Associated Production of a Top Pair and a Heavy Colorless Boson at the LHC at NLO+NNLL

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Top Quark Physics at the
Precision Frontier
Fermilab
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Outline

- Focus on the associated production of a top-pair and a Higgs
- Factorization of the partonic cross section in the partonic threshold limit and resummation
- Preliminary NLO QCD + NNLL + NLO EW results for

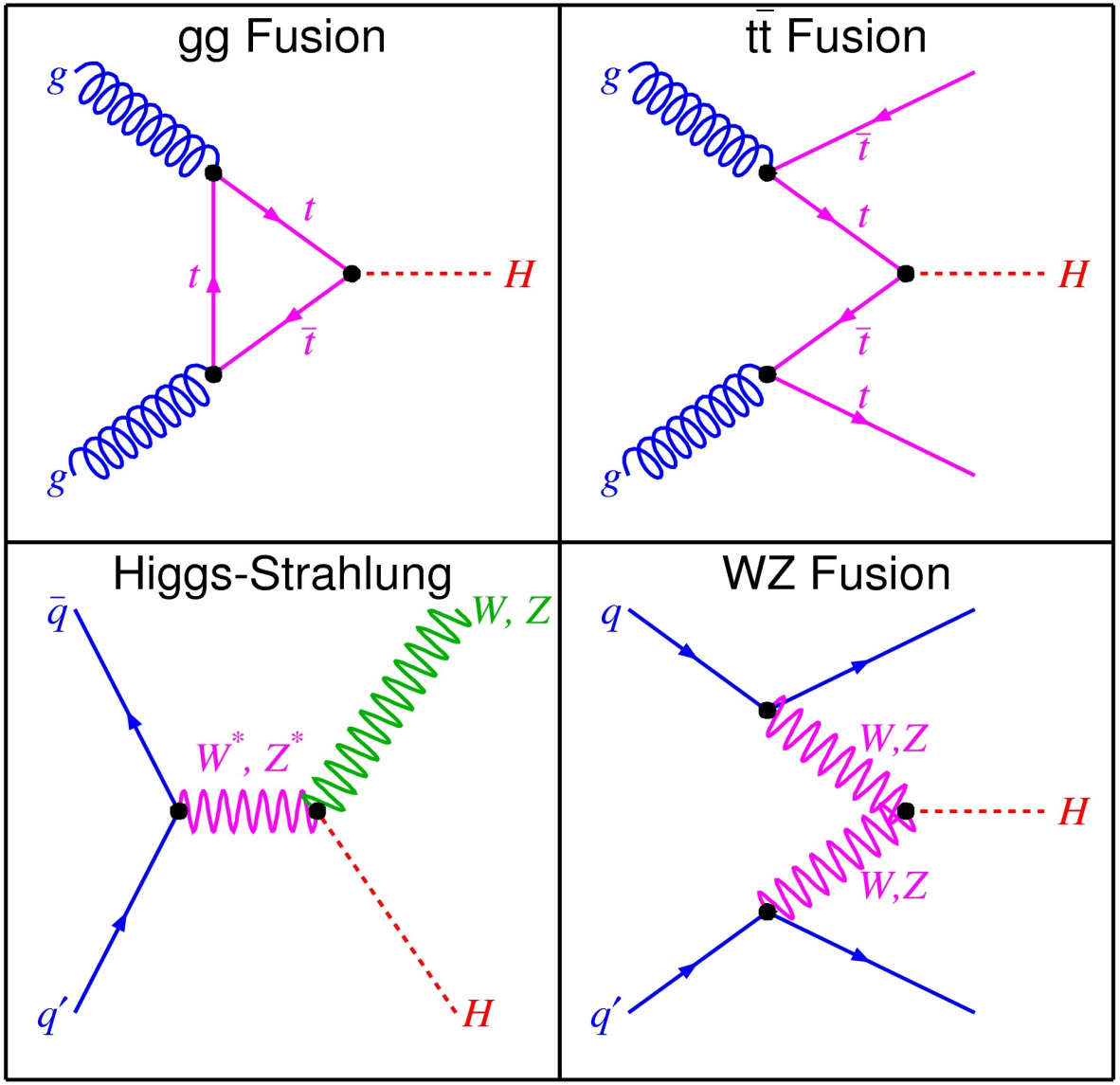
$$p\,p \longrightarrow t\,\bar{t}\,H$$

(results for ttW and ttZ production are also available)

Higgs boson production channels

$\sigma \sim 49\,\mathrm{pb}$

$\sigma \sim 0.6\,\mathrm{pb}$



$\sigma_W \sim 1.5\,\mathrm{pb}$

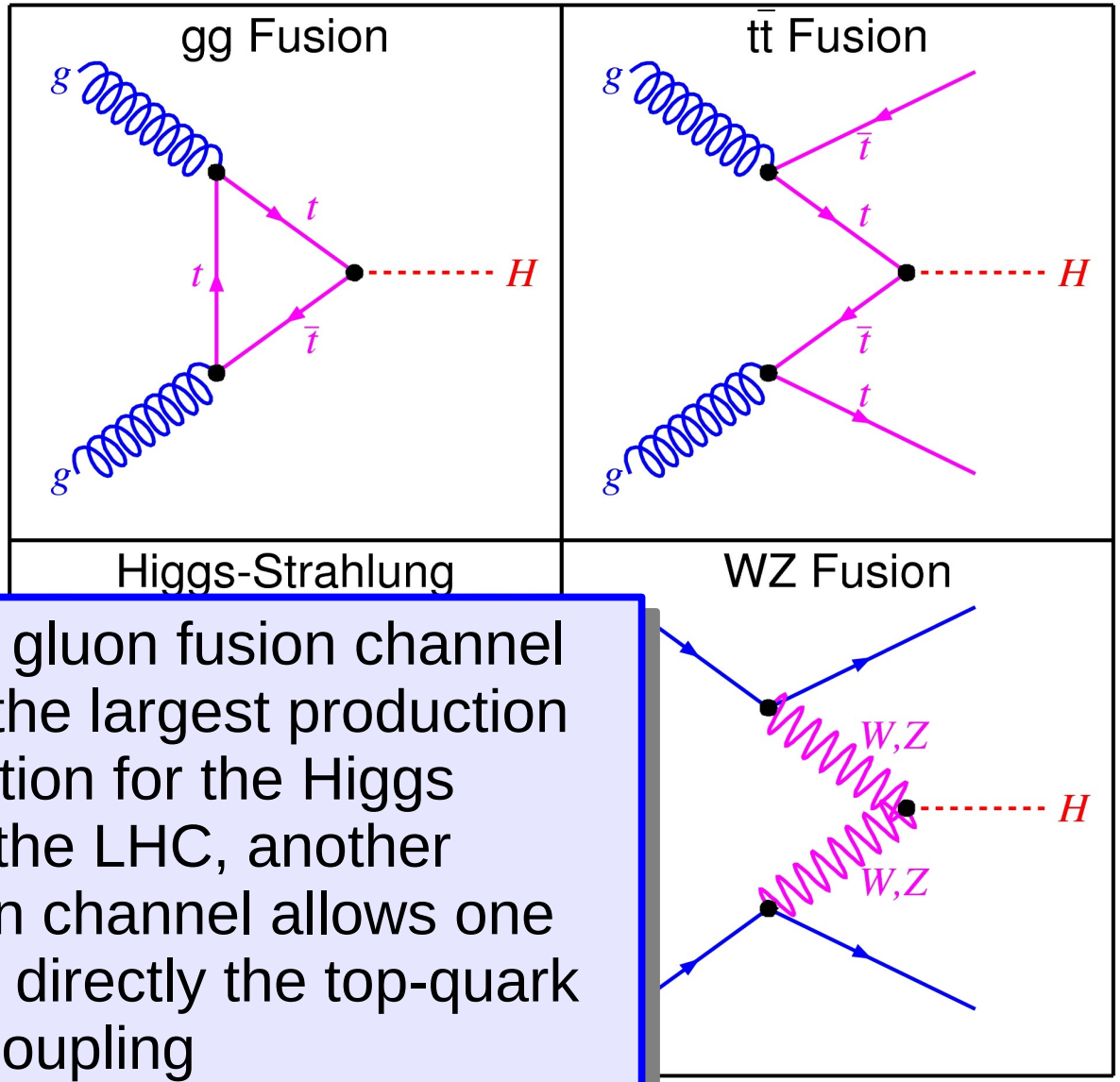
$\sigma_Z \sim 0.97\,\mathrm{pb}$

$\sigma \sim 4.2\,\mathrm{pb}$

LHC @ 14 TeV

Higgs boson production channels

$\sigma \sim 49\,\mathrm{pb}$



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$\sigma \sim 4.2\,\mathrm{pb}$

σ_W

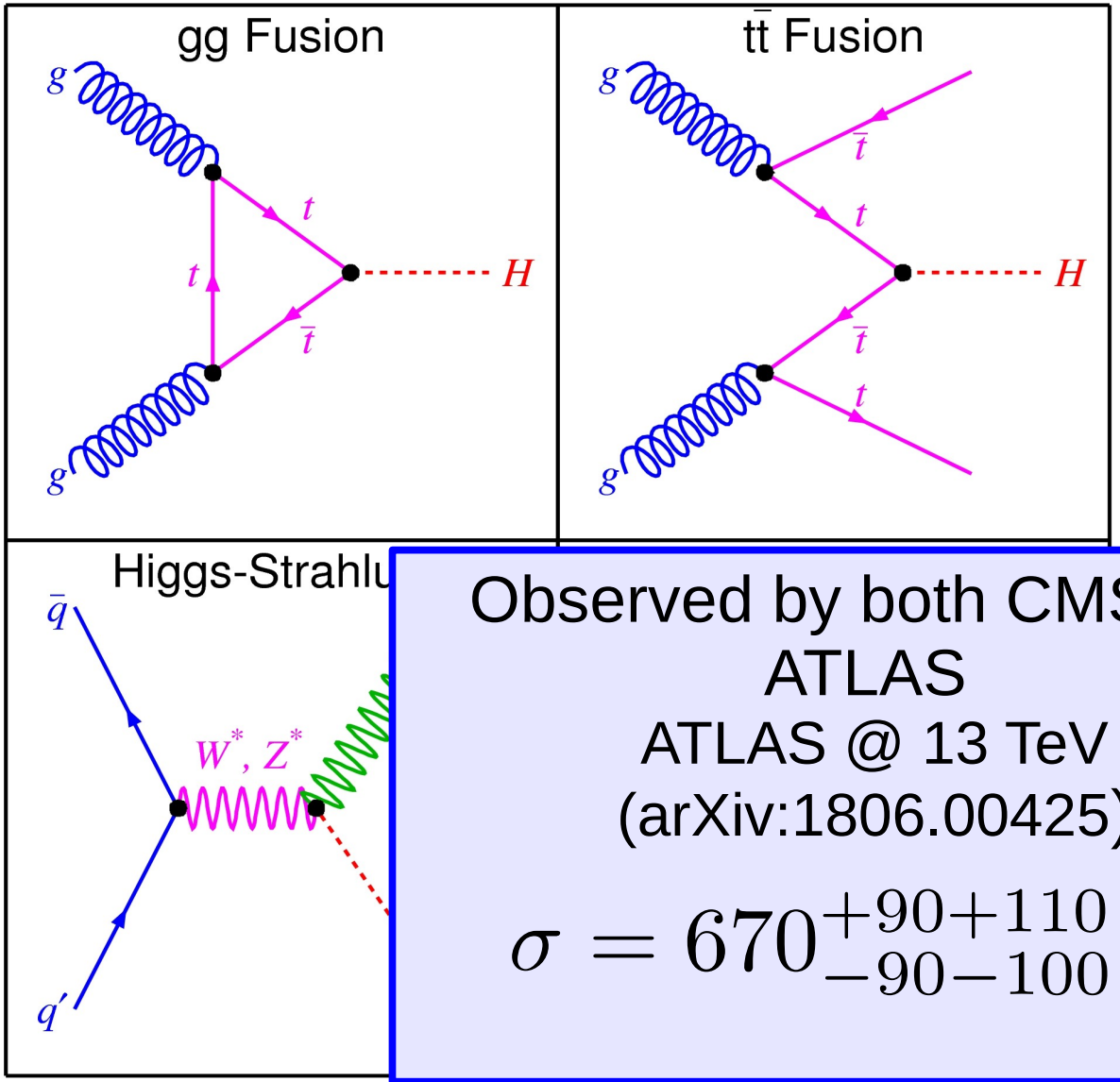
σ_Z

While the gluon fusion channel provides the largest production cross section for the Higgs boson at the LHC, another production channel allows one to access directly the top-quark Yukawa coupling

LHC @ 14 TeV

Higgs boson production channels

$\sigma \sim 49\,\mathrm{pb}$



$\sigma \sim 0.6\,\mathrm{pb}$

$\sigma_W \sim 1.5\,\mathrm{pb}$

$\sigma_Z \sim 0.97\,\mathrm{pb}$

2 pb

Observed by both CMS and ATLAS

ATLAS @ 13 TeV

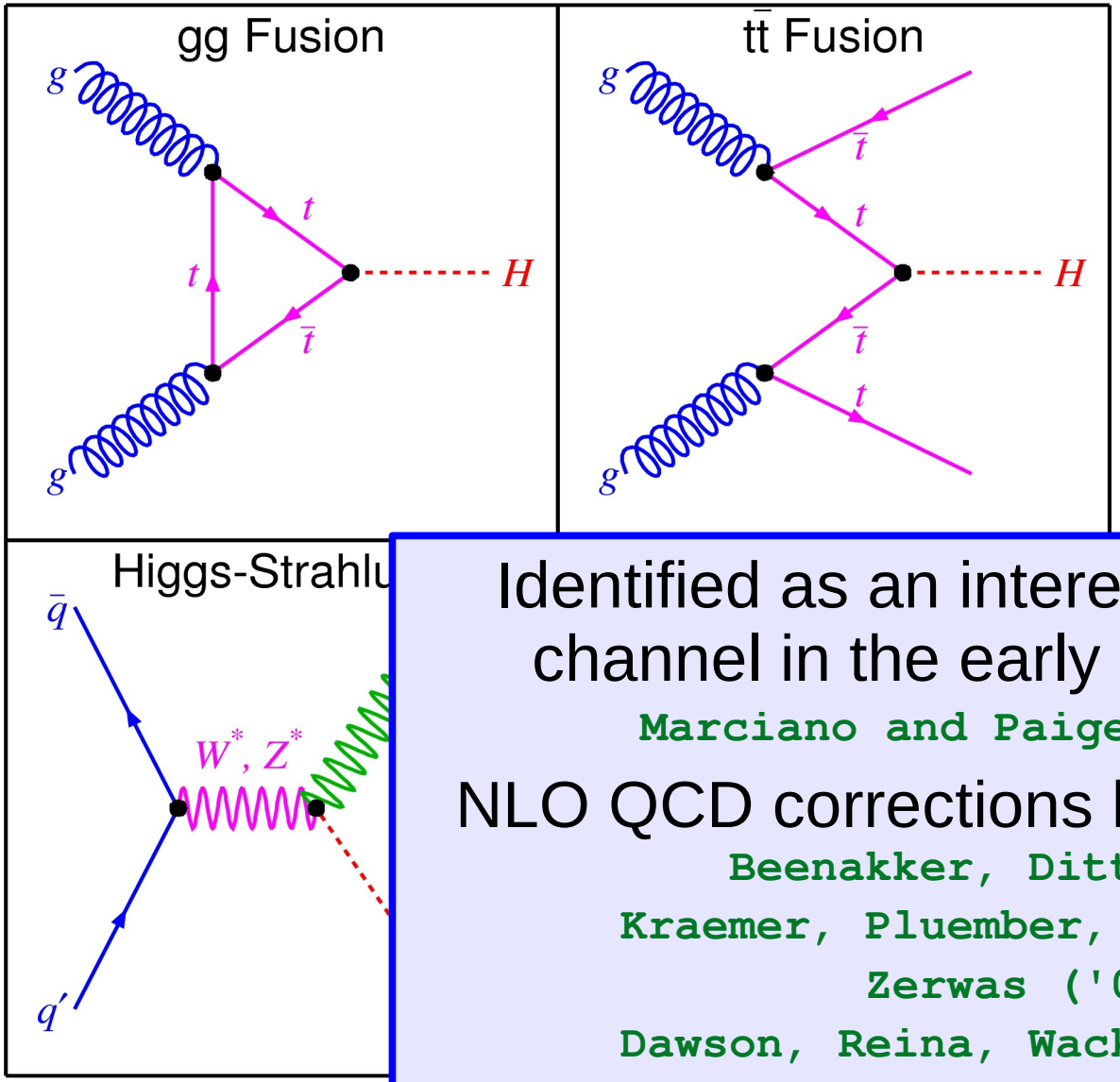
(arXiv:1806.00425)

$$\sigma = 670^{+90+110}_{-90-100}\ \mathrm{fb}$$

LHC @ 14 TeV

Higgs boson production channels

$\sigma \sim 49\,\mathrm{pb}$



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2 pb

LH

Identified as an interesting channel in the early 90s

Marciano and Paige (`91)

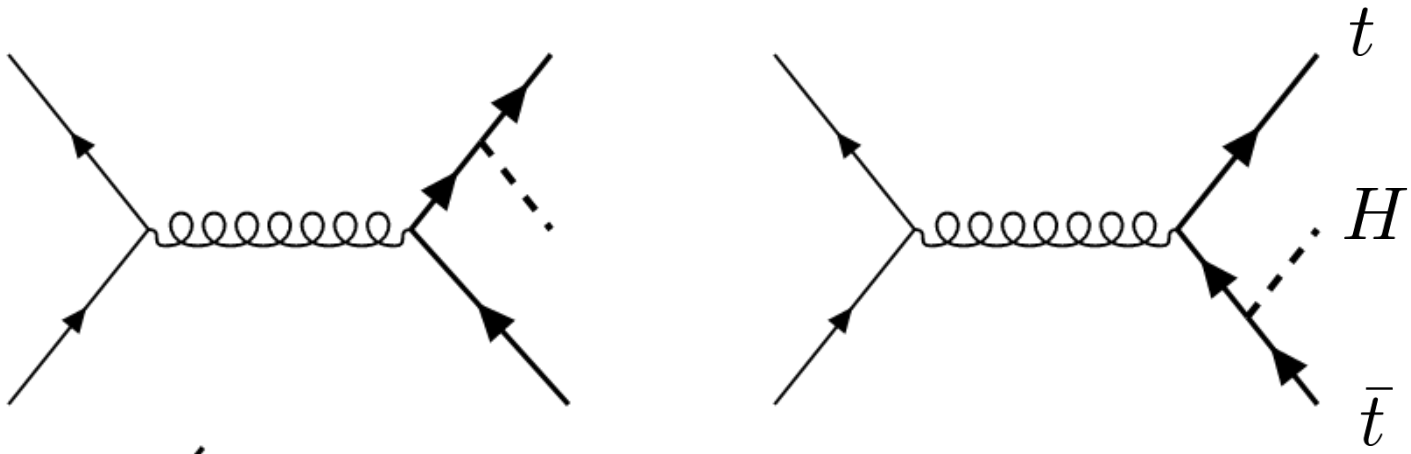
NLO QCD corrections known

Beenakker, Dittmaier, Kraemer, Pluember, Spira, Zerwas ('01–'02)

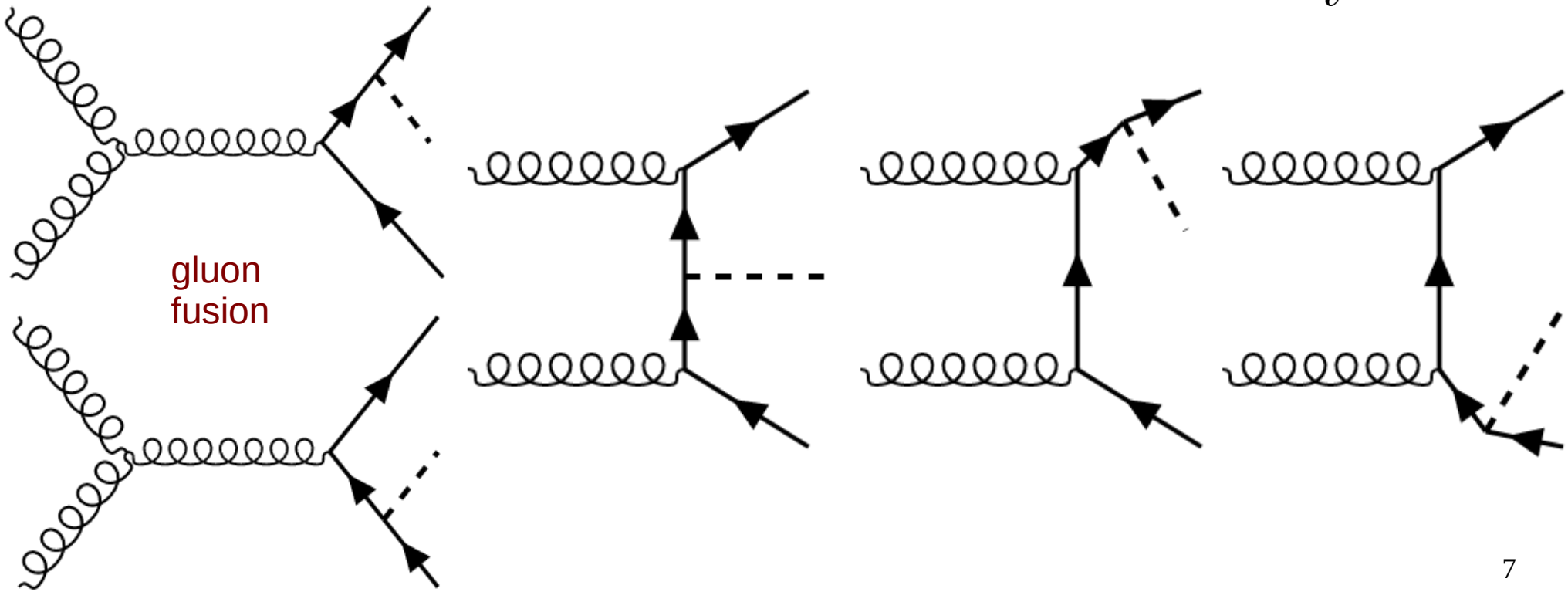
Dawson, Reina, Wackeroth, Orr,Jackson ('01,'03)

Top pair + H - tree level diagrams

quark
annihilation



gluon
fusion



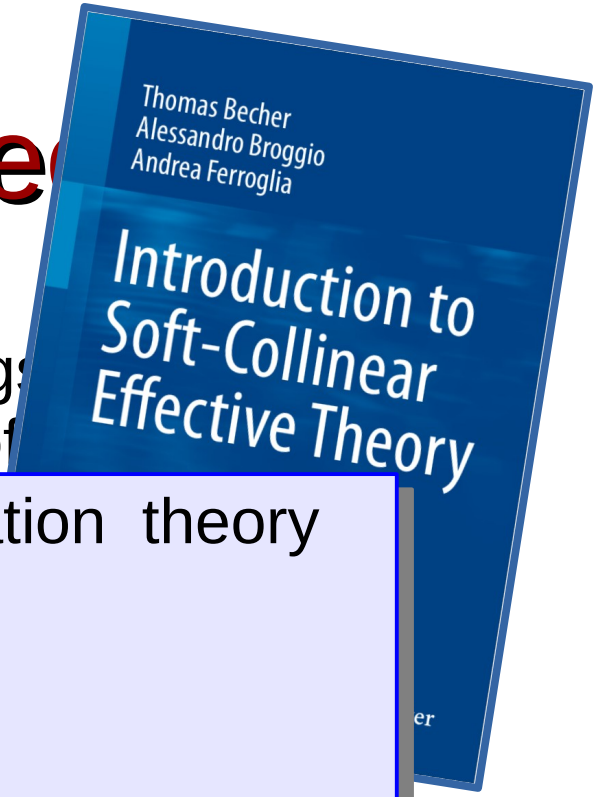
Large logarithmic corrections

- The partonic cross section for top pair (+Higgs,W or Z) production receives potentially large corrections from soft gluon emission diagrams
- Schematically, the partonic cross section depends on logarithms of the ratio of two different scales:

$$L \equiv \ln\left(\frac{\text{“hard” scale}}{\text{“soft” scale}}\right)$$

- It can be that $\alpha_s L \sim 1$
- One needs to reorganize the perturbative series: Resummation
- The resummation of soft emission corrections can be carried out by means of effective field theory methods

Large logarithmic corre



- The partonic cross section for top pair (+Higgs
  receives potentially large corrections from sof
  dia
- So
  log

Renormalization group improved perturbation theory schematically:

➔ Separation of scales ↔ factorization

➔ Evaluate each (single-scale) factor in fixed order perturbation theory at a scale for which it is free of large logs

➔ Use Renormalization Group Equations to evolve the factors to a common scale

- It c
- Or tion
- Th d
  out by means of effective field theory methods

Resummation

From a lecture by E. Laenen

("Direct QCD" approach)

Resummation = (re-)arrangement of large logarithms in perturbative expansion

$$\hat{O} = \overbrace{\overbrace{1 + \alpha_s(L^2 + L + 1)}^{\text{NLO}} + \alpha_s^2(L^4 + L^3 + L^2 + L + 1)}^{\text{NNLO}} + \mathcal{O}(\alpha_s^3)$$

$$= \exp\Big(\underbrace{\underbrace{Lg_1(\alpha_s L)}_{\text{LL}} + g_2(\alpha_s L)}_{\text{NLL}} + \alpha_s g_3(\alpha_s L) + \cdots\Big)\ \underbrace{C(\alpha_s)}_{\text{constants}}$$

$$+ \text{suppressed terms}$$

$$Lg_1 \longrightarrow \alpha_s^n L^{n+1}\,, \quad g_2 \longrightarrow \alpha_s^n L^n\,, \quad \alpha_s g_3 \longrightarrow \alpha_s^{n+1} L^n$$

Resummation reduces the theoretical uncertainty on a given observable

Goal

We want to analyze the factorization properties of

$$p + p \longrightarrow t + \bar{t} + H(\text{or } W, Z) + X$$

in the soft emission limit in order to

i. Obtain NNLL resummation formulas for these processes

ii. Evaluate the total cross section and differential distributions depending on the 4-momenta of the final state particles

iii. Match NLO and NNLL calculations to obtain NLO+NNLL predictions

- Associated production of top pair and Higgs boson: **A. Broggio,AF, B.D. Pecjak, A. Signer, L.L. Yang**

  JHEP 1603 (2016) 124 [arXiv:1510.01914] JHEP 1702 (2017) 127 [arXiv:1611.00049]

- Associated production of a top pair and W or Z boson **A. Broggio, AF, B.D. Pecjak, G. Ossola, R.D. Sameshima**

  JHEP 1609 (2016) 089 [arXiv:1607.05303] JHEP 1704 (2017) 105 [arXiv:1702.00800]

Soft limit & factorization

(for top pair + H production)

“Pair” Invariant Mass kinematics

- For top pair + Higgs production, we have two tree-level partonic processes

$$q(p_1) + \bar{q}(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$
$$g(p_1) + g(p_2) \to t(p_3) + \bar{t}(p_4) + H(p_5)$$

- Define the invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4 + p_5)^2$$

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Partonic center of mass energy (squared)

Invariant mass of the heavy particles in the final state

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- Define the invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = (p_3 + p_4 + p_5)^2$$

If real radiation in the final state is present, $\hat{s} \neq M^2$

$$z = \frac{M^2}{\hat{s}}$$

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- Define the invariants

$$\hat{s} = (p_1 + p_2)^2 \qquad M^2 = ($$

Partonic threshold limit

$$z \longrightarrow 1$$

No constraint is imposed on the velocity of the massive particles

If real radiation in the final state is

$$z = \frac{M^2}{\hat{s}}$$

Factorization in a nutshell

Differential cross section:

$$\frac{d\sigma}{dM^2} = \mathit{ff}(z) \otimes C(z)$$

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Partonic luminosity

Hard scattering kernel
(partonic cross section)

Factorization in a nutshell

Differential cross section:

$$\frac{d\sigma}{dM^2} = \mathit{ff}(z) \otimes C(z)$$

In the soft emission limit a clear scale hierarchy emerges:

$$\hat{s}, M^2, m_t^2 \gg \hat{s}(1-z)^2 \gg \Lambda_{\mathrm{QCD}}^2$$

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Hard scales

Soft scale

Factorization in a nutshell

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In this limit, the partonic cross section factors into two parts:

$$C_{ij} = \mathrm{Tr}\left[\mathbf{H}_{ij}\left(M, \{p_i\}, \mu\right) \mathbf{S}_{ij}\left(\sqrt{\hat{s}}(1-z), \{p_i\}, \mu\right)\right]$$

Channels involving qg initial state partons are subleading in the threshold limit

Factorization in a nutshell

Differential cross section:

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Hard function
(virtual corrections)

Soft function
(real soft emission)

Mellin space

- The resummation can be carried out in Mellin space (by taking the Mellin transform of the factorized cross section), similar to “direct QCD” resummation

$$\tilde{c}(N,\mu) = \int_0^1 dz z^{N-1} \int d\mathrm{PS}_{t\bar{t}H} \mathrm{Tr}\left[\mathbf{H}\left(\{p\},\mu\right)\mathbf{S}\left(\sqrt{\hat{s}}(1-z),\{p\},\mu\right)\right]$$

- The soft limit $z \to 1$ corresponds to the limit $N \to \infty$ in moment space

- The total cross section can be then recovered with an inverse Mellin transform

$$\sigma = \frac{1}{2s}\int_{\tau_{\min}}^1 \frac{d\tau}{\tau}\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN \tau^{-N} \widetilde{ff}(N,\mu) \int d\mathrm{PS}_{t\bar{t}H}\, \widetilde{c}\,(N,\mu)$$

+ Minimal Prescription (Catani et al, ‘96) + Mellin space lum. (Bonvini and Marzani ‘12, ‘14)

Renormalization group equations

- The hard and soft functions satisfy RGEs regulated by known anomalous dimensions
- By solving the RGEs one can resum large corrections depending on the ratio of hard and soft scales
- After taking care of several technicalities (Mellin space etc):

$$\sigma(s, m_t, m_H) = \frac{1}{2s} \int_{\tau_{\min}}^{1} \frac{d\tau}{\tau} \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dN \tau^{-N} \sum_{ij} \widetilde{f\!\!f}_{ij}(N, \mu) \int d\mathrm{PS}_{t\bar{t}H}\, \widetilde{c}_{ij}(N, \mu)$$

$$\widetilde{c}_{ij}(N, \mu_f) = \mathrm{Tr}\Bigg[\widetilde{\mathbf{U}}_{ij}(\bar{N}, \{p\}, \mu_f, \mu_h, \mu_s)\, \mathbf{H}_{ij}(\{p\}, \mu_h)\, \widetilde{\mathbf{U}}^{\dagger}_{ij}(\bar{N}, \{p\}, \mu_f, \mu_h, \mu_s) \times \widetilde{\mathbf{s}}_{ij}\left(\ln \frac{M^2}{\bar{N}^2 \mu_s^2}, \{p\}, \mu_s \right) \Bigg].$$

Renormalization group equations

- The hard and soft functions satisfy RGEs regulated by known anomalous dimensions
- By solving the RGEs one can resum large corrections depending on the ratio of hard and soft scales
- Afte echnicalities (Mellin space, etc.):

$$\sigma(s, m_t, m_H \quad dN\tau^{-N} \sum_{ij} \widetilde{f}_{ij} ($$

Evolution factor: Depends on all scales, resums large logs

Hard function: Free from large logs at $\mu_h \sim M$

$$\widetilde{c}_{ij}(N, \mu_f) = \mathrm{Tr}\left[\widetilde{\mathbf{U}}_{ij}(\bar{N}, \{p\}, \mu_f, \mu_h, \mu_s)\, \mathbf{H}_{ij}(\{p\}, \mu_h)\, \widetilde{\mathbf{U}}^{\dagger}_{ij}(\bar{N}, \{p\}, \mu_f, \mu_h, \mu_s) \times \widetilde{\mathbf{s}}_{ij}\left(\ln \frac{M^2}{\bar{N}^2 \mu_s^2}, \{p\}, \mu_s\right)\right]$$

Soft function: Free from large logs at $\mu_s \sim M/\bar{N}$

Complete NLO calculations

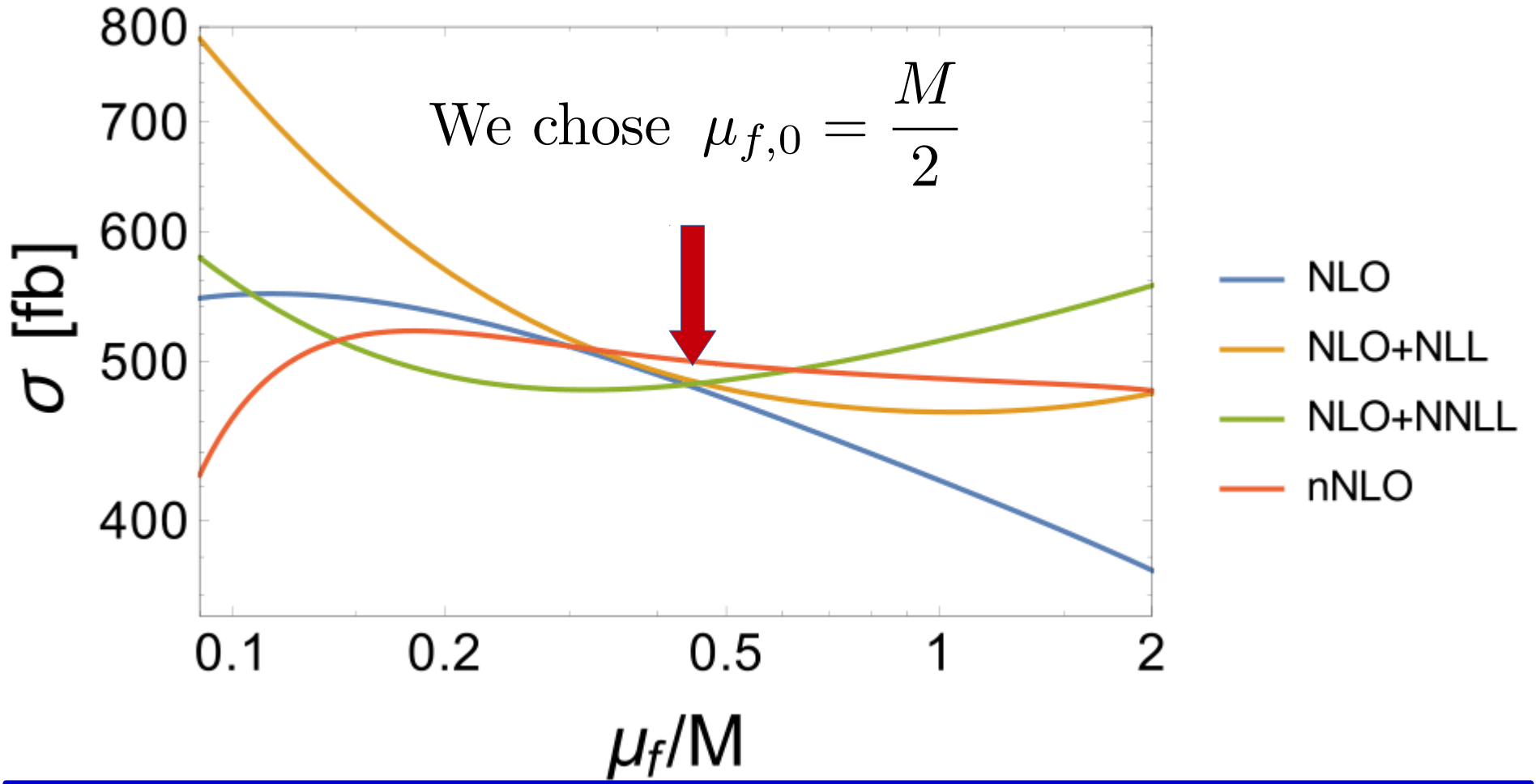
We need complete NLO results for the total cross section and the differential distributions we are interested in, both to validate the approximate formulas and to match results to the full NLO:

MadGraph5_aMC@NLO

Precise theoretical predictions are obtained by combining NNLL resummation and NLO calculation. The matching procedure allows one to avoid the double counting of terms included in both approaches

$$d\sigma^{\mathrm{NLO+NNLL}} = d\sigma^{\mathrm{NNLL}}\Big|_{\mu_h,\mu_s,\mu_f} + \left(d\sigma^{\mathrm{NLO}} - d\sigma^{\mathrm{NNLL}}\Big|_{\mu_s=\mu_h=\mu_f}\right)$$

ttH production scale dependence



The factorization scale should be chosen such in such a way that logarithms of the ratio μ_f /M are not large. Since we are working in the partonic threshold limit it is natural to choose a dynamical value for the factorization scale which is correlated with M

Combining NLO+NNLL QCD with NLO EW

- **A. Broggio, AF, R. Frederix, D. Pagani, B. Pecjak, and I. Tsinikos**

Work in progress

NLO EW corrections

R. Frederix, S. Frixione, V. Hirschi D. Pagani,

H.-S. Shao, and M.Zaro (2015-2018)

NLO electroweak corrections to ttV can be phenomenologically important

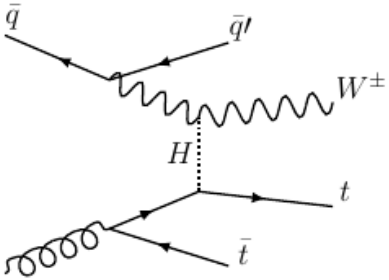
$$\Sigma_{\rm NLO}(\alpha_s,\alpha) = \alpha_s^3\alpha\Sigma_{4,0} + \alpha_s^2\alpha^2\Sigma_{4,1} + \alpha_s\alpha^3\Sigma_{4,2} + \alpha^4\Sigma_{4,3}$$

$$\equiv \Sigma_{\rm NLO_1} + \Sigma_{\rm NLO_2} + \Sigma_{\rm NLO_3} + \Sigma_{\rm NLO_4}$$

NLO QCD

Dominant NLO EW corrections for ttZ and ttH

Dominant EW correction in ttW



$m_t = 173.34$ GeV, $m_W = 80.385$ GeV, $m_Z = 91.1876$ GeV, $m_H = 125$ GeV $G_\mu = 1.16639 \cdot 10^{-5}$ GeV^{-2}

Total cross section - ttH

13 TeV

Scale choices based on M = m(ttH), (like in the older QCD only papers)

$t\bar{t}H$, PDFs=LUXQED17 (82200)	
$\mu_f^0 = \mu_r^0 = m(t\bar{t}H)/2, \mu_h^0 = m(t\bar{t}H), \mu_s^0 = \frac{m(t\bar{t}H)}{\bar{N}}$	
Order	σ [fb]
LO QCD	$327.65(4)^{+94.18(+28.7\%)}_{-68.46(-20.9\%)}{}^{+7.11(+2.2\%)}_{-7.11(-2.2\%)}$
NLO QCD	$463.70(8)^{+45.1(+9.7\%)}_{-49.72(-10.7\%)}{}^{+11.08(+2.4\%)}_{-11.08(-2.4\%)}$
(complete) NLO	$475.68(8)^{+46.94(+9.9\%)}_{-51.11(-10.7\%)}{}^{+11.21(+2.4\%)}_{-11.21(-2.4\%)}$
NLO QCD+NNLL	$479.1(1)^{+29.0(+6.1\%)}_{-24.2(-5.0\%)}{}^{+11.5(+2.4\%)}_{-11.5(-2.4\%)}$
NLO+NNLL	$491.1(1)^{+27.8(+5.7\%)}_{-24.0(-4.9\%)}{}^{+11.6(+2.4\%)}_{-11.6(-2.4\%)}$

NLO complete → QCD+EW

Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)

Total cross section - ttH

13 TeV

$t\bar{t}H$, PDFs=I	
$\mu_f^0 = \mu_r^0 = m(t\bar{t}H)/2$	
Order	
LO QCD	327
NLO QCD	463.
(complete) NLO	475.
NLO QCD+NNLL	$479.1(1)^{+29.0(+6.1\%)}_{-24.2(-5.0\%)}\ ^{+11.5(+2.4\%)}_{-11.5(-2.4\%)}$
NLO+NNLL	$491.1(1)^{+27.8(+5.7\%)}_{-24.0(-4.9\%)}\ ^{+11.6(+2.4\%)}_{-11.6(-2.4\%)}$

ATLAS (arXiv:1806.00425)

$$\sigma = 670^{+90+110}_{-90-100}\ \text{fb}$$

CMS (arXiv:1804.02610)

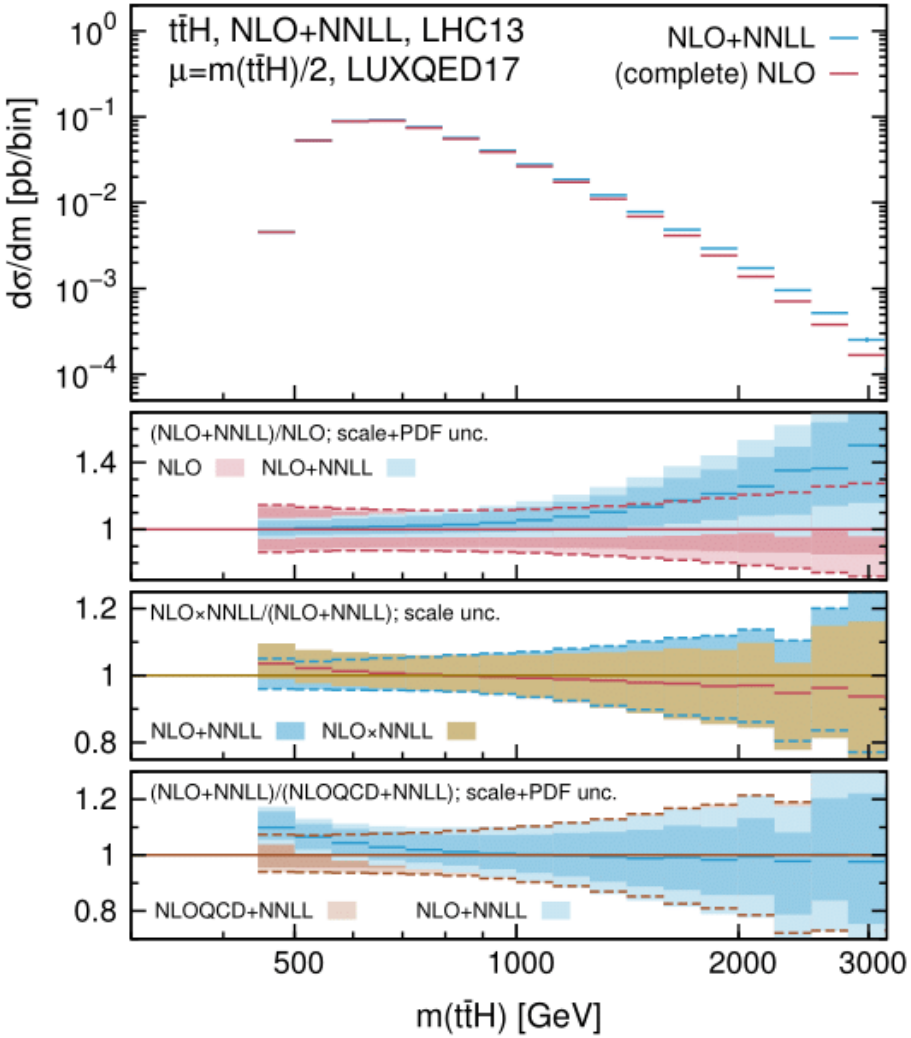
$$\frac{\sigma_{\text{exp}}}{\sigma_{\text{SM}}} = 1.26^{+0.31}_{-0.26}$$

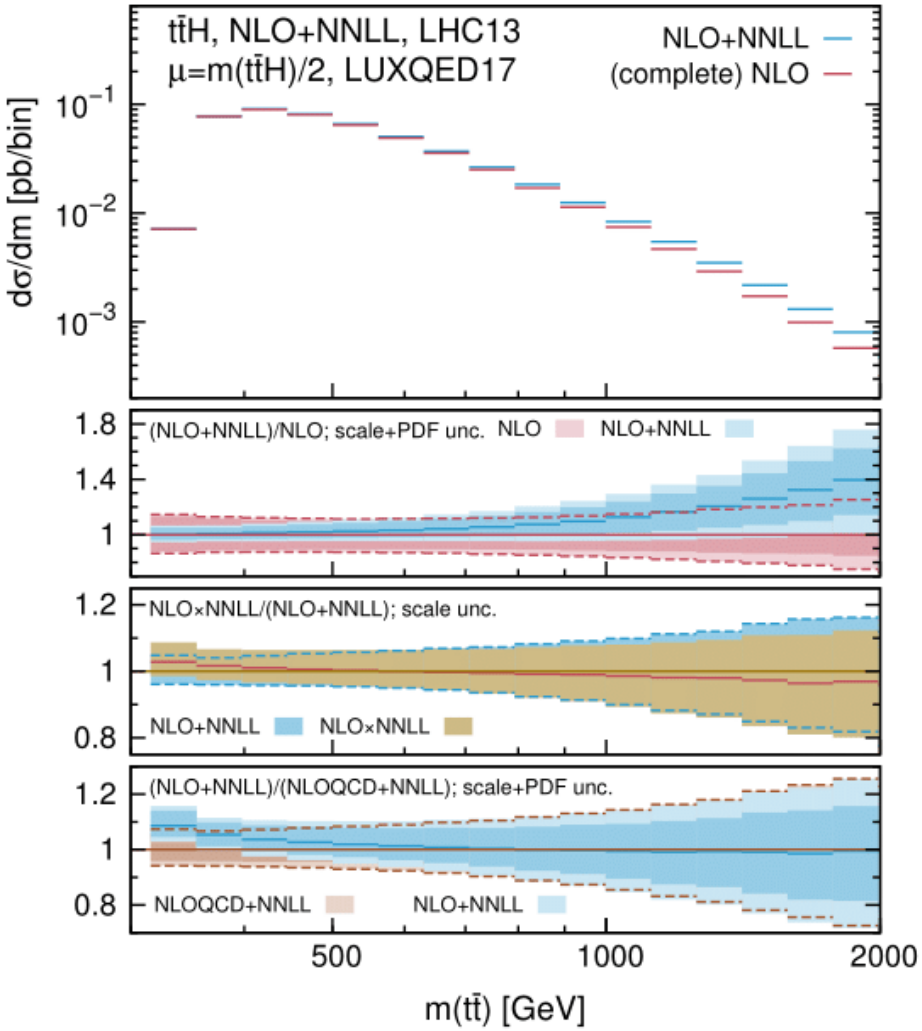
NLO complete → QCD+EW

Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)

Invariant mass distributions - ttH

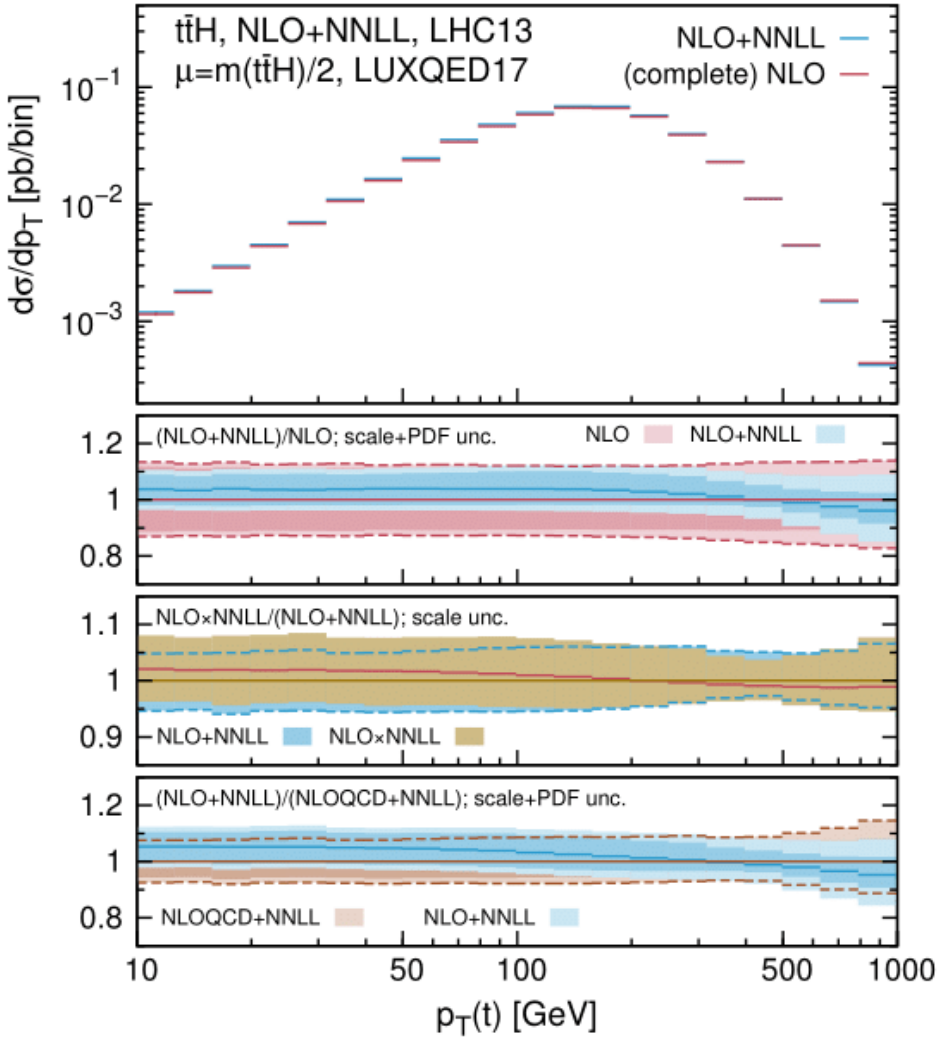
Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)

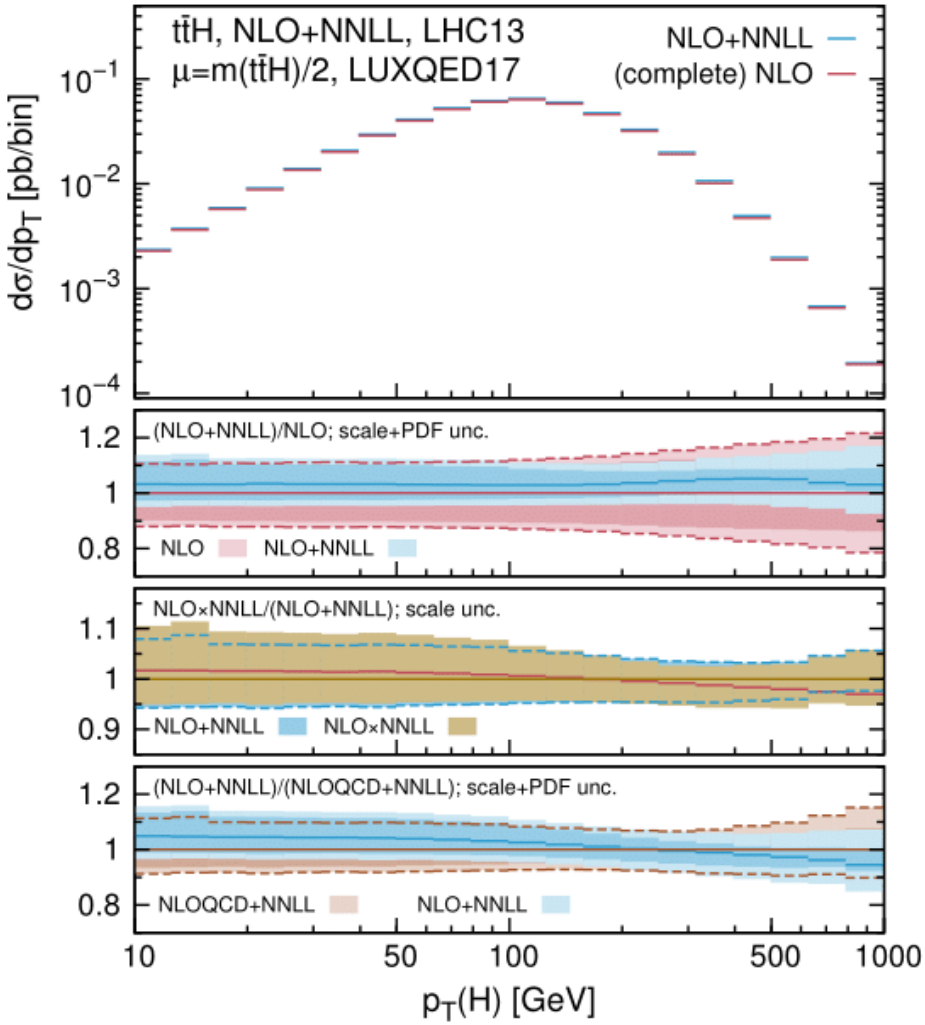




Transverse momentum distributions - ttH

Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)





Scale choices in top-pair production

The hard function include terms proportional to $\ln\left(\frac{\mu^2}{-t_1}\right)$

$$|t_1| = \frac{M_{t\bar{t}}}{\ldots}\left(1 - \ldots\sqrt{1 - \frac{4m_t^2}{\ldots}}\cos\theta\right)$$

The calculation of the top pair productions at fixed order in perturbation theory converges better with a factorization scale equal to H_T/4 than with a factorization scale set equal to M/2.

Also for resummation in “boosted” top pair production there are arguments to prefer a scale choice related to H_T rather than M

if $\mu \sim$ ... log!

One ca...

$$\mu \sim H_T \equiv \sqrt{p_{T,t}^2 + m_t^2} + \sqrt{p_{T,\bar{t}}^2 + m_t^2}$$

M. Czakon, D. Heymes, A.Mitov
arXiv:1606.03350
M. Czakon,AF, D. Heymes, A.Mitov, B.D. Pecjak, D.J. Scott, X. Wang, and L.L. Yang
arXiv:1803.07623

Scale choices in top-pair production

The hard function include terms proportional to $\ln\left(\frac{\mu^2}{-t_1}\right)$

$$|t_1| = \frac{M_{t\bar{t}}}{2}\left(1 - \sqrt{1 - \frac{4m_t^2}{M_{t\bar{t}}^2}}\cos\theta\right)$$

$$|t_1| \xrightarrow{\theta\to 0, m_t\to 0} p_T^2 + m_t^2 \sim m_t^2$$

$$\text{if} \quad \mu \sim M \longrightarrow \ln\left(\frac{\mu^2}{-t_1}\right) \xrightarrow{\theta\to 0, m_t\to 0} \ln\left(\frac{M^2}{m_t^2}\right) \quad \text{large log!}$$

One can choose instead

$$\mu \sim H_T \equiv \sqrt{p_{T,t}^2 + m_t^2} + \sqrt{p_{T,\bar{t}}^2 + m_t^2}$$

So that $\ln\left(\frac{\mu^2}{-t_1}\right)$ remains small always

H_T based scale choices

An alternative approach for ttH production (and in general ttV production), based on the work on top pair production, consists on parameterizing the scale choices in terms of H_T rather than on the invariant mass of the three massive objects in the final state

$$H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2} + \sqrt{m_H^2 + p_{T,H}^2}$$

$$\mu_f^0 = \frac{H_T}{2}\,, \qquad \mu_h^0 = \frac{H_T}{2}\,, \qquad \mu_s^0 = \frac{H_T}{\bar{N}}$$

A conservative (and safe) approach consists in taking the scale uncertainty as the envelope of the scale uncertainties obtained by varying the M and H_T based scales

ttH total cross section: H_T vs M combined

	$m(t\bar{t}H)/2$-based scales		σ [fb]
NLO+NNLL	491.1(1)	$^{+27.8(+5.7\%)}_{-24.0(-4.9\%)}$	$^{+11.6(+2.4\%)}_{-11.6(-2.4\%)}$
	$H_T/2$-based scales		
NLO+NNLL	501.67(9)	$^{+33.34(+6.6\%)}_{-22.54(-4.5\%)}$	$^{+12.05(+2.4\%)}_{-12.05(-2.4\%)}$
	Combined scales		
NLO+NNLL	496.36(7)	$^{+38.64(+7.8\%)}_{-29.35(-5.9\%)}$	$^{+11.92(+2.4\%)}_{-11.92(-2.4\%)}$

ttH total cross section: H_T vs M combined

	$m(t\bar{t}H)/2$-based scales		σ [fb]
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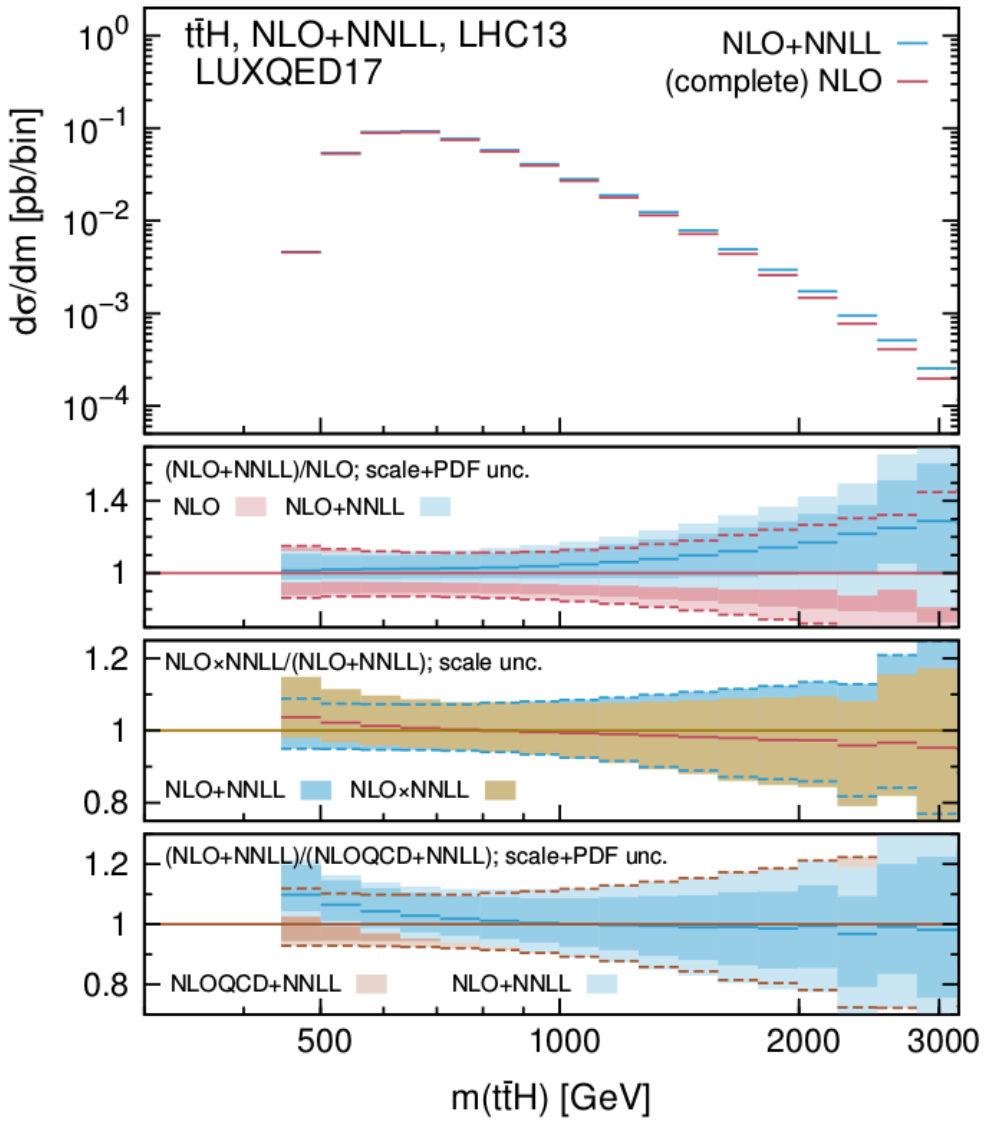
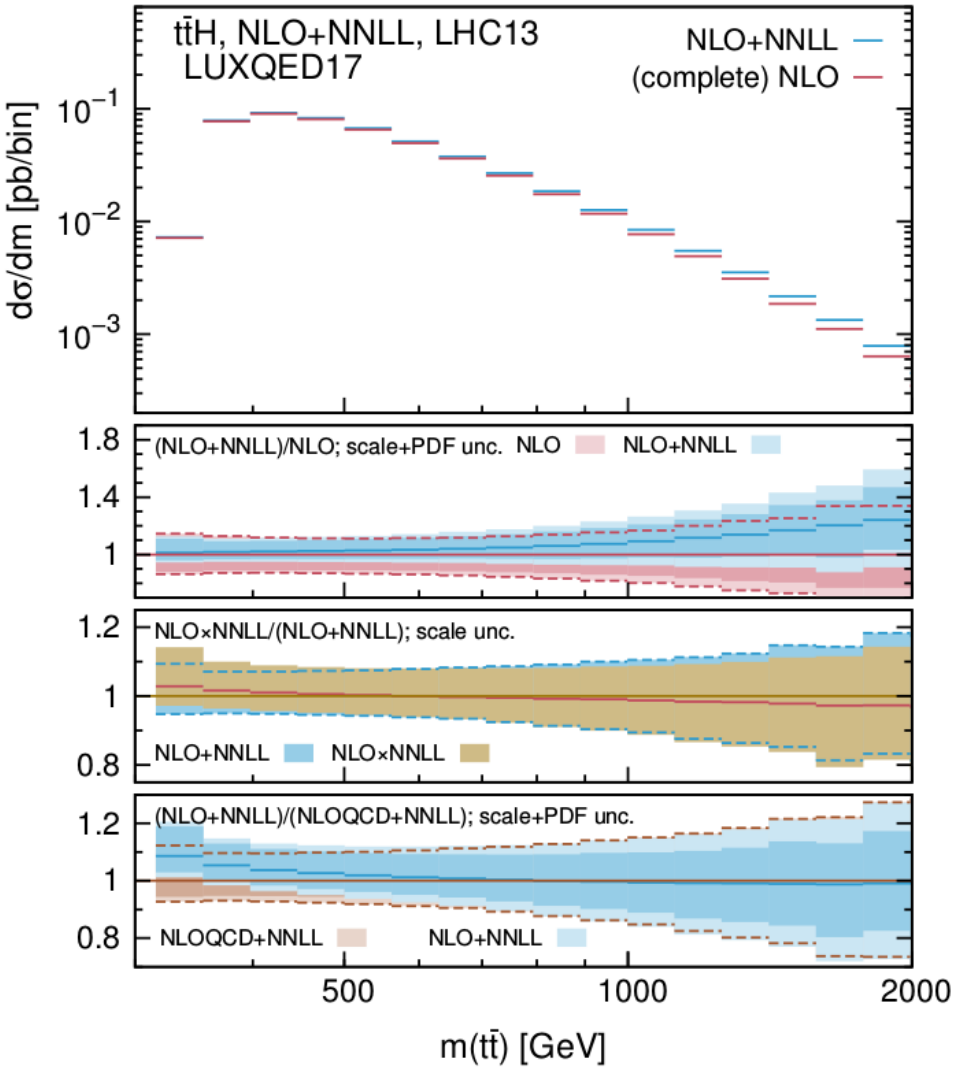
NLO+NNLL	501.67(9)	$^{+33.34(+6.6\%)}_{-22.54(-4.5\%)}$	$^{+12.05(+2.4\%)}_{-12.05(-2.4\%)}$
	Combined scales		
NLO+NNLL	496.36(7)	$^{+38.64(+7.8\%)}_{-29.35(-5.9\%)}$	$^{+11.92(+2.4\%)}_{-11.92(-2.4\%)}$
$\mathrm{NLO_{QCD}}$	467.96(5)	$^{+45.57(+9.7\%)}_{-53.98(-11.5\%)}$	$^{+11.31(+2.4\%)}_{-11.31(-2.4\%)}$

Invariant mass distributions - ttH

Combined scales

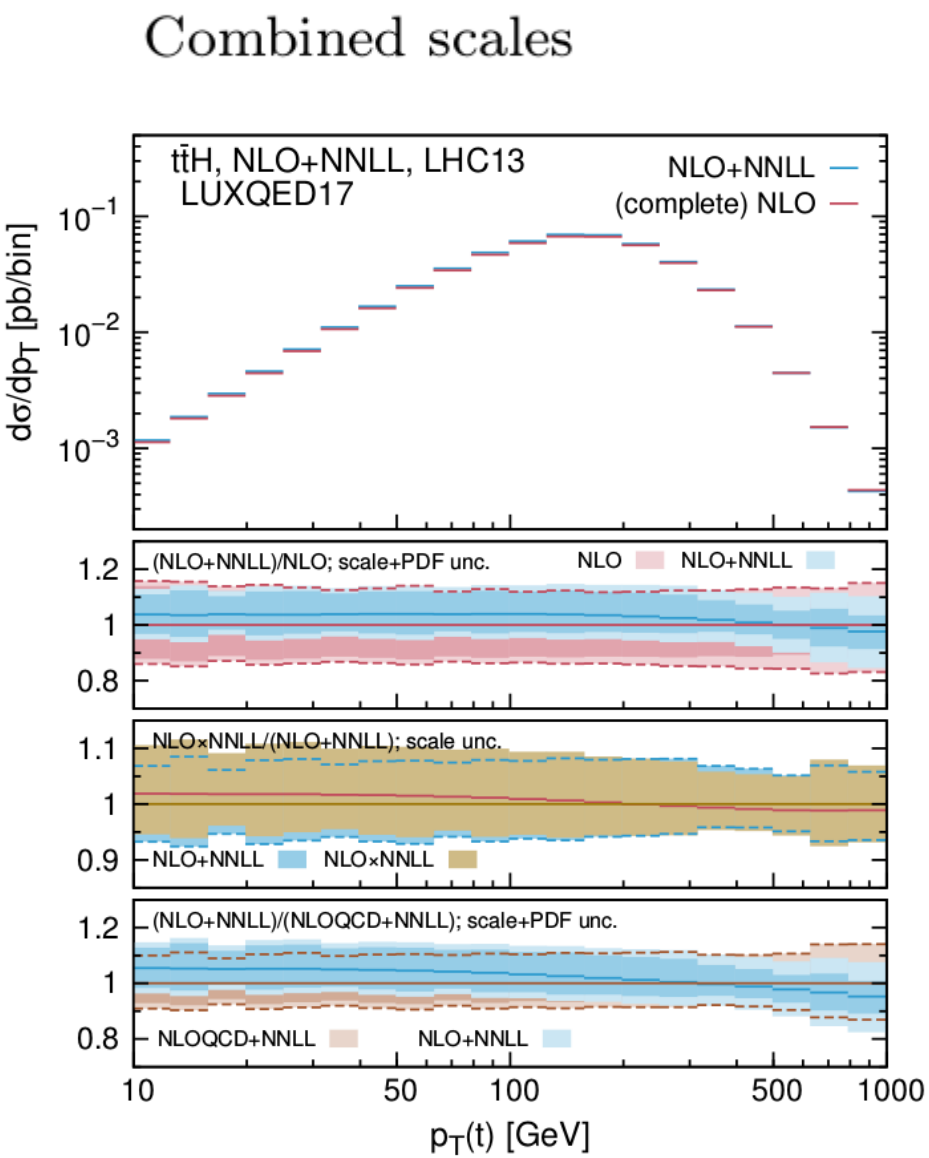


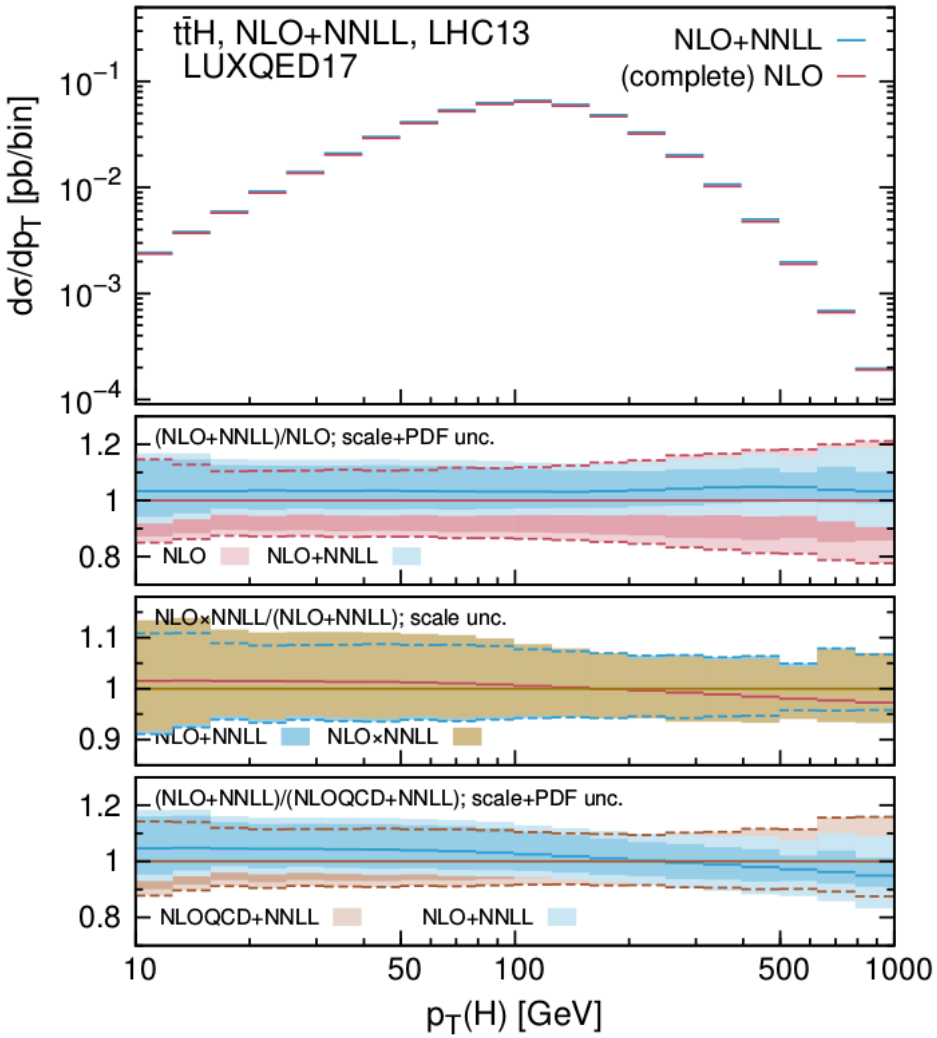
Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)



Transverse momentum distributions - ttH

Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)





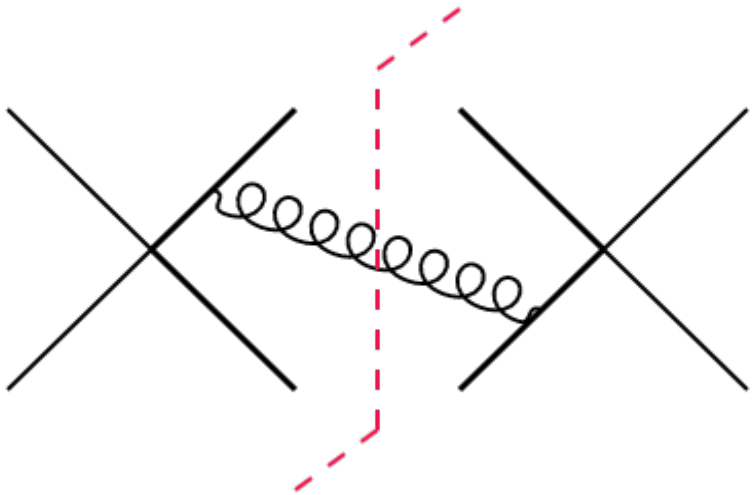
Conclusions and Outlook

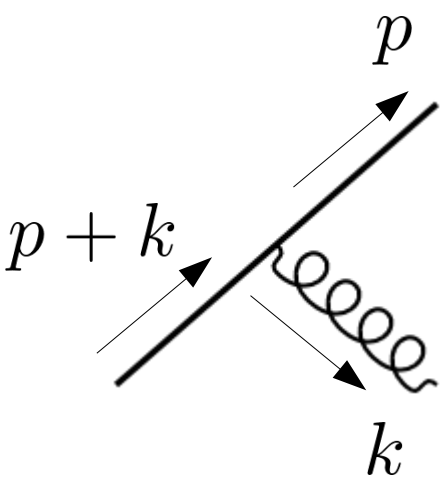
- We implemented a method to study partonic threshold corrections to top pair + H/W/Z boson production
- NLO QCD+NNLL results are available for top pair + W, top pair + H, top pair + Z production (total cross section + diff. Distributions)
- In progress: NLO (QCD & EW)+NNLL for ttW/H/Z (in collaboration with A. Broggio, R. Frederix, D. Pagani, B.D. Pecjak, I. Tsinikos)

Back up material

Soft function at NLO

The soft function can be calculated by evaluating diagrams involving the emission of soft gluons from the external legs





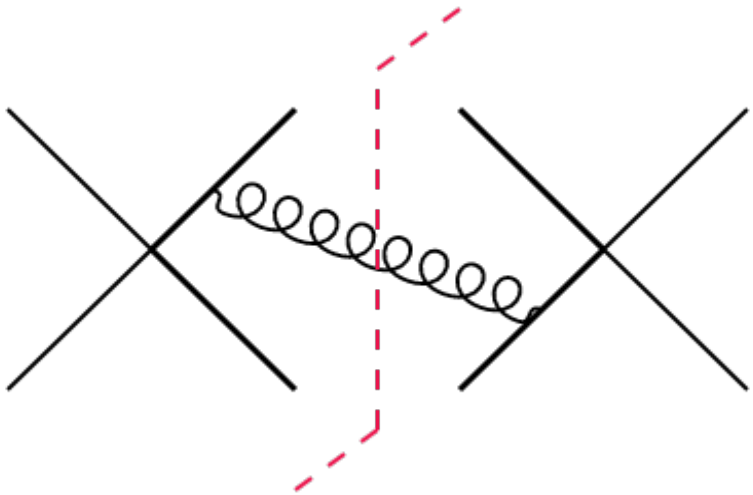
$$i\frac{\not{p}+\not{k}+m}{(p+k)^2-m^2}\gamma^\mu \xrightarrow{k\to 0} i\frac{p^\mu}{p\cdot k}$$

Eikonal vertex

$$\mathcal{I}_{ij}(\epsilon,x_0,\mu) = -\frac{(4\pi\mu^2)^\epsilon}{\pi^{2-\epsilon}}\, v_i\cdot v_j \int d^dk\, \frac{e^{-ik^0x_0}}{v_i\cdot k\, v_j\cdot k}\,(2\pi)\,\delta(k^2)\,\theta(k^0)$$

Soft function at NLO

The soft function can be calculated by evaluating diagrams involving the emission of soft gluons from the external legs



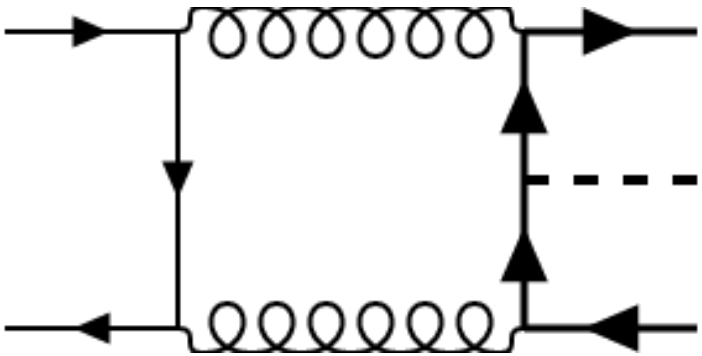
In momentum space the soft function depends on z-dependent plus distributions

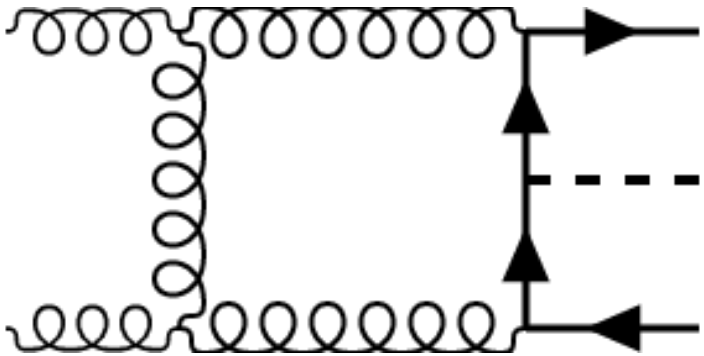
$$P'_n(z) \equiv \left[\frac{1}{1-z}\ln^n\left(\frac{M^2(1-z)^2}{\mu^2 z}\right)\right]_+$$

$$\int_0^1 dz\left[\frac{\ln^n(1-z)}{1-z}\right]_+ f(z) = \int_0^1 dz\frac{\ln^n(1-z)}{1-z}\left(f(z)-f(1)\right)$$

Hard function at NLO

In order to evaluate the NLO hard function one needs to calculate one-loop QCD amplitudes. In doing this one need to separate the various components of the amplitude in color space. Some examples are:

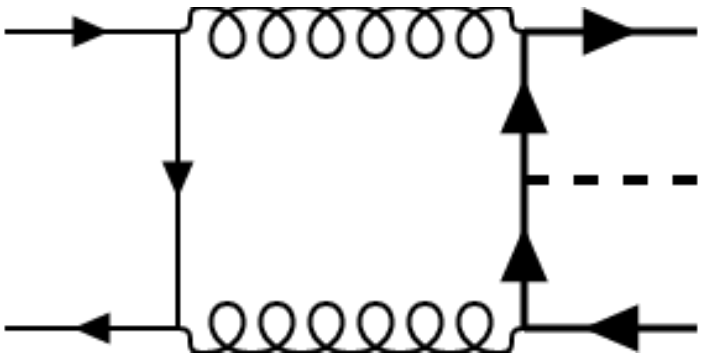


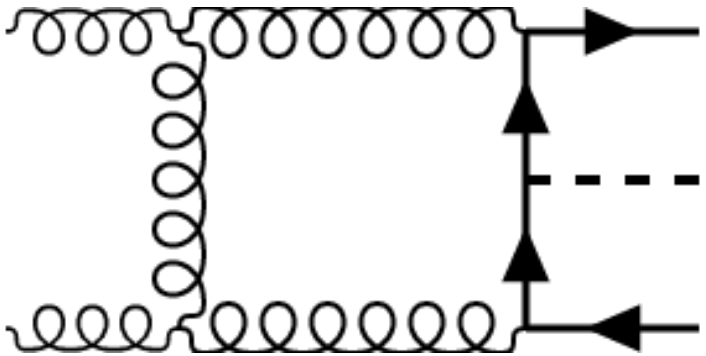


For top-quark pair production, the NLO matrix elements can still be calculated analytically: Things are more complicated for top-pair+Higgs, which is a 2 to 3 process

Hard function at NLO

In order to evaluate the NLO hard function one needs to calculate one-loop QCD amplitudes. In doing this one need to separate the various components of the amplitude in color space. Some examples are:





Hard Functions evaluated by means of

modified versions of GoSam and Openloops (+ Collier)

Cullen, Greiner, Heinrich, Luisoni, Mastrolia, Ossola,et al. ('12-'14)

Cascioli, Maierhofer, Pozzorini ('12) Denner, Dittmaier, Hofer (`16)

Final state phase space

- The final state phase space is written as the convolution of two two-particle phase spaces:

$$\int d\Phi_{t\bar{t}H} = \int \frac{ds_{t\bar{t}}}{2\pi} \frac{1}{2M^2} \frac{d\Omega}{16\pi^2} K\left(M^2, s_{t\bar{t}}, m_H^2\right) \frac{1}{2s_{t\bar{t}}} \frac{d\Omega^*}{16\pi^2} K\left(s_{t\bar{t}}, m_t^2, m_t^2\right)$$

$$K(x,y,z) = \sqrt{x^2 + y^2 + z^2}$$

- Five integrations left in the final state phase space
- Three integrations for the initial state (τ, N, and the luminosity variable x)
- One needs to build a Monte Carlo integration over 8 variables
- The 8 integration variables determine the top, antitop, Higgs (or W/Z) and incoming parton momenta: one can bin events and plot distributions

Evolution Factors

- Hard and soft scales are evaluated at values of the scale where large corrections are absent

$$\mu_h = M \quad \mu_s = M/\bar{N}$$

- RG evolution is used to obtain $\widetilde{c}$ at the scale μ_f

$$\widetilde{c}(\mu_f) = \mathrm{Tr}\left[\widetilde{\mathbf{U}}(\mu_f, \mu_h, \mu_s)\, \mathbf{H}(\mu_h)\, \widetilde{\mathbf{U}}^\dagger(\mu_f, \mu_h, \mu_s)\widetilde{\mathbf{s}}\,(\mu_s)\right]$$

- By rewriting $\alpha_s(\mu_f)$ and $\alpha_s(\mu_s)$ as a function of $\alpha_s(\mu_h)$

$$\widetilde{\mathbf{U}} = \exp\left\{\frac{4\pi}{\alpha_s(\mu_h)} g_1\left(\lambda, \lambda_f\right) + g_2\left(\lambda, \lambda_f\right) + \frac{\alpha_s(\mu_h)}{4\pi} g_3\left(\lambda, \lambda_f\right) + \cdots\right\}$$

$$\times\, \mathbf{u}(\{p\}, \mu_h, \mu_s)$$

$$\lambda = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0 \ln\frac{\mu_h}{\mu_s}\,, \qquad \lambda_f = \frac{\alpha_s(\mu_h)}{2\pi}\beta_0 \ln\frac{\mu_h}{\mu_f}$$

$$\lambda, \lambda_f \sim 1$$

Minimal Prescription

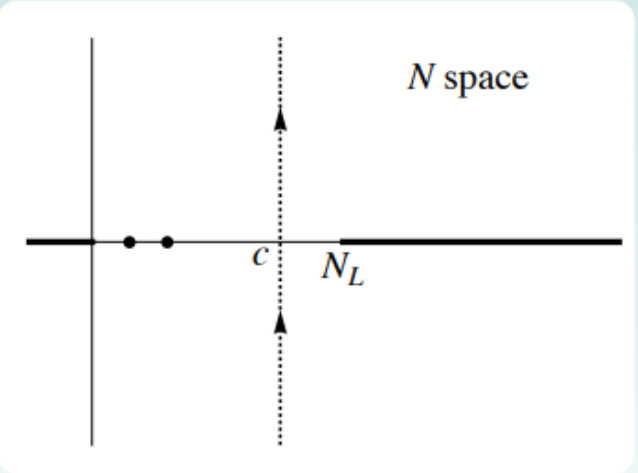
From M.Bonvini slides, 2009

Minimal prescription

Proposed by S.Catani, M.Mangano, P.Nason, L.Trentadue:

$$\sigma^{\mathrm{MP}}(x,Q^2) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dN\, x^{-N}\, \mathcal{L}(N,Q^2)\, \hat{\sigma}^{\mathrm{res}}\left(N,\alpha_s(Q^2)\right)$$

with $c < N_L$, as in the figure.



Good properties:

- well defined for all x
- exact for invertible functions
- asymptotic to the original divergent series

But...

- a non-physical region of the parton cross-section contributes
- problems in numerical implementation

Scale uncertainty

- In fixed order results, the scale uncertainty is evaluated by varying $\mu_f \in [\mu_{f,0}/2, 2\mu_{f,0}]$ with $\mu_{f,0} = M/2$

- For resummed results, we vary all scales (hard, soft and factorization) independently in the range $\mu_i \in [\mu_{i,0}/2, 2\mu_{i,0}]$

- For an observable O (the total cross section, or the value of a differential cross section in a given bin) one evaluates (for $i = s, f, h$ and $\kappa_i = \mu_i/\mu_{i,0}$)

$$\Delta O_i^+ = \max\{O(\kappa_i = 1/2), O(\kappa_i = 1), O(\kappa_i = 2)\} - O(\kappa_i = 1)$$

$$\Delta O_i^- = \min\{O(\kappa_i = 1/2), O(\kappa_i = 1), O(\kappa_i = 2)\} - O(\kappa_i = 1)$$

- The quantities $\Delta O_i^+ (\Delta O_i^-)$ are then combined in quadrature in order to obtain the scale uncertainty above (below) the central value

Total cross section - ttZ

13 TeV

$t\bar{t}Z$, PDFs=LUXQED17 (82200) $\mu_f^0 = \mu_r^0 = m(t\bar{t}Z)/2, \mu_h^0 = m(t\bar{t}Z), \mu_s^0 = \frac{m(t\bar{t}Z)}{\bar{N}}$	
Order	σ [fb]
LO QCD	$463.90(4)^{+133.53(+28.8\%)}_{-96.96(-20.9\%)}{}^{+10.30(+2.2\%)}_{-10.30(-2.2\%)}$
NLO QCD	$732.9(1)^{+92.7(+12.6\%)}_{-90.1(-12.3\%)}{}^{+17.0(+2.3\%)}_{-17.0(-2.3\%)}$
(complete) NLO	$741.5(1)^{+92.3(+12.4\%)}_{-89.9(-12.1\%)}{}^{+17.2(+2.3\%)}_{-17.2(-2.3\%)}$
NLO QCD+NNLL	$790.7(2)^{+61.5(+7.8\%)}_{-66.2(-8.4\%)}{}^{+18.4(+2.3\%)}_{-18.4(-2.3\%)}$
NLO+NNLL	$799.3(2)^{+61.7(+7.7\%)}_{-66.3(-8.3\%)}{}^{+18.6(+2.3\%)}_{-18.6(-2.3\%)}$

Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)

Total cross section - ttZ

13 TeV

$t\bar{t}Z$, PDFs= $\mu_f^0 = \mu_r^0 = m(t\bar{t}Z)$	
Order	
LO QCD	46
NLO QCD	
(complete) NLO	$741.5(1)^{+}_{-89.9(-12.1\%)}\ {}^{-17.2(-2.3\%)}$
NLO QCD+NNLL	$790.7(2)^{+61.5(+7.8\%)}_{-66.2(-8.4\%)}\ {}^{+18.4(+2.3\%)}_{-18.4(-2.3\%)}$
NLO+NNLL	$799.3(2)^{+61.7(+7.7\%)}_{-66.3(-8.3\%)}\ {}^{+18.6(+2.3\%)}_{-18.6(-2.3\%)}$

ATLAS CONF 2018-047

$$\sigma_{t\bar{t}Z} = 950^{+130}_{-130}\ \text{fb}$$

CMS JHEP 08 (2018) 011
arXiv: 1711.02547

$$\sigma_{t\bar{t}Z} = 990^{+90+120}_{-80-100}\ \text{fb}$$

Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)

Total cross section – ttW+

13 TeV

$t\bar{t}W^+$, PDFs=LUXQED17 (82200)	
$\mu_f^0 = \mu_r^0 = m(t\bar{t}W^+)/2, \mu_h^0 = m(t\bar{t}W^+), \mu_s^0 = \frac{m(t\bar{t}W^+)}{\bar{N}}$	
Order	σ [fb]
LO QCD	$225.45(1)^{+51.61(+22.9\%)}_{-39.41(-17.5\%)}\ ^{+5.85(+2.6\%)}_{-5.85(-2.6\%)}$
NLO QCD	$355.69(4)^{+43.50(+12.2\%)}_{-39.29(-11.0\%)}\ ^{+8.12(+2.3\%)}_{-8.12(-2.3\%)}$
(complete) NLO	$376.58(5)^{+46.52(+12.4\%)}_{-41.73(-11.1\%)}\ ^{+8.02(+2.1\%)}_{-8.02(-2.1\%)}$
NLO QCD+NNLL	$347.1(1)^{+23.9(+6.9\%)}_{-14.4(-4.2\%)}\ ^{+7.9(+2.3\%)}_{-7.9(-2.3\%)}$
NLO+NNLL	$368.0(1)^{+26.5(+7.2\%)}_{-16.2(-4.4\%)}\ ^{+7.8(+2.1\%)}_{-7.8(-2.1\%)}$

Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)

Total cross section – ttW-

13 TeV

$t\bar{t}W^-$, PDFs=LUXQED17 (82200)	
$\mu_f^0 = \mu_r^0 = m(t\bar{t}W^-)/2, \mu_h^0 = m(t\bar{t}W^-), \mu_s^0 = \frac{m(t\bar{t}W^-)}{\bar{N}}$	
Order	σ [fb]
LO QCD	$114.305(6)^{+26.261(+23.0\%)}_{-20.056(-17.5\%)}{}^{+3.563(+3.1\%)}_{-3.563(-3.1\%)}$
NLO QCD	$181.65(2)^{+22.74(+12.5\%)}_{-20.36(-11.2\%)}{}^{+5.20(+2.9\%)}_{-5.20(-2.9\%)}$
(complete) NLO	$193.26(2)^{+24.55(+12.7\%)}_{-21.81(-11.3\%)}{}^{+5.29(+2.7\%)}_{-5.29(-2.7\%)}$
NLO QCD+NNLL	$178.16(4)^{+12.29(+6.9\%)}_{-7.13(-4.0\%)}{}^{+5.09(+2.9\%)}_{-5.09(-2.9\%)}$
NLO+NNLL	$189.77(5)^{+13.82(+7.3\%)}_{-8.09(-4.3\%)}{}^{+5.19(+2.7\%)}_{-5.19(-2.7\%)}$

Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)

Total cross section – ttW-

13 TeV

ATLAS CONF 2018-047

$$\sigma_{t\bar{t}W} = 870^{+190}_{-190}\,\mathrm{fb}$$

CMS JHEP 08 (2018) 011
arXiv: 1711.02547

$$\sigma_{t\bar{t}W} = 770^{+120+130}_{-110-120}\,\mathrm{fb}$$

Combined prediction NLO+NNLL

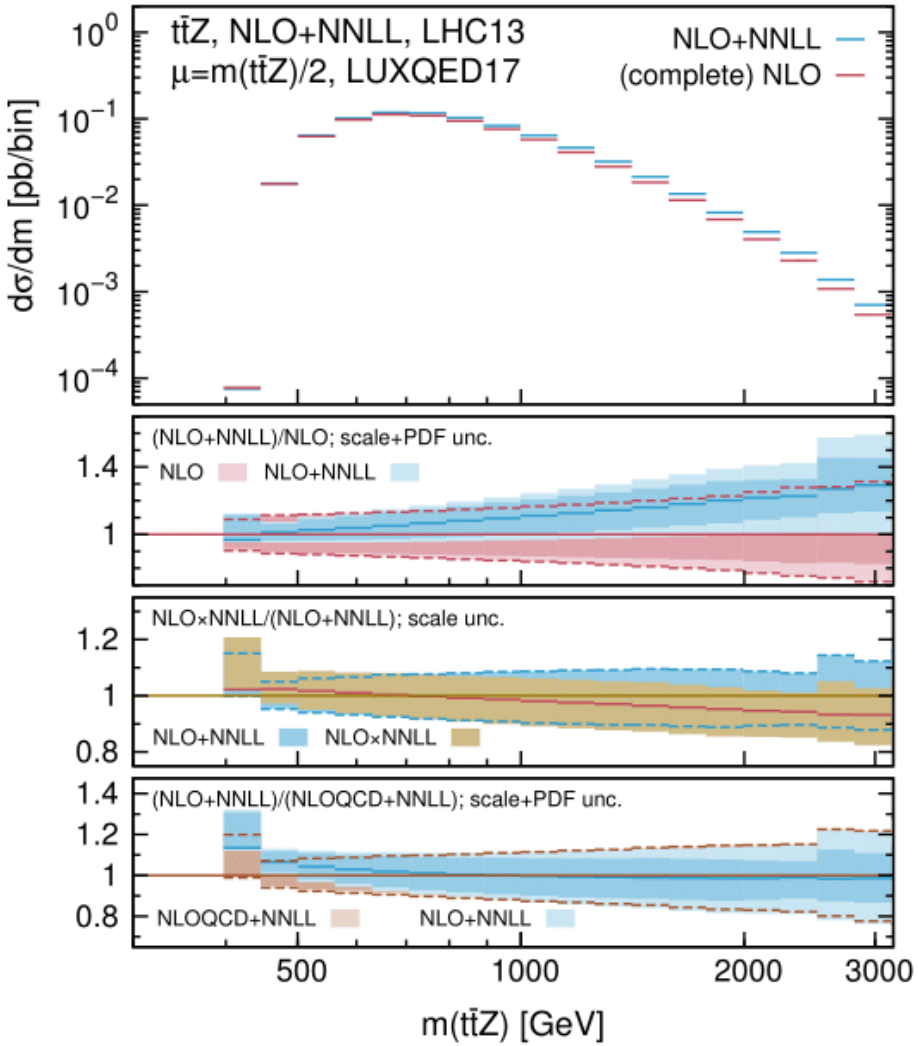
$$\sigma_{t\bar{t}W} = 558^{+28}_{-42}\,\mathrm{fb}$$

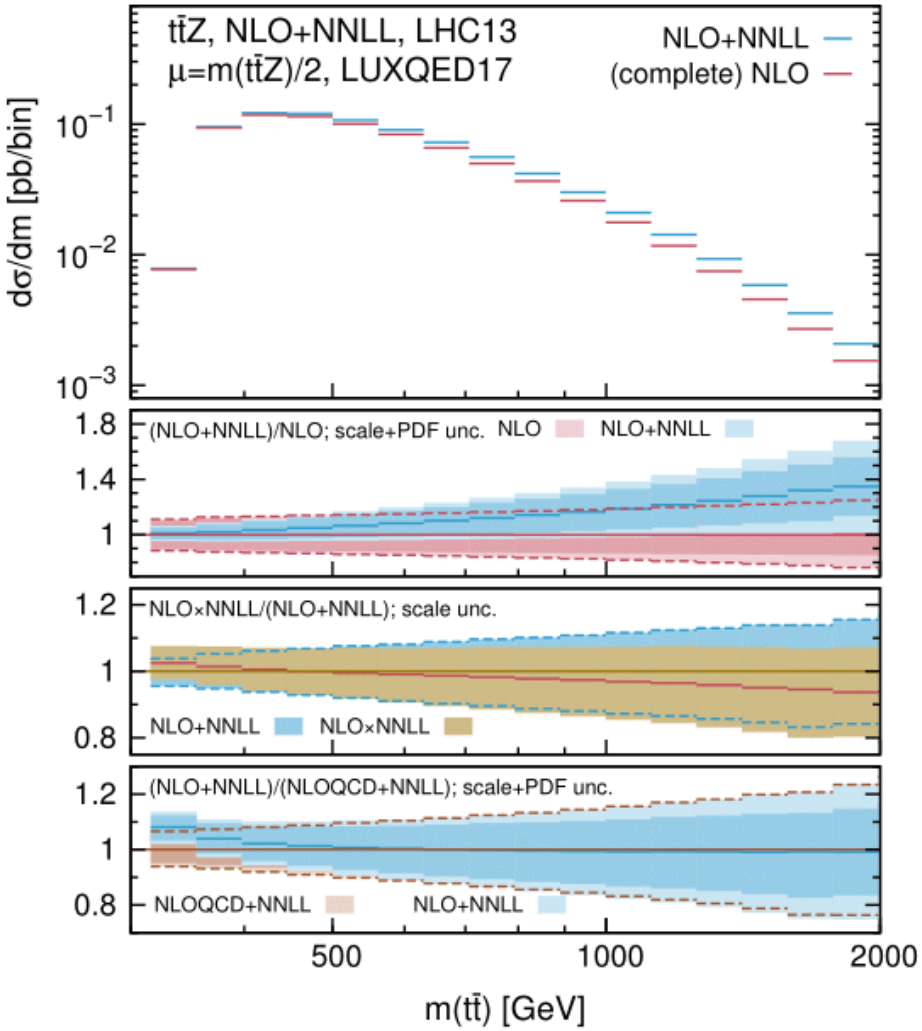
$t\bar{t}$	)
$\mu_f^0 = \mu_r^0 =$	$= \frac{m(t\bar{t}W^-)}{\bar{N}}$
Order	
LO QCD	$+3.563(+3.1\%)$ $-3.563(-3.1\%)$
NLO QCD	$+5.20(+2.9\%)$ $-5.20(-2.9\%)$
(complete) N	$+5.29(+2.7\%)$ $-5.29(-2.7\%)$
NLO QCD+N	$+5.09(+2.9\%)$ $-5.09(-2.9\%)$
NLO+NNLL	$+5.19(+2.7\%)$ $-5.19(-2.7\%)$

Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)

Invariant mass distributions - ttZ

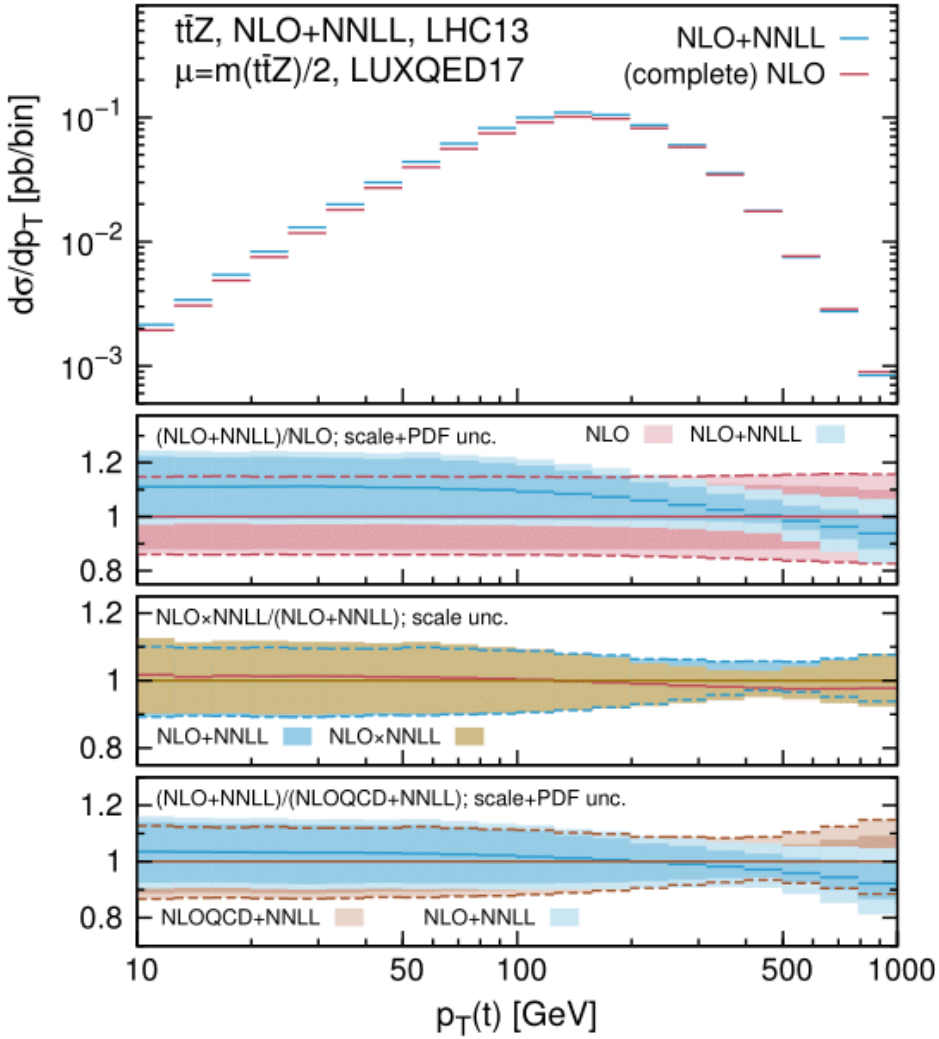
Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)

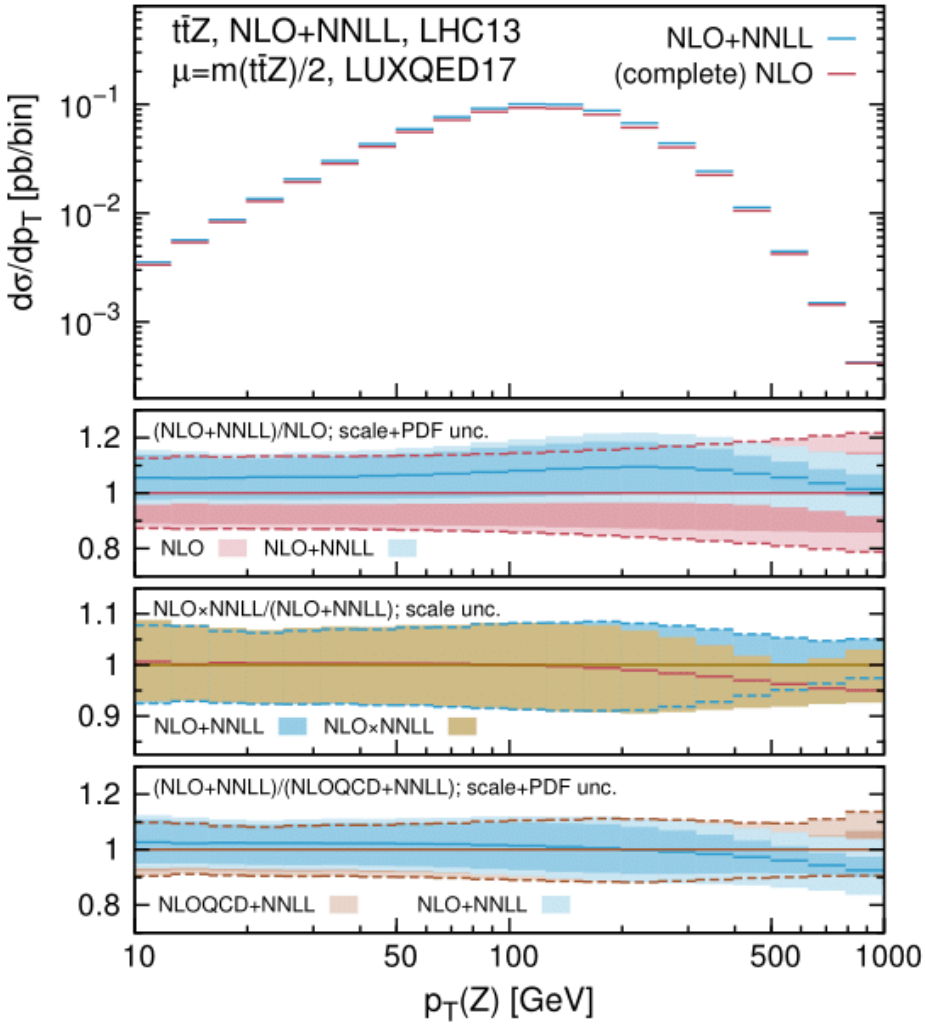




Transverse momentum distributions - ttZ

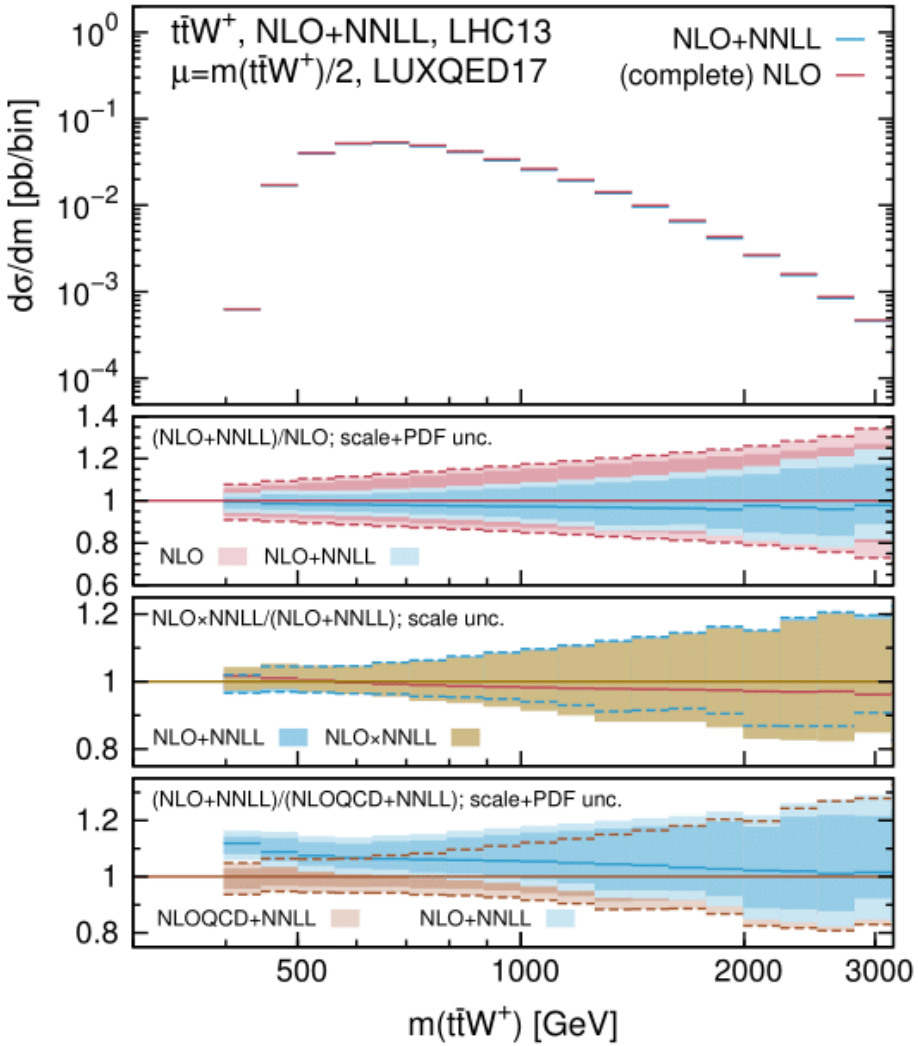
Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)

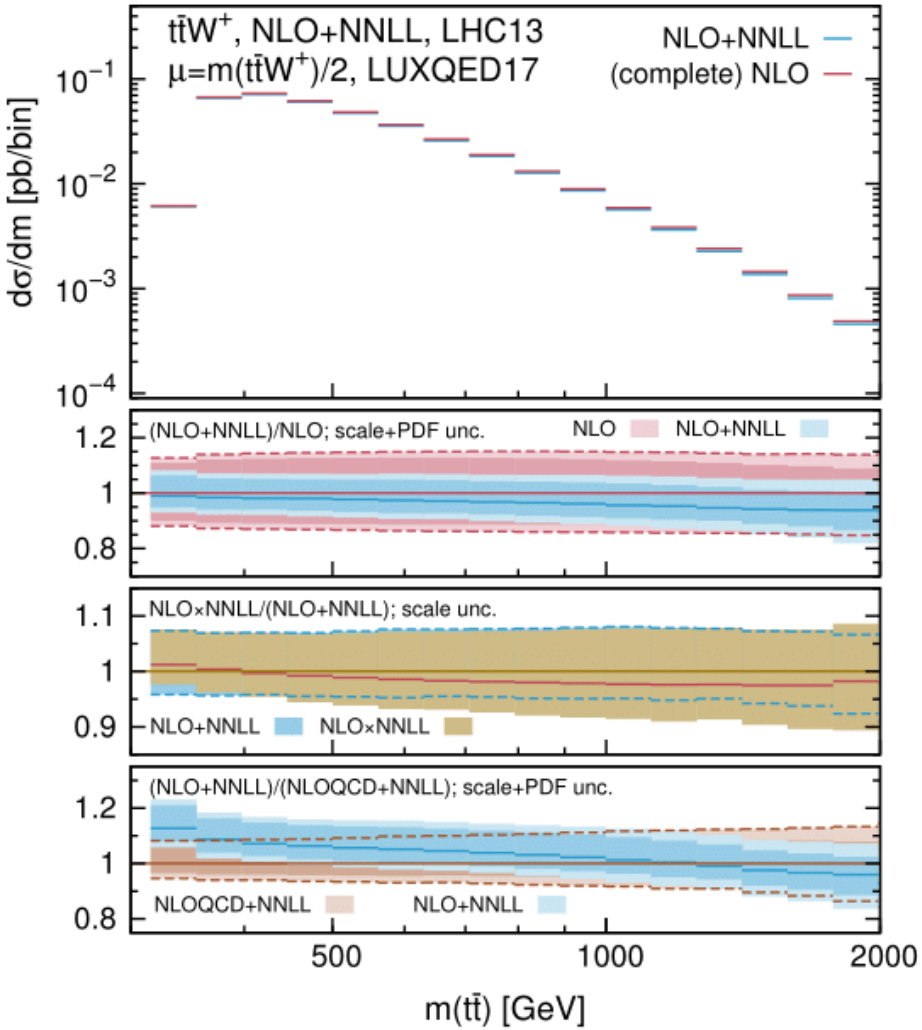




Invariant mass distributions - ttW+

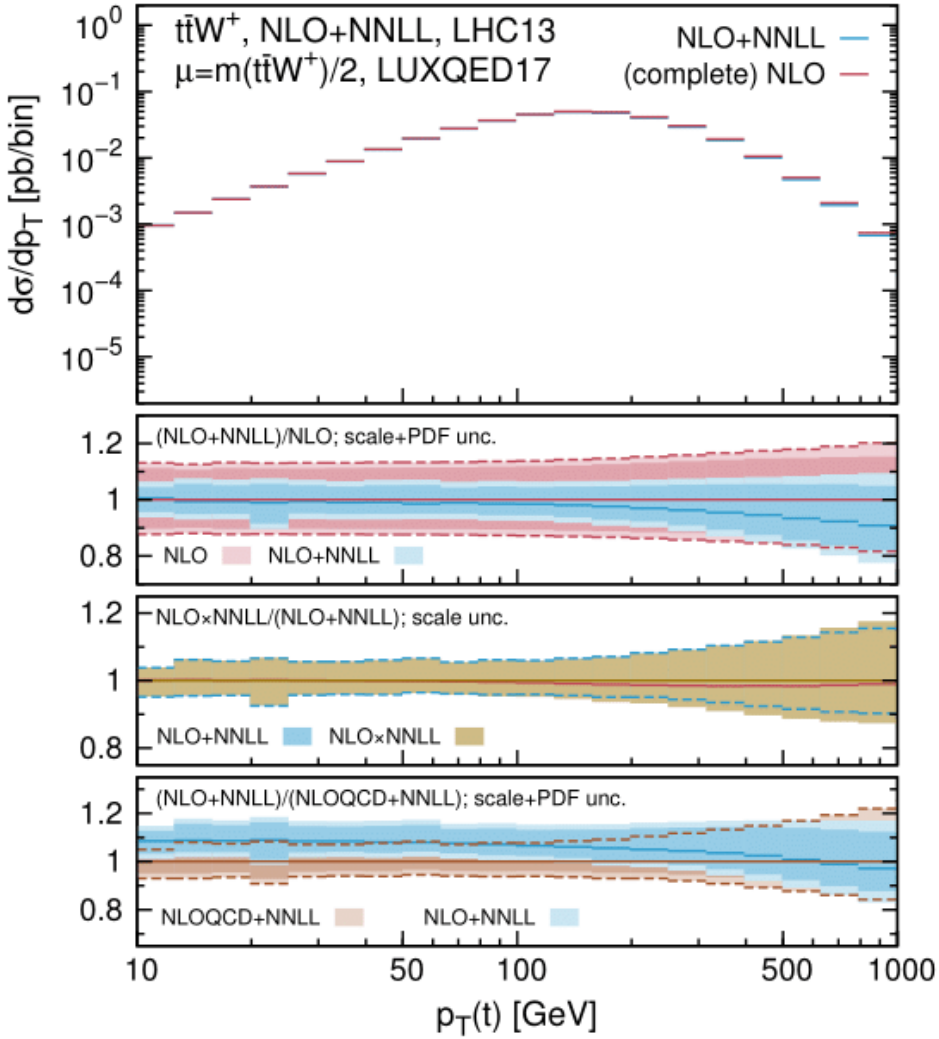
Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)

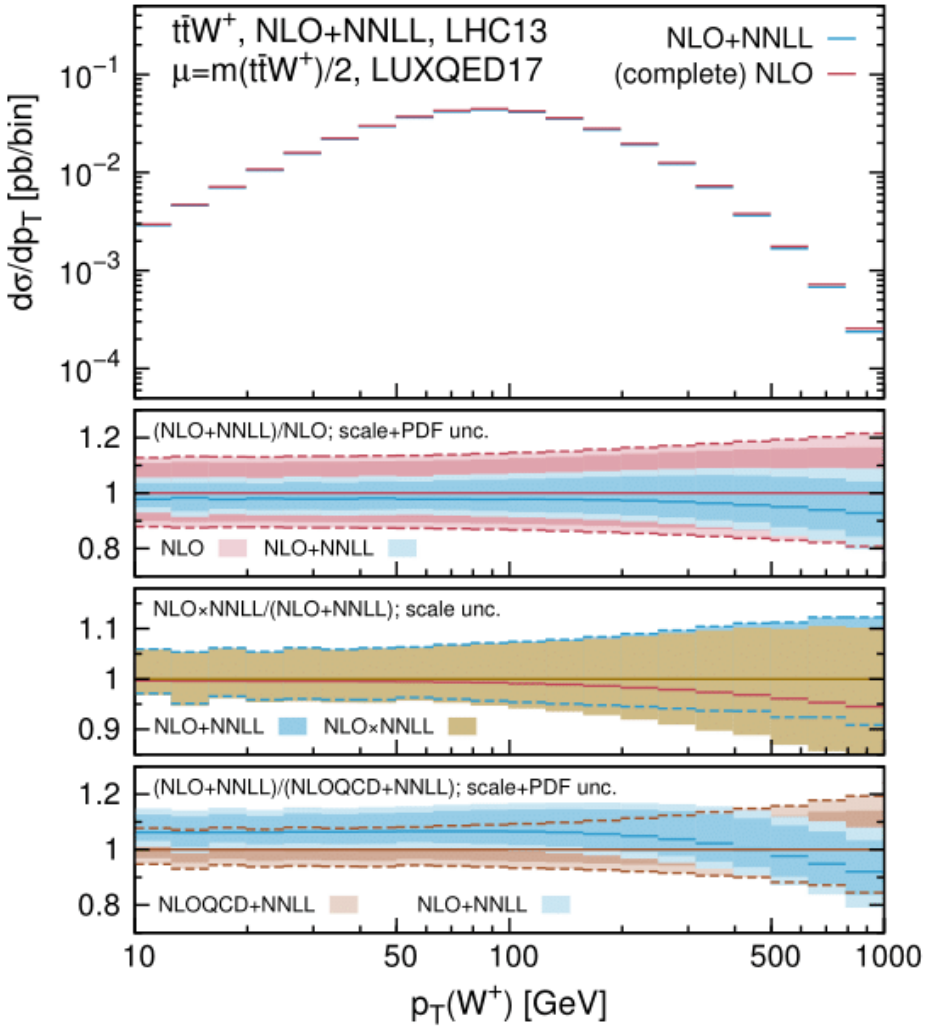




Transverse momentum distributions - ttW+

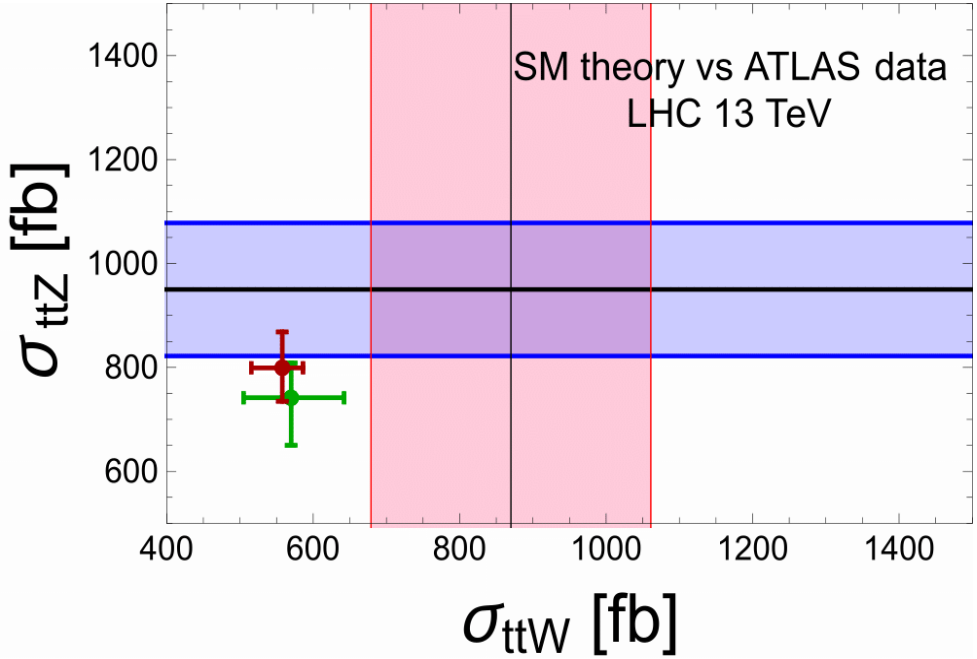
Preliminary (courtesy of R. Frederix, D. Pagani, I. Tsinikos)



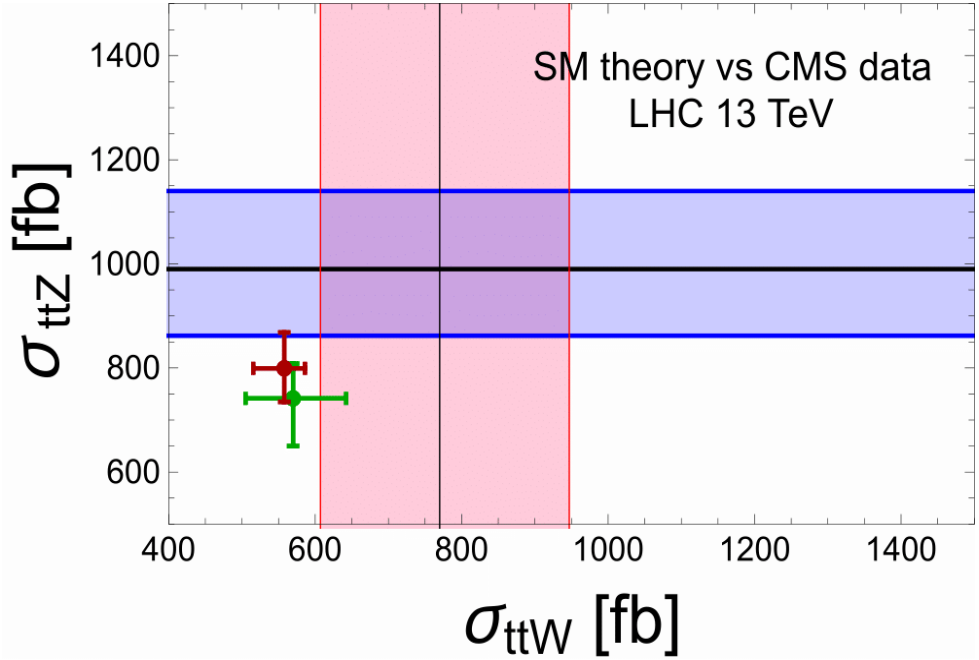


ttZ/ttW cross section vs data

13 TeV - ATLAS data
ATLAS-CONF-2018-047



13 TeV - CMS data
arXiv:1711.02547



$$\mu_f = M/2$$

Total cross section at **NLO** (QCD + EW green cross) and **NLO+NNLL** (red cross) compared to the ATLAS measurement (13 TeV) and CMS measurement (13 TeV).