QCD

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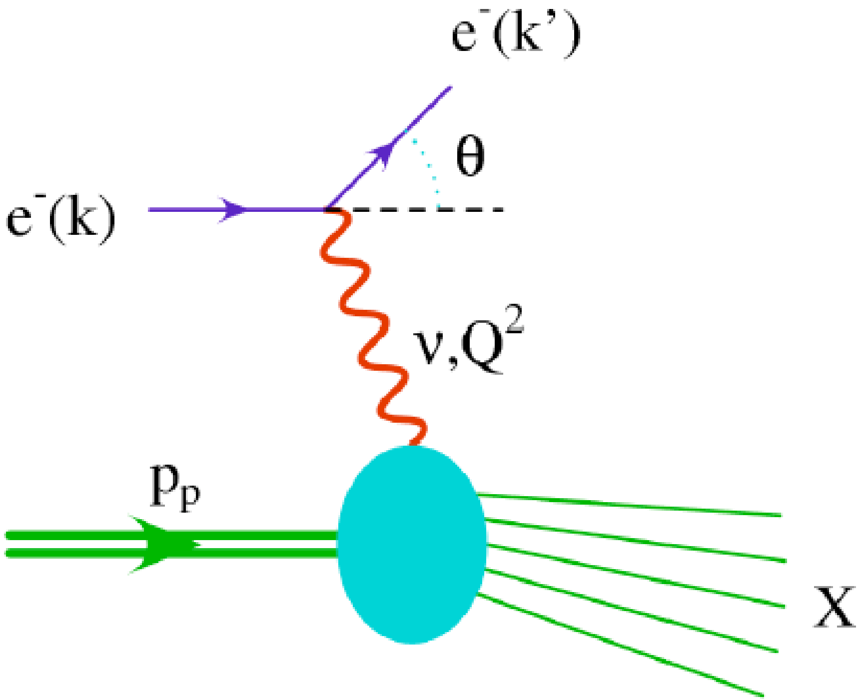
Recap of first lecture

- Color "explains" hadron spectrum : charge of QCD
- QCD Lagrangian derived from gauge principle with non-abelian group SU(3) : Feynman rules for perturbative calculations
- There are UV divergences dealt by renormalization : as a result running coupling constant
- Two faces of QCD : asymptotically free and consistent with confinement
- There are also IR divergences that cancel when adding real and virtual contributions
- Jet algorithm is relevant to define IR safe observables
- QCD at work in e+e- : test the nature of SU(3) OK!

Outline of the lecture 2

- Deep Inelastic Scattering
- Parton Model
- Scaling Violations and Evolution
- Factorization
- Parton Distribution Functions

Deep inelastic scattering



$s = (P+k)^2$ cms energy^2

$Q^2 = -(k-k')^2$ momentum transfer^2

$x = Q^2/2(P\cdot q)$ scaling variable

$\nu = (P\cdot q)/M = E - E'$ energy loss

$y = (P\cdot q)/(P\cdot k) = 1 - E'/E$ rel. energy loss

$W^2 = (P+q)^2 = M^2 + \frac{1-x}{x}Q^2$ recoil mass

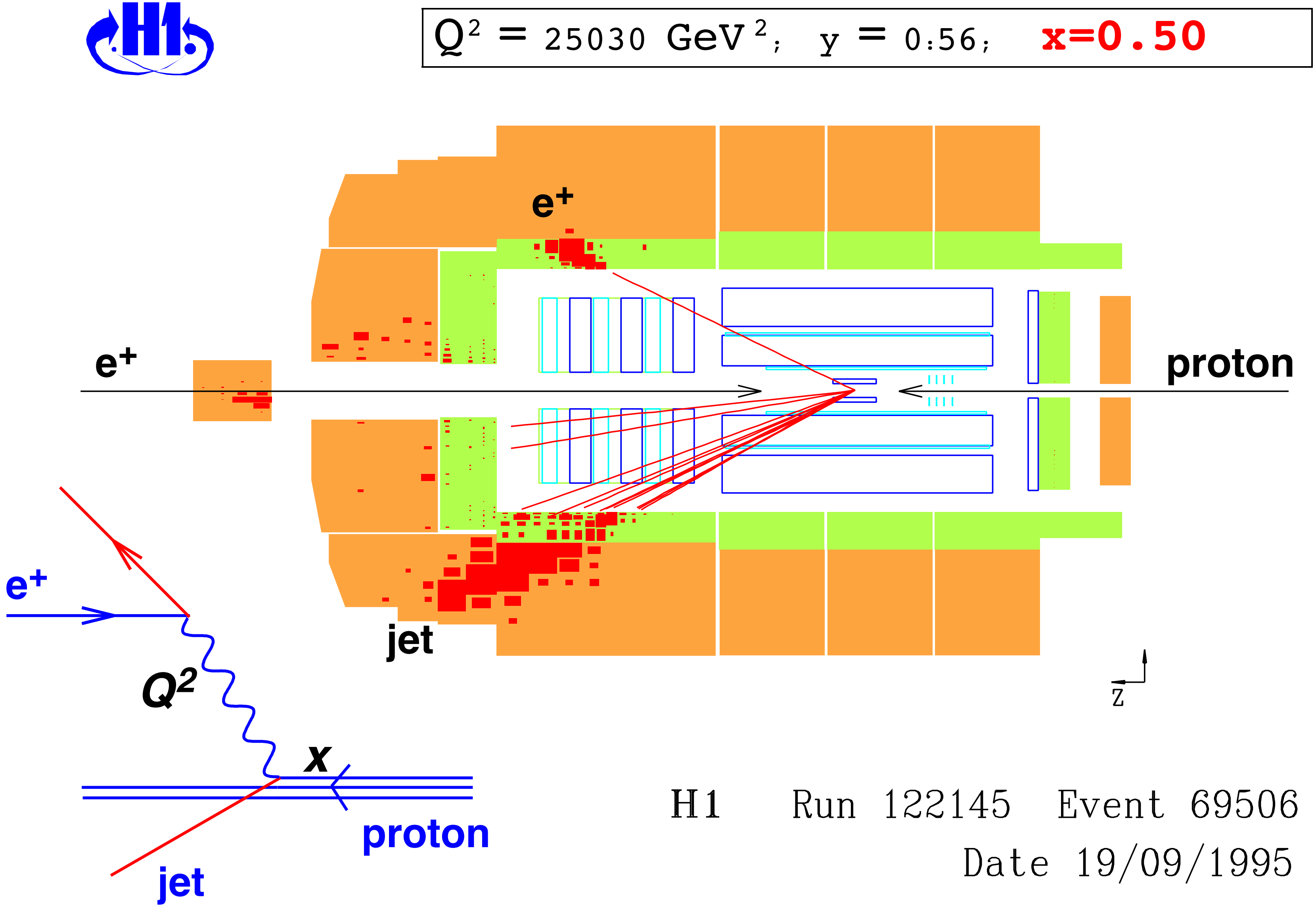
★ Photon virtuality : transverse resolution at which it probes proton structure (quantum wavelength) $\lambda \sim 1/Q$

large virtuality ~ better resolution

"Deep Inelastic" $Q, W \gg M$

Large virtuality and recoil mass (inelastic)

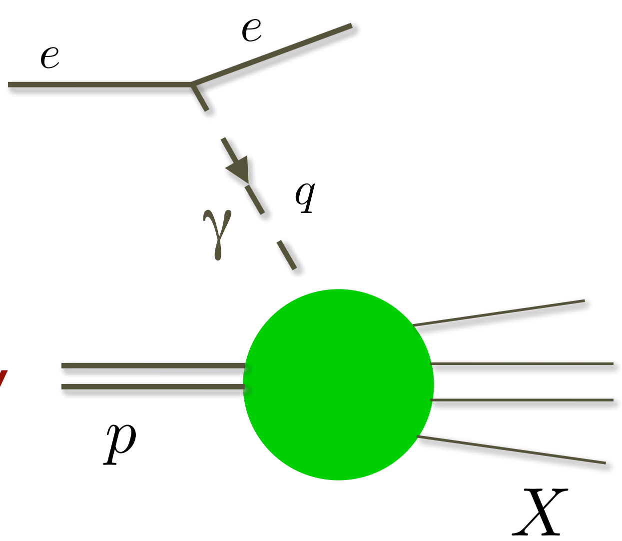
One example at HERA e@27.5 GeV x p@920 GeV



Deep inelastic scattering

$$ep \to eX$$

$$Q^2 = -q^2 \qquad x = \frac{Q^2}{2\, p \cdot q}$$



If $Q^2 < M_Z^2$ the cross section is dominated by one-photon exchange

$$k_0' \frac{d\sigma}{d^3k'} = \frac{1}{k \cdot p} \left(\frac{\alpha}{q^2}\right)^2 L^{\mu\nu} W_{\mu\nu}$$

sum over final states

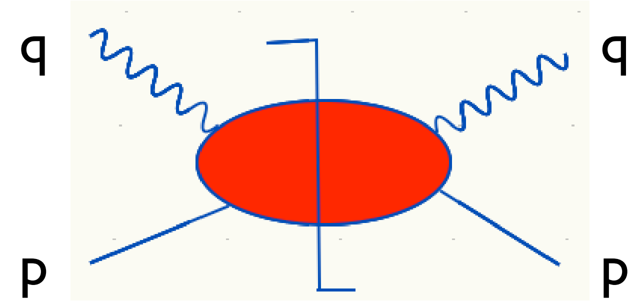
Leptonic tensor: computable QED

Hadronic tensor

$$L^{\mu\nu} = \frac{1}{4}\mathrm{tr}[\not{k}\gamma^\mu \not{k}'\gamma^\nu] = k^\mu k'^\nu + k'^\mu k^\nu - g^{\mu\nu} k \cdot k'$$

Hadronic tensor can not be computed perturbatively : involves proton

$$W_{\mu\nu} = \frac{1}{2\pi} \int d^4y \, e^{iqy} \, \langle p| J_\mu(y) J_\nu(0) |p\rangle$$



We can construct the most general tensor: parameterized by several structures but there are restrictions from parity, current conservation

$$\partial_\mu J^\mu = 0 \quad \Rightarrow \quad q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$$

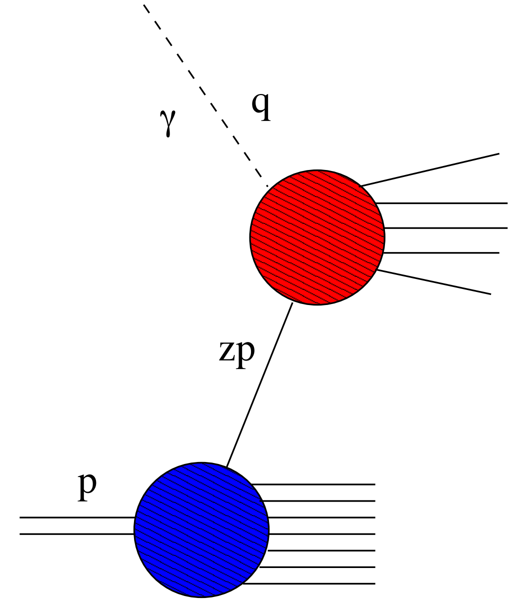
Only two structures survive (photon exchange, no spin)

$$= F_1 \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) + F_2 \frac{1}{p\cdot q} \left(p_\mu - \frac{p\cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p\cdot q}{q^2} q_\nu \right)$$

Structure functions $F_i(x, Q^2)$

Structure functions contain information on proton structure

Parton Model



Proton made up of pointlike particles : partons

★ Photon virtuality sets resolution $\lambda \sim 1/Q$

★ Photon-quark Interaction $t_{hard} \sim 1/Q$

★ Interaction between partons $t \sim 1/\Lambda_{QCD}$

As $Q >> \Lambda_{QCD}$

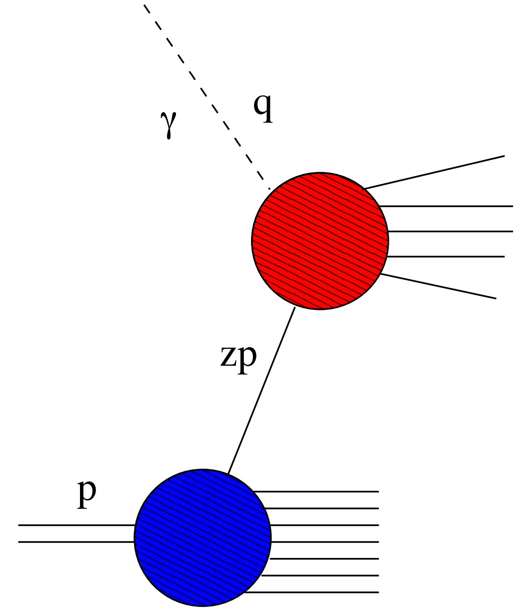
During "hard interaction", partons don't have time to interact among them, behave as if they were free (snapshot of the proton)

➡ Scattering is incoherent on the single partons

Hadron is a jet of partons moving in the same direction and sharing the momentum and energy (fraction z) Infinite momentum frame

(Naive) Parton Model

Factorization



$$\sigma(ep \to eX) = \int_0^1 dz \sum_i f_i(z)\,\hat{\sigma}(eq_i \to eX)$$

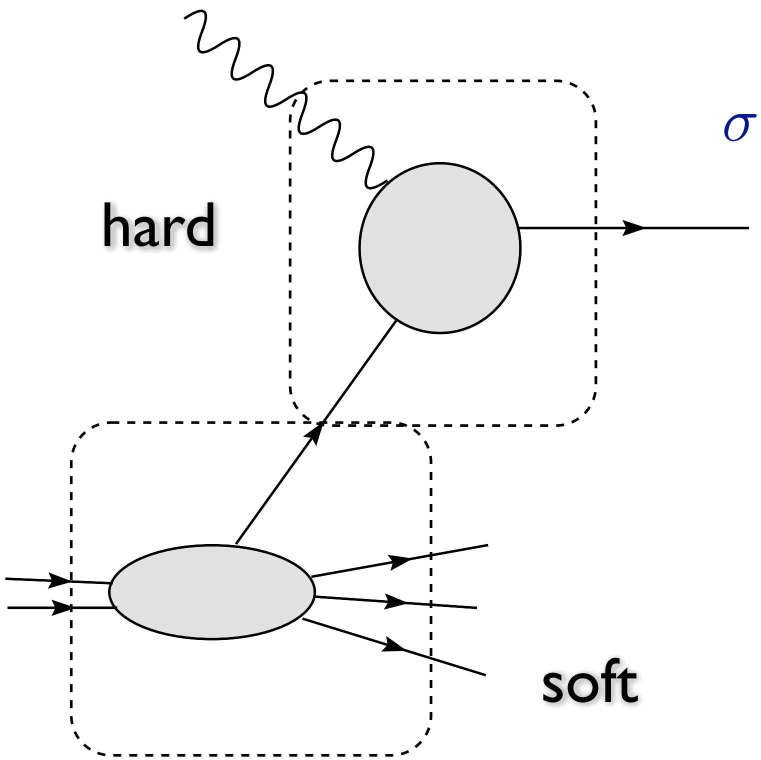
large distances ($f_i(z)$)

small distances ($\hat{\sigma}$)

- ☑ Probability to find parton "i" with momentum fraction z in proton
- ☑ $\hat{\sigma}$ ="partonic" cross section → computed perturbatively
- ☑ Parton distributions (PDF) are ***universal :*** the same for any process

Universality is the key, we can not compute them perturbatively (no way to compute it precisely enough even with other methods) but we can extract it from known processes and use if for predictions

Factorization (Naive parton model)



large distances

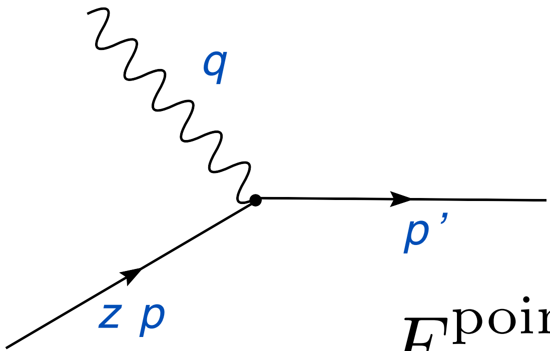
$$\sigma(ep \to eX) = \int_0^1 dz \sum_i f_i(z)\,\hat{\sigma}(eq_i \to eX)$$

small distance

At this order separation between hard and soft component is unambiguous. Things become much more complicated when higher order corrections are accounted for.

At lowest order

What happens if photon interacts with pointlike particle?



$$(p')^2 = (zp+q)^2 = 2z\; p\cdot q - Q^2 = 0 \quad \rightarrow \quad z = x$$

only couples to quark with mom. fraction *x*!

$$F_2^{\text{pointlike}} \sim e_q^2\, x\, \delta(z-x)$$

no Q : scaling!

- Point-like interaction → scaling (and "direct" access to *x*)

$$F_2(x, \cancel{Q}^2) = \sum_q e_q^2\, x\, f_q(x)$$

- Quarks are fermions → no coupling to longitudinal photons, only transverse polarization (Callan-Gross relation)

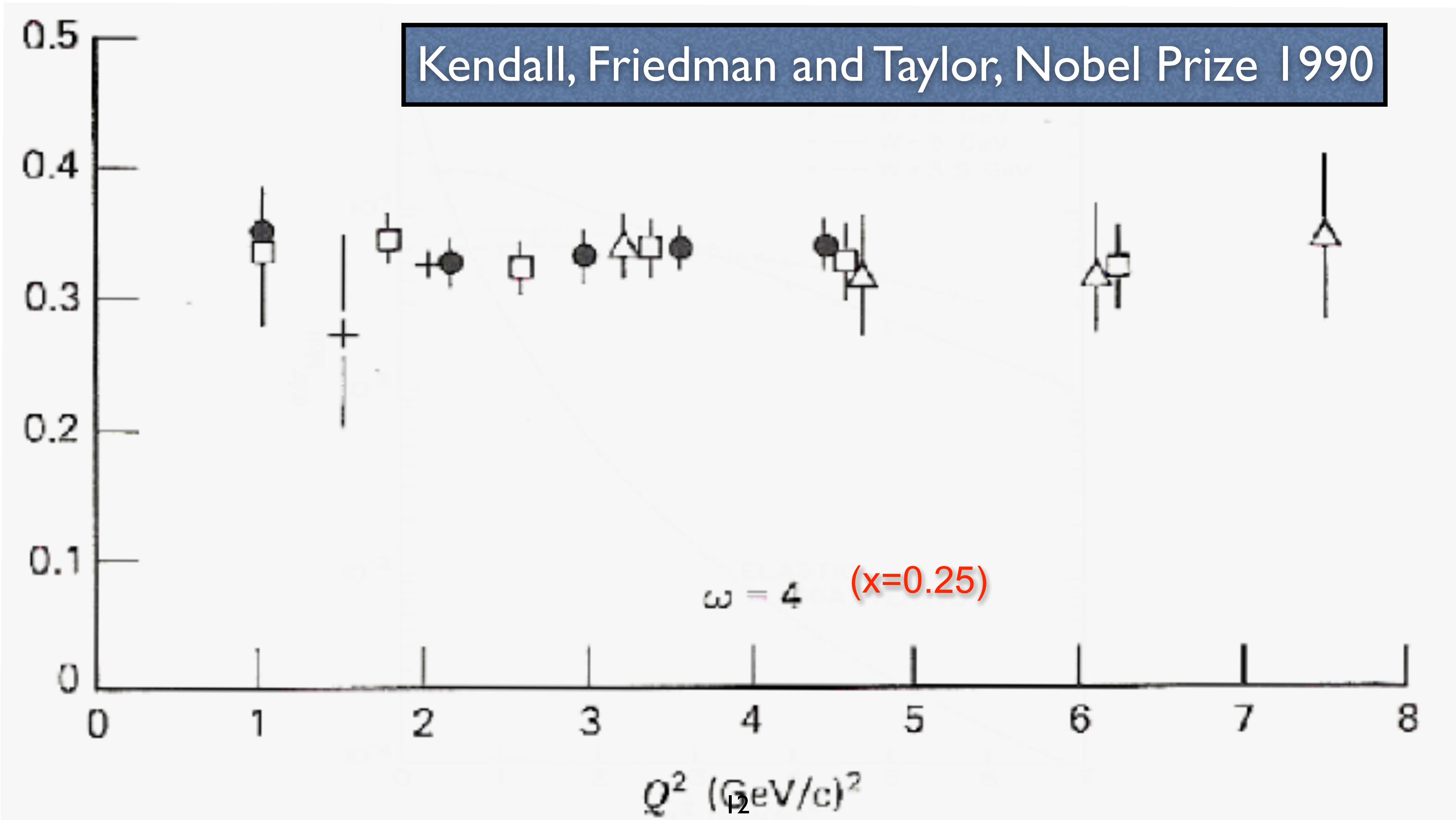
$$F_L(x,Q^2) = F_2(x,Q^2) - 2xF_1(x,Q^2) = 0\,!$$

If quarks were scalars $F_1=0$

Cross section at lowest order: only F_2

$$\frac{d^2\sigma}{dx dQ^2} = \frac{2\pi\alpha^2}{xQ^4}[(1+(1-y)^2)F_2(x) - y^2 F_L(x)]$$

Scaling (Bjorken 1968, SLAC data)



Proton structure function (with electron scattering) is

$$F_2^{ep}/x = \frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x) + \frac{1}{9}s(x) + \frac{1}{9}\bar{s}(x) + \frac{4}{9}c(x) + \frac{4}{9}\bar{c}(x)$$

Same applies for neutron but with "neutron parton distributions"

Actually, can relate neutron to proton PDFs using isospin symmetry

$$f_{u/n}(x) = f_{d/p}(x) \equiv d(x)$$
$$f_{\bar{u}/n}(x) = f_{\bar{d}/p}(x) \equiv \bar{d}(x)$$
$$f_{d/n}(x) = f_{u/p}(x) \equiv u(x)$$
$$f_{s/n}(x) = f_{s/p}(x) \equiv s(x)$$

(p ↔ n)

(usually better than % accuracy)

$$F_2^{en}/x = \frac{1}{9}u(x) + \frac{4}{9}d(x) + \frac{1}{9}\bar{u}(x) + \frac{4}{9}\bar{d}(x) + \frac{1}{9}s(x) + \frac{1}{9}\bar{s}(x) + \frac{4}{9}c(x) + \frac{4}{9}\bar{c}(x)$$

In real life one measures deuteron (p+n) structure functions

But ep/en DIS does not provide access to $q - \bar{q}$

Photon interacts the same way with quarks and antiquarks ~e_q^2

$$F_2^{ep}/x = \frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\bar{u}(x) + \frac{1}{9}\bar{d}(x) + \frac{1}{9}s(x) + \frac{1}{9}\bar{s}(x) + \frac{4}{9}c(x) + \frac{4}{9}\bar{c}(x)$$

W's interact differently with quarks and antiquarks

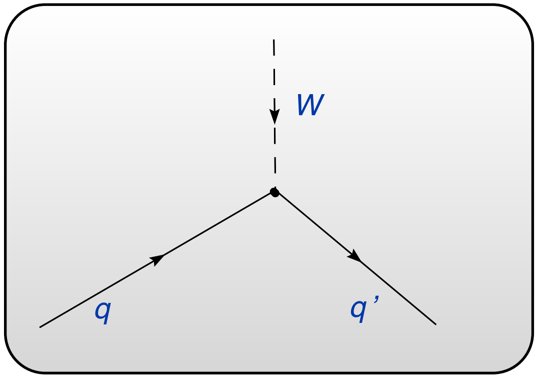
For weak interactions: parity violation, extra term in hadronic tensor

→ F_3

$$\frac{d^2\sigma(^{\nu}_{\bar{\nu}} + p)}{dx\,dQ^2} = \frac{G_F^2}{4\pi x}\left(\frac{M_w^2}{Q^2 + M_w^2}\right)^2 \left[\left(1 + (1-y)^2\right) F_2^{^{\nu}_{\bar{\nu}}} - y^2 F_L^{^{\nu}_{\bar{\nu}}} \pm \left(1 - (1-y)^2\right) x F_3^{^{\nu}_{\bar{\nu}}}\right]$$

$$\begin{aligned} F_2^{\nu p}/x &= 2d(x) + 2\bar{u}(x) + 2s(x) + 2\bar{c}(x) \\ F_3^{\nu p} &= 2d(x) - 2\bar{u}(x) + 2s(x) - 2\bar{c}(x) \\ F_2^{\bar{\nu} p}/x &= 2u(x) + 2\bar{d}(x) + 2c(x) + 2\bar{s}(x) \\ F_3^{\bar{\nu} p} &= 2u(x) - 2\bar{d}(x) + 2c(x) - 2\bar{s}(x) \end{aligned}$$

W^+ (for νp), W^- (for $\bar{\nu} p$)



Measuring several DIS cross-sections

→ Extraction of quark distributions possible

What does it mean that proton has two up and one down quark?

Valence distributions

$$u_v(x) = u(x) - \bar{u}(x)$$

$$d_v(x) = d(x) - \bar{d}(x)$$

Sum Rules

$$\int_0^1 dx\, u_v(x) = 2$$

$$\int_0^1 dx\, d_v(x) = 1 \qquad \int_0^1 dx\, [u(x) + \bar{u}(x)] = \infty$$

$$s(x) \neq \bar{s}(x) \qquad \int_0^1 dx\, s_v(x) = 0$$

Notice that number of quarks plus antiquarks can be infinity!

Momentum of the proton distributed among components

50%

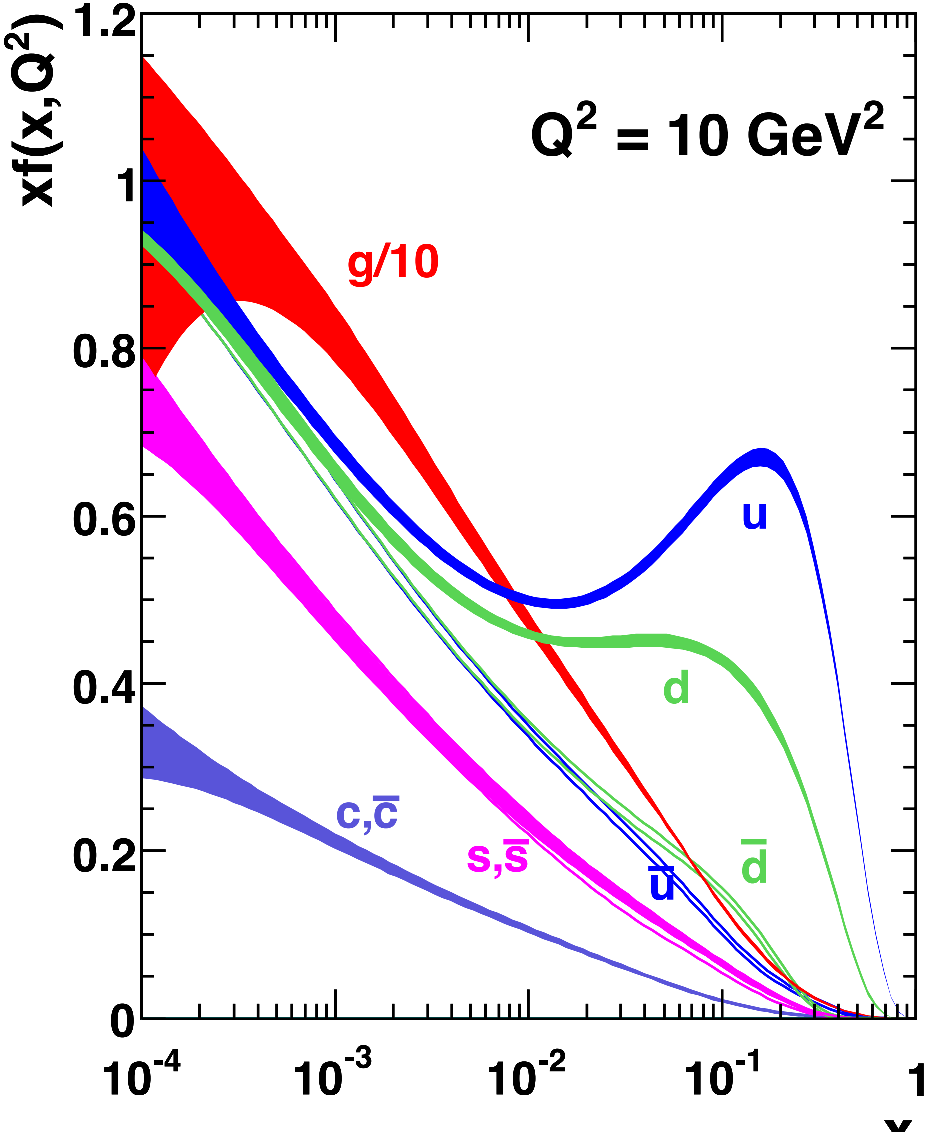
$$\int_0^1 dx \sum_q [x\, q(x) + x\, \bar{q}(x)] + \int_0^1 dx\, x\, g(x) = 1$$

Parton Distributions

How do they look like?

- Vanish when $x \to 1$
- "Quark" peak at $x \sim 1/3$
- Gluon and "sea" rise as $x \to 0$

radiation of soft particles



Notice gluon divided by 10 ! : gluon distribution is huge in kinematical region relevant for LHC

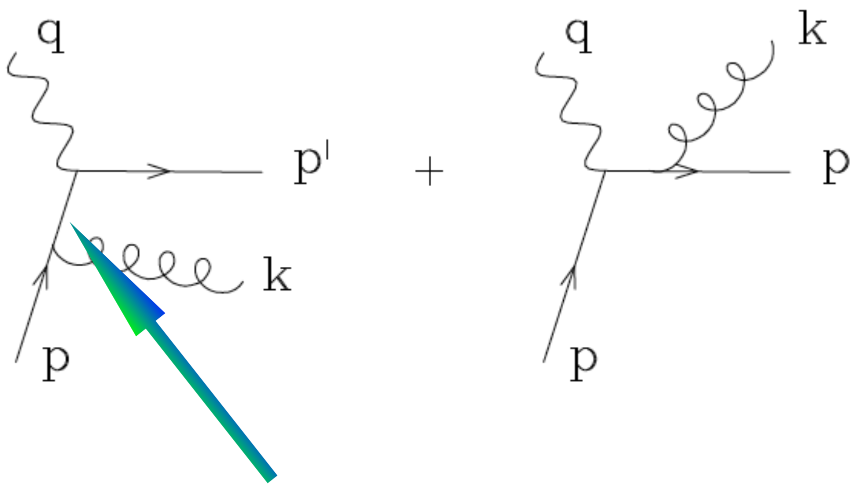
LHC is a "gluon Collider"

QCD corrections and scaling violation

Does simple parton model survive at higher orders?

Quarks can radiate gluons : real corrections

which x?



Divergences again ...

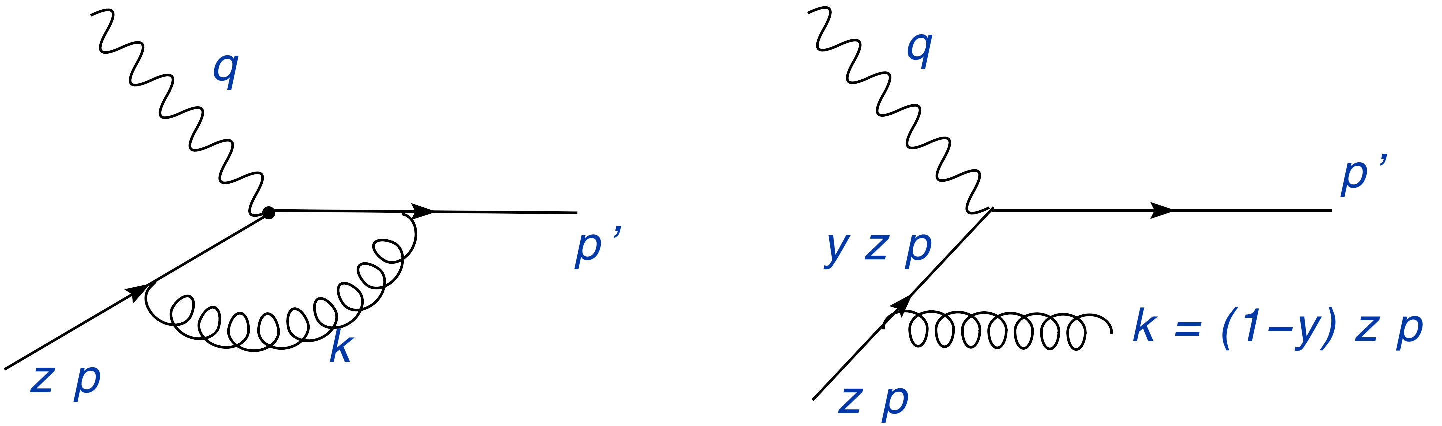
$$\frac{-1}{(p-k)^2} = \frac{1}{2\,p\cdot k} = \frac{1}{2E_qE_g(1-\cos\theta)}$$

when gluon has no transverse momentum $k_T \sim E_g \sin\theta \to 0$

Will virtual contributions solve the problem again?

No (not all of them)!!!

Virtual and Real contribute to different kinematics



virtual $$(p')^2 = (zp+q)^2 = 2z\ p\cdot q - Q^2 = 0 \quad \longrightarrow \quad z = x$$

real $$(p')^2 = (zp+q-k)^2 \sim 2zy\, p\cdot q - Q^2 = 0 \quad \longrightarrow \quad zy = x$$

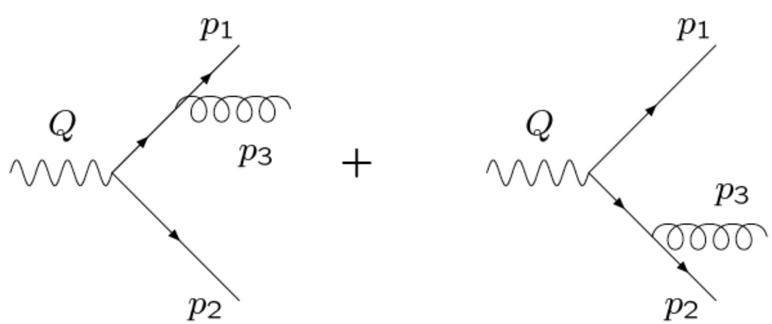
Sum of real + virtual: soft singularities cancelled (*y*=1)

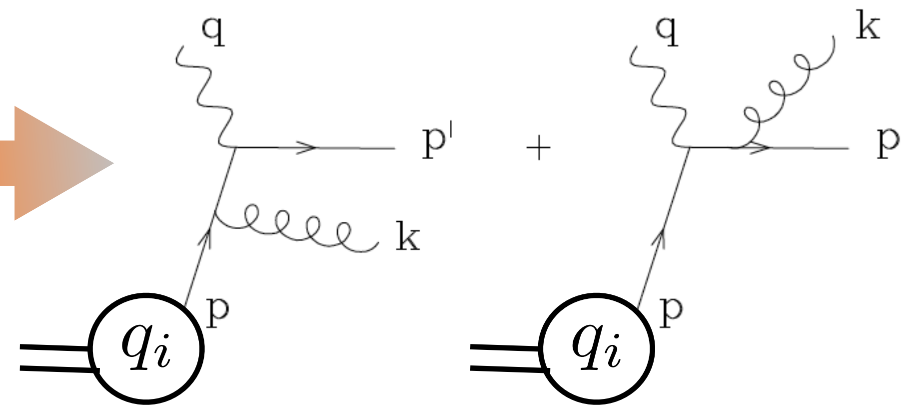
But for other values of *y*, singularities (collinear) remain ...

Why cancellation does not occur?

Feynman diagrams are the same as in $e^+e^- \to \text{hadrons}$

rotation!



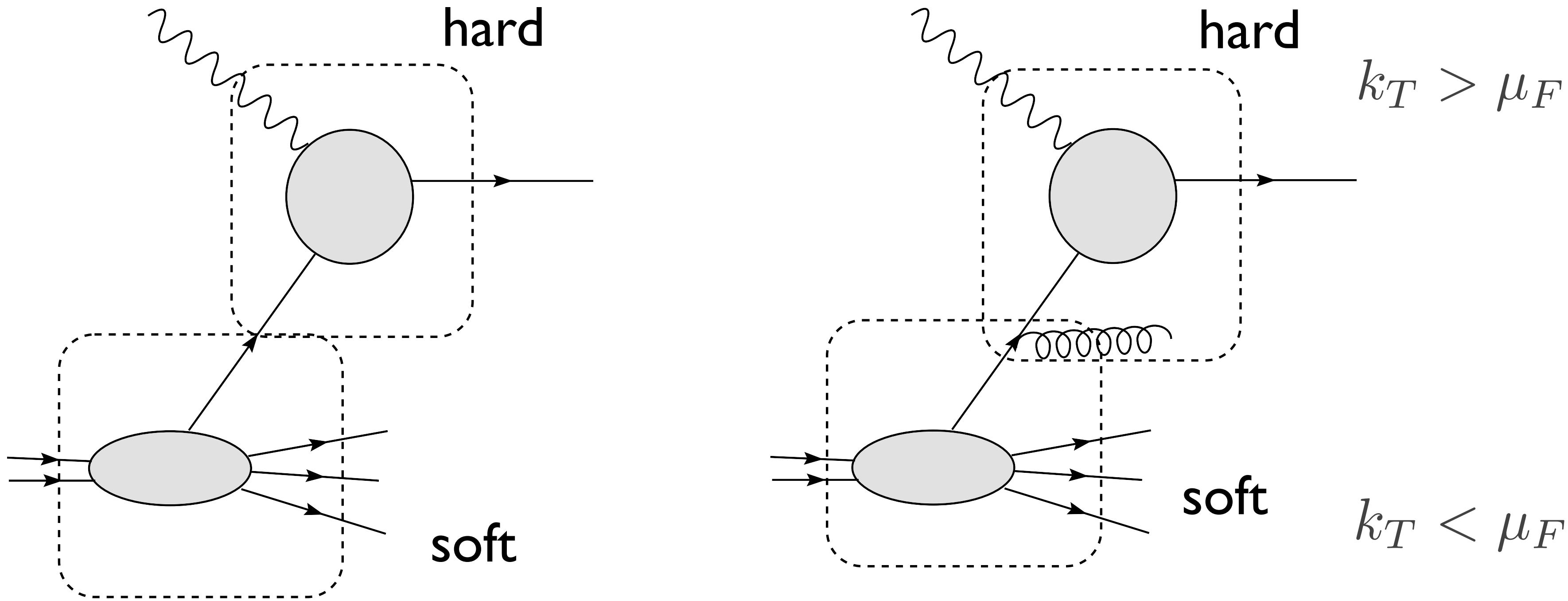


KLN: Infrared singularities in massless theory cancel out after a sum over degenerate (initial and final) states. But here we are not adding over initial states, we assume "identified and free" colored parton attached to proton with corresponding pdf

Cross section with incoming parton is collinear unsafe

Collinear (IR) configuration corresponds to non-perturbative regime

Parton model: separation between soft and hard physics

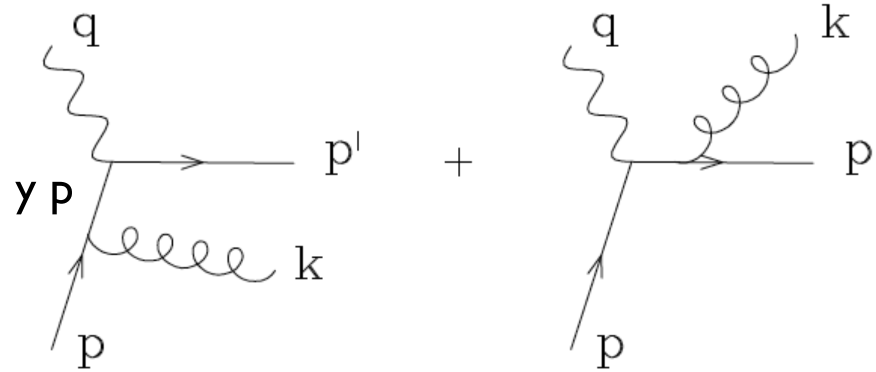


Define "hardness" of contributions by using some kinematical variable (virtuality of quark/ transverse momentum of gluon)

Introduce new (ad-hoc) factorization scale to separate hard from soft

$$\mu_F$$

Real contribution



$$\sum_q e_q^2\, x\, \frac{\alpha_s}{2\pi} \int_0^{\sim Q^2} \frac{dk_T^2}{k_T^2} \int_x^1 \frac{dy}{y} \underbrace{\left[C_F \frac{1+y^2}{1-y}\right]}_{P_{qq}(y)} q\left(\frac{x}{y}\right) + \text{finite}$$

$P_{qq}(y)$ Prob. collinear emission

Regularize the divergence with a cut-off $\mu_0^2 \lesssim k_T^2 < Q^2$

$$F_2^{cor}(x,Q^2) = \sum_q e_q^2\, x\, \frac{\alpha_s}{2\pi} \log\left(\frac{Q^2}{\mu_0^2}\right) \int_x^1 \frac{dy}{y} P_{qq}(y)\, q\left(\frac{x}{y}\right) + \text{finite}$$

First thing to notice: scaling broken due to gluon radiation

$$F_2^{cor}(x,Q^2) = \sum_q e_q^2\, x\, \frac{\alpha_s}{2\pi} \log\left(\frac{Q^2}{\mu_0^2}\right) \int_x^1 \frac{dy}{y} P_{qq}(y)\, q\left(\frac{x}{y}\right) + \text{finite}$$

$$\log\left(\frac{Q^2}{\mu_0^2}\right) = \log\left(\frac{\mu_F^2}{\mu_0^2}\right) + \log\left(\frac{Q^2}{\mu_F^2}\right)$$

soft (and divergent) to PDF

Hard (and finite)

Factorization (in pdfs) IR equivalent to UV renormalization

$$q(x,\mu_F^2) = q(x) + \frac{\alpha_s}{2\pi} \log\left(\frac{\mu_F^2}{\mu_0^2}\right) \int_x^1 \frac{dy}{y} P_{qq}(y)\, q\left(\frac{x}{y}\right)$$

Factorization scale unphysical, typically chosen as $\mu_F = \mu_R = Q$

fixed order calculation shows "spurious" factorization scale dependence

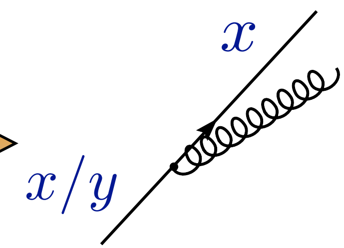
Scaling broken, but we can predict dependence on virtuality perturbatively (not on *x*)

$$q(x,\mu_F^2) = q(x) + \frac{\alpha_s}{2\pi}\log\left(\frac{\mu_F^2}{\mu_0^2}\right)\int_x^1 \frac{dy}{y} P_{qq}(y)\, q\left(\frac{x}{y}\right)$$

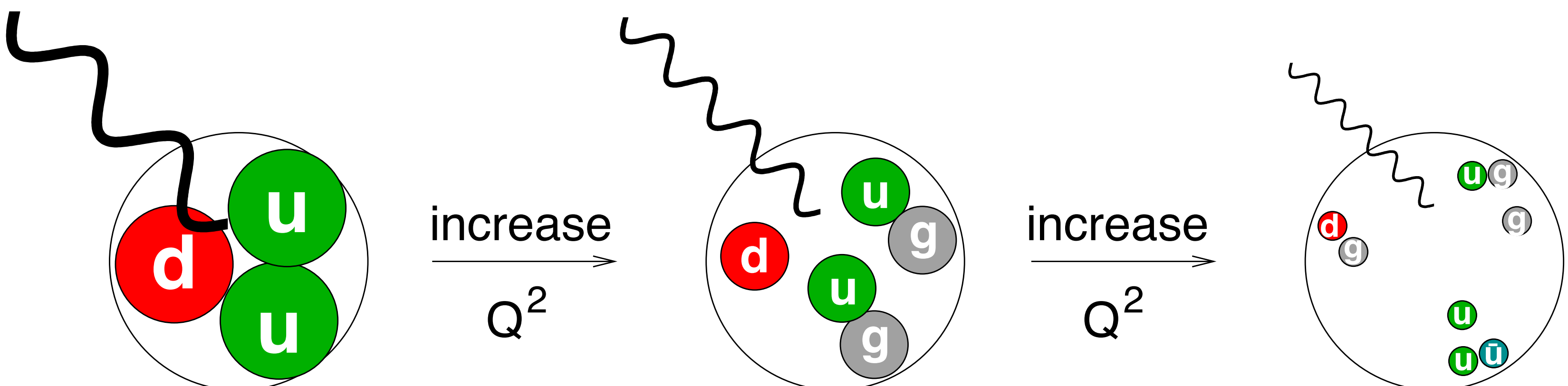
Altarelli-Parisi equation (RGE like: resummation of collinear logs)

DGLAP : Dokshitzer, Grivov, Lipatov, Altarelli, Parisi

$$\frac{\partial q(x,\mu_F^2)}{\partial \log(\mu_F^2)} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} P_{qq}(y)\, q\left(\frac{x}{y},\mu_F^2\right)$$



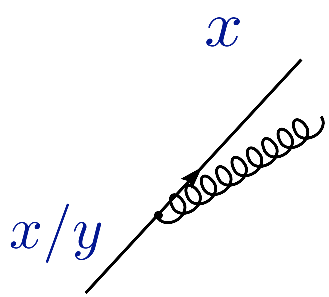
Increase "resolution" scale: resolve more details of "partonic structure"

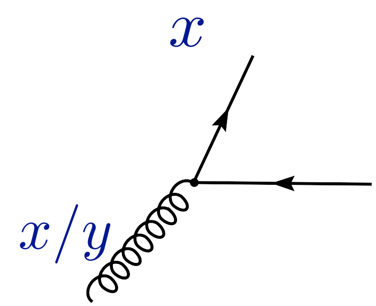


To have the complete picture we have to account for contributions initiated by gluons in the proton

$$\frac{\partial q(x,\mu_F^2)}{\partial \log(\mu_F^2)} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} P_{qq}(y)\, q\left(\frac{x}{y},\mu_F^2\right) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} P_{qg}(y)\, g\left(\frac{x}{y},\mu_F^2\right)$$

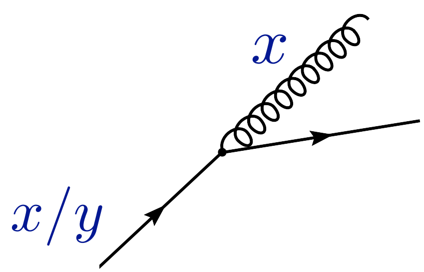
Probabilistic interpretation

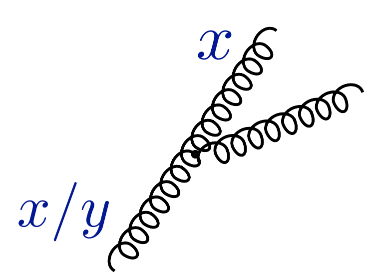




Similarly for gluon distribution

$$\frac{\partial g(x,\mu_F^2)}{\partial \log(\mu_F^2)} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} P_{gq}(y) \sum_q q\left(\frac{x}{y},\mu_F^2\right) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} P_{gg}(y)\, g\left(\frac{x}{y},\mu_F^2\right)$$





Not trivial to solve AP equations in x-space due to its nature and convolutions. But much simpler with moments

$$\frac{\partial q(x,\mu_F^2)}{\partial \log(\mu_F^2)} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} P_{qq}(y)\, q\left(\frac{x}{y},\mu_F^2\right) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y} P_{qg}(y)\, g\left(\frac{x}{y},\mu_F^2\right)$$

Mellin moments

$$F_n \equiv \int_0^1 \frac{\mathrm{d}x}{x}\, x^n\; F(x)$$

Mellin space : convolutions turn into products $\int f \otimes g \to f^n \times g^n$

$$\frac{\partial q(N,\mu_F^2)}{\partial \log(\mu_F^2)} = \frac{\alpha_s}{2\pi} P_{qq}(N)\, q\left(N,\mu_F^2\right) + \frac{\alpha_s}{2\pi} P_{qg}(N)\, g\left(N,\mu_F^2\right)$$

Convenient to split into non-singlet (valence-like) and singlet

non-singlet
$$V(x) = \sum_i f_i(x) - \sum_{\bar{\imath}} f_{\bar{\imath}}(x)$$

singlet
$$\Sigma(x) = \sum_i f_i(x) + \sum_{\bar{\imath}} f_{\bar{\imath}}(x)$$

Evolution equations become:

$$\frac{dV^{(n)}}{dt} = \frac{\alpha_s}{2\pi} P_{qq}^{(n)} V^{(n)}$$

$$\frac{d\Sigma^{(n)}}{dt} = \frac{\alpha_s}{2\pi} \left[P_{qq}^{(n)} \Sigma^{(n)} + 2n_f P_{qg}^{(n)} f_g^{(n)} \right]$$

$$\frac{df_g^{(n)}}{dt} = \frac{\alpha_s}{2\pi} \left[P_{gq}^{(n)} \Sigma^{(n)} + P_{gg}^{(n)} f_g^{(n)} \right]$$

They have analytical solution in Mellin space, specially simpler for non-singlet, driven by coupling constant and anomalous dimensions

non-singlet $$q_{NS}(N,Q^2) = \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right]^{-2P_{qq}(N)/\beta_0} q_{NS}(N,Q_0^2)$$

anomalous dimension

Evolution performed in Mellin space and the inverted back to x

$$F(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \mathrm{d}n \, x^{-n} F_n$$

- Valence Quark number conservation is also simple in Mellin space

First moment $$V^{(1)} = \int_0^1 dx\, V(x)$$

Valence Number conservation $$\frac{dV^{(1)}}{dt} \equiv 0 = \frac{\alpha_s}{2\pi} P_{qq}^{(1)} V^{(1)} = 0$$

- Helps to fix virtual contribution at *z=1*

$$P_{qq}^{(0)} = C_F \frac{1+z^2}{1-z} \quad \Rightarrow \quad C_F \left[\frac{1+z^2}{(1-z)_+} + A\,\delta(1-z) \right]$$

"+" Distribution $$\int_0^1 \frac{f(z)}{(1-z)_+} \equiv \int_0^1 \frac{f(z)-f(1)}{1-z}$$

$$P_{qq}^{(0)}(z) = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z) \right]$$

- Momentum conservation is also simple in Mellin space

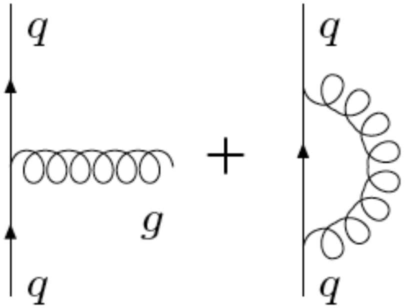
$$\int_0^1 dx\, x \left[\sum_{i,\bar{i}} f_i(x) + f_g(x) \right] \equiv \Sigma^{(2)} + f_g^{(2)} = 1$$

- Due to quark and gluon evolution two conditions must be fulfilled for the second moment of the splitting functions

$$\begin{aligned} P_{qq}^{(2)} + P_{gq}^{(2)} &= 0 \\ P_{gg}^{(2)} + 2n_f\, P_{qg}^{(2)} &= 0 \end{aligned}$$

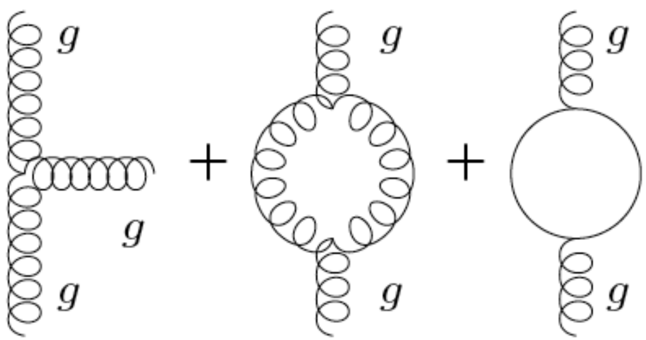
First one confirms result from fermion number in qq kernel

Second one used to fix z=1 behavior of gg kernel

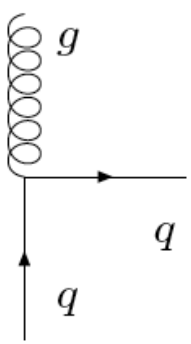


$$P^{(0)}_{qq}(z) = C_F\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

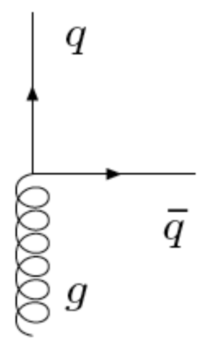
Altarelli and Parisi, NPB126 (1977) 298



$$P^{(0)}_{gg}(z) = 2C_A\left[\frac{z}{(1-z)_+} + \frac{1-z}{z} + z(1-z)\right] + \left(\frac{11}{6}C_A - \frac{2}{3}T_R\, n_F\right)\delta(1-z)$$



$$P^{(0)}_{gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right]$$



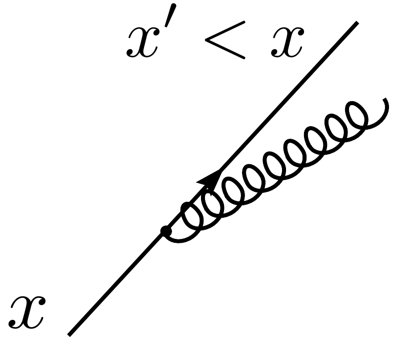
$$P^{(0)}_{qg}(z) = T_R\left[z^2 + (1-z)^2\right]$$

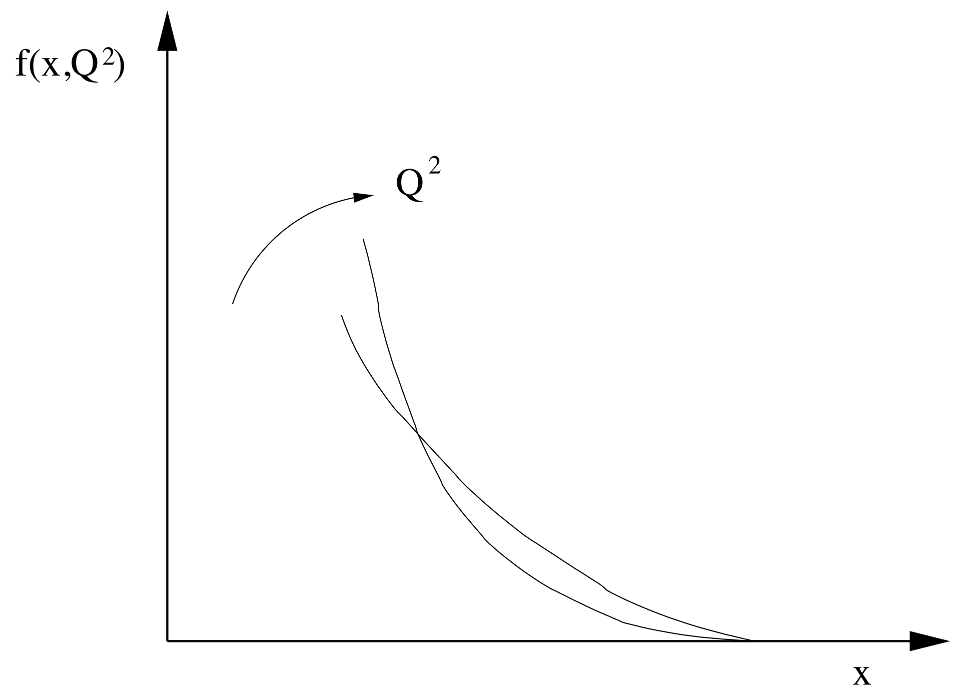
Scaling violations are:

- Positive at small *x* *(more partons with smaller energy)*
- Slightly negative at large *x*

Main effect of increase in Q^2 is shift of partons from larger to smaller *x*

Resolve shorter distances in the proton: quark with fraction x can be resolved as a qg pair (quark with smaller momentum)

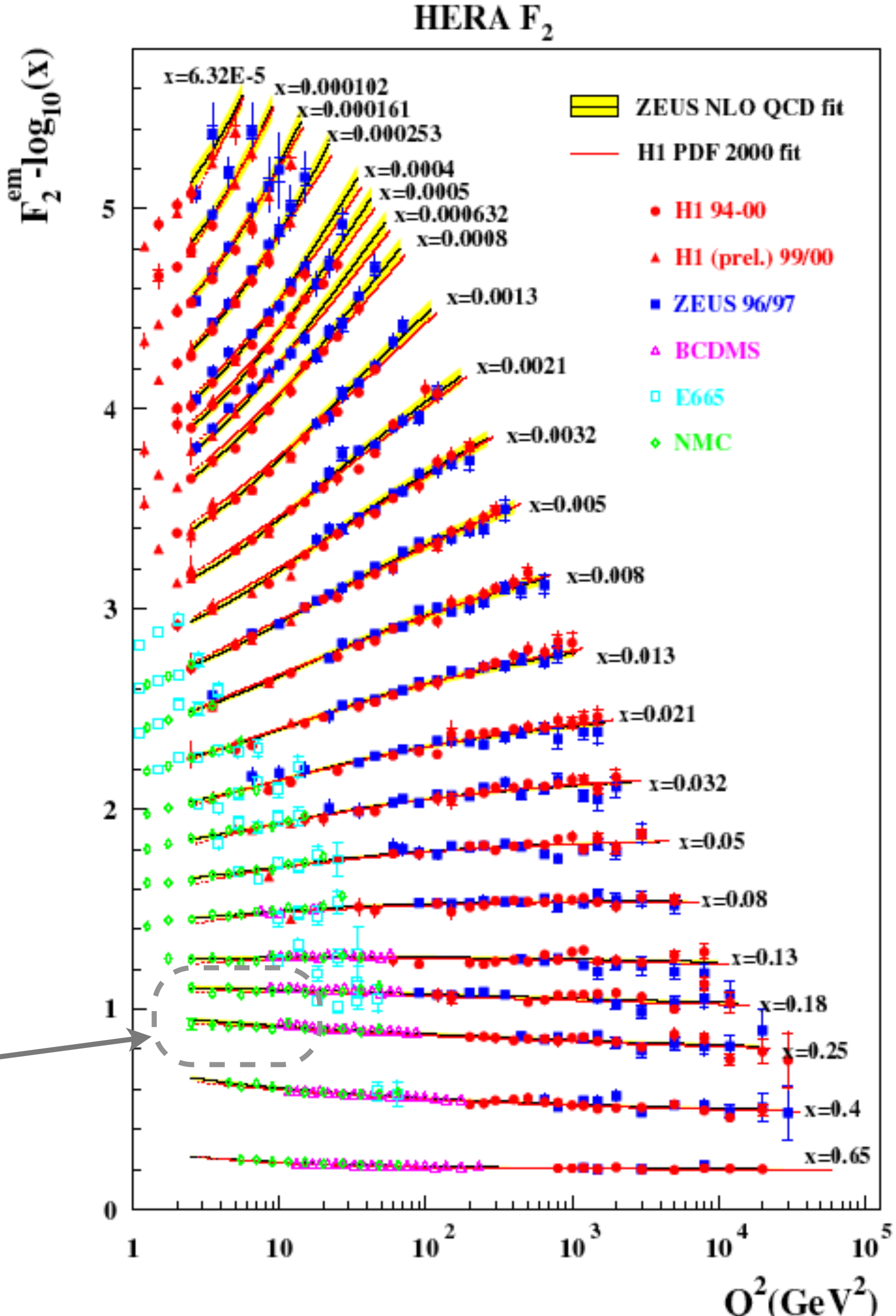




AP Evolution equations allow to predict the Q^2 dependence of DIS data

And very well!

Region studied to find scaling!



pQCD vocabulary: LO-NLO-NNLO-...

Improved (factorized) Parton Model

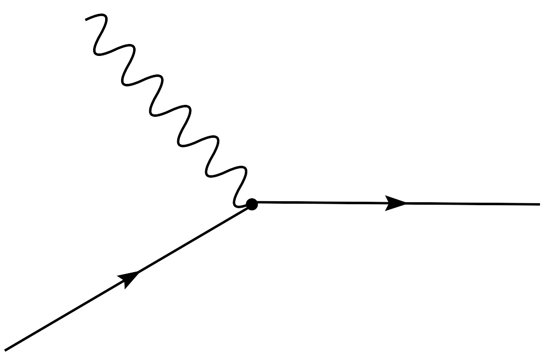
$$\sigma(ep \to eX) = \int_0^1 dz \sum_{i=q,\bar{q},g} f_i(z, \mu_F^2)\ \hat{\sigma}^{\mathrm{hard}}(ei \to eX)$$

Factorized

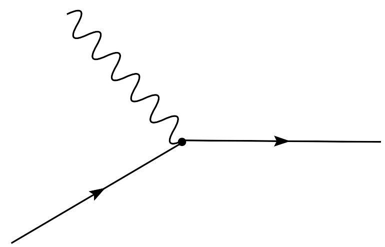
LO Leading Order: Born partonic cross-section

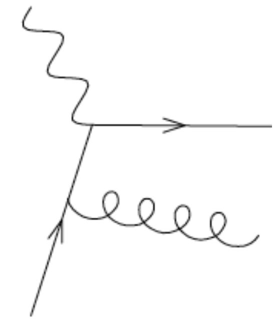
+ LO evolution of pdfs

$$F_2(x, Q^2) = \sum_q e_q^2\, x\, f_q(x, Q^2)$$



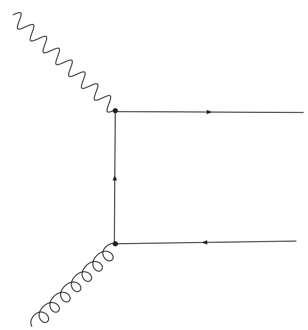
NLO Next-to-Leading Order: Born + $\mathcal{O}(\alpha_s)$ (finite) cross-section + NLO evolution of pdfs





$$F_2(x,Q^2) = \sum_q e_q^2\, x\, f_q(x,Q^2) + \alpha_s \sum_q e_q^2 \int_x^1 \frac{dy}{y}\, C_q^{(1)}(y)\, f_q(x/y,Q^2)$$

$$+\alpha_s \sum_q e_q^2 \int_x^1 \frac{dy}{y}\, C_g^{(1)}(y)\, f_g(x/y,Q^2)$$



NNLO Next-to-Next-to-Leading Order: ...+ $\mathcal{O}(\alpha_s^2)$ (finite) cross-section + NNLO evolution of pdfs

$$+\,\alpha_s^2\; C_i^{(2)}(y)$$

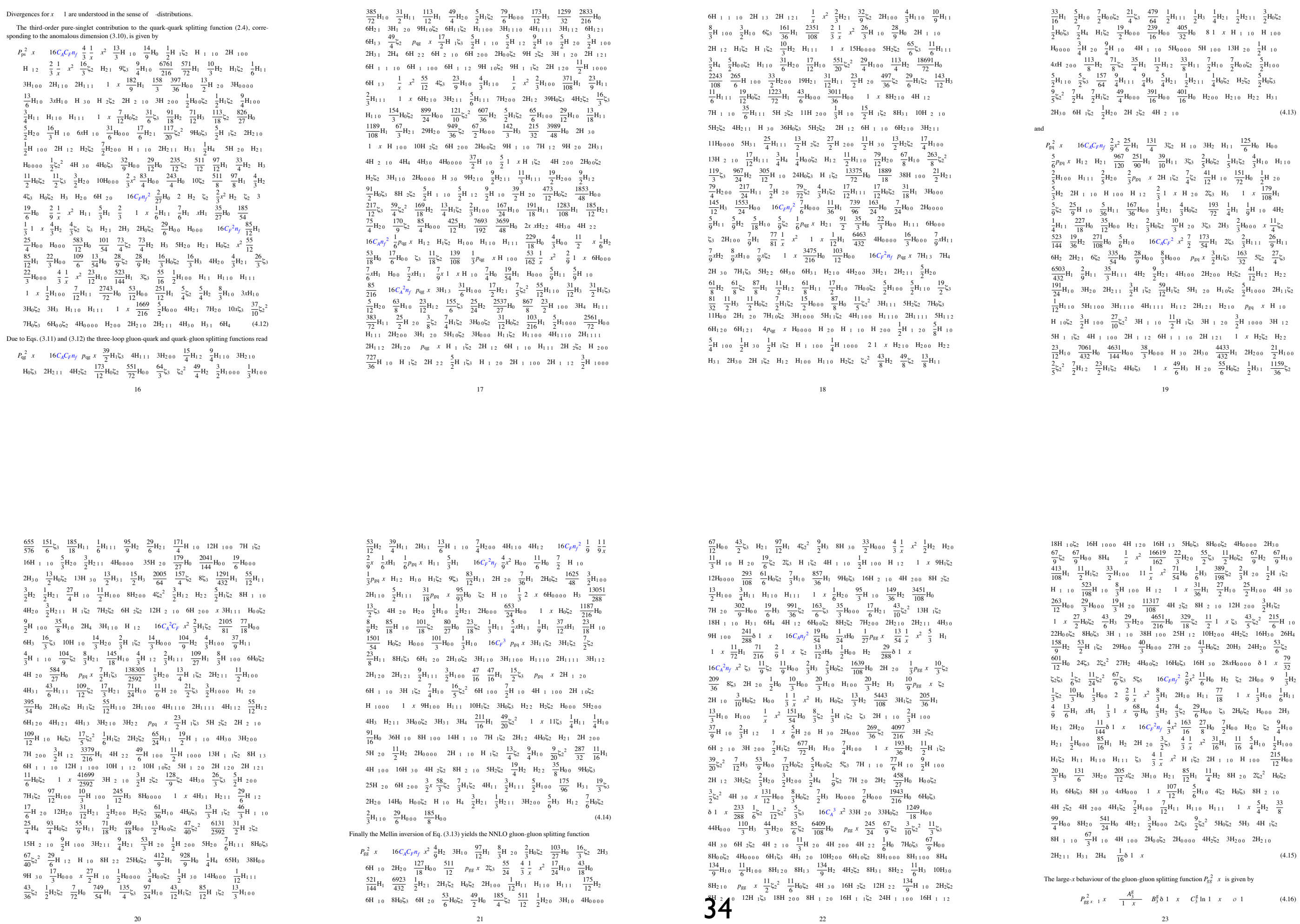
Higher order Altarelli-Parisi kernels known (NNLO)

→ three-loop

Moch, Vermaseren, Vogt (2004)

and working on the 4-loop now!!

9607 (3-loop) Feynman diagrams: 20 man-year work !!

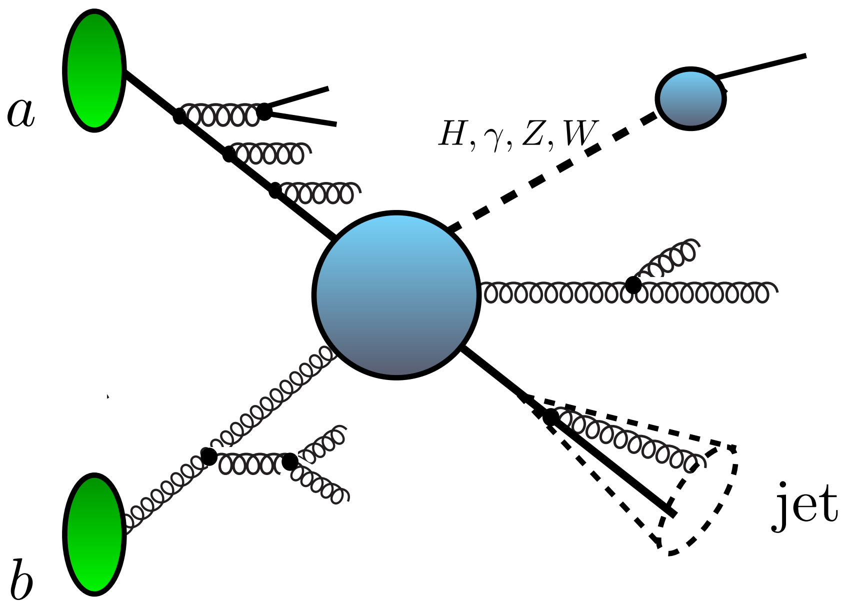


Factorization Formula

non-perturbative parton distributions

$$d\sigma = \sum_{ab} \int dx_a \int dx_b\, f_a(x_a, \mu_F^2) f_b(x_b, \mu_F^2) \times d\hat{\sigma}_{ab}(x_a, x_b, Q^2, \alpha_s(\mu_R^2)) + \mathcal{O}\left(\left(\frac{\Lambda}{Q}\right)^m\right)$$

perturbative partonic cross-section



Partonic cross-section:
expansion in $\alpha_s(\mu_R^2) \ll 1$

$$d\hat{\sigma} = \alpha_s^n\, d\hat{\sigma}^{(0)} + \alpha_s^{n+1}\, d\hat{\sigma}^{(1)} + \ldots$$

(next lecture)

Expression relies on factorization theorem : HT, mass corrections, etc. not trivial

Need precision for both perturbative and non-perturbative components!

Status of PDFs

Parton distributions are determined by performing global fits:

- ✓ Parametrize distributions at input scale $Q_0 = 1 - 4\ \mathrm{GeV}$

$$xf(x, Q_0^2) = Ax^{\alpha}(1-x)^{\beta}(1 + \epsilon\sqrt{x} + \gamma x + ...)$$

- ✓ Impose sum rules (momentum)

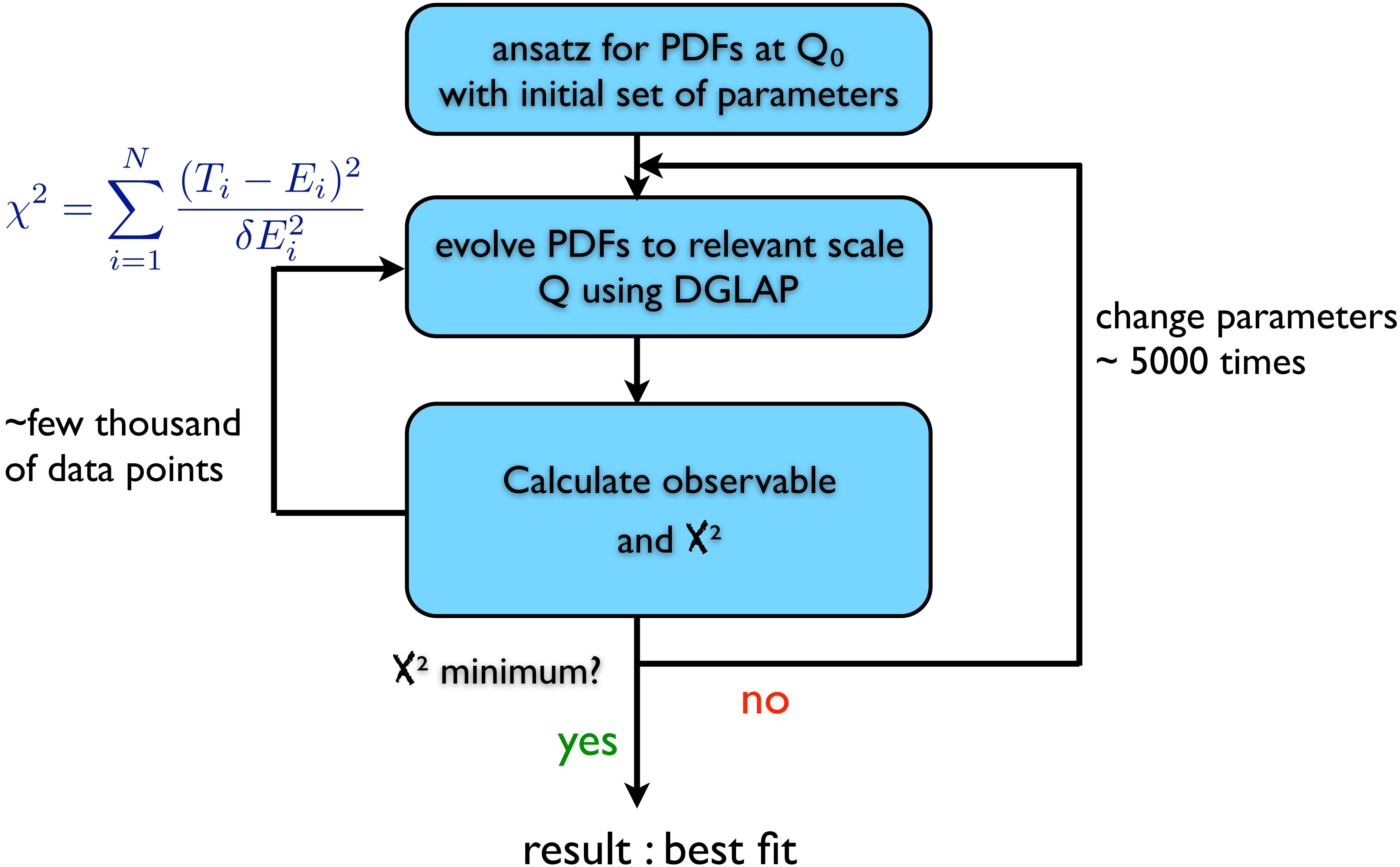
$$\int_0^1 dx \sum_q [x\, q(x, Q_0^2) + x\, \bar{q}(x, Q_0^2)] + \int_0^1 dx\, x\, g(x, Q_0^2) = 1$$

- ✓ Evolve PDF to physical scale and compute observable
- ✓ Compute χ^2 and search for the best parameters

$$\chi^2 = \sum_{i=1}^{N} \frac{(T_i - E_i)^2}{\delta E_i^2}$$

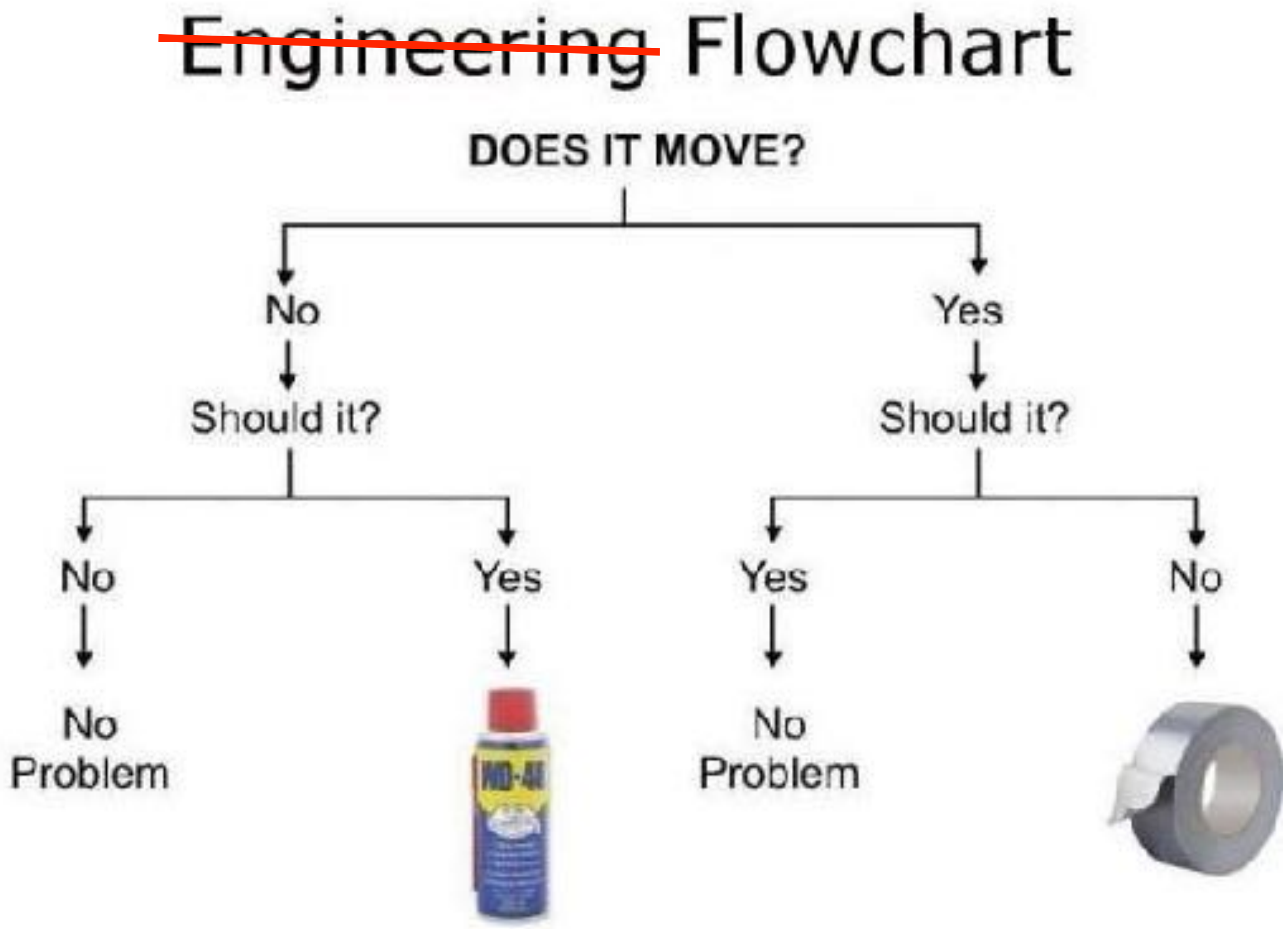
Several groups working on global fits of pdfs

PDFs obtained by global fit : χ^2 minimization



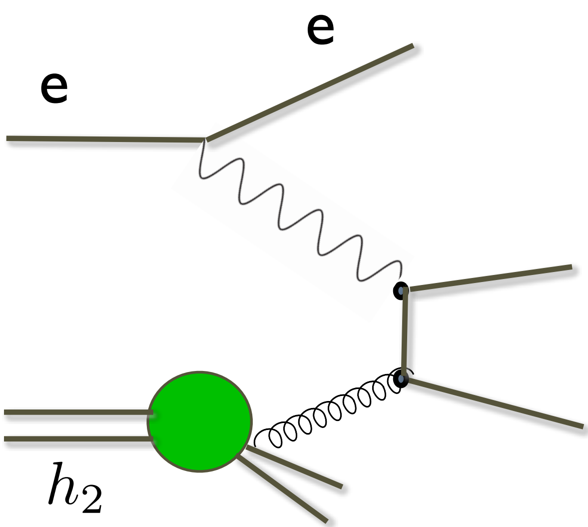
$$\chi^2 = \sum_{i=1}^{N} \frac{(T_i - E_i)^2}{\delta E_i^2}$$

PDF fit



Engineering Flowchart
DOES IT MOVE?
No
Yes
Should it?
Should it?
No
Yes
Yes
No
No Problem
WD-40
No Problem

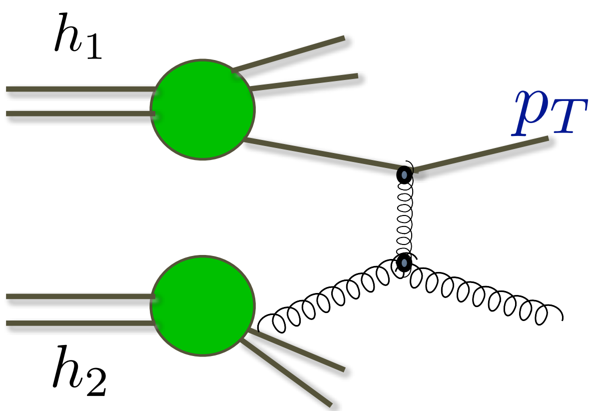
Apart from inclusive DIS, some relevant processes are



Jets and charm production in DIS: complementary information from inclusive DIS

Sensitive to $g(x),\ c(x)$

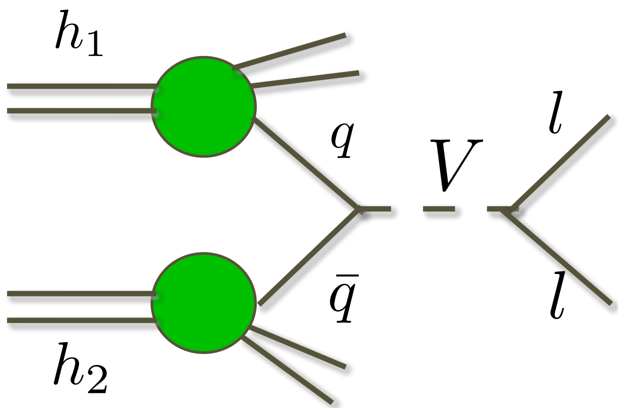
And in hadronic collisions



Jet production : sensitive to many channels gluons enter at lowest order

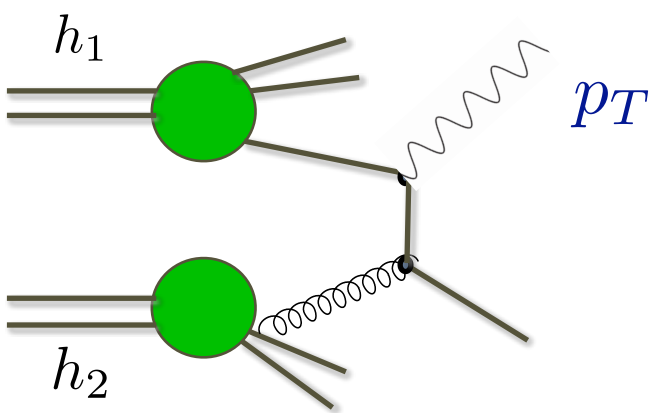
Transverse momentum q_T and rapidity distributions $y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z}$

More in hadronic collisions



Drell-Yan: main production mechanism for Gauge bosons (lepton pair)

Sensitive to $q(x_1)\,\bar{q}(x_2)$



Prompt-Photons: "clean" in principle, but some exp/th issues for fixed target

Sensitive to $q(x_1)\,g(x_2)$

Not much used...

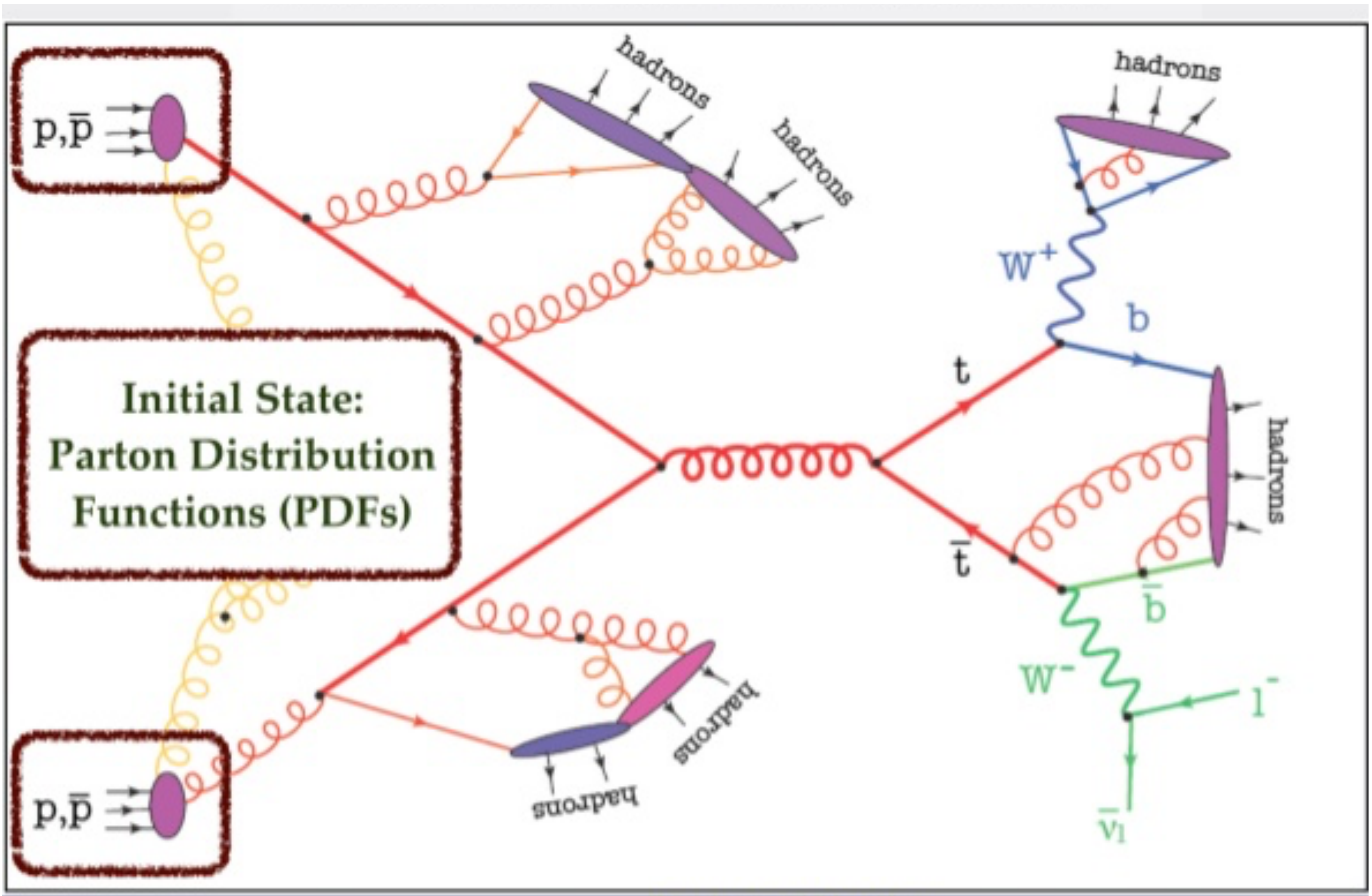
- Include all observables where pQCD is under control : each one helps to constrain a combination of pdfs at certain kinematics

	Process	Subprocess	Partons	x range
Fixed target : $l p$ and DY	$\ell^\pm \{p,n\} \to \ell^\pm X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
	$\ell^\pm n/p \to \ell^\pm X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
	$pp \to \mu^+\mu^- X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
	$pn/pp \to \mu^+\mu^- X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
	$\nu(\bar{\nu}) N \to \mu^-(\mu^+) X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
	$\nu N \to \mu^-\mu^+ X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
	$\bar{\nu} N \to \mu^+\mu^- X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
HERA	$e^\pm p \to e^\pm X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
	$e^+ p \to \bar{\nu} X$	$W^+ \{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
	$e^\pm p \to e^\pm c\bar{c} X$	$\gamma^* c \to c,\ \gamma^* g \to c\bar{c}$	$c,\ g$	$0.0001 \lesssim x \lesssim 0.01$
	$e^\pm p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
Tevatron	$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
	$p\bar{p} \to (W^\pm \to \ell^\pm \nu) X$	$ud \to W, \bar{u}\bar{d} \to W$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
	$p\bar{p} \to (Z \to \ell^+\ell^-) X$	$uu, dd \to Z$	d	$x \gtrsim 0.05$

MSTW

PDFs

Main ingredient of any high-energy observable in Hadronic Colliders



fitting is 50% science plus 50% art Morfin & Tung (1991)

Some issues: 1% inspiration plus 99% transpiration.. Einstein

- **Selection of data**
  - which observables (no prompt photon)
  - "incompatible" data sets (W lepton asymmetries)
  - open bins/combined data (Hera)
- **Weights for some experiments**
  - enhance the relevance of some data set
  - enhance some "parton distribution"
  - reduce effect of inconsistent data sets
- **"Aesthetic" requirements**
  - unphysical behavior of pdfs at $x=0$ and 1 : penalty terms
- **Theoretical issues**
  - HQ treatment and masses
  - Parametrization of pdfs
  - Selection of factorization/renormalization scales
  - TH improvements for some observables (resummation)
  - Solution of evolution equations and precision (speed!)
  - α_s from fit or external value? which value/uncertainty?
- **Uncertainties**
  - what is 1sigma in a global fit? $\Delta\chi^2 = ?$

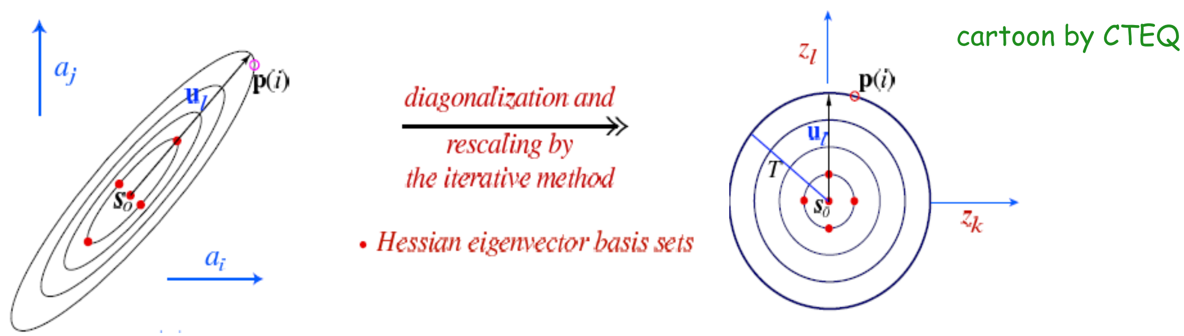
Traditional Uncertainties : Hessian approach

- Assume quadratic dependence on parameters around minimum

$$\Delta\chi^2(a) = \chi^2 - \chi_0^2 = \sum_{i,j} H_{ij}\delta a_i \delta a_j + \cdots$$

- Diag. to eigenvectors : optimized orthonormal basis near minimum

1 per parameter



- Allow for some $\Delta\chi^2$ to define extreme sets (+/-) for each eigenvector:
2 full pdf sets $S_k^{\pm}$ for each eigenvector to compute uncertainties for each observable

$$\Delta\mathcal{O}_i = \frac{1}{2}\left(\sum_{k=1}^{N_{\text{par}}}\left[\mathcal{O}_i(S_k^+) - \mathcal{O}_i(S_k^-)\right]^2\right)^{1/2}$$

provide ~50 eigenvector sets to compute uncertainties for any observable

Neural Network approach

NNPDF, Ball et al

- Construct a set of MonteCarlo replicas of the original data set where the replicas fluctuate about central data

- Split data sets into training and validation sets

- Fit to the data replicas obtaining PDF replicas

- PDFs generated using a neural net to find the best fit. Eliminates largely dependence on parameterization. Still includes pre-processing factor to constrain kinematic limits

$$f(x,\mu_0^2) = A\,x^{\alpha}(1-x)^{\beta}\,NN(x)$$

- Statistical definition of mean value and standard deviation for observable

$$N_{\mathbf{rep}} = 100\,\mathbf{or}\,1000$$

$$\langle \mathcal{F}[\{q\}]\rangle = \frac{1}{N_{\rm rep}}\sum_{k=1}^{N_{\rm rep}} \mathcal{F}[\{q^{(k)}\}] \qquad \sigma_{\mathcal{F}} = \left(\frac{1}{N_{\rm rep}-1}\sum_{k=1}^{N_{\rm rep}}\left(\mathcal{F}[\{q^{(k)}\}] - \langle \mathcal{F}[\{q\}]\rangle\right)^2\right)^{1/2}$$

PDFs

- Several groups provide pdf fits + uncertainties
- Differ by: data input, TH/bias, HQ treatment, coupling, etc

set	H.O.	data	$\alpha_s(M_Z)@NNLO$	uncertainty	HQ
MMHT14	NNLO	DIS+DY+Jets+LHC	0,118	Hessian (dynamical tolerance)	GM-VFN (ACOT+TR')
CT14	NNLO	DIS+DY+Jets+LHC	0,118	Hessian (dynamical tolerance)	GM-VFN (SACOT-X)
NNPDF 3	NNLO	DIS+DY+Jets+LHC	0,118	Monte Carlo	GM-VFN (FONLL)
ABM	NNLO	DIS+DY(f.t.)+DY-tT(LHC)	0,1132	Hessian	FFN BMSN
(G)JR	NNLO	DIS+DY(f.t.)+ some jet	0,1124	Hessian	FFN (VFN massless)
HERA PDF	NNLO	only DIS HERA	0,1176	Hessian	GM-VFN (ACOT+TR')

Find all PDFs in http://durpdg.dur.ac.uk/hepdata/pdf.html

Parton Distribution Functions

Unpolarized Parton Distributions

Access the parton distribution code, on-line calculation and graphical display of the distributions, ffrom CTEQ, GRV, MRS and Alekhin.

CTEQ distributions, fortran code and grids
GRV distributions, fortran code and grids
MRST distributions, fortran code and grids, C++ code
ALEKHIN distributions, fortran,C++ and Mathematica code, and grids

On-line Parton Distribution Calculator with Graphical Display.
- now includes PDF error calculations from MRST2001E and CTEQ6.

Public access to the ZEUS 2002 PDFs , ZEUS 2005 jet fit PDFs and H1 PDF 2000 sets.

J. Bluemlein, H. Boettcher and A.Guffanti - hep-ph/0607200 BBG06_NS

Polarized Parton Distributions

Currently available parametrizations:

E.Leader, A.V.Sidorov and D.B.Stamenov, Eur.Phys.J.C23 (2002) 479: LSS2001
E.Leader, A.V.Sidorov and D.B.Stamenov, Phys.Rev.D73 (2006) 034023: LSS2005
M. Glueck, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D53 (1996) 4775: GRSV
M. Glueck, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D63 (2001) 094005: GRSV2000
T. Gehrmann and W.J. Stirling, Phys. Rev. D53 (1996) 6100: GS
J. Bluemlein and H. Boettcher - Nucl.Phys.B636(2002)225: BB
Asymmetry Analysis Collaboration - M. Hirai et al- Phys. Rev. D69 (2004) 054021: AAC
D. de Florian and R. Sassot, Phys. Rev. D62 (2000) 094025: DS2000
D. de Florian, G.A. Navarro and R. Sassot, Phys. Rev. D71 (2005) 094018: DNS2005

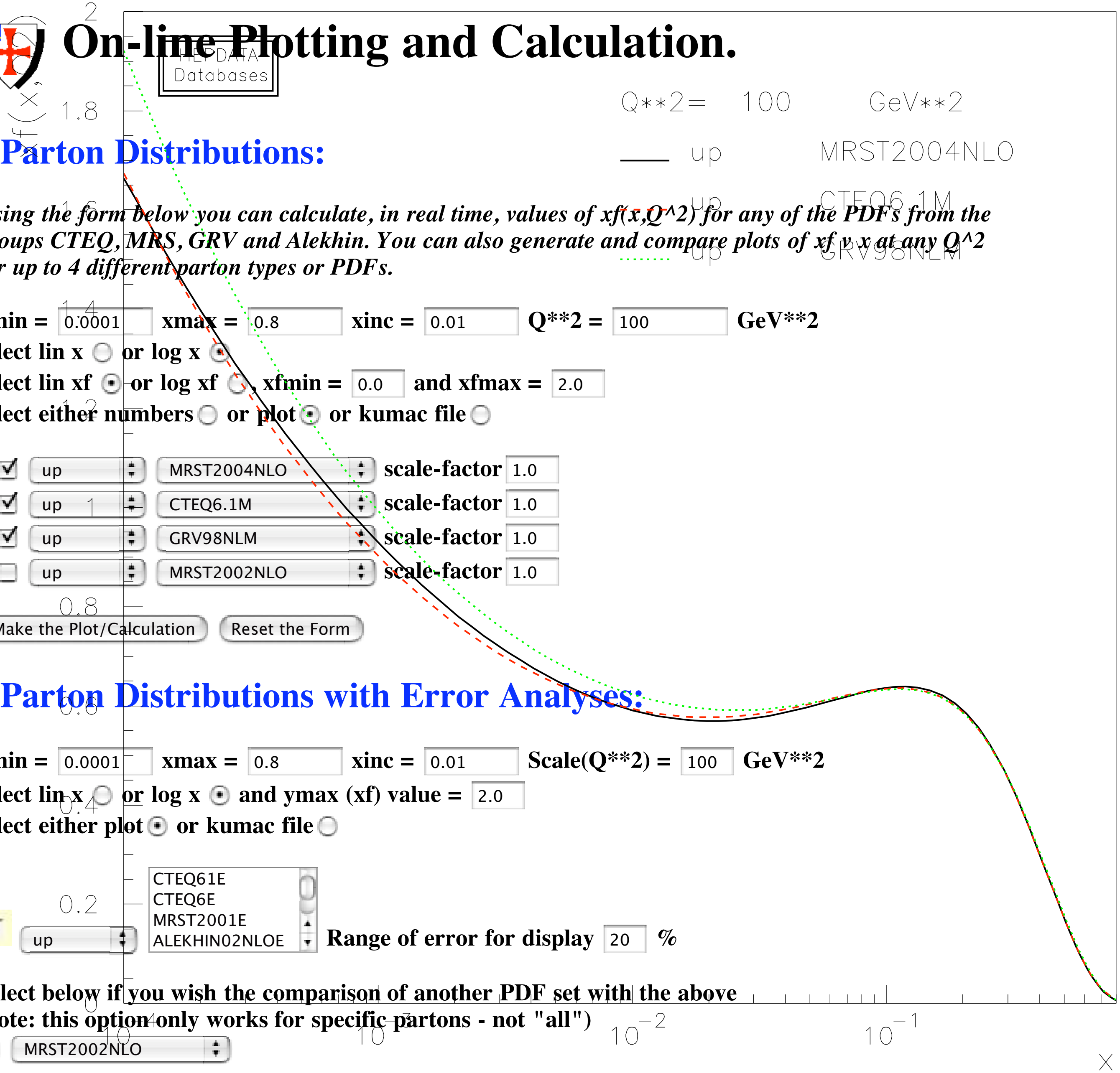
Diffractive Parton Distributions

A.D.Martin, M.G.Ryskin and G.Watt: MRW2006.

Pion Parton Distributions

Access the parton distribution code for pions

On-line Plotting and Calculation.



Parton Distributions:

Using the form below you can calculate, in real time, values of xf(x,Q^2) for any of the PDFs from the groups CTEQ, MRS, GRV and Alekhin. You can also generate and compare plots of xf v x at any Q^2 for up to 4 different parton types or PDFs.

xmin = 0.0001 **xmax =** 0.8 **xinc =** 0.01 **Q**2 =** 100 **GeV**2**

select lin x ○ **or log x** ◉

select lin xf ◉ **or log xf** ○ **, xfmin =** 0.0 **and xfmax =** 2.0

select either numbers ○ **or plot** ◉ **or kumac file** ○

1	☑	up	MRST2004NLO	**scale-factor**	1.0
2	☑	up	CTEQ6.1M	**scale-factor**	1.0
3	☑	up	GRV98NLM	**scale-factor**	1.0
4	☐	up	MRST2002NLO	**scale-factor**	1.0

[Make the Plot/Calculation] [Reset the Form]

Parton Distributions with Error Analyses:

xmin = 0.0001 **xmax =** 0.8 **xinc =** 0.01 **Scale(Q**2) =** 100 **GeV**2**

select lin x ○ **or log x** ◉ **and ymax (xf) value =** 2.0

select either plot ◉ **or kumac file** ○

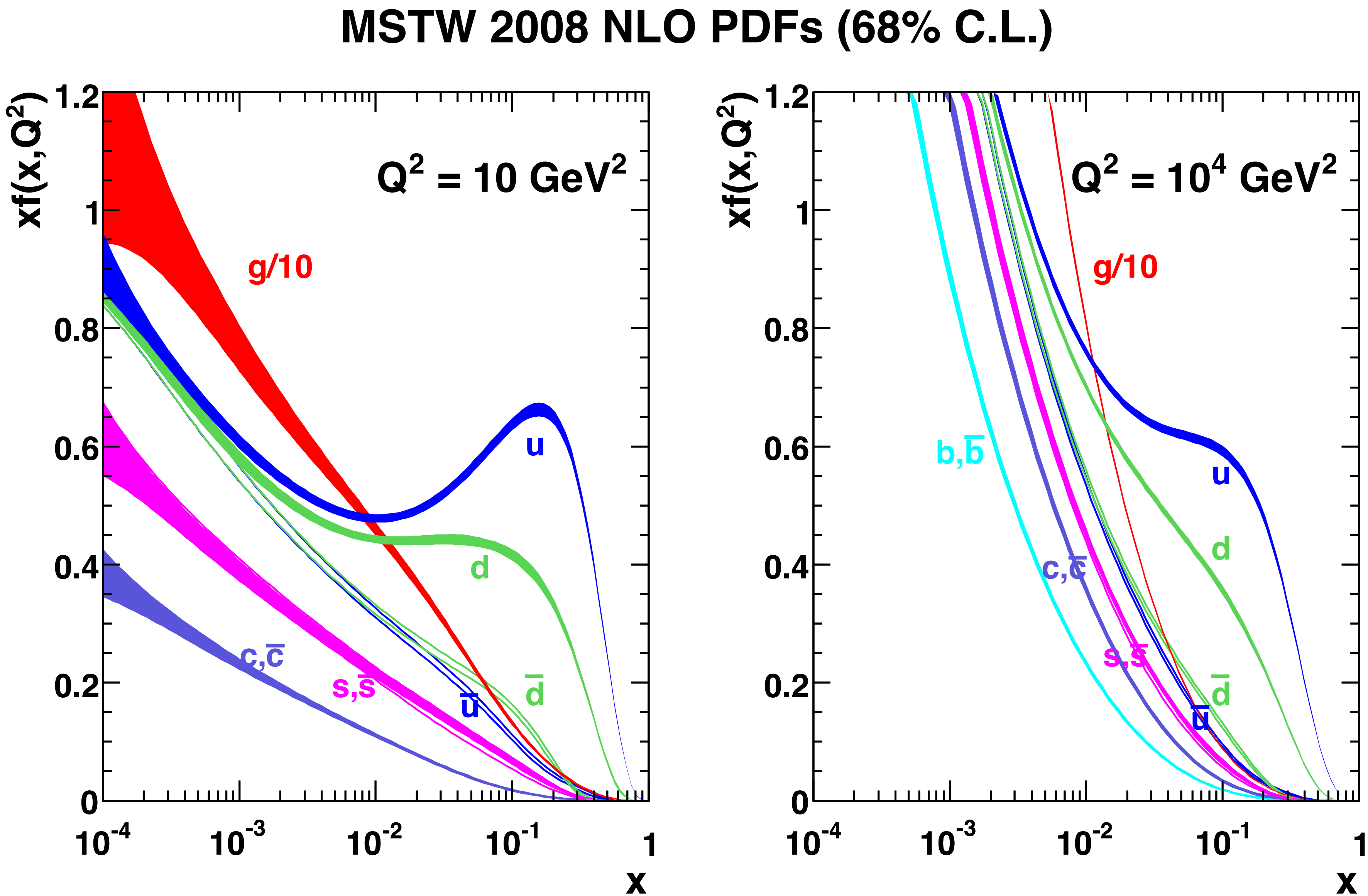
☑ up | CTEQ61E, CTEQ6E, MRST2001E, ALEKHIN02NLOE | **Range of error for display** 20 **%**

Select below if you wish the comparison of another PDF set with the above (note: this option only works for specific partons - not "all")

☐ MRST2002NLO

[Make the Plot/Calculation] [Reset the Form]

How do they look like?



Main Issues

Heavy quark treatment : different TH approaches with some ad-hoc procedures

→ Not only affects HQ distributions but substantially modifies the gluon density

Coupling constant : affects evolution and evaluation of cross sections!

$\alpha_s(M_Z)@NNLO$

0,1171
0,118
0,1174
0,1132
0,1124
0,1176

Until recently

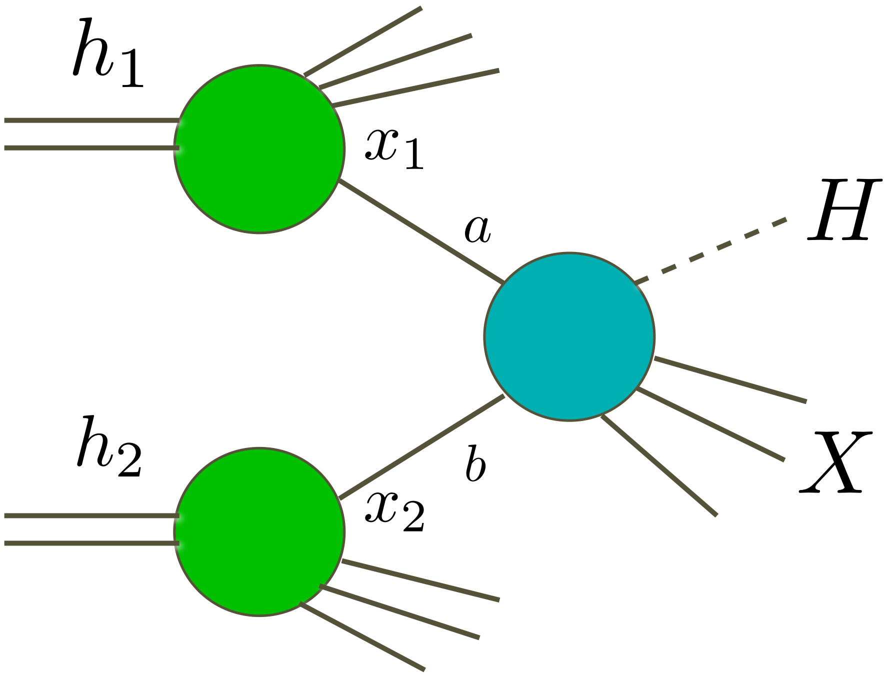
← up to 5% !

>15% in Higgs cross

PDF4LHC recommendation

Now the 3 main sets agree on common coupling 0.118 ± 0.0015

At hadron colliders more than PDFs it is interesting to look at Luminosities for each channel



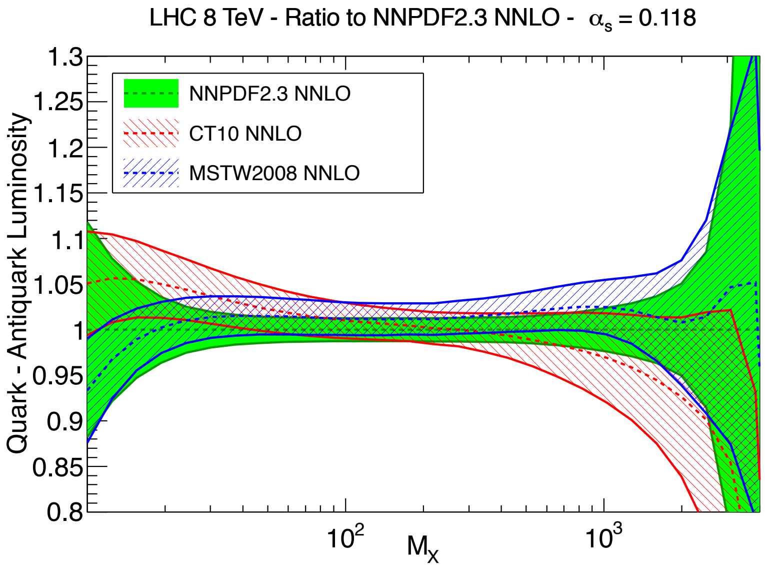
$$\sigma(S) = \sum_{i,j} \int d\tau \left[\frac{1}{S} \frac{dL_{ij}}{d\tau} \right] [\hat{s}\hat{\sigma_{ij}}]$$

$$\tau \frac{dL_{ij}}{d\tau} = \int_0^1 dx_1 dx_2 x_1 f_i(x_1, \mu_F^2) \times x_2 f_j(x_2, \mu_F^2) \delta(\tau - x_1 x_2)$$

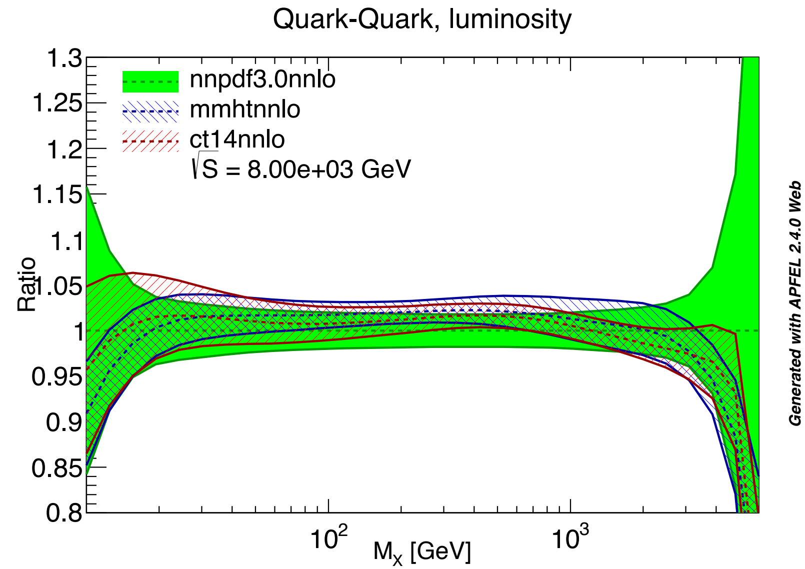
Continuous improvements: TH (NNLO), more data (LHC), coupling

QUARK-QUARK

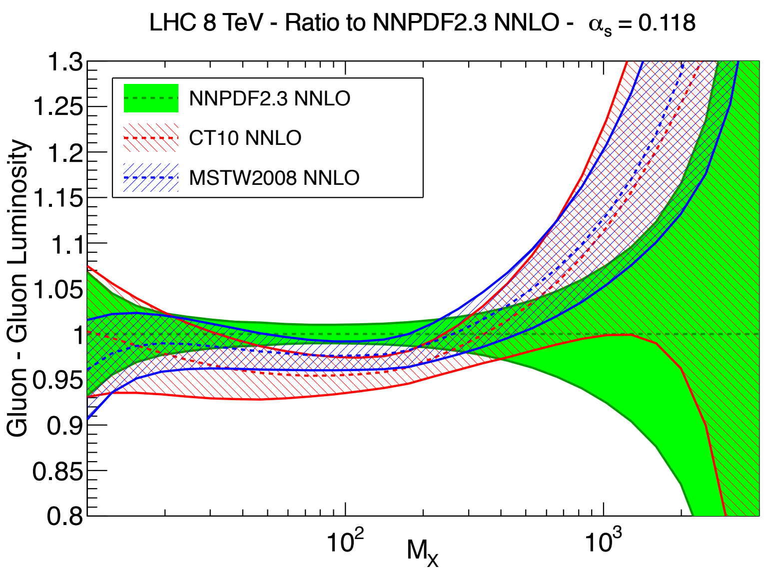
2012

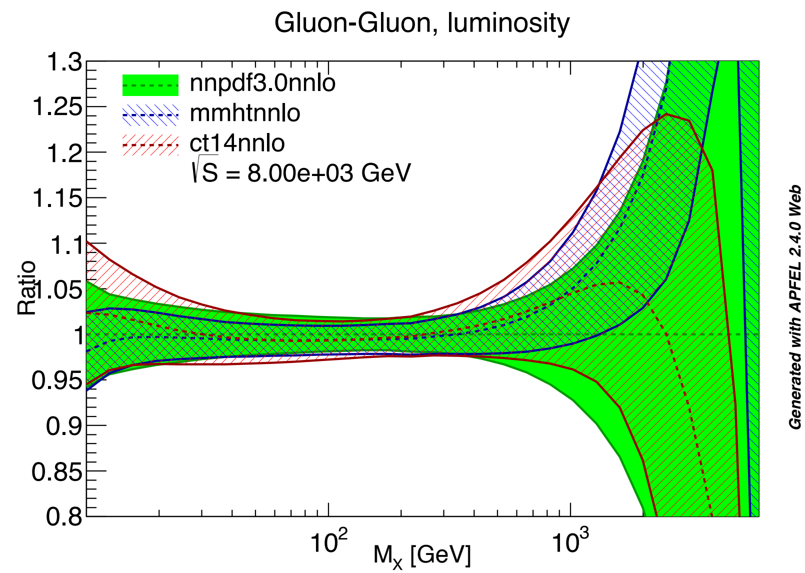


2015



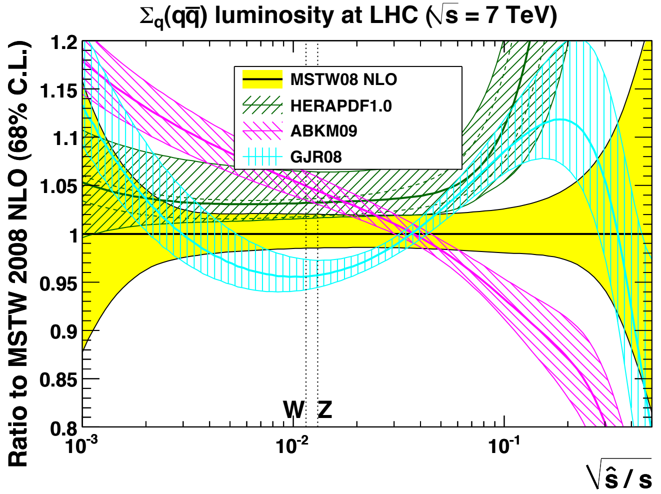
GLUON-GLUON

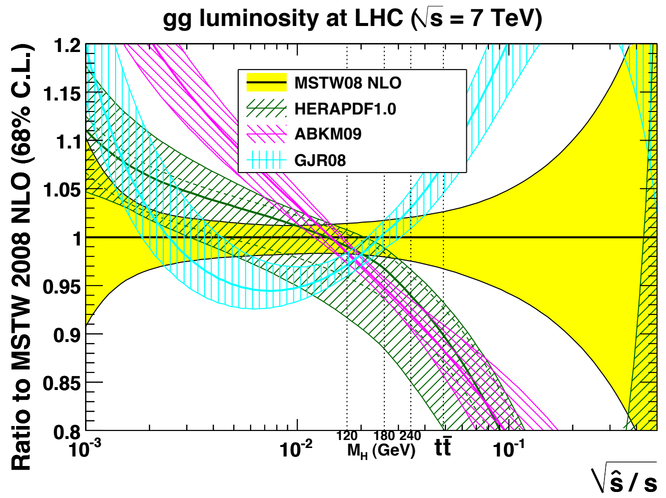




Still larger disagreement with "non-global" fits

$\mu_F^2 = \hat{s}$



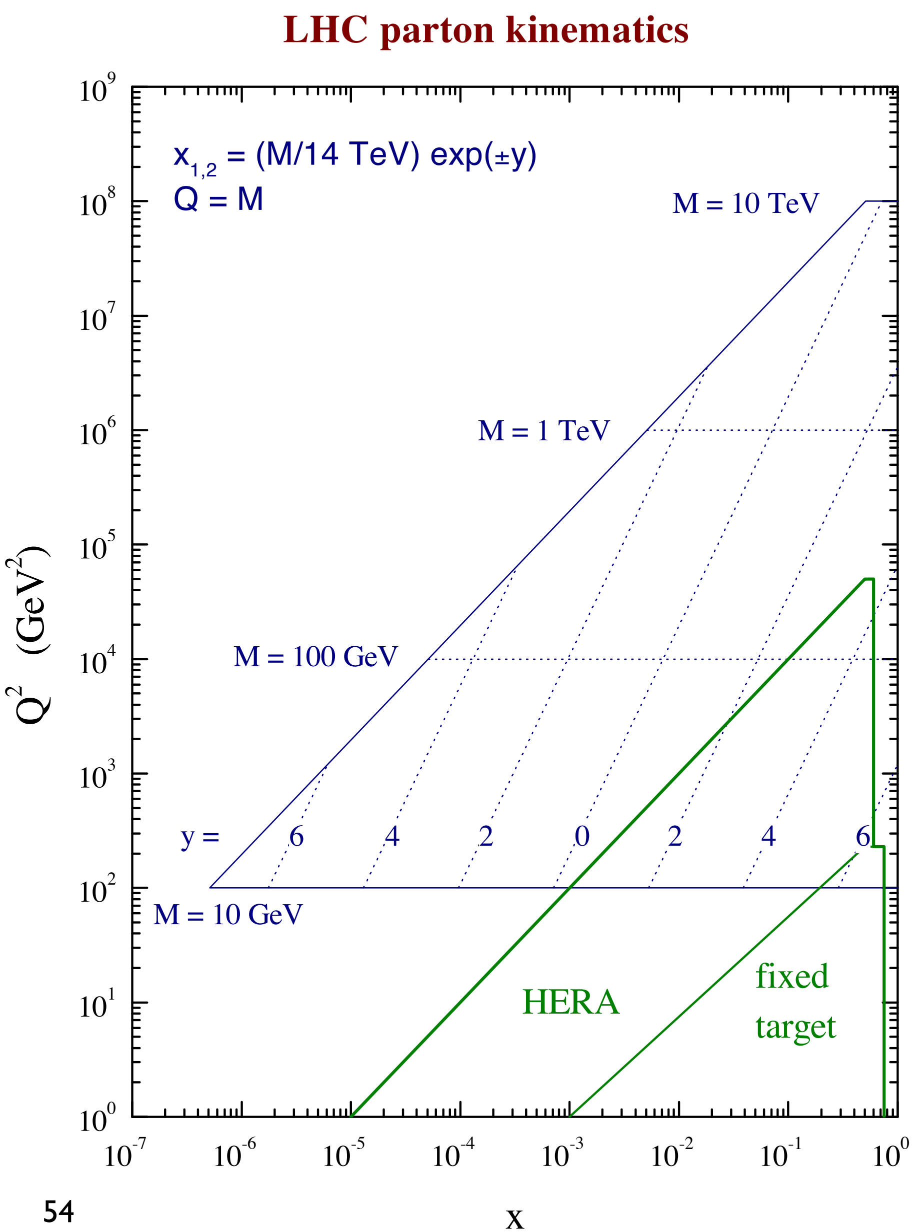


much bigger differences for non-global pdfs!

LHC helps (and will help more)

- Precise LHC data needed for validation & improvement

Still missing full Jet calculation at NNLO (on the way)



THE NEW PDF4LHC PRESCRIPTION

- PERFORM MONTE CARLO COMBINATION OF UNDERLYING PDF SETS
- SETS ENTERING THE COMBINATION MUST SATISFY COMMON REQUIREMENTS
- DELIVER A SINGLE COMBINED PDF SET THROUGH SUITABLE TOOLS
- A PDF4LHC15 PDF SET WILL BE RELEASED
- NLO AND NNLO GRIDS WILL BE AVAILABLE
- REPLICA/ERROR SETS FOR PDF UNCERTAINTY, SEPARATE UPPER AND LOWER SETS FOR α_s UNCERTAINTY;
  $\alpha_s = 0.118 \pm 0.002$ OR $\alpha_s = 0.118 \pm 0.0015$ FOR NLO,
  $\alpha_s = 0.118 \pm 0.001$ OR $\alpha_s = 0.118 \pm 0.0015$ FOR NNLO
- THREE VERSIONS (DIFFERENT DELIVERIES OF SAME PDF SET):
  - PDF4LHC15_100 HESSIAN100 WHEN GAUSSIAN ACCURATE PREDICTION REQUIRED (EXAMPLE: HIGGS IN GLUON FUSION SIGNAL STRENGTH)
  - PDF4LHC15_30 HESSIAN30 WHEN FAST CALCULATION NEEDED (EXAMPLE: ACCEPTANCE)
  - PDF4LHC15_MC MONTECARLO WHEN MC DESIRABLE OR NONGAUSSIAN EFFECTS IMPORTANT (EXAMPLE: SEARCHES)

PV: looks a bit too optimistic…

arXiv.org > hep-ph > arXiv:1510.03865

High Energy Physics – Phenomenology

PDF4LHC recommendations for LHC Run II

Jon Butterworth, Stefano Carrazza, Amanda Cooper-Sarkar, Albert De Roeck, Joel Feltesse, Stefano Forte, Jun Gao, Sasha Glazov, Joey Huston, Zahari Kassabov, Ronan McNulty, Andreas Morsch, Pavel Nadolsky, Voica Radescu, Juan Rojo, Robert Thorne

(Submitted on 13 Oct 2015 (v1), last revised 12 Nov 2015 (this version, v2))

We provide an updated recommendation for the usage of sets of parton distribution functions (PDFs) and the assessment of PDF and PDF+α_s uncertainties suitable for applications at the LHC Run II. We review developments since the previous PDF4LHC recommendation, and discuss and compare the new generation of PDFs, which include substantial information from experimental data from the Run I of the LHC. We then propose a new prescription for the combination of a suitable subset of the available PDF sets, which is presented in terms of a single combined PDF set. We finally discuss tools which allow for the delivery of this combined set in terms of optimized sets of Hessian eigenvectors or Monte Carlo replicas, and their usage, and provide some examples of their application to LHC phenomenology.

arXiv.org > hep-ph > arXiv:1603.08906

High Energy Physics – Phenomenology

Recommendations for PDF usage in LHC predictions

A. Accardi, S. Alekhin, J. Blümlein, M.V. Garzelli, K. Lipka, W. Melnitchouk, S. Moch, R. Placakyte, J.F. Owens, E. Reya, N. Sato, A. Vogt, O. Zenaiev

(Submitted on 29 Mar 2016)

We review the present status of the determination of parton distribution functions (PDFs) in the light of the precision requirements for the LHC in Run 2 and other future hadron colliders. We provide brief reviews of all currently available PDF sets and use them to compute cross sections for a number of benchmark processes, including Higgs boson production in gluon-gluon fusion at the LHC. We show that the differences in the predictions obtained with the various PDFs are due to particular theory assumptions made in the fits of those PDFs. We discuss PDF uncertainties in the kinematic region covered by the LHC and on averaging procedures for PDFs, such as advocated by the PDF4LHC15 sets, and provide recommendations for the usage of PDF sets for theory predictions at the LHC.

Recap of second lecture

- DIS provides the best scenario to study proton structure
- Parton Model : scattering is an incoherent sum of partonic cross sections
- Factorization allows us to compute the partonic cross section perturbatively and at the same time implies that parton distributions are universal
- IR divergences appear again but do not cancel completely : must be factorized in parton distributions
- Parton distributions are scale dependent. Evolution perturbatively determined by DGLAP equations
- PDFs are extracted by global analysis. Also statistical uncertainties are determined
- Still some issues in PDF extraction : uncertainties, coupling constant, but continuous improvements