## QCD

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## DISCLAIMER(S)

Purpose(s) of these lectures:
Introduction to QCD (Quantum ChromoDynamics)
Refresh your knowledge on QCD (another view)
Understand the vocabulary!
New developments in the field (Lectures 3 and 4)


## QCCD <br> QCD

3

- In the LHC era, QCD is everywhere!

- In these lectures : pQCD as precision QCD for Colliders

LHC incredibly successful at $7,8 \& 13 \mathrm{TeV}$ (Runs I and II)

## Standard Model Total Production Cross Section Measurements <br> Status: August 2016



Everything SM like (including Higgs)


But.... there should be Physics Beyond the Standard Model (BSM)

- Lacks description of Quantum Gravity
- Hierarchy, naturalness problems

Gravity is $\sim 40$ orders of magnitude weaker than EM in atom
I3 orders of magnitude between lightest and heaviest particle

Finer tuning in Higgs sector
No candidate for Dark Matter !!
$>20 \%$ of universe

- Matter-antimatter asymmetry


There are(?) TH candidates, but search is DRIVEN BY EXPERIMENTS now

- Most direct searches for new physics have been carried out with approx. $35 \mathrm{fb}^{-1}$, so only $1 \%$ of the data of the entire LHC program
-There is plenty of room for discoveries yet
- It will take time (doubling time of the luminosity should be counted in several years)
from M. Kado




Search for new states
Resonances "Descriptive TH"


Search for new interactions
Deviations from TH "Precision TH"


Need for precision ~ 1\% EXP-TH accuracy

- Very likely: New physics might show up in the detail
- Flavor Physics
- Contribution from new particles at loop level

- Need to be precise on cross-sections and SM parameters

$$
\mathrm{EW} \text { vacuum stability } \quad m_{H}, m_{t}, \alpha_{s}, \ldots
$$

- Explore Higgs sector with precision
- Multiple Gauge boson and HQ production (gauge/couplings to new physics)

Precision is the name of the game

## These Lectures <br> Toolkit for precise TH predictions at the LHC

## Outline of the lecture I

© Basics of QCD : Lagrangian and Feynman rules
$\mathscr{Q}$ QCD at work: beta function and running coupling
\& QCD at work in $e^{+} e^{-}$
\& Infrared Safety in QCD
\& Jets in QCD

Everything starts by organizing hadron spectrum to show some pattern of symmetry (such as Mendeleev did for atoms in periodic table)

$$
\begin{aligned}
& s=0 \\
& s=-1 \\
& s=-2
\end{aligned}
$$



One still missing by that time, but predicted following pattern
Then one asks ... what is the reason for this pattern?

## Quarks (1964)

Gell-Mann and Zweig propose the existence of elementary (spin I/2) particles named quarks : with 3 of them (plus antiquarks) can explain the composition of all known hadrons


Bound states are only made by 3 quarks (baryon) Baryon $q q q$ or by a quark+antiquark (meson). No other structure observed.

Meson $q \bar{q}$


$$
J / \Psi=(c \bar{c})
$$

(1974) Discovered at SLAC and Brookhaven. Expected due to strong theoretical arguments (GIM mechanism)

$\infty$

$$
\Upsilon=(b \bar{b})
$$

$m_{b} \approx 4.0-4.4 \mathrm{GeV}$
(1977) Discovered at Fermilab (E288) 3rd family of quarks needed to account for CP violation

t

$$
m_{t} \approx 171 G e V
$$

(1995) Discovered at Tevatron EW precision measurements predicted mass with accuracy

Several orders of magnitude in masses


| quark | charge | mass (approx.) |
| :---: | :---: | :---: |
| u | $2 / 3$ | $\sim 4 \mathrm{MeV}$ |
| $d$ | $-1 / 3$ | $\sim 7 \mathrm{MeV}$ |
| c | $2 / 3$ | $\sim 1.3 \mathrm{GeV}$ |
| s | $-1 / 3$ | $\sim 150 \mathrm{MeV}$ |
| t | $2 / 3$ | $\sim 171 \mathrm{GeV}$ |
| b | $-1 / 3$ | $\sim 4.4 \mathrm{GeV}$ |

## Spin-statistics issue



$$
\Delta^{++}=u \uparrow u \uparrow u \uparrow
$$

Wave function (flavor+spin) completely symmetric : forbidden by Pauli exclusion principle

Introduce new additional quantum number : color

$$
\Delta^{++}=\epsilon_{i j k} u_{i} \uparrow u_{j} \uparrow u_{k} \uparrow
$$

wave function becomes antisymmetric

Will see that experiment directly confirms 3 colors

$$
\frac{\sigma\left(e^{+} e^{-} \rightarrow \text { hadrons }\right)}{\sigma\left(e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}\right)}
$$

Upgrade color to "charge of the strong interactions"
So strong that only hadrons observed in nature are those combinations of quarks that result in color singlets!

|  | Baryon |
| :--- | :--- |
| Only | $q q q$ |
| Meson | $q \bar{q}$ |

3 colors explain observed spectrum of hadrons!
$S U(3)_{\text {color }}$ is an exact symmetry of nature


## QCD: non-abelian gauge theory under $\mathrm{SU}(3)$

$$
\mathcal{L}_{Q C D}=-\frac{1}{4} F_{\mu \nu}^{(a)} F^{(a) \mu \nu}+\sum_{q} \bar{\psi}_{i}^{q}\left(i \gamma^{\mu}\left(D_{\mu}\right)_{i j}-m_{q} \delta_{i j}\right) \psi_{j}^{q}
$$

$$
\left(D_{\mu}\right)_{i j}=\delta_{i j} \partial_{\mu}-i g_{s} T_{i j}^{a} A_{\mu}^{a}
$$

$$
F_{\mu \nu}^{(a)}=\partial_{\mu} A_{\nu}^{a}-\partial_{\nu} A_{\mu}^{a}+g_{s} f_{a b c} A_{\mu}^{b} A_{\nu}^{c}
$$

one single coupling constant

$$
\alpha_{\mathrm{S}} \equiv \frac{g_{s}^{2}}{4 \pi}
$$

8 (32-1) generators obeying $\quad\left[T^{a}, T^{b}\right]=i f^{a b c} T^{c}$

$$
\begin{array}{rlrl}
t^{A}=\frac{1}{2} \lambda^{A} & \lambda^{1} & =\left(\begin{array}{lll}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{array}\right), \lambda^{2}=\left(\begin{array}{ccc}
0 & -i & 0 \\
i & 0 & 0 \\
0 & 0 & 0
\end{array}\right), \lambda^{3}=\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 0
\end{array}\right), \lambda^{4}=\left(\begin{array}{ccc}
0 & 0 & 1 \\
0 & 0 & 0 \\
1 & 0 & 0
\end{array}\right), \\
& \lambda^{5}=\left(\begin{array}{ccc}
0 & 0 & -i \\
0 & 0 & 0 \\
i & 0 & 0
\end{array}\right), \lambda^{6}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{array}\right), \lambda^{7}=\left(\begin{array}{ccc}
0 & 0 & 0 \\
0 & 0 & -i \\
0 & i & 0
\end{array}\right), \lambda^{8}=\frac{1}{\sqrt{3}}\left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -2
\end{array}\right)
\end{array}
$$

$\mathcal{L}_{\text {free }}+\mathcal{L}_{\text {int }}$

$$
\mathcal{L}_{i n t}=g \sum_{f=1}^{N_{f}} \bar{\psi}_{f}^{i} \gamma^{\mu} t_{i j}^{a} A_{\mu}^{a} \psi_{f}^{i} \quad q \bar{q} g \text { vertex }
$$

Feynman rules

$$
-g f^{a b c} \partial^{\mu} A_{\nu}^{a} A_{\mu}^{b} A^{\nu c} \quad g g g \text { vertex }
$$

$$
-\frac{1}{4} g^{2} f^{a b c} f^{a d e} A_{\mu}^{b} A_{\nu}^{c} A^{\mu d} A^{\nu e} \quad g g g g \text { vertex }
$$

## Propagators



## Quark


spin polarization tensor

$$
d^{\mu \nu}(p)=\sum_{\lambda} \varepsilon_{(\lambda)}^{\mu}(p) \varepsilon_{(\lambda)}^{\nu^{*}}(p)
$$

## Explicit expression depends on gauge

propagation of physical and

## Color algebra

Conventional normalization

$$
\operatorname{Tr}\left(t^{a} t^{b}\right)=T_{R} \delta_{a b} \quad T_{R}=1 / 2
$$

Fundamental representation 3

$$
\left(t^{a} t^{a}\right)_{i l}=C_{F} \delta_{i l} \quad C_{F}=\frac{N_{c}^{2}-1}{2 N_{c}} \quad \begin{aligned}
& \mathrm{i}, \mathrm{j}, \ldots \text { quark } \\
& \mathrm{a}, \mathrm{~b}, . . \text { gluon }
\end{aligned}
$$

Adjoint representation 8

$$
f^{a d c} f^{b d c}=C_{A} \delta^{a b} \quad C_{A}=N_{c}
$$

Very useful Fierz identity

$$
\begin{aligned}
& \mathrm{z} \text { identity } \quad\left(t^{a}\right)_{k}^{i}\left(t^{a}\right)_{j}^{l}=\frac{1}{2} \delta_{j}^{i} \delta_{k}^{l}-\frac{1}{2 N_{c}} \delta_{k}^{i} \delta_{j}^{l} \\
& \rightarrow-\frac{1}{2} \rightarrow+
\end{aligned}
$$



Gluon emission changes quark color


Gluon carries color + anticolor

Most relevant color structures
Compute those!


$$
\begin{align*}
& t_{i j}^{a} t_{j l}^{a}=C_{F} \delta_{i l} \\
& \text { quark }
\end{align*}>\text { gluon }
$$



$$
\operatorname{Tr}\left(t^{a} t^{b}\right)=T_{R} \delta_{a b} \quad 1 / 2
$$

$$
\text { gluon } \Longrightarrow \text { quark }
$$



$$
f^{a d c} f^{b d c}=C_{A} \delta^{a b}
$$

$$
\text { gluon } \Longrightarrow \text { gluon }
$$

## QCD at work

Q QCD can not be solved exactly: use perturbation theory

$$
\sigma=\sigma^{(0)}+\alpha_{s}(\mu) \sigma^{(1)}+\alpha_{s}^{2}(\mu) \sigma^{(2)}+\ldots
$$

Coupling constant "large" : many orders needed for precision
ISeveral problems appear in the calculation of perturbative corrections

Ultraviolet (UV) and InfraRed (IR) divergences
\& QFT has problems with loops: ultraviolet divergences originate from integration over very large momentum


A manifestation that QFT FAILS at very large energies!

To be able to use QFT, search for a procedure to isolate the "large" energy regime were it fails $\longrightarrow$ renormalization
I. Regularize the divergency
2. "Absorb" it by redefinition of "bare" ( $\mathrm{g}, \mathrm{m}, \mathrm{A}, \Psi$ ) parameters in Lagrangian (thanks to gauge symmetry!)

## Example



Regularization $\Lambda_{c u t} \quad \sim \alpha_{B}\left\{1+\alpha_{B} \beta_{0} \int_{p^{2}}^{\Lambda_{c u t}^{2}} \frac{d^{4} k}{\left(k^{2}\right)^{2}}+\right.$ finite $\}$
Renormalization scale $\mu$

$$
\sim \alpha_{B}\left\{1+\alpha_{B} \beta_{0}\left(\log \frac{\Lambda_{c u t}^{2}}{\mu^{2}}+\log \frac{\mu^{2}}{p^{2}}\right)+\quad \text { finite }\right\}
$$

Renormalized coupling

$$
\alpha_{s}\left(\mu^{2}\right) \equiv \alpha_{B}\left(1+\beta_{0} \alpha_{B} \log \frac{\Lambda_{c u t}^{2}}{\mu^{2}}\right)
$$

Renormalized cross-section

$$
=\alpha\left(\mu^{2}\right)\left\{1 \quad+\quad \beta_{0} \alpha\left(\mu^{2}\right) \log \frac{\mu^{2}}{p^{2}}+\quad \text { finite }\right\}
$$

Renormalized (running) coupling constant : $\mu$ dependent $\mathrm{RGE} \quad \frac{d \alpha_{s}\left(\mu^{2}\right)}{d \log \mu^{2}}=-\beta\left(\alpha_{s}\right)_{26} \quad \beta\left(\alpha_{s}\right)=\beta_{0} \alpha_{s}^{2}+\ldots$

QED


Gross,Wilczek, Politzer (1973)

QCD $\beta_{0}=\frac{11 C_{A}-2 n_{F}}{12 \pi}>0 \quad\left(n_{F}<16\right)$
Coupling constant DEcreases with energy

The two faces of QCD


Quarks do not show up as "free particles"

The Nobel Prize in Physics 2004

## David J. Gross, H. David Politzer, Frank Wilczek


"for the discovery of asymptotic freedom in the theory of the strong interaction".



## asymptotic freedom

It is a prediction of perturbation theory and allows to use it at high energies

## confinement

Perturbation theory breaks down: no rigorous proof yet ...

World Average

$$
\left(\alpha_{s}\left(M_{Z}^{2}\right)=0.105\right.
$$

$$
0.1181 \pm 0.0011
$$

Dominated by Lattice
disçyssion about "optimistic" uncertainties

RGE $\quad \frac{d \alpha_{s}\left(\mu^{2}\right)}{d \log \mu^{2}}=-\beta\left(\alpha_{s}\right) \quad$ at leading order (LO)

$$
\frac{d \alpha_{s}\left(\mu^{2}\right)}{d \log \mu^{2}}=-\beta_{0} \alpha_{s}^{2}\left(\mu^{2}\right) \square \alpha_{s}\left(\mu^{2}\right)=\frac{\alpha_{s}\left(\mu_{0}^{2}\right)}{1+\beta_{0} \alpha_{s}\left(\mu_{0}^{2}\right) \log \frac{\mu^{2}}{\mu_{0}^{2}}}
$$

This expression allows to compute coupling at any scale by knowing it at a reference value, e.g. $\mu_{0}=M_{Z}$

But it is convenient to introduce the fundamental parameter of QCD

$$
\Lambda_{Q C D}
$$

Such as
$\Lambda_{Q C D}=\mu_{0} \exp \left[-\frac{1}{2 \beta_{0} \alpha_{s}\left(\mu_{0}^{2}\right)}\right]$

$$
\alpha_{s}\left(\mu^{2}\right)=\frac{1}{\beta_{0} \log \frac{\mu^{2}}{\Lambda_{Q C D}^{2}}}
$$

- Scale at which coupling becomes large -Scale that control hadron masses

$$
\Lambda_{Q C D} \sim 200 \mathrm{MeV}
$$

In real life:
Dimensional regularization $4 \leadsto$ D dimensions,
"divergences" appear as I/(D-4) poles
Finite terms can be subtracted: renormalization scheme
Next-to-Next-to-Leading Order (NNLO) in $\overline{M S}$ scheme

$$
\overline{M S} \text { scheme. Subtract } \longrightarrow \frac{2}{4-D}+\ln (4 \pi)-\gamma_{E}
$$

$$
\alpha_{s}(\mu)=\frac{4 \pi}{\beta_{0} \ln \left(\mu^{2} / \Lambda^{2}\right)}\left[1-\frac{2 \beta_{1}}{\beta_{0}^{2}} \frac{\ln \left[\ln \left(\mu^{2} / \Lambda^{2}\right)\right]}{\ln \left(\mu^{2} / \Lambda^{2}\right)}+\frac{4 \beta_{1}^{2}}{\beta_{0}^{4} \ln ^{2}\left(\mu^{2} / \Lambda^{2}\right)}\right.
$$

$$
\left.\times\left(\left(\ln \left[\ln \left(\mu^{2} / \Lambda^{2}\right)\right]-\frac{1}{2}\right)^{2}+\frac{\beta_{2} \beta_{0}}{8 \beta_{1}^{2}}-\frac{5}{4}\right)\right] .
$$

$$
\beta_{0}=\frac{11}{3} C_{A}-\frac{4}{3} T_{F} n_{f}
$$

$$
\beta_{1}=\frac{34}{3} C_{A}^{2}-4 C_{F} T_{F} n_{f}-\frac{20}{3} C_{A} T_{F} n_{f}
$$

$$
\begin{aligned}
\beta_{2}= & \frac{2857}{54} C_{A}^{3}+2 C_{F}^{2} T_{F} n_{f}-\frac{205}{9} C_{F} C_{A} T_{F} n_{f} \\
& -\frac{1415}{27} C_{A}^{2} T_{F} n_{f}+\frac{44}{9} C_{F} T_{F}^{2} n_{f}^{2}+\frac{158}{27} C_{A} T_{F}^{2} n_{f}^{2} \\
\beta_{3}= & C_{A}^{4}\left(\frac{150653}{486}-\frac{44}{9} \zeta_{3}\right)+C_{A}^{3} T_{F} n_{f}\left(-\frac{39143}{81}+\frac{136}{3} \zeta_{3}\right) \\
& +C_{A}^{2} C_{F} T_{F} n_{f}\left(\frac{7073}{243}-\frac{656}{9} \zeta_{3}\right)+C_{A} C_{F}^{2} T_{F} n_{f}\left(-\frac{4204}{27}+\frac{352}{9} \zeta_{3}\right) \\
& +46 C_{F}^{3} T_{F} n_{f}+C_{A}^{2} T_{F}^{2} n_{f}^{2}\left(\frac{7930}{81}+\frac{224}{9} \zeta_{3}\right)+C_{F}^{2} T_{F}^{2} n_{f}^{2}\left(\frac{1352}{27}-\frac{704}{9} \zeta_{3}\right) \\
& +C_{A} C_{F} T_{F}^{2} n_{f}^{2}\left(\frac{17152}{243}+\frac{448}{9} \zeta_{3}\right)+\frac{424}{243} C_{A} T_{F}^{3} n_{f}^{3}+\frac{1232}{243} C_{F} T_{F}^{3} n_{f}^{3} \\
& +\frac{d_{A}^{a b c d} d_{A}^{a b c d}}{N_{A}}\left(-\frac{80}{9}+\frac{704}{3} \zeta_{3}\right)+n_{f} \frac{d_{F}^{a b c d} d_{A}^{a b c d}}{N_{A}}\left(\frac{512}{9}-\frac{1664}{3} \zeta_{3}\right) \\
& +n_{f}^{2} \frac{d_{F}^{a b c d} d_{F}^{a b c d}}{N_{A}}\left(-\frac{704}{9}+\frac{512}{3} \zeta_{3}\right)
\end{aligned}
$$

$$
\begin{aligned}
\beta_{4} & =\frac{8157455}{16}+\frac{621885}{2} \zeta_{3}-\frac{88209}{2} \zeta_{4}-288090 \zeta_{5} \\
& +n_{f}\left(-\frac{336460813}{1944}-\frac{4811164}{81} \zeta_{3}+\frac{33935}{6} \zeta_{4}+\frac{1358995}{27} \zeta_{5}\right) \\
& +n_{f}^{2}\left(\frac{25960913}{1944}+\frac{698531}{81} \zeta_{3}-\frac{10526}{9} \zeta_{4}-\frac{381760}{81} \zeta_{5}\right) \\
& +n_{f}^{3}\left(-\frac{630559}{5832}-\frac{48722}{243} \zeta_{3}+\frac{1618}{27} \zeta_{4}+\frac{460}{9} \zeta_{5}\right)+n_{f}^{4}\left(\frac{1205}{2916}-\frac{152}{81} \zeta_{3}\right)
\end{aligned}
$$

## 5 loop also known!



rather good convergence starting at NNLO

## QCD at work

Observable computed as an expansion in strong coupling constant

$$
\sigma=\sigma^{(0)}+\alpha_{s}(\mu) \sigma^{(1)}+\alpha_{s}^{2}(\mu) \sigma^{(2)}+\ldots
$$

Example: $\quad e^{+} e^{-} \rightarrow$ hadrons

We can not compute "hadrons" but can assume that once there are partons in the final state they will form hadrons. If we neglect some hadronization effects then "hadrons $\sim$ partons"

LO: $\quad \sigma\left(e^{+} e^{-} \rightarrow\right.$ hadrons $) \approx \sigma\left(e^{+} e^{-} \rightarrow\right.$ quarks $)$


$$
R_{\mathrm{had}} \equiv \frac{\sigma\left(e^{+} e^{-} \rightarrow \text { hadrons }\right)}{\sigma\left(e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}\right)}
$$



## R is Sensitive to number of colors!

$$
R_{\mathrm{had}} \equiv \frac{\sigma\left(e^{+} e^{-} \rightarrow \text { hadrons }\right)}{\sigma\left(e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}\right)}=\sum_{q} e_{q}^{2} N_{c}
$$

( $\mathrm{Nc}=3$

■ Quark Flavor thresholds
Compare TH to experimental data

n
2
10/3
3
4

| I/3
5

What about the next term in the expansion? $\mathcal{O}\left(\alpha_{s}\right)$
Coupling constant not so small : can lead to visible effect
Two contributions: real and virtual gluon emission
Real included because we are interested in inclusive cross section, not in cross section with a fixed number of partons in final state (which by the way can not be computed...see later..)


## Real gluon emission (massless)

Best variables to describe the process

$$
\begin{gathered}
x_{i}=\frac{2 p_{i} \cdot Q}{Q^{2}} \equiv \frac{2 E_{i}}{Q} \\
0 \leq x_{i} \leq 1
\end{gathered}
$$



Exercise: compute this!

$$
\left|M_{\text {real }}\left(x_{1,2}, x_{3}\right)\right|^{2}=C_{F} \frac{\alpha_{s}}{2 \pi} \frac{x_{1}^{2}+x_{2}^{2}}{\left(1-x_{1}\right)\left(1-x_{2}\right)}
$$

Some more kinematics (angles between final state partons)

$$
\begin{aligned}
1-x_{1} & =\frac{1}{2} x_{2} x_{3}\left(1-\cos \theta_{q g}\right) \\
1-x_{2} & =\frac{1}{2} x_{1} x_{3}\left(1-\cos \theta_{\bar{q} g}\right)_{38}
\end{aligned} \quad x_{1}+x_{2}+x_{3}=2
$$

Integrate over phase space real contribution to cross-section

Exercise: do this!

$$
\begin{gathered}
\sigma^{R}=\int_{0}^{1} d x_{1} d x_{2} d x_{3} \delta\left(2-x_{1}-x_{2}-x_{3}\right)\left|M_{\text {real }}\left(x_{1}, x_{2}, x_{3}\right)\right|^{2} \quad \text { singular at } \quad x_{i}=1 \\
\left|M_{\text {real }}\left(x_{1}, 2, x_{3}\right)\right|^{2}=C_{F} \frac{\alpha_{s}}{2 \pi} \frac{x_{1}^{2}+x_{2}^{2}}{\left(1-x_{1}\right)\left(1-x_{2}\right)}
\end{gathered}
$$

Origin of singular contributions: soft and collinear emission

$\begin{aligned} &\left|M_{\text {real }}\left(x_{1,2}, x_{3}\right)\right|^{2} \rightarrow \frac{1}{\left(1-x_{1}\right)} \underbrace{x_{1}}_{\substack{3 \pi} \frac{\alpha_{s}}{2 \pi} C_{F} \frac{1+x_{2}^{2}}{\left(1-x_{2}\right)}} \quad \begin{aligned} \text { universal splitti } \\ q \rightarrow q g\end{aligned} \\ & q \rightarrow q\end{aligned}$


$$
\sigma^{V}=\int_{0}^{1} d x_{1} d x_{2} \delta\left(2-x_{1}-x_{2}\right) \int_{0}^{\infty} d x_{3}\left|M_{\text {virtual }}\left(x_{1}, x_{2}, x_{3}\right)\right|^{2}
$$

Different phase space due to virtual gluon (instead of real)

$$
\begin{aligned}
& \int_{0}^{\infty} d x_{3} \ldots= \int_{1}^{\infty} d x_{3} \ldots+ \\
& \text { IR finite } \int_{0}^{1} d x_{3} \ldots \\
& \text { IR divergent }
\end{aligned}
$$

Looks similar to Real contribution (different kinematics)

$$
\sigma^{R}=\int_{0}^{1} d x_{1} d x_{2} d x_{3} \delta\left(2-x_{1}-x_{2}-x_{3}\right)\left|M_{\text {real }}\left(x_{1}, x_{2}, x_{3}\right)\right|^{2}
$$

and also divergent...not UV, again due to soft and collinear emission

Looks bad: computing a physical quantity ... and diverges..
Lets regularize it by introducing a gluon mass $m m_{g}$

$$
\sigma^{R}=\sigma^{(0)} C_{F} \frac{\alpha_{s}}{2 \pi}\left(\log ^{2} \frac{m_{g}^{2}}{Q^{2}}+3 \log \frac{m_{g}^{2}}{Q^{2}}+7-\frac{\pi^{2}}{3}\right)
$$

Double (log) singularities due to soft and collinear emission, one "log"per each
b) Add virtual contribution


$$
\sigma^{(N L O)}=\sigma^{(0)}\left(1+\frac{\alpha_{s}}{\pi}\right)
$$

Same singularities but opposite sign!

$$
\sigma^{V}=\sigma^{(0)} C_{F} \frac{\alpha_{s}}{2 \pi}\left(-\log ^{2} \frac{m_{g}^{2}}{Q^{2}}-3 \log \frac{m_{g}^{2}}{Q^{2}}-\frac{11}{2}+\frac{\pi^{2}}{3}\right)
$$

## Lets regularize by using dimensional regularization

Phase space and matrix elements computed in $d=4-2 \epsilon$

## phase space

$$
\int_{0}^{1} \frac{1}{1-x} d x=\infty \quad \int_{0}^{1} \frac{(1-x)^{-2 \epsilon}}{1-x} d x=-\frac{1}{2 \epsilon}
$$

$$
\sigma^{R}=\sigma^{(0)} C_{F} \frac{\alpha_{S}}{2 \pi}\left(\frac{2}{\epsilon^{2}}+\frac{3}{\epsilon}+\frac{19}{2}-\pi^{2}\right)
$$

finite real and virtual terms very different from previous slide (unphysical), but sum must be the same

$$
\sigma^{V}=\sigma^{(0)} C_{F} \frac{\alpha_{S}}{2 \pi}\left(-\frac{2}{\epsilon^{2}}-\frac{3}{\epsilon}-8+\pi^{2}\right)
$$

## Cancellation not by miracle

Since (Feynman, yes blame him!) we compute virtual and real separately: regularization needed until achieve cancellation

IR worse than UV!
Real and Virtual diagrams have very similar structure: cuts (dashed line)


In the infrared region: virtual and real are kinematically equivalent (-I) from Unitarity

## KLN Theorem

Cancellation is a general feature: Kinoshita-Lee-Nauenberg theorem Infrared singularities in massless theory cancel out after a sum over degenerate (initial and final) states


Physically a hard parton can not be distinguish from a parton plus a soft gluon or two collinear partons : degenerate states. One should add over them (to some extent/resolution) to obtain a physically sound observable

## KLN Theorem

In QED: Bloch-Nordsieck (only needs sum over final states), proved to all orders


Solution of the well-known "infrared catastrophe" in QED (soft photon emission)

We can use QCD to compute observables corresponding "inclusive enough" processes
$\rightarrow$ InfraRed safe (IRS)

Observable "insensitive" to collinear and soft emission
$e^{+} e^{-} \rightarrow q \bar{q}$ is not IRS while $\quad e^{+} e^{-} \rightarrow 2$ jets is


IR safe: KLN works
cancellation not as complete as for fully inclusive: some logs remain


$$
\alpha_{s} \log R
$$



## Infrared-safe observables (beyond total cross sections)

Definition insensitive to soft and collinear branching

## Event shape variables in e+e-




Thrust to determine spin of the gluon


Non-Abelian nature : 4 jets


Abelian


Non-Abelian contribution


Bengtsson-Zerwas: angle between the planes containing the two highest and lowest energy jets

## Color Factors

From combinations of 4-jet events \& event shapes

$\underline{Q C D}$
1.33

$$
\begin{aligned}
& C_{A}=2.89 \pm 0.21 \\
& C_{F}=1.30 \pm 0.09 \\
& \hline
\end{aligned}
$$

Multiple jet production number of jets $\sim \alpha_{S}{ }^{n-I}$


See what happens in a more complicated "environment"

Jets : several definitions available
I. How do you group particles together in a common jet? : jet algorithm 2. How do you combine the momenta of particles inside the jet? : recombination scheme

$$
\begin{aligned}
& \text { (E-scheme) add } 4 \text {-vectors } \\
& \text { (P-scheme) add } 3 \text {-vectors (E from m=0) }
\end{aligned}
$$



2-jets

number of jets depends on algorithm

First jet algorithm: Sterman-Weinberg (1977)
To study jets, we congider the partial cross section $o(E, B, \Omega, C, \delta)$ For $e^{+} e^{-}$hadron production events, in which all but
a Eraction $\mathrm{E} \ll 1$ of the total $\mathrm{e}^{+} \mathrm{e}^{-}$energy E is emitted within some pair of oppositely directed cones of half-angle $\delta$ ex 1 ,
lying within two fixed cones of solid angle $\boldsymbol{A}$ (with $\pi \delta^{2} \ll \Omega \ll 1$ ) at an angle $\theta$ to the $e^{+} e^{-}$beam line. We expect this to be measur-

$$
\begin{equation*}
\sigma(E, \theta, \Omega, \varepsilon, \delta)=(d \sigma / d \Omega)_{0} \Omega\left[1-\left(g_{E}^{2} / 3 \pi^{2}\right)\left\{3 \ln \delta+4 \ln \delta \ln 2 \varepsilon+\frac{\pi^{3}}{3}-\right.\right. \tag{o}
\end{equation*}
$$

2-jets events if fraction $1-\epsilon$ of total energy contained in 2 cones of opening angle $\delta$


Many since then, some with problems.... like infinities... Don't find infinities in experiment, but IR unsafety spoils calculations from certain orders

## introduces large sensitivity on non-perturbative physics

1. Simple to implement in an experimental analysis;

Snowmass
accord (1990)
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross sections at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronisation.

|  | Last meaningful order |  |  |  |
| :--- | :---: | :--- | :--- | :--- |
|  | JetClu, ATLAS <br> cone $[\mathrm{CC}$-SM] | MidPoint <br> $\left[\mathrm{IC} \mathrm{C}_{\text {mp }}\right.$-SM] | CMS it. cone <br> $[\mathrm{IC}-\mathrm{PR}]$ | Known at |
| Inclusive jets | LO | NLO | NLO | NLO $(\rightarrow$ NNLO) |
| $W / Z+1$ jet | LO | NLO | NLO | NLO |
| 3 jets | none | LO | LO | NLO [nlojet++] |
| $W / Z+2$ jets | none | LO | LO | NLO [MCFM] |
| $m_{\text {jet }}$ in $2 j+X$ | none | none | none | LO |

Popular algorithms for hadron colliders: $\mathrm{k}_{T}$ and anti- $\mathrm{k}_{T}$
Sequential recombination (bottom-up approach)

$$
\mathrm{k}_{\mathrm{T}}
$$

$$
\begin{aligned}
& d_{i j}=\min \left(p_{t i}^{2}, p_{t j}^{2}\right) \frac{\Delta R_{i j}^{2}}{R^{2}} \text { distance parameter for pairs } \\
& \qquad \Delta R_{i j}^{2}=\left(\Delta \eta_{i j}\right)^{2}+\left(\Delta \phi_{i j}\right)^{2} \\
& d_{i B}=p_{t i}^{2} \quad \text { distance parameter to beam }
\end{aligned}
$$

Search for smallest distance among all possibilities
-if $\mathrm{d}_{\mathrm{i}}$ then particle i removed from list of particles and called a jet
-if $d_{i j}$ then particles $i$ and $j$ are recombined in a single particle


Jets irregular : soft particles recombine at the initial stages
-Acceptance corrections

- Underlying event corrections
- Energy calibration

Can "undo" clustering sequence and look inside the jet

## anti- $\mathrm{k}_{\mathrm{T}}$

"invert" distance measure

$$
\begin{aligned}
d_{i j} & =\frac{1}{\max \left(p_{t i}^{2}, p_{t j}^{2}\right)} \frac{\Delta R_{i j}^{2}}{R^{2}} \\
d_{i B} & =\frac{1}{p_{t i}^{2}}
\end{aligned}
$$



Soft particles recombine early but preferably with hard particles : jets grow in concentric circles (like cone)

Can not look inside jet

## Recap of first lecture

OColor "explains" hadron spectrum : charge of QCD
OQCD Lagrangian derived from gauge principle with non-abelian group $S U(3)$ : Feynman rules for perturbative calculations

OThere are UV divergences dealt by renormalization : as a result running coupling constant

OTwo faces of QCD : asymptotically free and consistent with confinement

OThere are also IR divergences that cancel when adding real and virtual contributions

OJet algorithm is relevant to define IR safe observables
OQCD at work in e+e- : test the nature of $\operatorname{SU}(3) \mathrm{OK}$ !

