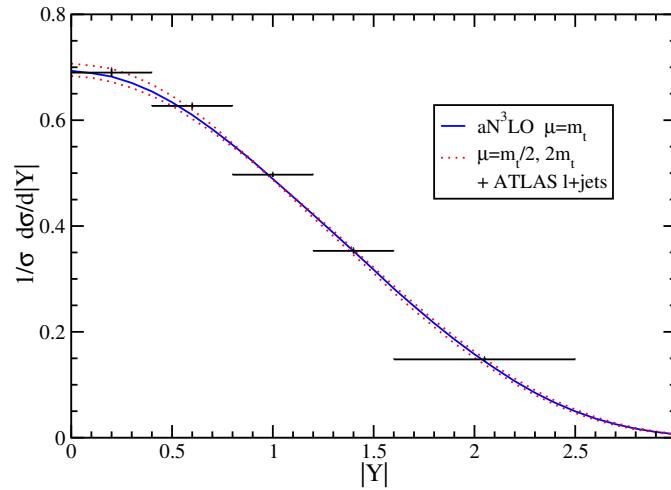
Normalized top rapidity distribution at LHC $S^{1/2}=8$ TeV
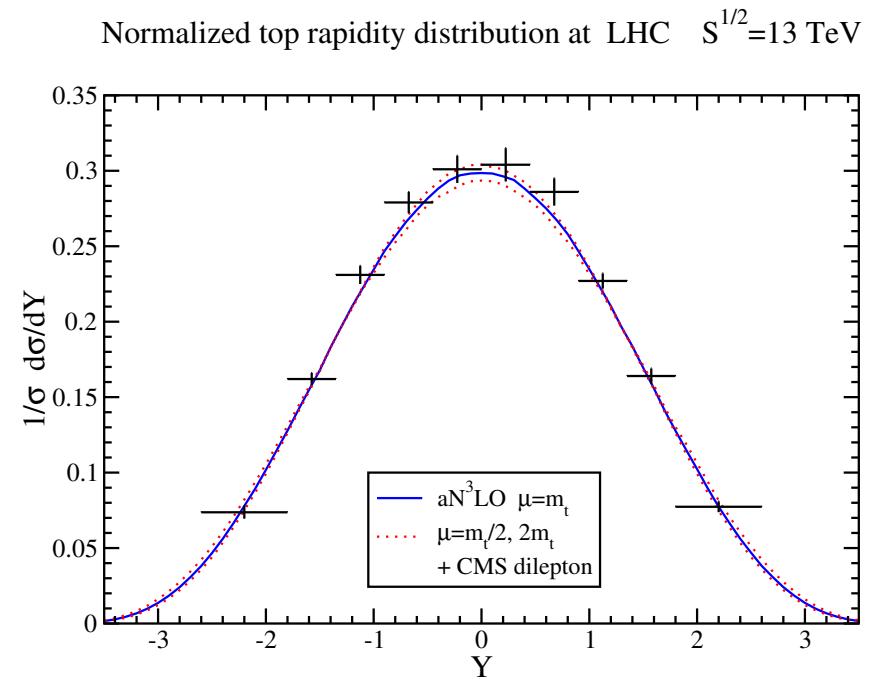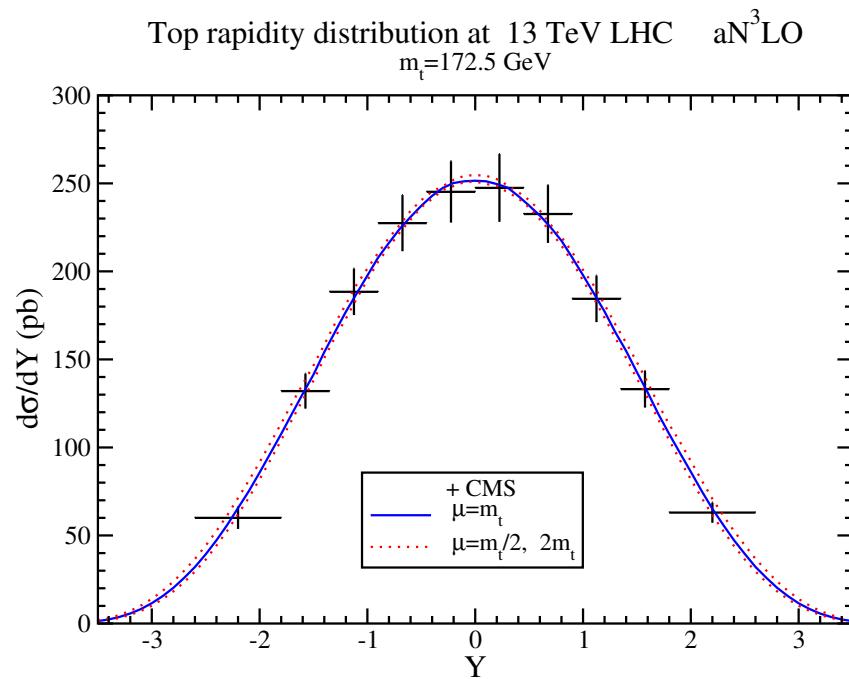
Normalized top rapidity distribution at LHC $S^{1/2}=8$ TeV
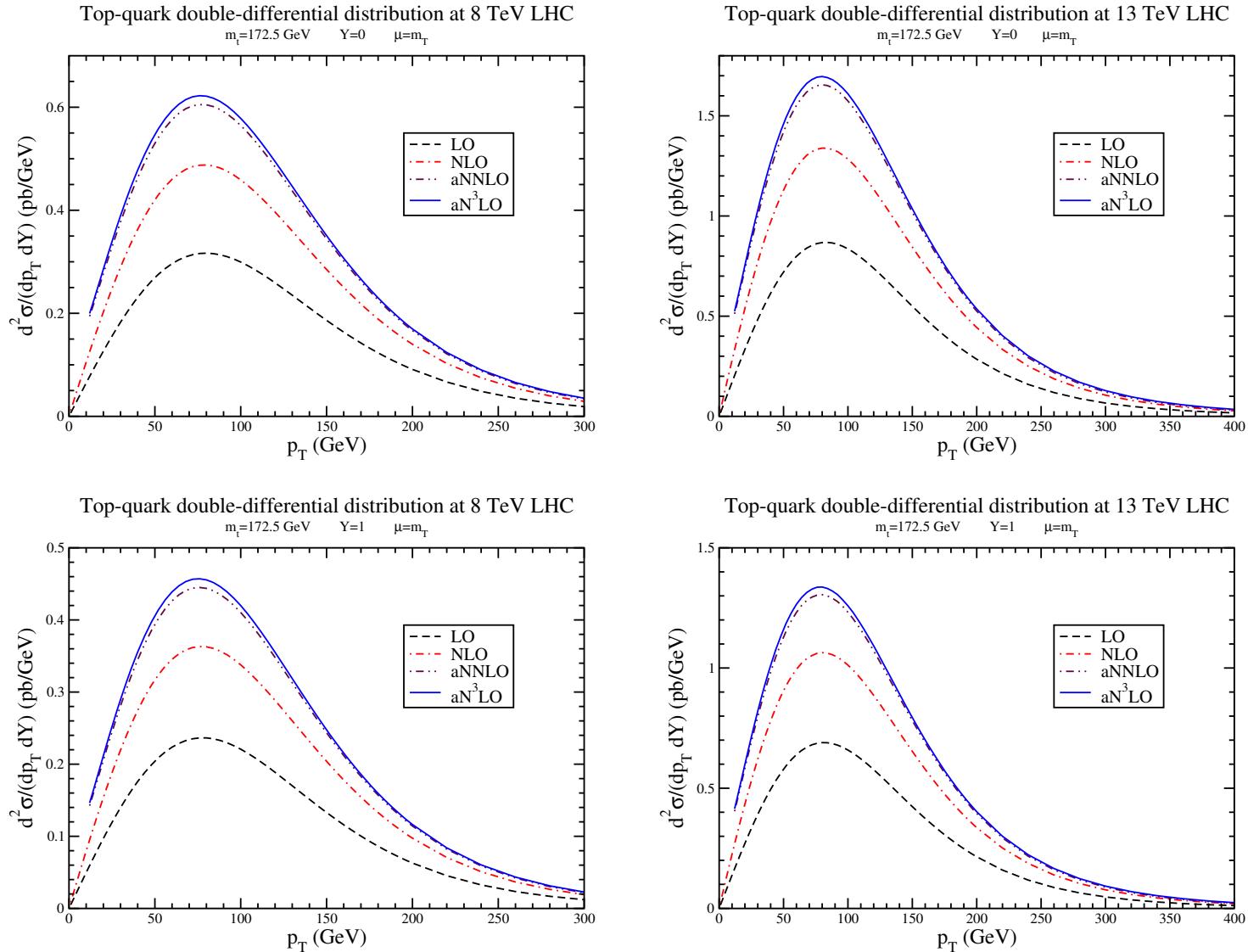
Normalized top rapidity distribution at LHC $S^{1/2}=8$ TeV

Top rapidity distributions in $t\bar{t}$ production

Top rapidity distribution at 13 TeV LHC   aN^{3}LO
m_t=172.5 GeV

Normalized top rapidity distribution at LHC   S$^{1/2}$=13 TeV

Top double-differential distributions in $t\bar{t}$ production

Top-quark double-differential distribution at 8 TeV LHC
m_t=172.5 GeV   Y=0 μ=m_T

Top-quark double-differential distribution at 13 TeV LHC
m_t=172.5 GeV   Y=0 μ=m_T

Top-quark double-differential distribution at 8 TeV LHC
m_t=172.5 GeV   Y=1 μ=m_T
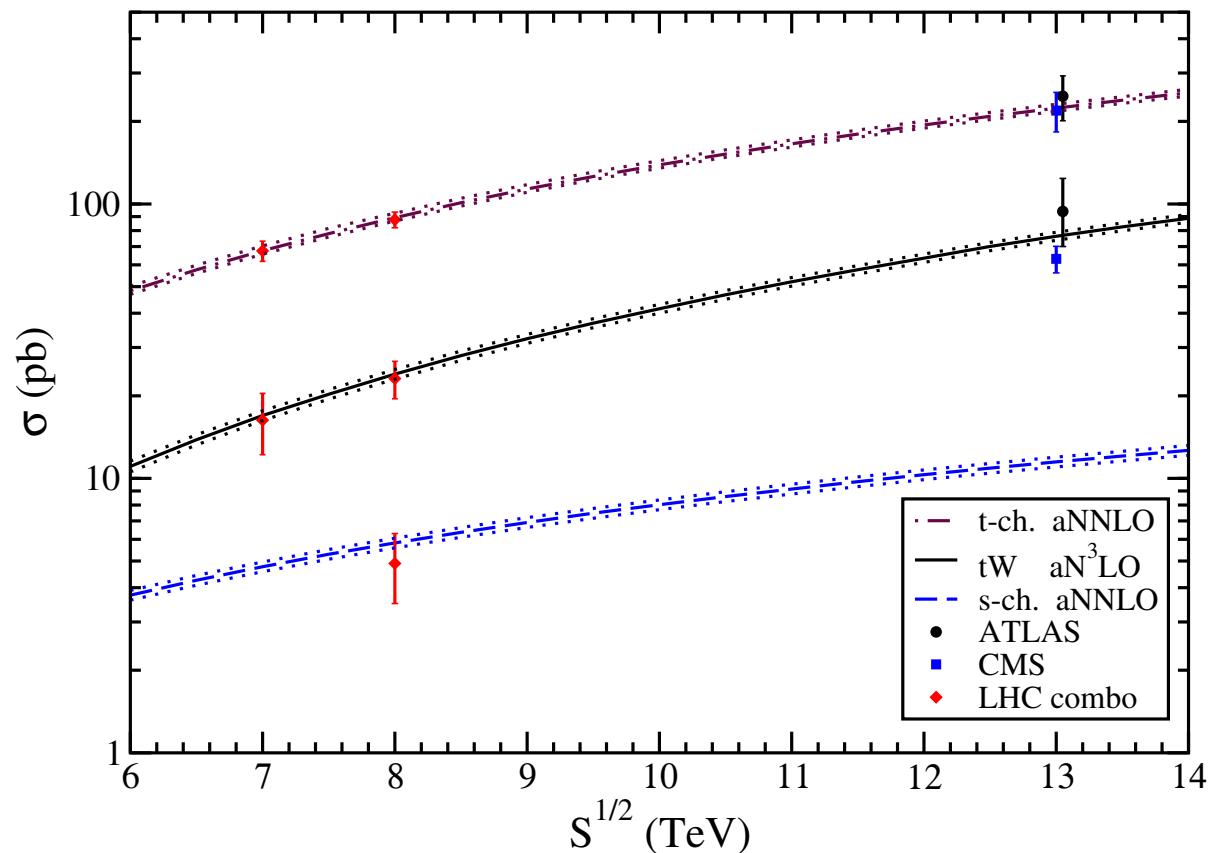
Top-quark double-differential distribution at 13 TeV LHC
m_t=172.5 GeV   Y=1 μ=m_T
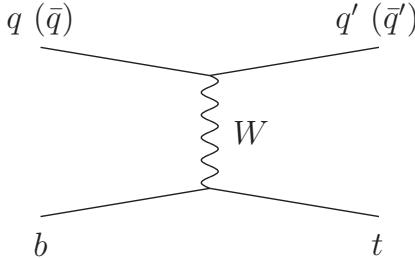
Single-top production

Single-top cross sections m_t=172.5 GeV

Single-top t-channel production

At one loop

$$\Gamma_{S\,11}^{t\,(1)} = C_F \left[\ln \left(\frac{t(t-m_t^2)}{m_t s^{3/2}} \right) - \frac{1}{2} \right], \qquad \Gamma_{S\,12}^{t\,(1)} = \frac{C_F}{2N} \ln \left(\frac{u(u-m_t^2)}{s(s-m_t^2)} \right), \qquad \Gamma_{S\,21}^{t\,(1)} = \ln \left(\frac{u(u-m_t^2)}{s(s-m_t^2)} \right)$$

$$\Gamma_{S\,22}^{t\,(1)} = C_F \left[\ln \left(\frac{t(t-m_t^2)}{m_t s^{3/2}} \right) - \frac{1}{2} \right] - \frac{1}{N} \ln \left(\frac{u(u-m_t^2)}{s(s-m_t^2)} \right) + \frac{N}{2} \ln \left(\frac{u(u-m_t^2)}{t(t-m_t^2)} \right)$$

At two loops
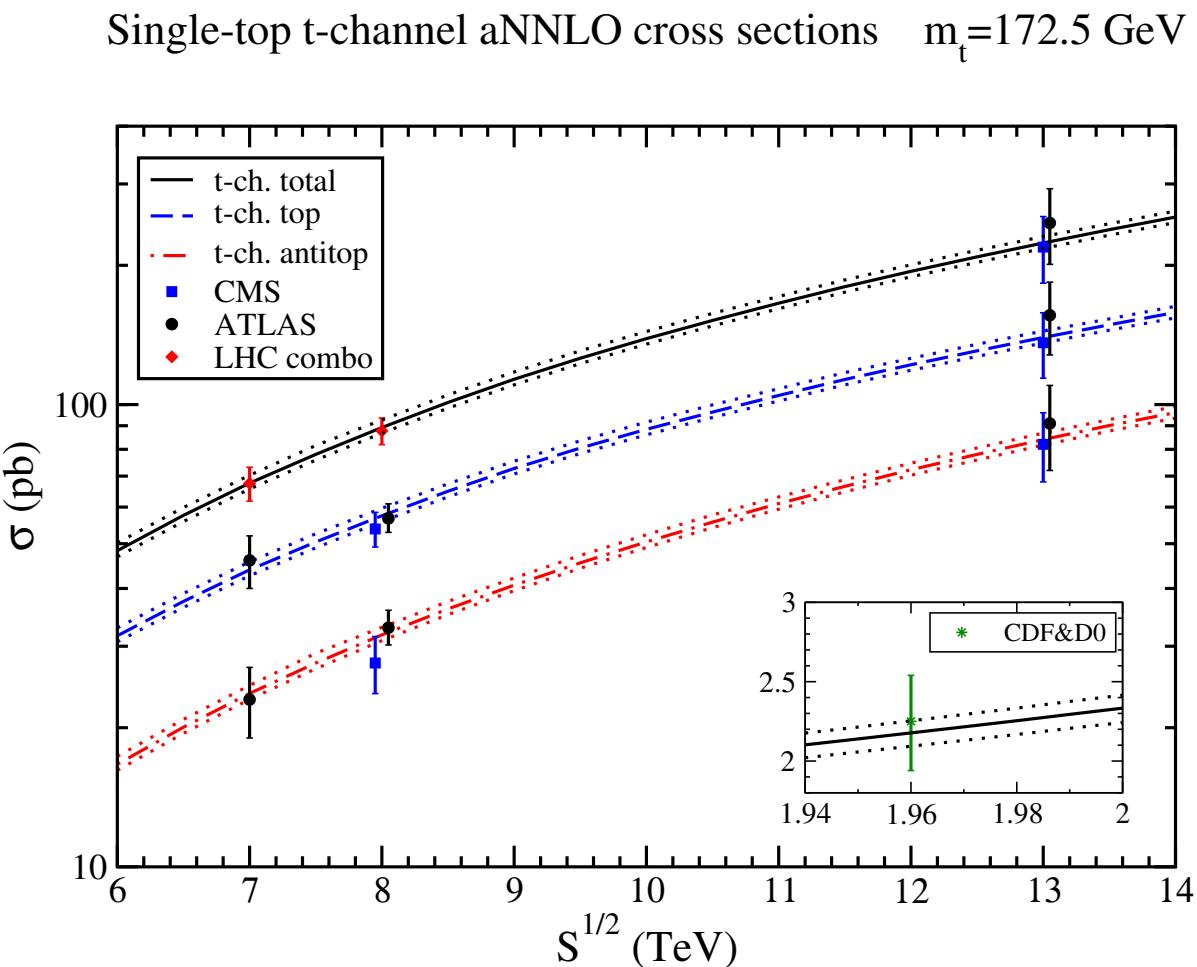
$$\Gamma_{S\,11}^{t\,(2)} = K'^{(2)} \Gamma_{S\,11}^{t\,(1)} + \frac{1}{4} C_F C_A (1 - \zeta_3), \qquad \Gamma_{S\,12}^{t\,(2)} = K'^{(2)} \Gamma_{S\,12}^{t\,(1)}$$

$$\Gamma_{S\,21}^{t\,(2)} = K'^{(2)} \Gamma_{S\,21}^{t\,(1)}, \qquad \Gamma_{S\,22}^{t\,(2)} = K'^{(2)} \Gamma_{S\,22}^{t\,(1)} + \frac{1}{4} C_F C_A (1 - \zeta_3)$$
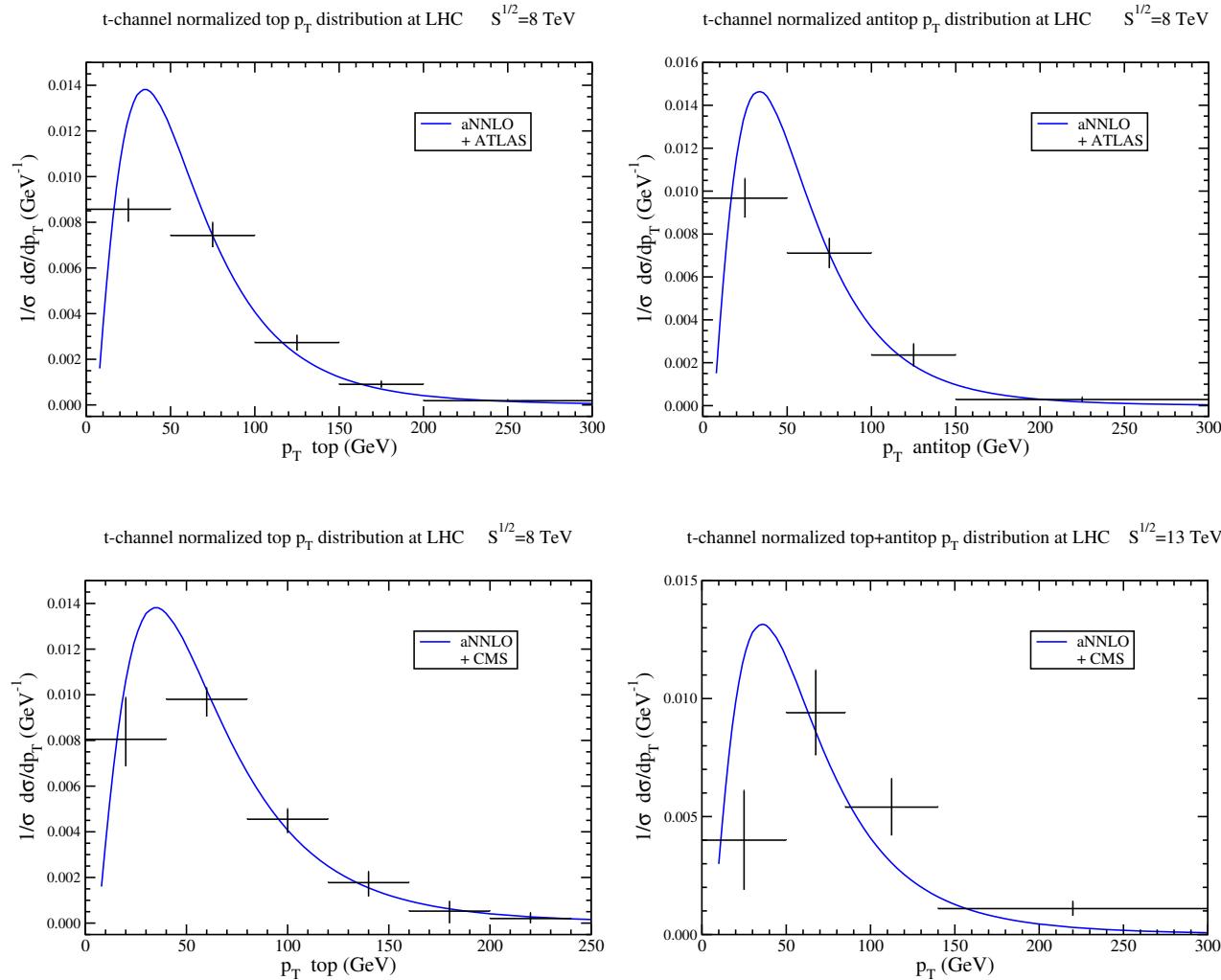
At three loops

$$\Gamma_{S\,11}^{t\,(3)} = K'^{(3)} \Gamma_{S\,11}^{t\,(1)} + \frac{1}{2} K^{(2)} C_A (1 - \zeta_3) + C^{(3)}$$

t-channel production at aNNLO with NNLL accuracy

Single-top t-channel aNNLO cross sections m_t=172.5 GeV
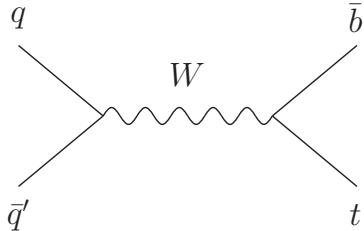
Top p_T distributions in t-channel production at the LHC

t-channel normalized top p_T distribution at LHC $S^{1/2}$=8 TeV

t-channel normalized antitop p_T distribution at LHC $S^{1/2}$=8 TeV

t-channel normalized top p_T distribution at LHC $S^{1/2}$=8 TeV

t-channel normalized top+antitop p_T distribution at LHC $S^{1/2}$=13 TeV

Single-top s-channel production

At one loop

$$\Gamma^{s\,(1)}_{S\,11} = C_F \left[\ln\left(\frac{s - m_t^2}{m_t\sqrt{s}} \right) - \frac{1}{2} \right], \qquad \Gamma^{s\,(1)}_{S\,12} = \frac{C_F}{2N} \ln\left(\frac{t(t - m_t^2)}{u(u - m_t^2)} \right), \qquad \Gamma^{s\,(1)}_{S\,21} = \ln\left(\frac{t(t - m_t^2)}{u(u - m_t^2)} \right)$$

$$\Gamma^{s\,(1)}_{S\,22} = C_F \left[\ln\left(\frac{s - m_t^2}{m_t\sqrt{s}} \right) - \frac{1}{2} \right] - \frac{1}{N} \ln\left(\frac{t(t - m_t^2)}{u(u - m_t^2)} \right) + \frac{N}{2} \ln\left(\frac{t(t - m_t^2)}{s(s - m_t^2)} \right)$$

At two loops
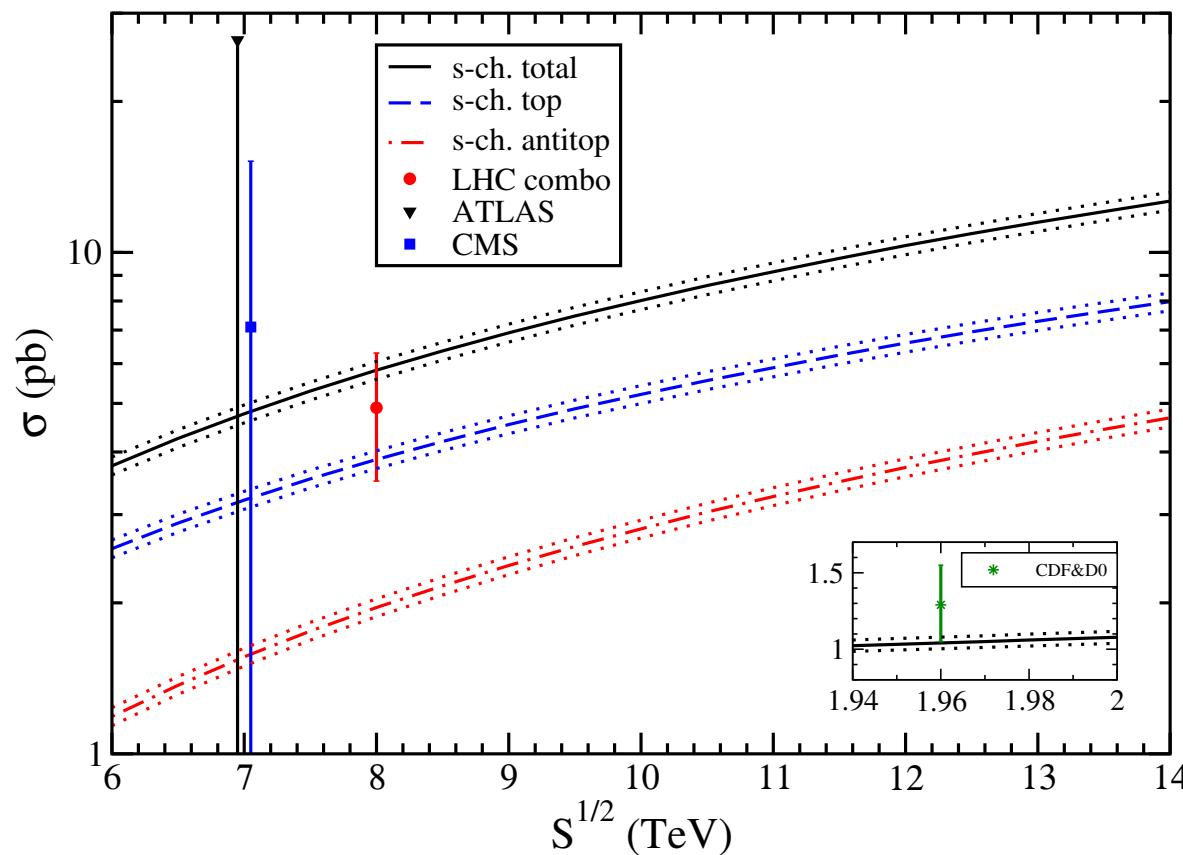
$$\Gamma^{s\,(2)}_{S\,11} = K^{'(2)} \Gamma^{s\,(1)}_{S\,11} + \frac{1}{4} C_F C_A (1 - \zeta_3), \qquad \Gamma^{s\,(2)}_{S\,12} = K^{'(2)} \Gamma^{s\,(1)}_{S\,12}$$

$$\Gamma^{s\,(2)}_{S\,21} = K^{'(2)} \Gamma^{s\,(1)}_{S\,21}, \qquad \Gamma^{s\,(2)}_{S\,22} = K^{'(2)} \Gamma^{s\,(1)}_{S\,22} + \frac{1}{4} C_F C_A (1 - \zeta_3)$$
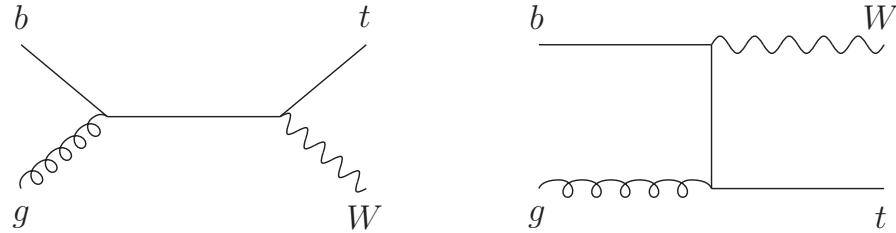
At three loops

$$\Gamma^{s\,(3)}_{11} = K^{'(3)} \Gamma^{s\,(1)}_{11} + \frac{1}{2} K^{(2)} C_A (1 - \zeta_3) + C^{(3)}$$

s-channel production at aNNLO with NNLL accuracy

Single-top s-channel aNNLO cross sections m_t=172.5 GeV

Associated tW production

At one loop

$$\Gamma_S^{tW\,(1)} = C_F \left[\ln\left(\frac{m_t^2 - t}{m_t \sqrt{s}} \right) - \frac{1}{2} \right] + \frac{C_A}{2} \ln\left(\frac{u - m_t^2}{t - m_t^2} \right)$$
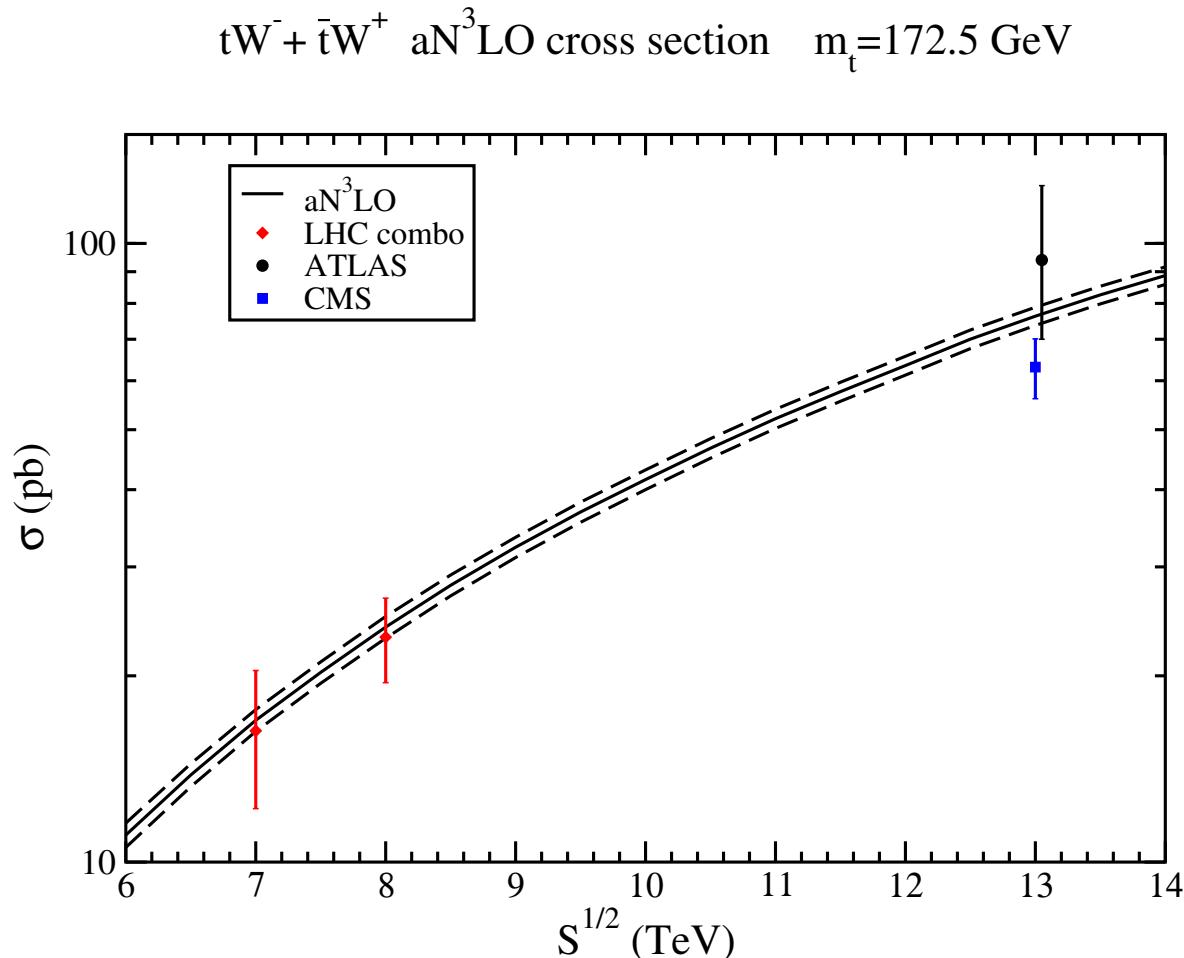
At two loops

$$\Gamma_S^{tW\,(2)} = K'^{(2)} \Gamma_S^{tW\,(1)} + \frac{1}{4} C_F C_A (1 - \zeta_3)$$

At three loops
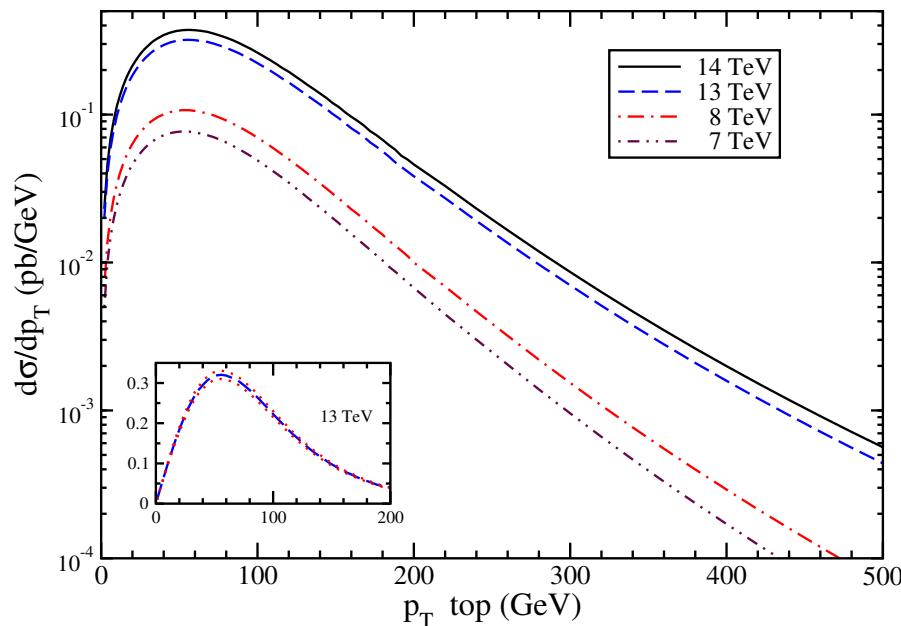
$$\Gamma_S^{tW\,(3)} = K'^{(3)} \Gamma_S^{tW\,(1)} + \frac{1}{2} K^{(2)} C_A (1 - \zeta_3) + C^{(3)}$$

tW production at aN^{3}LO with NNLL accuracy

$tW^- + \bar{t}W^+$ aN^{3}LO cross section m_t=172.5 GeV
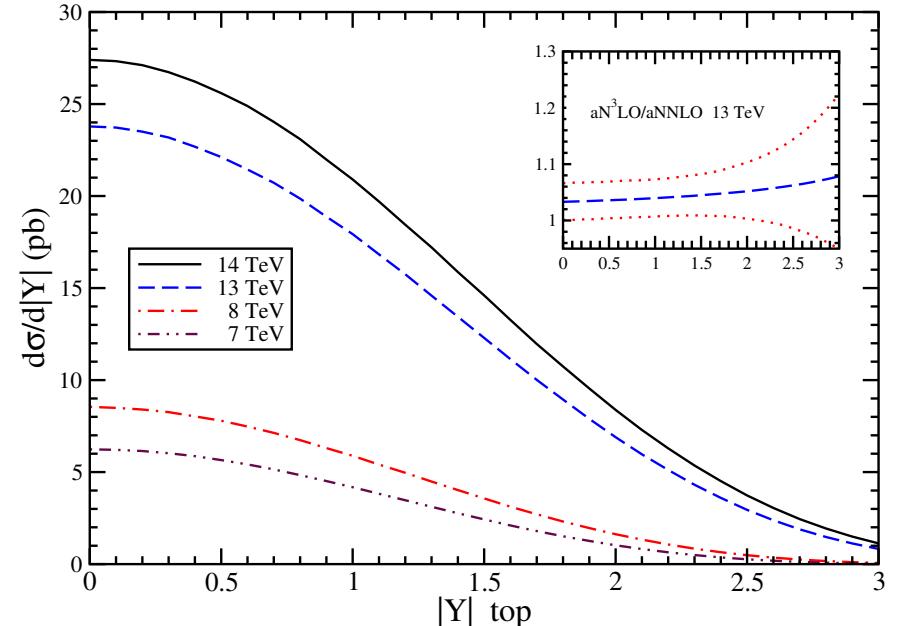
Top quark p_T and rapidity distributions in tW production
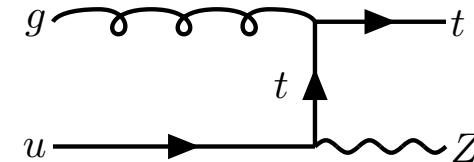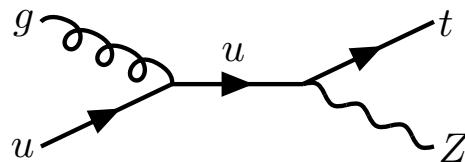
Top p_T distribution in tW$^-$ production at LHC   aN^{3}LO m_t=173.3 GeV
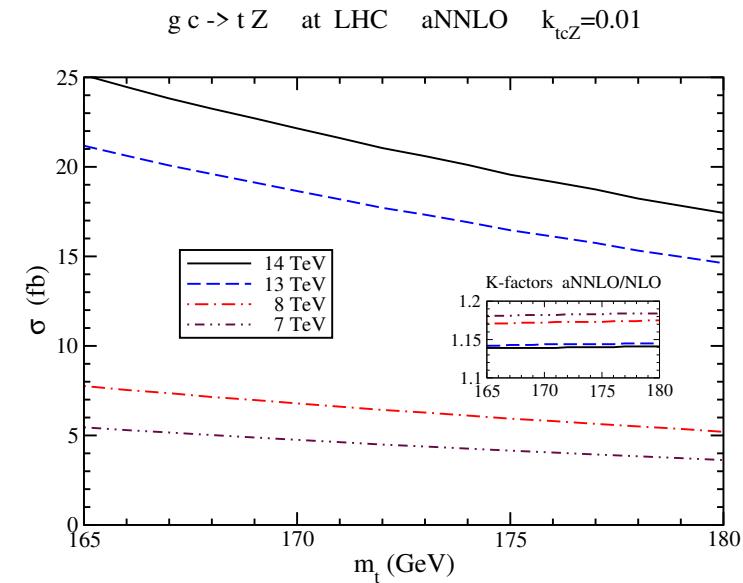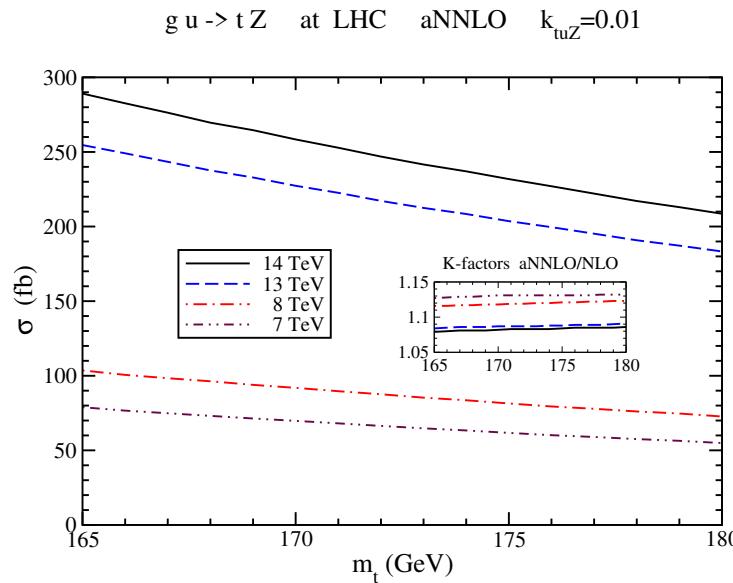
Top rapidity distribution in tW$^-$ production at LHC   aN^{3}LO m_t=173.3 GeV

tZ production via anomalous couplings

$$\Delta \mathcal{L}^{eff} = \frac{1}{\Lambda} \kappa_{tqZ} e \bar{t} (i/2)(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) q F_Z^{\mu\nu} + h.c.$$

g u -> t Z   at LHC   aNNLO   k$_{tuZ}$=0.01

g c -> t Z   at LHC   aNNLO   k$_{tcZ}$=0.01
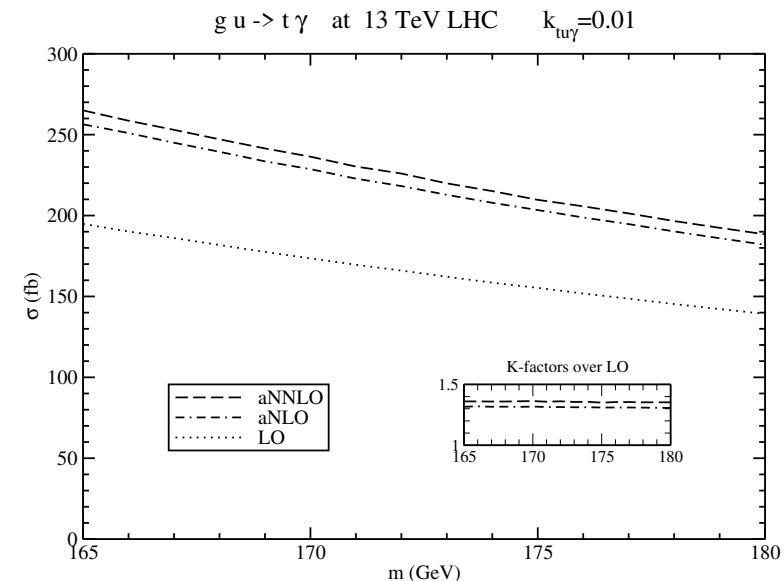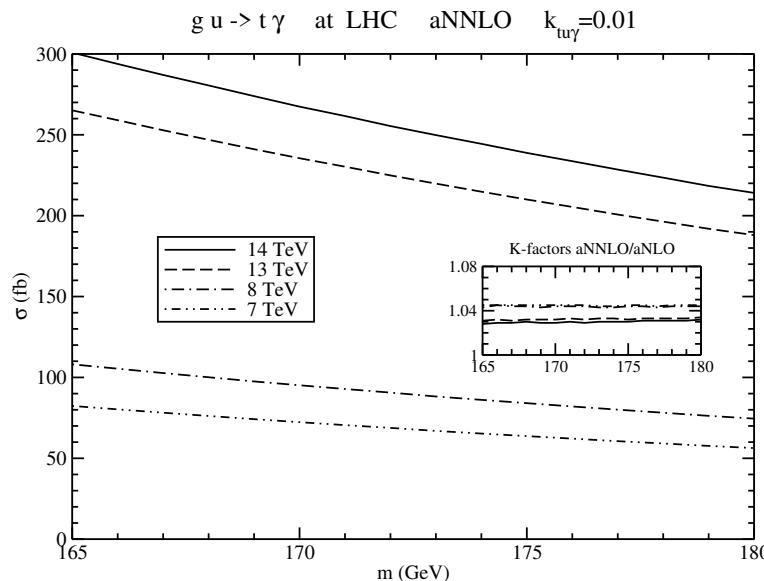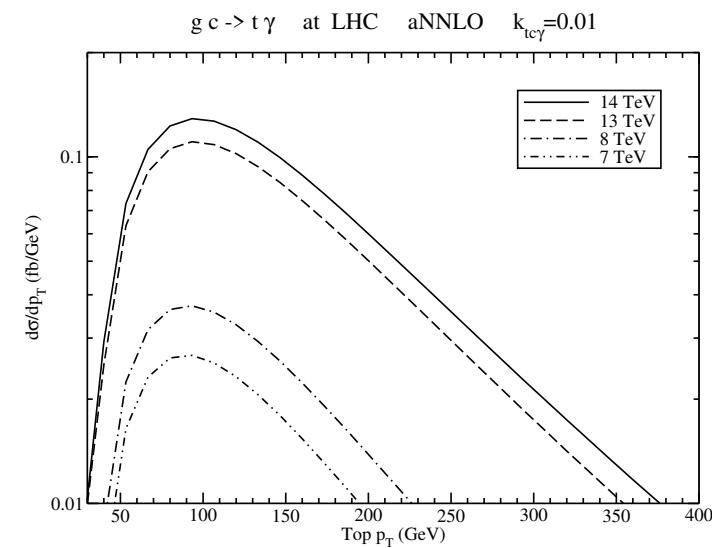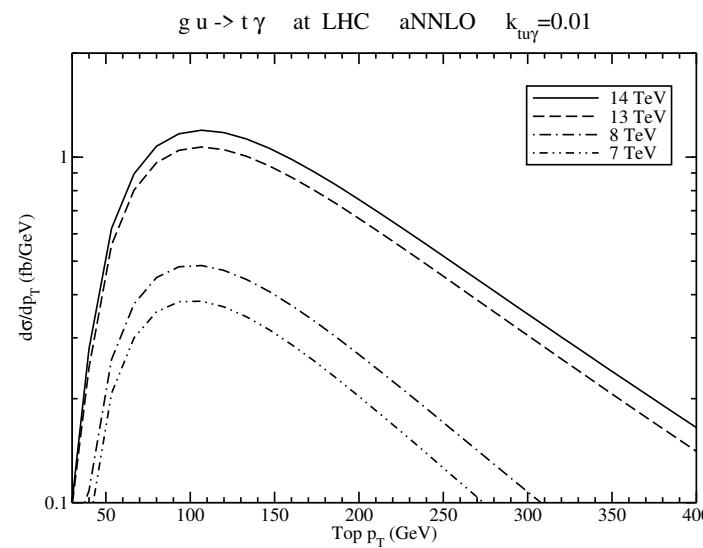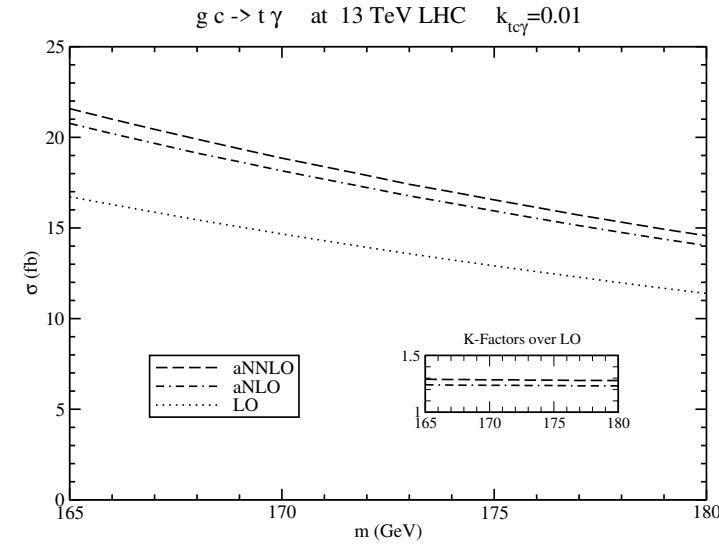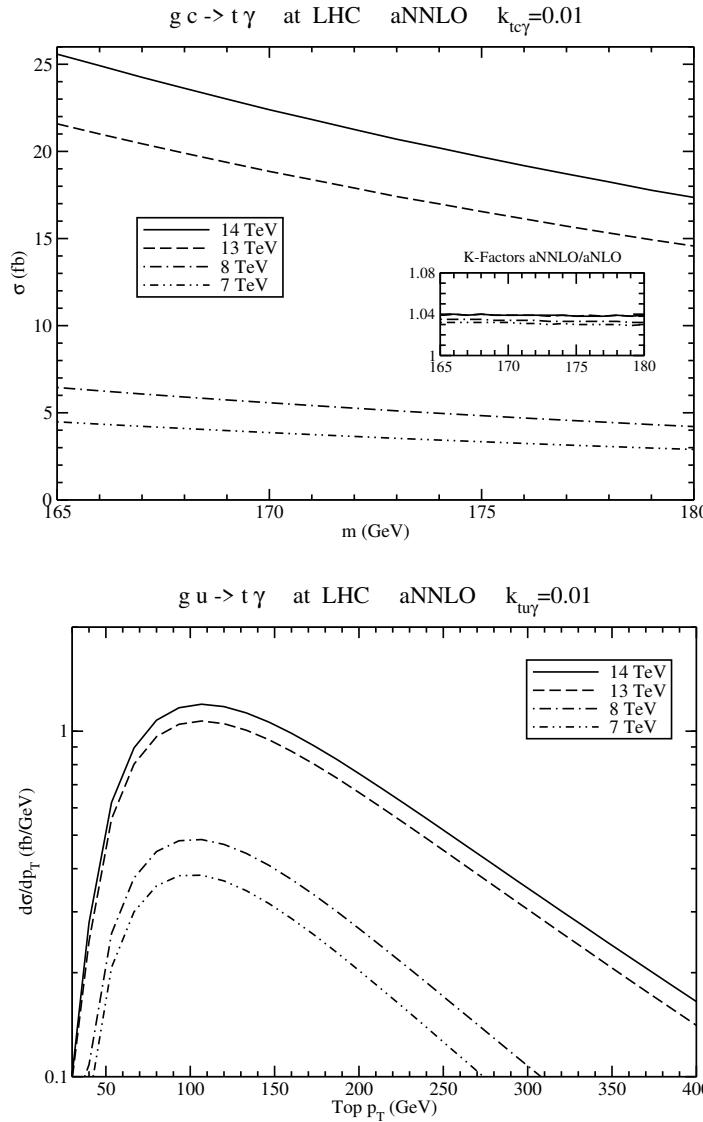
$t\gamma$ production via anomalous couplings
(with Matthew Forslund)

$$\Delta\mathcal{L}^{eff} = \frac{1}{\Lambda}\,\kappa_{tq\gamma}\,e\,\bar{t}\,(i/2)(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)\,q\,F_\gamma^{\mu\nu} + h.c.$$

$t\gamma$ production via anomalous couplings

g c -> t γ at LHC   aNNLO $k_{tc\gamma}$=0.01

g c -> t γ at 13 TeV LHC $k_{tc\gamma}$=0.01

g u -> t γ at LHC   aNNLO $k_{tu\gamma}$=0.01

g c -> t γ at LHC   aNNLO $k_{tc\gamma}$=0.01
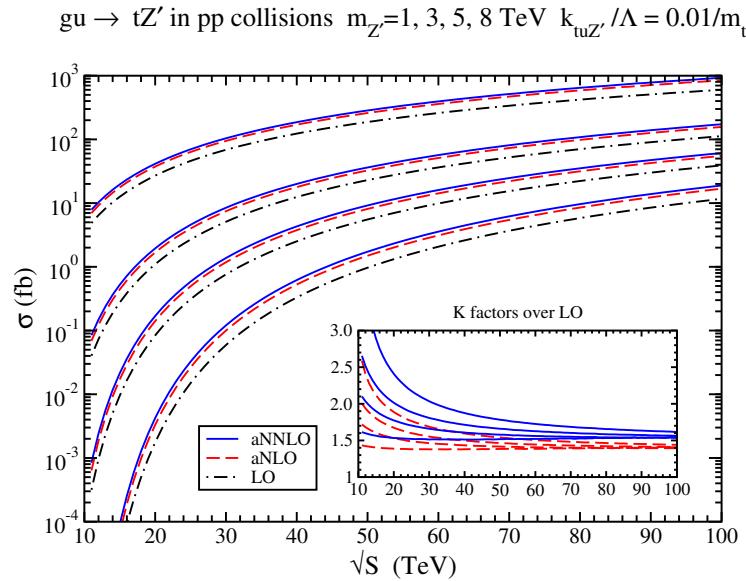
tZ' **production**

## (with Marco Guzzi)
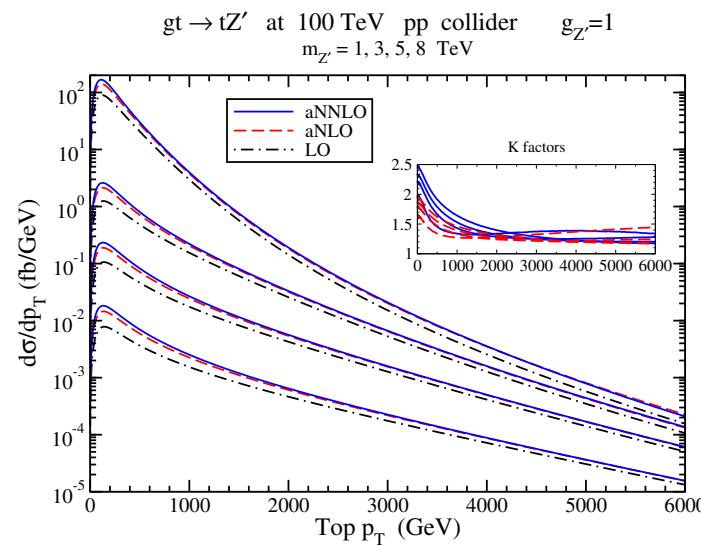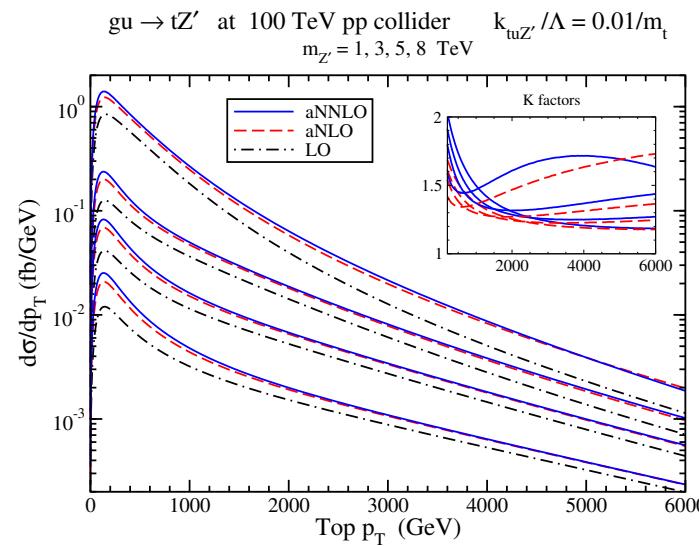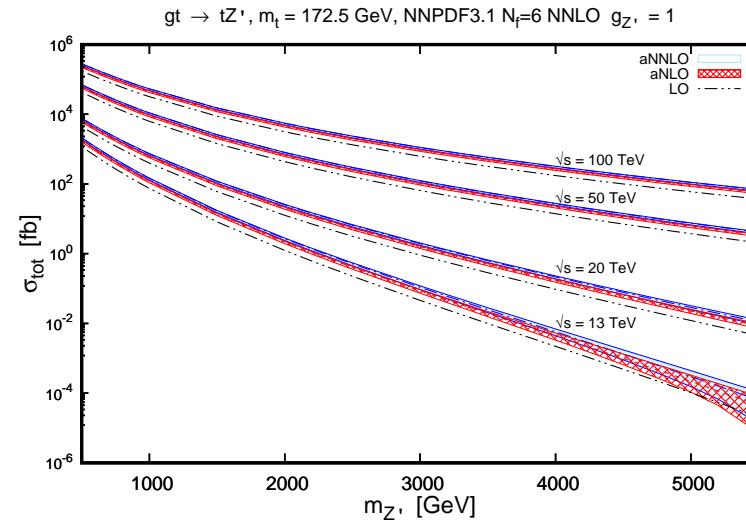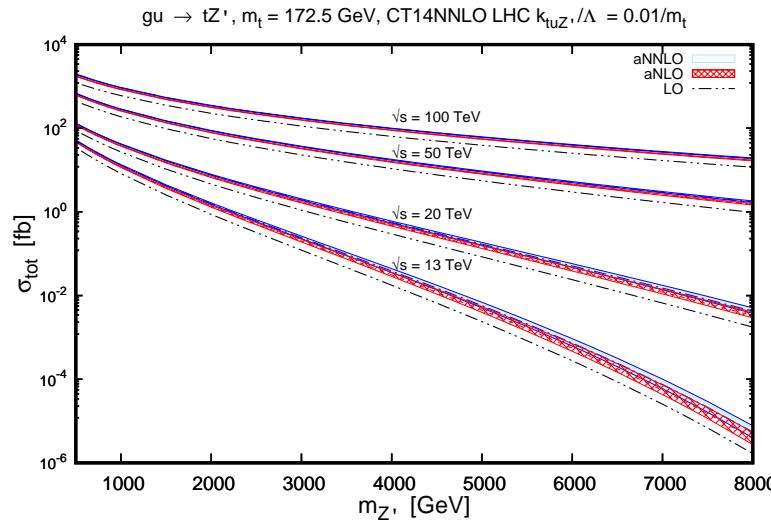


via anomalous couplings

with initial-state top

gu → tZ′ in pp collisions $m_{Z'}$=1, 3, 5, 8 TeV $k_{tuZ'}/\Lambda = 0.01/m_t$

gt → tZ′ in pp collisions $m_{Z'}$=1, 3, 5, 8 TeV $g_{Z'}$=1

tZ' production

gu → tZ', m_t = 172.5 GeV, CT14NNLO LHC $k_{tuZ'}/\Lambda$ = 0.01/m_t

gt → tZ', m_t = 172.5 GeV, NNPDF3.1 N_f=6 NNLO $g_{Z'}$ = 1

gu → tZ′ at 100 TeV pp collider $k_{tuZ'}/\Lambda$ = 0.01/m_t
$m_{Z'}$ = 1, 3, 5, 8 TeV

gt → tZ′ at 100 TeV pp collider $g_{Z'}$=1
$m_{Z'}$ = 1, 3, 5, 8 TeV
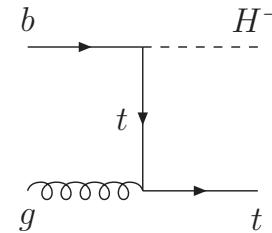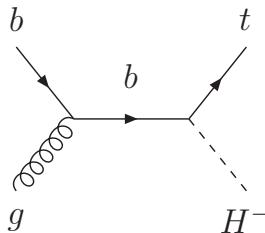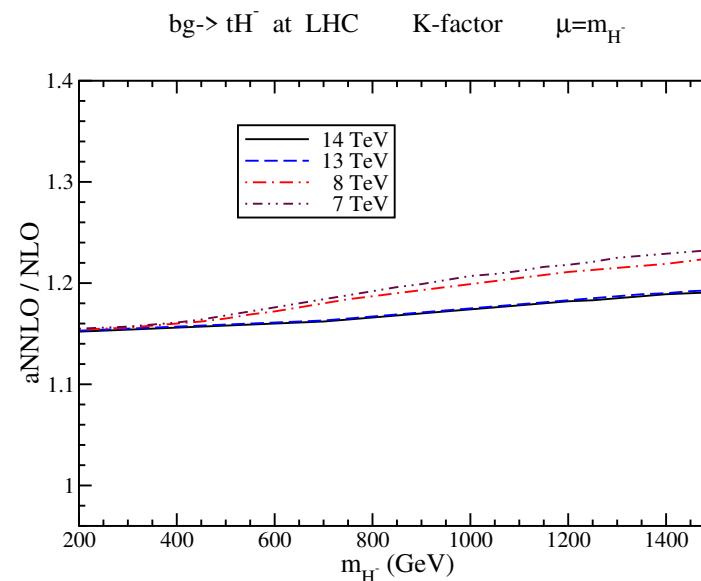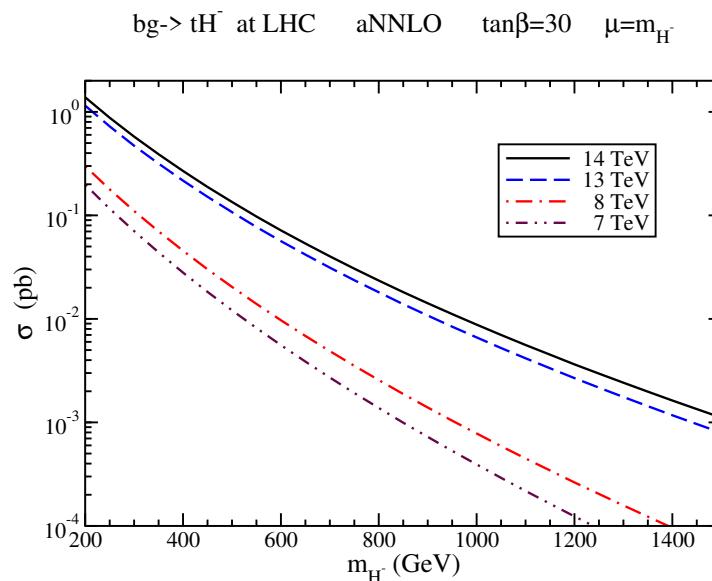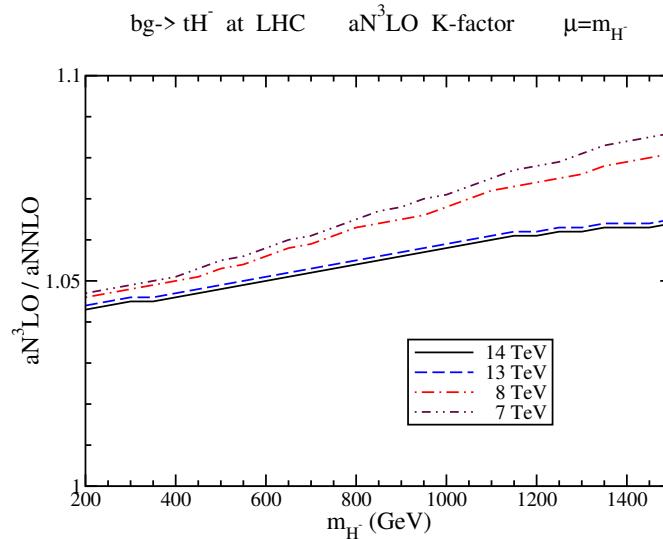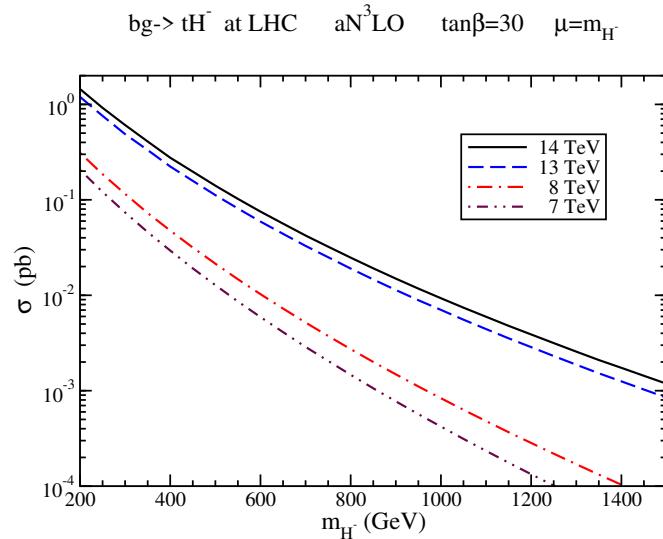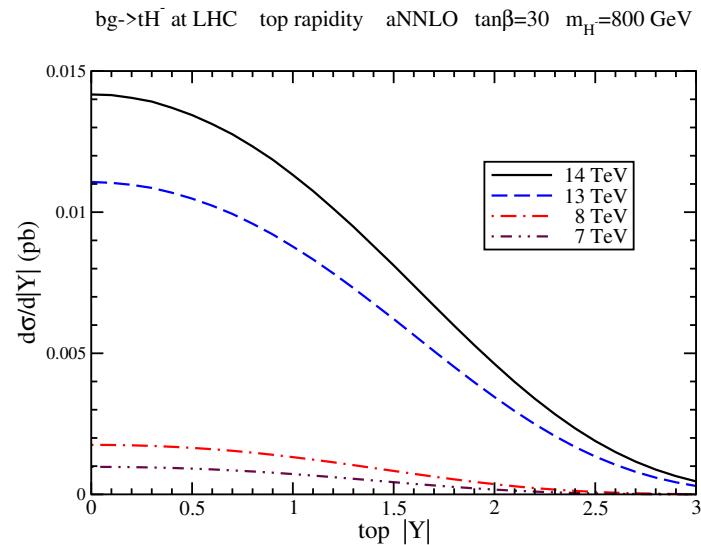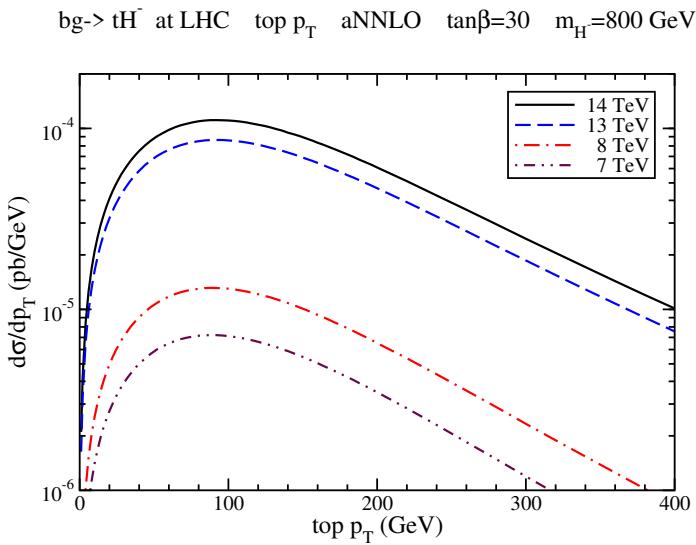
tH^- **production**

Leading-order cross section for $bg \to tH^- \propto \alpha\alpha_s(m_b^2 \tan^2\beta + m_t^2 \cot^2\beta)$

$\tan\beta = v_2/v_1$ **ratio of vevs of two Higgs doublets**

tH^- production

bg-> tH$^-$ at LHC   top p$_T$ aNNLO   tanβ=30   m$_{H^-}$=800 GeV

bg->tH$^-$ at LHC   top rapidity   aNNLO   tanβ=30   m$_{H^-}$=800 GeV

bg-> tH$^-$ at LHC   aN^{3}LO   tanβ=30 μ=m$_{H^-}$

bg-> tH$^-$ at LHC   aN^{3}LO K-factor μ=m$_{H^-}$

Summary

- **cross sections and distributions for top production**

- **soft-gluon corrections**

- $t\bar{t}$ **production at aN³LO**

- t**-channel and** s**-channel single top at aNNLO**

- tW **production at aN³LO**

- **excellent agreement with collider data**

- tZ **and** $t\gamma$ **production via anomalous couplings**

- tZ' **production in various models**

- tH^- **at aNNLO and beyond**

- **high-order corrections are very significant**