QCD

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Recap of second lecture

- DIS provides the best scenario to study proton structure
- Parton Model : scattering is an incoherent sum of partonic cross sections
- Factorization allows us to compute the partonic cross section perturbatively and at the same time implies that parton distributions are universal
- IR divergences appear again but do not cancel completely : must be factorized in parton distributions
- Parton distributions are scale dependent. Evolution perturbatively determined by DGLAP equations
- PDFs are extracted by global analysis. Also statistical uncertainties are determined
- Still some issues in PDF extraction : uncertainties, coupling constant, but continuous improvements

The perturbative toolkit for precision at colliders

State of the Art

NLO

Resummation

Automation

PS

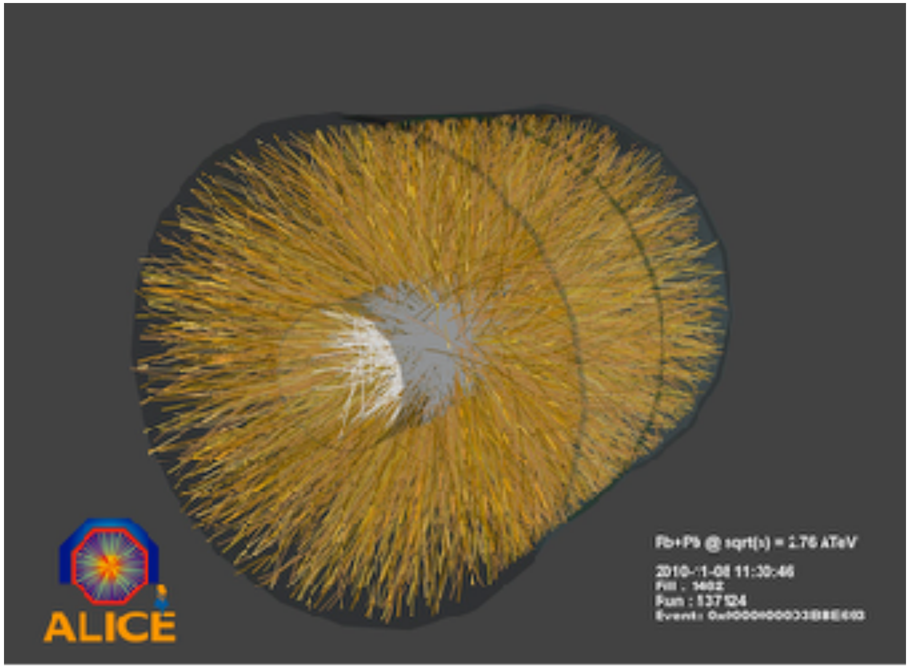
NNLO

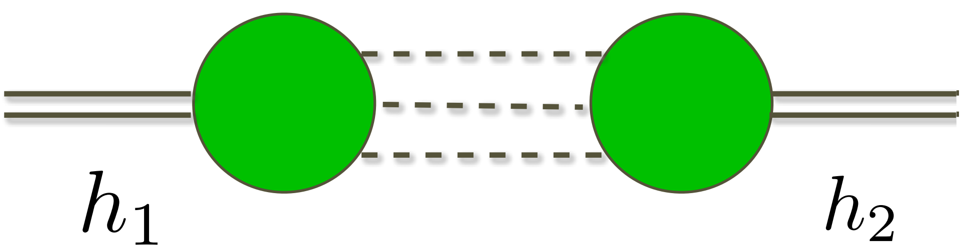
N^{3}LO…

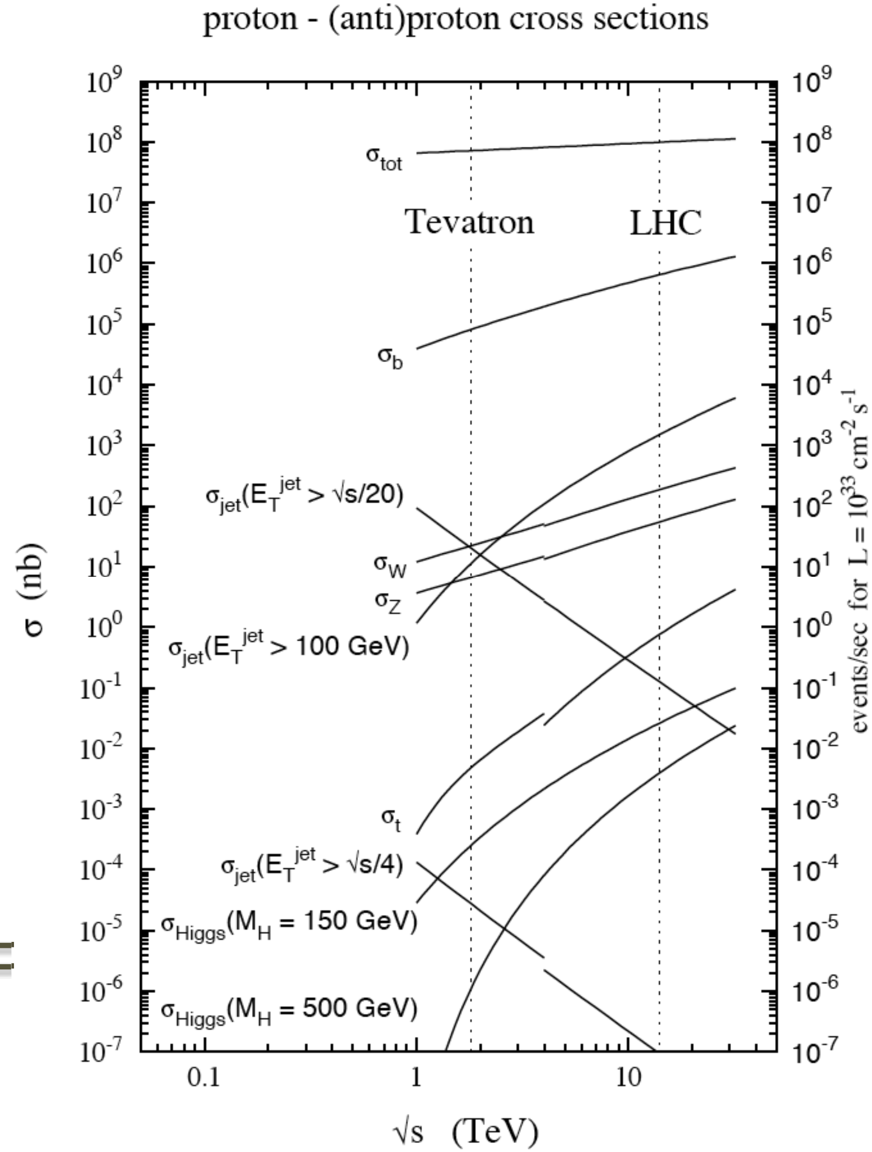
Outline of the lecture 3

- ✤ QCD at Colliders
- ✤ Why higher orders?
- ✤ How to do NLO
- ✤ Automated tools at NLO

QCD at Colliders







Most of the collisions correspond to soft physics: non-perturbative

Most interesting (new) physics involves large scales

Kinematics relevant in hadronic colliders

$$p^\mu = (E, p_x, p_y, p_z)$$ final state particle

$$p^\mu = (m_T \cosh y, p_T \sin\phi, p_T \cos\phi, m_T \sinh y)$$

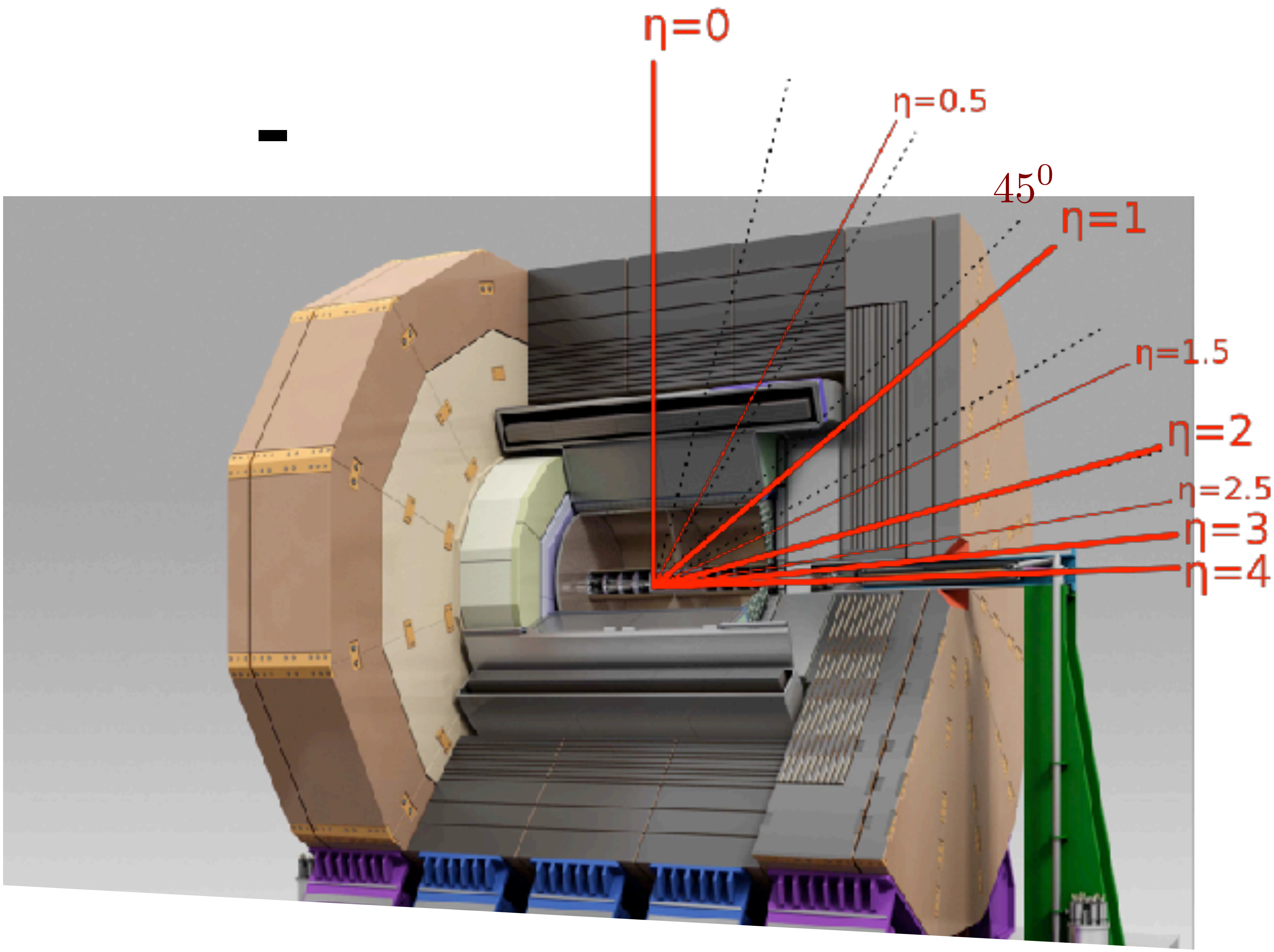
Transverse mass $m_T = \sqrt{p_T^2 + m^2}$

Rapidity | pseudo-rapidity

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \xrightarrow{\text{massless}} \eta = -\ln\tan(\theta/2)$$

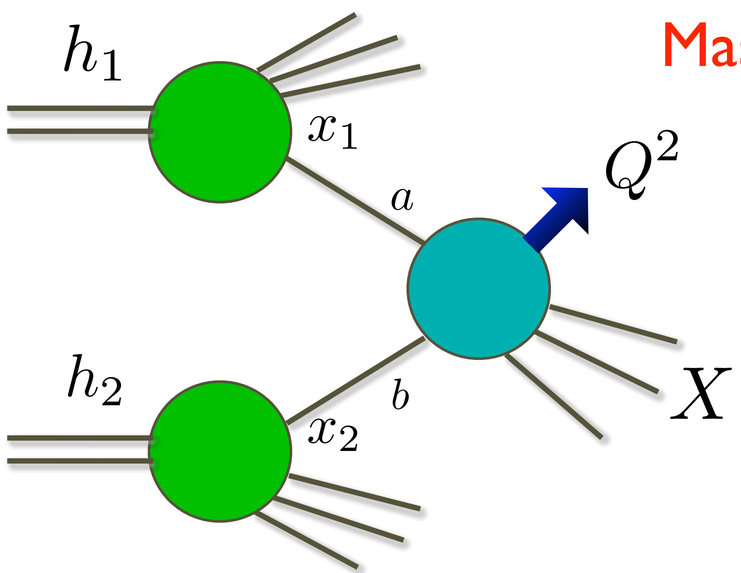
$$E_T = E \sin\theta \longrightarrow p_T$$

$$\frac{d^3p}{E} = p_T\, dp_T dy d\phi$$



$\eta=0$
$\eta=0.5$
45^0
$\eta=1$
$\eta=1.5$
$\eta=2$
$\eta=2.5$
$\eta=3$
$\eta=4$

Most interesting (new) physics involves large scales



Mass, invariant mass, transverse momentum

Hard/soft factorization still applies if large scale involved $+\,\mathcal{O}(\Lambda_{QCD}/Q)$

Factorization of singularities in parton distributions exactly as in DIS

$$\sigma(p_1,p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)$$

$$\times\ \hat{\sigma}_{ab}(x_1p_1, x_2p_2, \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

Two unphysical scales $\mu_R^2 \sim \mu_F^2 \sim Q^2$

LO : number of tools to compute tree level amplitudes

Fully automated calculations for very large multiplicities

MADGRAPH, HELAC-PHEGAS, ALPGEN, SHERPA, ComHep, COMIX,...

Pros of LO calculations

Fast (until recently the only option for many observables)
Simpler to integrate calculation to parton showers
Many tools available (tested!)

Cons of LO calculations

In most cases, not enough for precision physics : only qualitative
Large scale dependence
No control on normalization (poor on shapes)
No Control on uncertainties

Why higher order corrections?

Why higher order corrections?

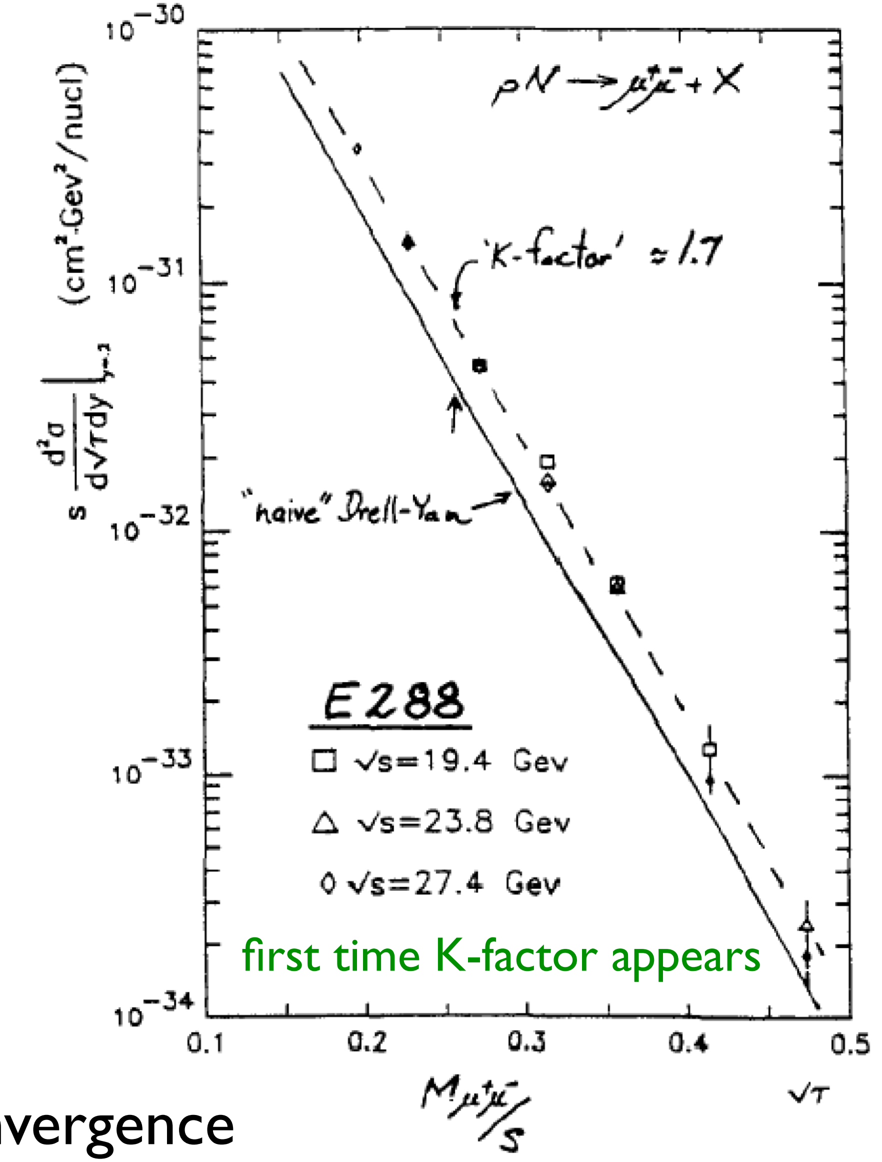
- Accurate Theoretical Predictions
  shape and normalization
- Large Corrections : check PT

First example: Drell-Yan

$$pp \to \mu^+\mu^-$$

- K-factor

$$K = \frac{\sigma^{N^i LO}}{\sigma^{LO}}$$



Even at LHC $\alpha_s \sim 0.1$ ➔ slow convergence

▸ Accurate Theoretical Predictions

Scale dependence: first error estimate

According to “master formula”

$$\sigma(p_1,p_2) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)$$
$$\times\ \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2, \alpha_s(\mu_R^2), \mu_R^2, \mu_F^2)$$

- 2 unphysical scales : dependence cancels if computed to all orders

μ_R Renormalization scale

μ_F Factorization scale

- after “perturbative” truncation: unphysical dependence remains

- (naive) estimate of size of missing higher orders

Go back to our "well-known" $R_{\rm had} \equiv \frac{\sigma(e^+e^- \to {\rm hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

At NLO the result is $$R_{\rm had} = R^{(0)}\left(1 + \frac{\alpha_s(\mu^2)}{\pi}\right)$$

Scale dependence: at which scale evaluate the coupling?

Scale is unphysical, in principle any value possible, but...

According to RGE, dependence cancels if observable computed to all orders in perturbation theory

The renormalization group equations tell us

$$\alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + \beta_0 \alpha_s(\mu_0^2) \log \frac{\mu^2}{\mu_0^2}}$$

Expanded to first order reads

$$\alpha_s(\mu^2) = \alpha_s(\mu_0^2) - \alpha_s^2(\mu_0^2)\, \beta_0 \, \log \frac{\mu^2}{\mu_0^2} + \ldots$$

$$R_{\mathrm{had}} = R^{(0)} \left(1 + \frac{\alpha_s(\mu_0^2)}{\pi} - \boxed{\alpha_s^2(\mu_0^2)\beta_0 \log\frac{\mu^2}{\mu_0^2}} + \ldots\right)$$

Notice that if computed to NLO, scale dependence appears at NNLO

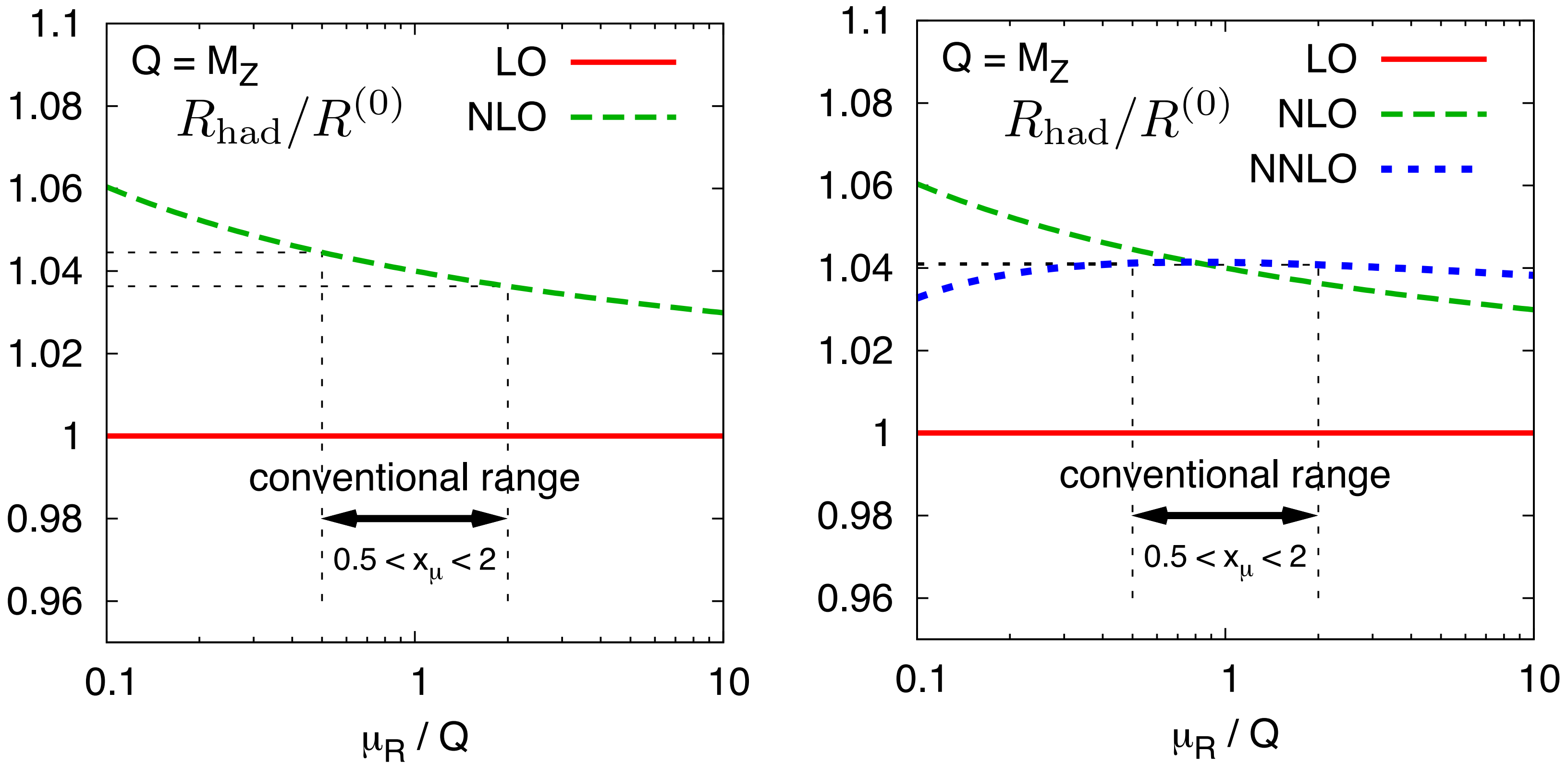
Coefficients in general depend on LOGS of ratios of energy scales

For single scale problems (as here), it is convenient to chose renormalization (and factorization) scales close to the energy scale of the process to avoid the appearance of large logarithmic terms that can spoil the convergence of the expansion $\mu \sim Q$

TH uncertainties are usually estimated by performing scale variations : provides a lower limit on the size of missing higher-order contributions

If scale dependence is large then large higher order corrections expected for sure (should cancel that!)

If scale dependence is small, might be that convergence if faster



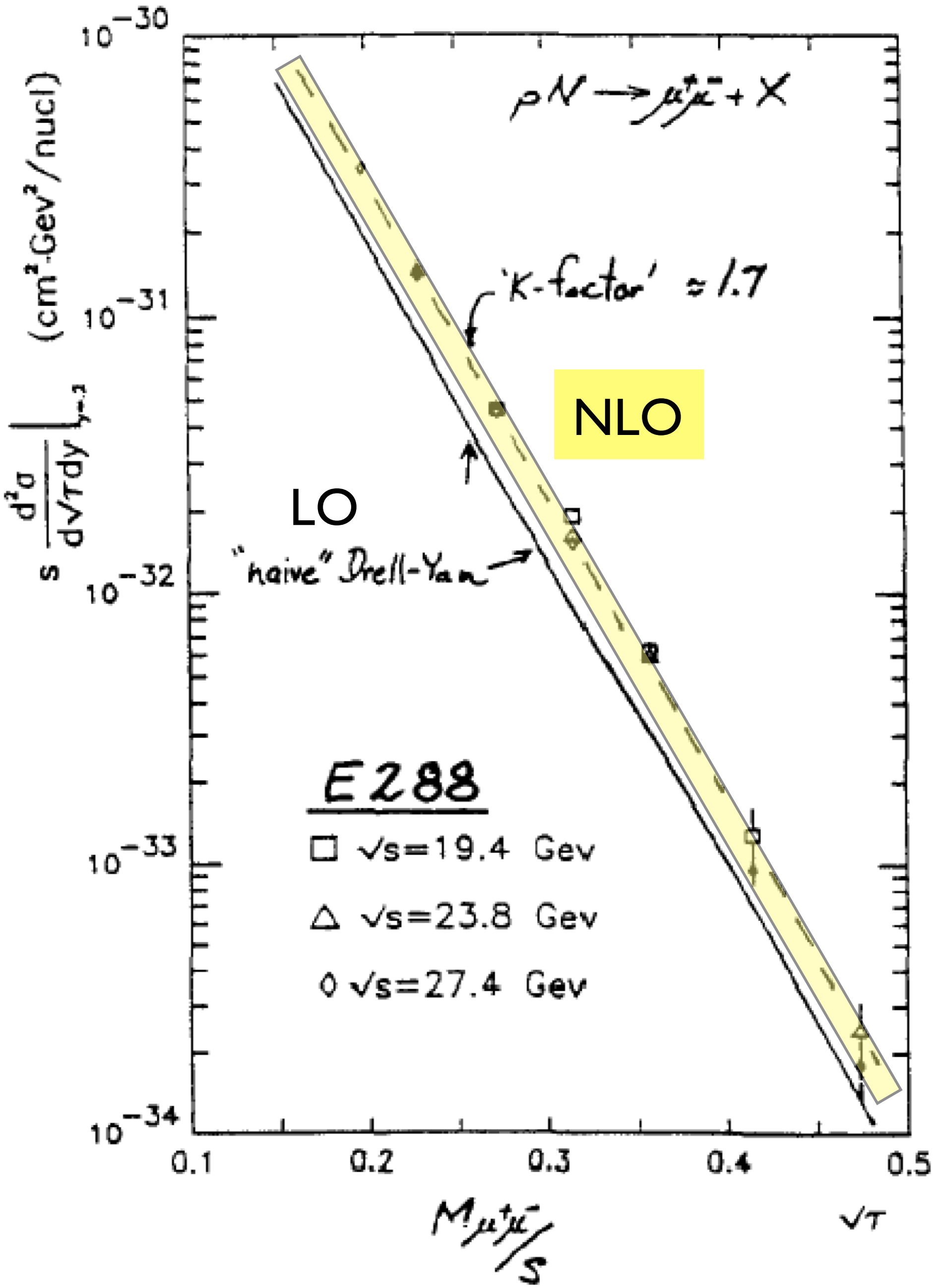
Use Q for central value and spread as "TH uncertainty"

$$R^{th} = \bar{R} \pm \Delta R$$

$\mu = Q$ $\qquad Q/2 < \mu < 2Q$

$$\alpha_s^n \Rightarrow \alpha_s^{n+1}$$

Uncertainty can only be reduced by explicit higher order calculation



Drell-Yan

- scale-dependence

Band instead of single line

$$\frac{M_{\mu^+\mu^-}}{2} \leq \mu_F \leq 2M_{\mu^+\mu^-}$$

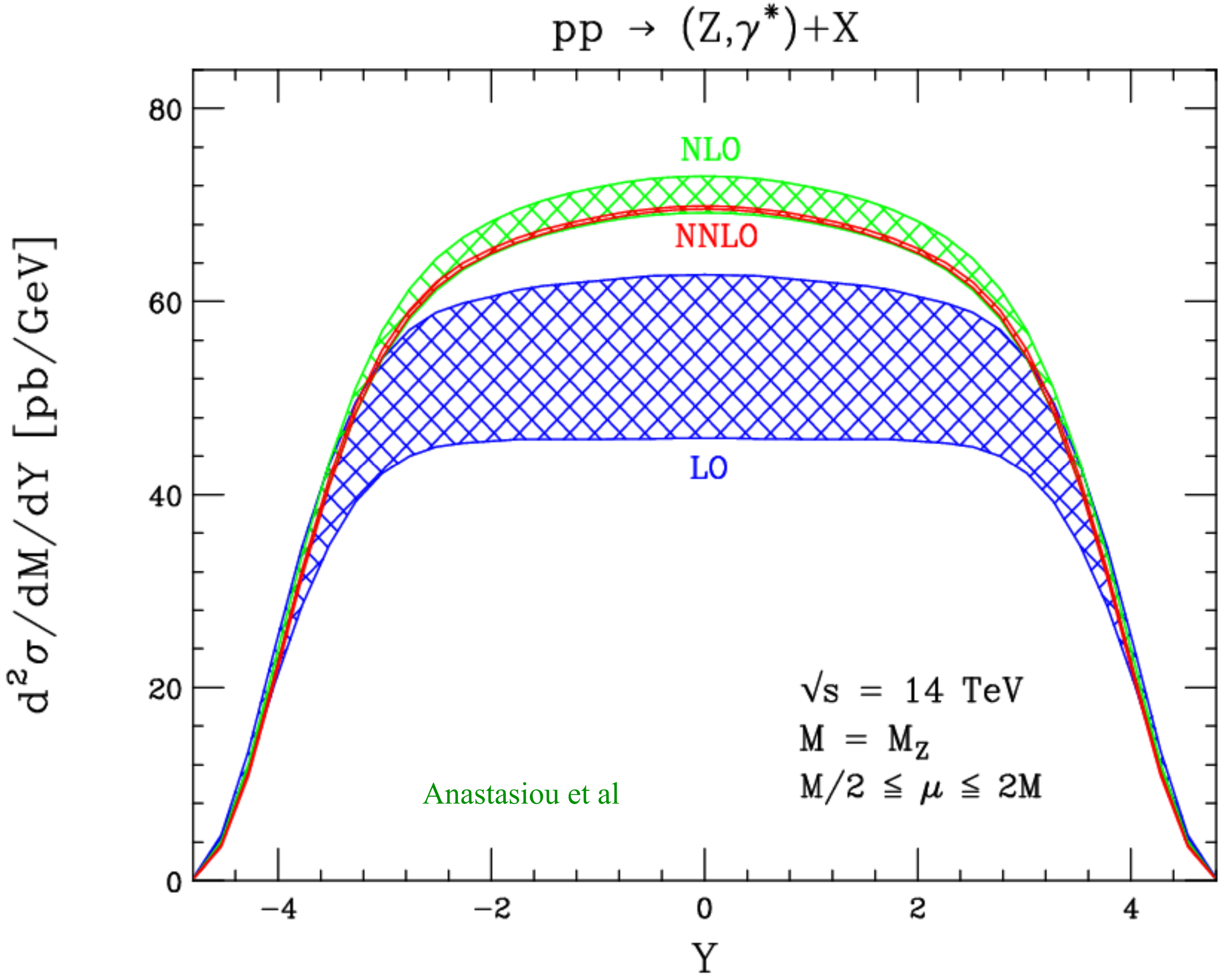
$$\frac{M_{\mu^+\mu^-}}{2} \leq \mu_R \leq 2M_{\mu^+\mu^-}$$

- factor of 2 conventional/historical
- usually "works" : anticipate higher order corrections
- Sometimes... it fails...

Scale dependence considerably reduced at higher orders

→ reduction in TH uncertainty

Drell-Yan

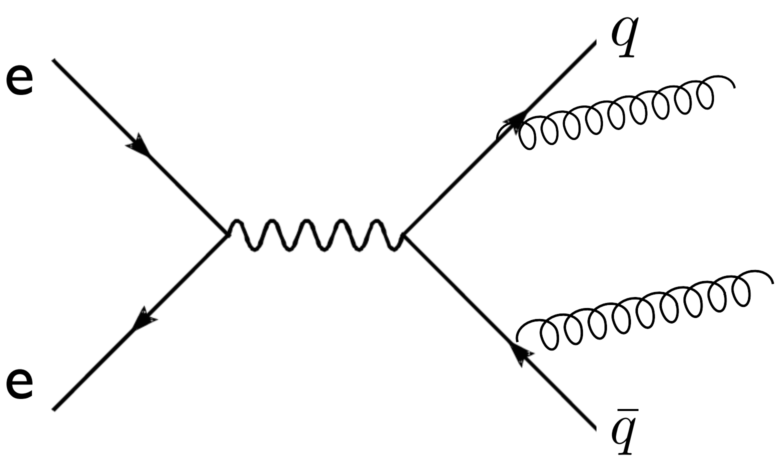


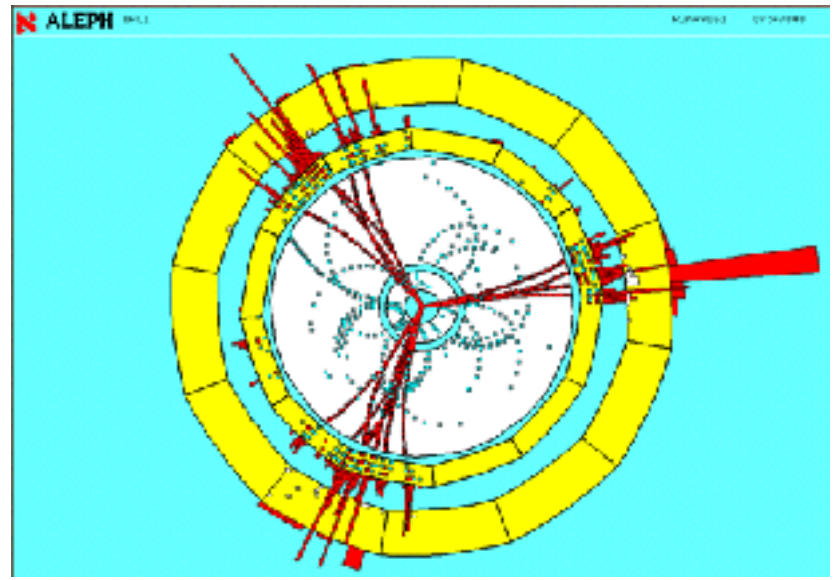
Notice that LO scale dependence fails to estimate NLO result!

- Effect of extra radiation : more partons → more realistic

Feynman Diagram vs real life

$e^+e^- \rightarrow$ hadrons



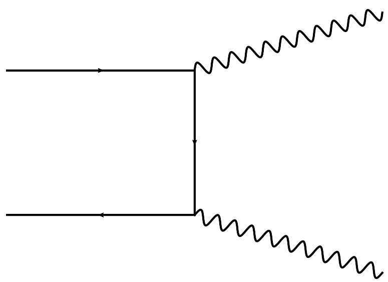


More accurate description of jet structure : first time appears at NLO (one extra parton)

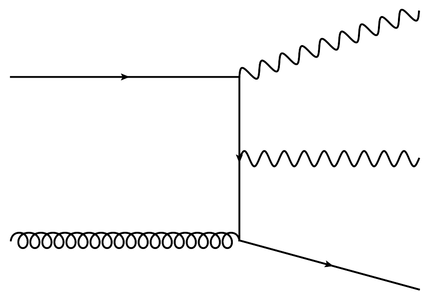
- Opening of new channels

Sometimes new channels at higher order provide large corrections due to parton luminosity (pdf, non-perturbative-pertubative interplay)

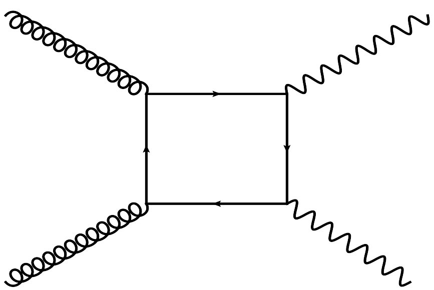
Diphoton production : main background to Higgs search



$\mathcal{O}(\alpha_s^0)$ but $q\bar{q}$ Luminosity



$\mathcal{O}(\alpha_s)$ but qg Luminosity



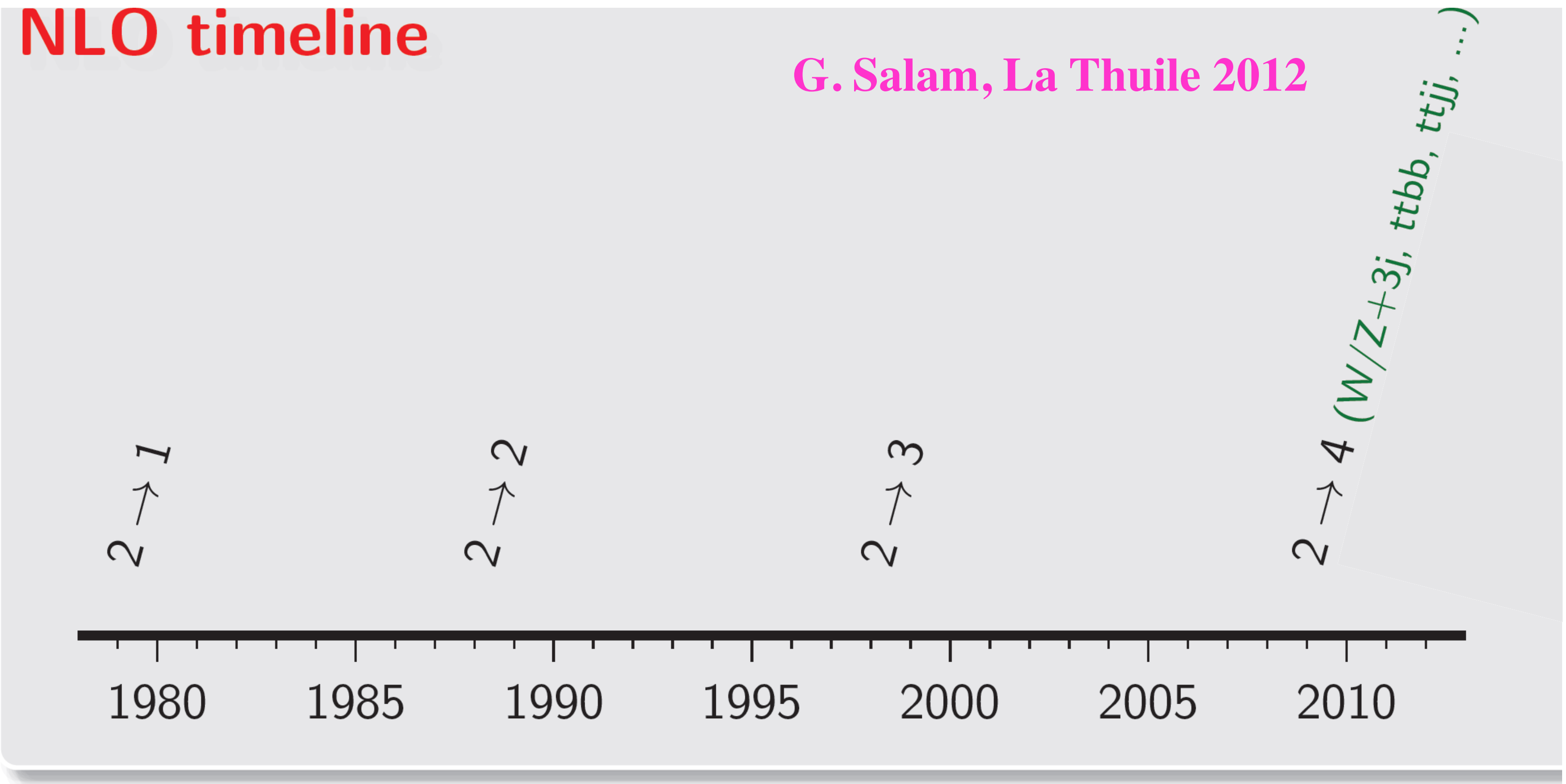
$\mathcal{O}(\alpha_s^2)$ but gg Luminosity

$\gamma\gamma$ production ➡ Box (subset of NNLO) known to be as large as Born!

Dicus, Willenbrock

How to do NLO?

Higher order calculations are difficult....



Slow progress during the first 30 years, one extra particle per decade.....

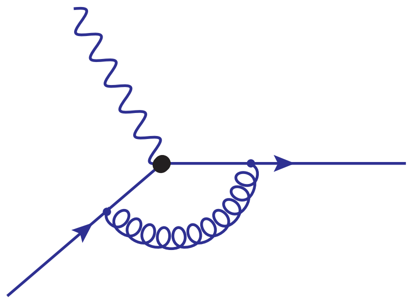
Experimenter's wish-list (Les Houches)

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \to VV$jet	WWjet completed by Dittmaier/Kallweit/Uwer [4,5]; Campbell/Ellis/Zanderighi [6]. ZZjet completed by Binoth/Gleisberg/Karg/Kauer/Sanguinetti [7]
2. $pp \to$ Higgs+2jets	NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [8]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [9,10]
3. $pp \to V\,V\,V$	ZZZ completed by Lazopoulos/Melnikov/Petriello [11] and WWZ by Hankele/Zeppenfeld [12] (see also Binoth/Ossola/Papadopoulos/Pittau [13])
4. $pp \to t\bar{t}\,b\bar{b}$	relevant for $t\bar{t}H$ computed by Bredenstein/Denner/Dittmaier/Pozzorini [14,15] and Bevilacqua/Czakon/Papadopoulos/Pittau/Worek [16]
5. $pp \to V$+3jets	calculated by the Blackhat/Sherpa [17] and Rocket [18] collaborations
Calculations remaining from Les Houches 2005	
6. $pp \to t\bar{t}$+2jets	relevant for $t\bar{t}H$ computed by Bevilacqua/Czakon/Papadopoulos/Worek [19]
7. $pp \to VV\,b\bar{b}$,	relevant for VBF $\to H \to VV$, $t\bar{t}H$
8. $pp \to VV$+2jets	relevant for VBF $\to H \to VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [20–22]
NLO calculations added to list in 2007	
9. $pp \to b\bar{b}b\bar{b}$	$q\bar{q}$ channel calculated by Golem collaboration [23]
NLO calculations added to list in 2009	
10. $pp \to V$+4 jets	top pair production, various new physics signatures
11. $pp \to Wb\bar{b}j$	top, new physics signatures
12. $pp \to t\bar{t}t\bar{t}$	various new physics signatures
Calculations beyond NLO added in 2007	
13. $gg \to W^*W^*$ $\mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs
14. NNLO $pp \to t\bar{t}$	normalization of a benchmark process
15. NNLO to VBF and Z/γ+jet	Higgs couplings and SM benchmark
Calculations including electroweak effects	
16. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

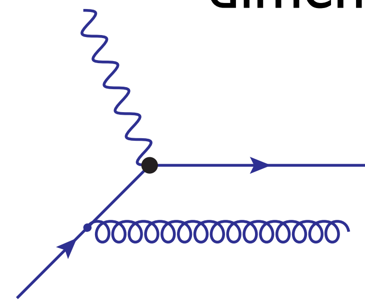
Real and virtual contributions : separately divergent

dimensional regularization

$$\frac{1}{\epsilon^2}$$



1 loop
IR + UV divergent



1 extra parton
IR in soft/collinear configurations

At Next-to-Leading Order (NLO)

$$\sigma_{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

+ renormalization
and factorization

Here one more parton
singular over PS

Here same number of partons
but one-loop matrix element
singular at loop level

But don't want to (or can't) repeat the calculation every time the definition of the observable changes: try to avoid analytical calculations

How NLO in general : subtraction method

Subtract (and add!) a term with the same singularities as the real contribution but much simpler to integrate (analytically) and universal (valid for any process)

$$d\sigma_{NLO} = \int_{d\Phi_{n+1}} \left(d\sigma^{R}_{NLO} - d\sigma^{S}_{NLO} \right) + \left[\int_{d\Phi_n} d\sigma^{V}_{NLO} + \int_{d\Phi_n} \left(\int_{d\Phi_1} d\sigma^{S}_{NLO} \right) \right]$$

finite compute numerically

until recently needed dedicated analytical calculation for virtual

Only need to analytically integrate the subtraction term (and virtual)

Subtraction term can be constructed because we understand the singular (soft and collinear) structure of QCD amplitudes

$$d\sigma_{NLO} = \int_{d\Phi_{n+1}} \left(d\sigma^R_{NLO} - d\sigma^S_{NLO} \right) + \left[\int_{d\Phi_n} d\sigma^V_{NLO} + \int_{d\Phi_n} \left(\int_{d\Phi_1} d\sigma^S_{NLO} \right) \right]$$

finite compute numerically

until recently needed dedicated analytical calculation for virtual

we understand the (universal) infrared behavior of amplitudes

→ collinear and soft limits

Different implementations for the subtraction term

- Dipole — Catani, Seymour
- Antenna — Kosower
- FKS — Frixione, Kunszt, Signer

Resulting code can compute several observables at once : change of measurement function (IR safe observables)

Many individual calculations but very complicated for large multiplicities

Real radiation automated using subtraction + tree level techniques

- SHERPA — Gleisberg, Krauss
- MadDipole — Frederix, Greiner, Gerhmann
- Helac/Phegas — Czakon, Papadopoulos, Worek
- TeVJet — Seymour, Tevlin
- AutoDipole — Hasegawa, Moch, Uwer
- MadFKS — Frederix, Frixione, Maltoni, Stelzer

Problem "conceptually" solved during the 90's

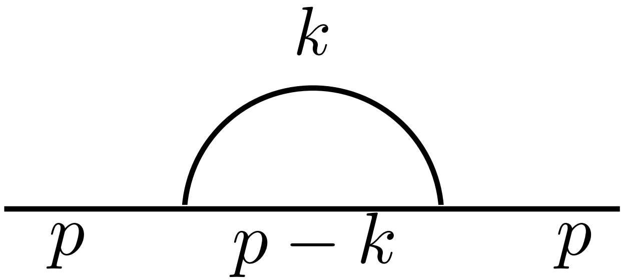
Bottleneck was in the virtual contribution

$2 \to 2$ all known

$2 \to 3$ almost all known

$2 \to 4$ many in 2010-2012

$2 \to 5$ new from 2013



$$\int d^d k \frac{N(p,k)}{(k^2-m^2)((p-k)^2-m^2)}$$

$N(p,k)=1$ Scalar integral : all known at one loop!

$N(p,k)=k^\mu k^\nu$ Tensor integral

$$\int \ldots = A\, p^\mu p^\nu + B\, g^{\mu\nu}$$

Contract both sides with

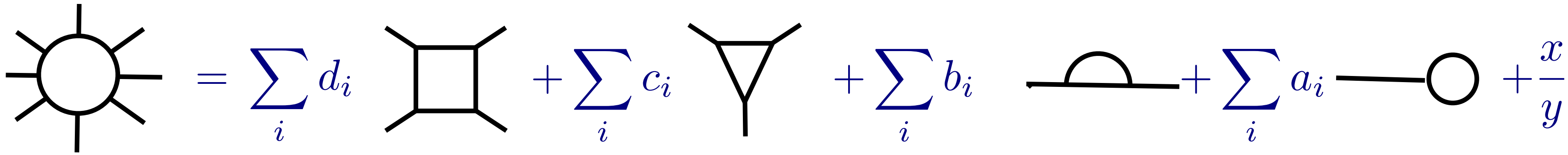
$p_\mu p_\nu\,,\ g_{\mu\nu}$

$$k^\mu k^\nu g_{\mu\nu} = (k^2-m^2)+m^2$$

$$k^\mu p_\mu = k\cdot p = \frac{1}{2}(k^2-((p-k)^2-m^2))$$

Obtain a very simple set of algebraic equations from where can extract *A* and *B* as combinations of scalar integrals

Feynmanian approach



- Perform tensorial decomposition from Feynman diagrams
- Compute coefficients (Passarino-Veltman)
- Scalar integrals known (analytical and numerical evaluation)

But for large multiplicities...

- Large number of diagrams (>1000)
- Growing number of terms in tensor reduction
- Numerical stability : vanishing of Gram determinant

$2 \to 4$ was considered an impossible task

But with clever ideas!

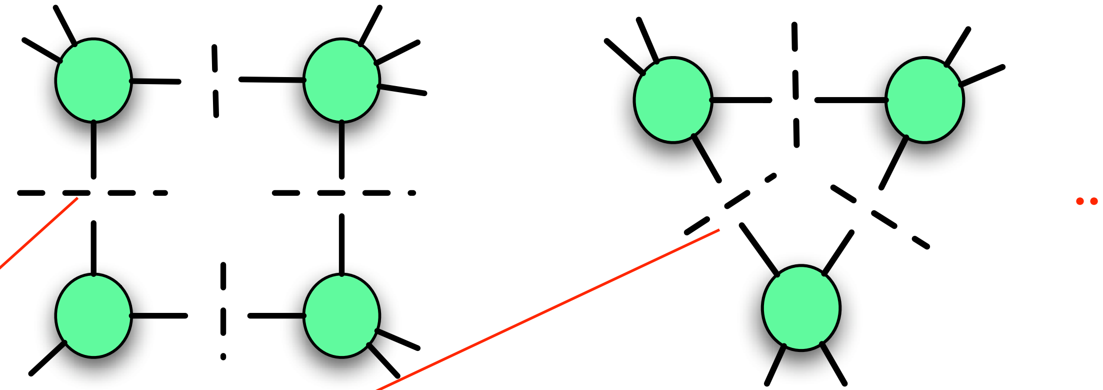
$pp \to t\bar{t}b\bar{b}$	Bredestein, Denner, Dittmaier, Pozzorini
$pp \to W^+W^-b\bar{b}$	Denner, Dittmaier, Kallweit, Pozzorini
$pp \to b\bar{b}b\bar{b}$	Cullen et al (Golem)

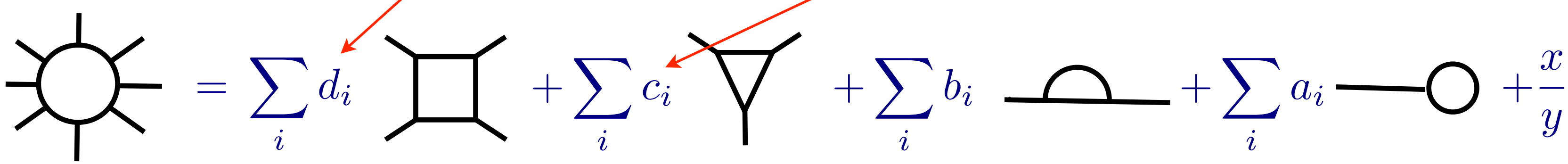
Unitarian approach

Britto, Cachazo, Feng (2004)

Find the coefficients using multi-particle cuts from generalized unitarity

replace propagator
by (on-shell) cut line
(delta, complex momentum)





Quadrupole cuts: 4 on-shell conditions on 4-dimensional loop momentum freezes the integration → box coefficient

No need of Feynman diagrams
Tree level amplitudes
Different methods for rational part
(D-dimensional) recursive relations

$$pp \to V + 5\,\mathrm{jets}$$

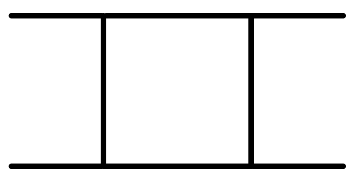
2 to 7 particles!

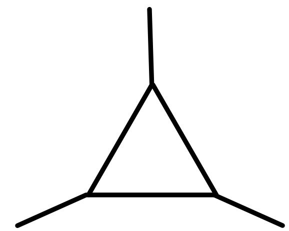
BlackHat Collaboration, Z.Bern et al

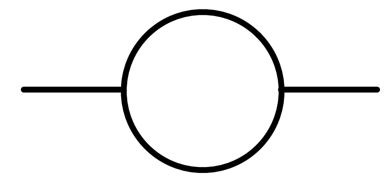
Very clever ideas include OPP: decomposition at the *integrand* level

Ossola, Papadopoulos, Pittau (2006)

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4}\ I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3}\ I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2}\ I^{(D)}_{i_1 i_2} \right)$$







Coefficients can be determined just by solving a system of equations : no loop, just algebra!

Universal - applicable to any process

Simple - based on basic algebraic properties

Ready for automation - easy to implement in a computer code

Combination of methods $\longrightarrow$ efficient numerical evaluation

Fully automated computation of 1-loop amplitudes!

- **CutTools** Ossola, Papadopoulos, Pittau
- **Rocket** Ellis, Giele, Kunszt, Melnikov, Zanderighi
- **Samurai** Mastrolia, Ossola, Reiter, Tramontano
- **Blackhat** Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower, Maître, Gleisberg
- **Golem** Binoth, Guillet, Heinrich, Pilon Reiter
- **Helac-1loop** Bevilacqua, Czakon, Garzelli, van Hameren, Kardos, Papadopoulos, Pittau, Worek
- **MadLoop** Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau
- **OpenLoops** Pozzorini, Cascioli, Maierhöfer, Buccioni, Zoller, Lang, Zhang, Lindert

and many others Lazopoulos; Giele, Kunszt, Winter, etc...

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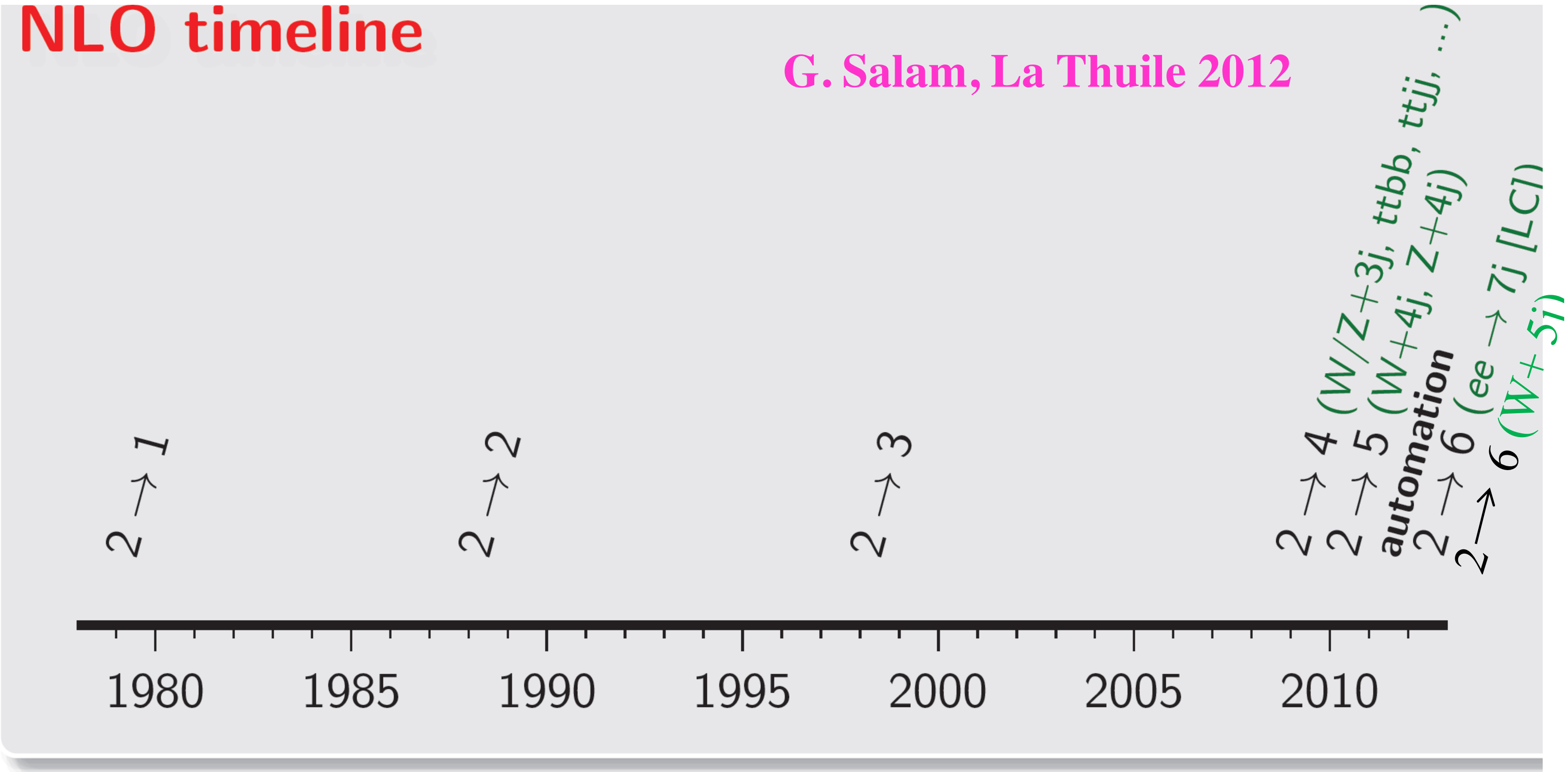
16. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

CLOSED



The NLO revolution

Combination of all the tools described before (and many others) allowed and amazing progress in the last few years

→ Large multiplicities relevant for LHC
Key is Automation!!

- Final goal: Really automatic NLO calculations

zero cost for humans

- Automatic NLO calculation "conceptually" solved

  - in a few years a number of codes

  ✓ compete on precision, flexibility, speed, stability, ...
  ✓ many features : uncertainties, ...

HELAC-NLO, Rocket, BlackHat+SHERPA, GoSam+SHERPA/MADGRAPH, NJet+SHERPA, Madgraph5-aMC@NLO, RECOLA, OpenLoops+SHERPA

MG5_aMC@NLO

[Alwall et al. arXiv:1405.0301]

Process		Syntax	Cross section (pb)			
Vector boson +jets			LO 13 TeV		NLO 13 TeV	
a.1	$pp \to W^{\pm}$	`p p > wpm`	$1.375 \pm 0.002 \cdot 10^{5}$	${}^{+15.4\%}_{-16.6\%}$ ${}^{+2.0\%}_{-1.6\%}$	$1.773 \pm 0.007 \cdot 10^{5}$	${}^{+5.2\%}_{-9.4\%}$ ${}^{+1.9\%}_{-1.6\%}$
a.2	$pp \to W^{\pm}j$	`p p > wpm j`	$2.045 \pm 0.001 \cdot 10^{4}$	${}^{+19.7\%}_{-17.2\%}$ ${}^{+1.4\%}_{-1.1\%}$	$2.843 \pm 0.010 \cdot 10^{4}$	${}^{+5.9\%}_{-8.0\%}$ ${}^{+1.3\%}_{-1.1\%}$
a.3	$pp \to W^{\pm}jj$	`p p > wpm j j`	$6.805 \pm 0.015 \cdot 10^{3}$	${}^{+24.5\%}_{-18.6\%}$ ${}^{+0.8\%}_{-0.7\%}$	$7.786 \pm 0.030 \cdot 10^{3}$	${}^{+2.4\%}_{-6.0\%}$ ${}^{+0.9\%}_{-0.8\%}$
a.4	$pp \to W^{\pm}jjj$	`p p > wpm j j j`	$1.821 \pm 0.002 \cdot 10^{3}$	${}^{+41.0\%}_{-27.1\%}$ ${}^{+0.5\%}_{-0.5\%}$	$2.005 \pm 0.008 \cdot 10^{3}$	${}^{+0.9\%}_{-6.7\%}$ ${}^{+0.6\%}_{-0.5\%}$
a.5	$pp \to Z$	`p p > z`	$4.248 \pm 0.005 \cdot 10^{4}$	${}^{+14.6\%}_{-15.8\%}$ ${}^{+2.0\%}_{-1.6\%}$	$5.410 \pm 0.022 \cdot 10^{4}$	${}^{+4.6\%}_{-8.6\%}$ ${}^{+1.9\%}_{-1.5\%}$
a.6	$pp \to Zj$	`p p > z j`	$7.209 \pm 0.005 \cdot 10^{3}$	${}^{+19.3\%}_{-17.0\%}$ ${}^{+1.2\%}_{-1.0\%}$	$9.742 \pm 0.035 \cdot 10^{3}$	${}^{+5.8\%}_{-7.8\%}$ ${}^{+1.2\%}_{-1.0\%}$
a.7	$pp \to Zjj$	`p p > z j j`	$2.348 \pm 0.006 \cdot 10^{3}$	${}^{+24.3\%}_{-18.5\%}$ ${}^{+0.6\%}_{-0.6\%}$	$2.665 \pm 0.010 \cdot 10^{3}$	${}^{+2.5\%}_{-6.0\%}$ ${}^{+0.7\%}_{-0.7\%}$
a.8	$pp \to Zjjj$	`p p > z j j j`	$6.314 \pm 0.008 \cdot 10^{2}$	${}^{+40.8\%}_{-27.0\%}$ ${}^{+0.5\%}_{-0.5\%}$	$6.996 \pm 0.028 \cdot 10^{2}$	${}^{+1.1\%}_{-6.8\%}$ ${}^{+0.5\%}_{-0.5\%}$
a.9	$pp \to \gamma j$	`p p > a j`	$1.964 \pm 0.001 \cdot 10^{4}$	${}^{+31.2\%}_{-26.0\%}$ ${}^{+1.7\%}_{-1.8\%}$	$5.218 \pm 0.025 \cdot 10^{4}$	${}^{+24.5\%}_{-21.4\%}$ ${}^{+1.4\%}_{-1.6\%}$
a.10	$pp \to \gamma jj$	`p p > a j j`	$7.815 \pm 0.008 \cdot 10^{3}$	${}^{+32.8\%}_{-24.2\%}$ ${}^{+0.9\%}_{-1.2\%}$	$1.004 \pm 0.004 \cdot 10^{4}$	${}^{+5.9\%}_{-10.9\%}$ ${}^{+0.8\%}_{-1.2\%}$

Process		Syntax	Cross section (pb)			
Vector-boson pair +jets			LO 13 TeV		NLO 13 TeV	
b.1	$pp \to W^+W^-$ (4f)	`p p > w+ w-`	$7.355 \pm 0.005 \cdot 10^{1}$	${}^{+5.0\%}_{-6.1\%}$ ${}^{+2.0\%}_{-1.5\%}$	$1.028 \pm 0.003 \cdot 10^{2}$	${}^{+4.0\%}_{-4.5\%}$ ${}^{+1.9\%}_{-1.4\%}$
b.2	$pp \to ZZ$	`p p > z z`	$1.097 \pm 0.002 \cdot 10^{1}$	${}^{+4.5\%}_{-5.6\%}$ ${}^{+1.9\%}_{-1.5\%}$	$1.415 \pm 0.005 \cdot 10^{1}$	${}^{+3.1\%}_{-3.7\%}$ ${}^{+1.8\%}_{-1.4\%}$
b.3	$pp \to ZW^{\pm}$	`p p > z wpm`	$2.777 \pm 0.003 \cdot 10^{1}$	${}^{+3.6\%}_{-4.7\%}$ ${}^{+2.0\%}_{-1.5\%}$	$4.487 \pm 0.013 \cdot 10^{1}$	${}^{+4.4\%}_{-4.4\%}$ ${}^{+1.7\%}_{-1.3\%}$
b.4	$pp \to \gamma\gamma$	`p p > a a`	$2.510 \pm 0.002 \cdot 10^{1}$	${}^{+22.1\%}_{-22.4\%}$ ${}^{+2.4\%}_{-2.1\%}$	$6.593 \pm 0.021 \cdot 10^{1}$	${}^{+17.6\%}_{-18.8\%}$ ${}^{+2.0\%}_{-1.9\%}$
b.5	$pp \to \gamma Z$	`p p > a z`	$2.523 \pm 0.004 \cdot 10^{1}$	${}^{+9.9\%}_{-11.2\%}$ ${}^{+2.0\%}_{-1.6\%}$	$3.695 \pm 0.013 \cdot 10^{1}$	${}^{+5.4\%}_{-7.1\%}$ ${}^{+1.8\%}_{-1.4\%}$
b.6	$pp \to \gamma W^{\pm}$	`p p > a wpm`	$2.954 \pm 0.005 \cdot 10^{1}$	${}^{+9.5\%}_{-11.0\%}$ ${}^{+2.0\%}_{-1.7\%}$	$7.124 \pm 0.026 \cdot 10^{1}$	${}^{+9.7\%}_{-9.9\%}$ ${}^{+1.5\%}_{-1.3\%}$
b.7	$pp \to W^+W^-j$ (4f)	`p p > w+ w- j`	$2.865 \pm 0.003 \cdot 10^{1}$	${}^{+11.6\%}_{-10.0\%}$ ${}^{+1.0\%}_{-0.8\%}$	$3.730 \pm 0.013 \cdot 10^{1}$	${}^{+4.9\%}_{-4.9\%}$ ${}^{+1.1\%}_{-0.8\%}$
b.8	$pp \to ZZj$	`p p > z z j`	$3.662 \pm 0.003 \cdot 10^{0}$	${}^{+10.9\%}_{-9.3\%}$ ${}^{+1.0\%}_{-0.8\%}$	$4.830 \pm 0.016 \cdot 10^{0}$	${}^{+5.0\%}_{-4.8\%}$ ${}^{+1.1\%}_{-0.9\%}$
b.9	$pp \to ZW^{\pm}j$	`p p > z wpm j`	$1.605 \pm 0.005 \cdot 10^{1}$	${}^{+11.6\%}_{-10.0\%}$ ${}^{+0.9\%}_{-0.7\%}$	$2.086 \pm 0.007 \cdot 10^{1}$	${}^{+4.9\%}_{-4.8\%}$ ${}^{+0.9\%}_{-0.7\%}$
b.10	$pp \to \gamma\gamma j$	`p p > a a j`	$1.022 \pm 0.001 \cdot 10^{1}$	${}^{+20.3\%}_{-17.7\%}$ ${}^{+1.2\%}_{-1.5\%}$	$2.292 \pm 0.010 \cdot 10^{1}$	${}^{+17.2\%}_{-15.1\%}$ ${}^{+1.0\%}_{-1.4\%}$
b.11*	$pp \to \gamma Zj$	`p p > a z j`	$8.310 \pm 0.017 \cdot 10^{0}$	${}^{+14.5\%}_{-12.8\%}$ ${}^{+1.0\%}_{-1.0\%}$	$1.220 \pm 0.005 \cdot 10^{1}$	${}^{+7.3\%}_{-7.4\%}$ ${}^{+0.9\%}_{-0.9\%}$
b.12*	$pp \to \gamma W^{\pm}j$	`p p > a wpm j`	$2.546 \pm 0.010 \cdot 10^{1}$	${}^{+13.7\%}_{-12.1\%}$ ${}^{+0.9\%}_{-1.0\%}$	$3.713 \pm 0.015 \cdot 10^{1}$	${}^{+7.2\%}_{-7.1\%}$ ${}^{+0.9\%}_{-1.0\%}$
b.13	$pp \to W^+W^+jj$	`p p > w+ w+ j j`	$1.484 \pm 0.006 \cdot 10^{-1}$	${}^{+25.4\%}_{-18.9\%}$ ${}^{+2.1\%}_{-1.5\%}$	$2.251 \pm 0.011 \cdot 10^{-1}$	${}^{+10.5\%}_{-10.6\%}$ ${}^{+2.2\%}_{-1.6\%}$
b.14	$pp \to W^-W^-jj$	`p p > w- w- j j`	$6.752 \pm 0.007 \cdot 10^{-2}$	${}^{+25.4\%}_{-18.9\%}$ ${}^{+2.4\%}_{-1.7\%}$	$1.003 \pm 0.003 \cdot 10^{-1}$	${}^{+10.1\%}_{-10.4\%}$ ${}^{+2.5\%}_{-1.8\%}$
b.15	$pp \to W^+W^-jj$ (4f)	`p p > w+ w- j j`	$1.144 \pm 0.002 \cdot 10^{1}$	${}^{+27.2\%}_{-19.9\%}$ ${}^{+0.7\%}_{-0.5\%}$	$1.396 \pm 0.005 \cdot 10^{1}$	${}^{+5.0\%}_{-6.8\%}$ ${}^{+0.7\%}_{-0.6\%}$
b.16	$pp \to ZZjj$	`p p > z z j j`	$1.344 \pm 0.002 \cdot 10^{0}$	${}^{+26.6\%}_{-19.6\%}$ ${}^{+0.7\%}_{-0.6\%}$	$1.706 \pm 0.011 \cdot 10^{0}$	${}^{+5.8\%}_{-7.2\%}$ ${}^{+0.8\%}_{-0.6\%}$
b.17	$pp \to ZW^{\pm}jj$	`p p > z wpm j j`	$8.038 \pm 0.009 \cdot 10^{0}$	${}^{+26.7\%}_{-19.7\%}$ ${}^{+0.7\%}_{-0.5\%}$	$9.139 \pm 0.031 \cdot 10^{0}$	${}^{+3.1\%}_{-5.1\%}$ ${}^{+0.7\%}_{-0.5\%}$
b.18	$pp \to \gamma\gamma jj$	`p p > a a j j`	$5.377 \pm 0.029 \cdot 10^{0}$	${}^{+26.2\%}_{-19.8\%}$ ${}^{+0.6\%}_{-1.0\%}$	$7.501 \pm 0.032 \cdot 10^{0}$	${}^{+8.8\%}_{-10.1\%}$ ${}^{+0.6\%}_{-1.0\%}$
b.19*	$pp \to \gamma Zjj$	`p p > a z j j`	$3.260 \pm 0.009 \cdot 10^{0}$	${}^{+24.3\%}_{-18.4\%}$ ${}^{+0.6\%}_{-0.6\%}$	$4.242 \pm 0.016 \cdot 10^{0}$	${}^{+6.5\%}_{-7.3\%}$ ${}^{+0.6\%}_{-0.6\%}$
b.20*	$pp \to \gamma W^{\pm}jj$	`p p > a wpm j j`	$1.233 \pm 0.002 \cdot 10^{1}$	${}^{+24.7\%}_{-18.6\%}$ ${}^{+0.6\%}_{-0.6\%}$	$1.448 \pm 0.005 \cdot 10^{1}$	${}^{+3.6\%}_{-5.4\%}$ ${}^{+0.6\%}_{-0.7\%}$

+... total of 172 processes up to 2→4

GoSam+Ninja

[van Duerzen et al. arXiv:1312.6678]

Benchmarks: GoSam + Ninja			
Process		# NLO diagrams	ms/event
$W+3j$	$d\bar{u} \to \bar{\nu}_e e^- ggg$	1 411	226
$Z+3j$	$d\bar{d} \to e^+e^- ggg$	2 928	1 911
$ZZZ+1j$	$u\bar{u} \to ZZZg$	915	*12 000
$WWZ+1j$	$u\bar{u} \to W^+W^-Zg$	779	*7 050
$WZZ+1j$	$u\bar{d} \to W^+ZZg$	756	*3 300
$WWW+1j$	$u\bar{d} \to W^+W^-W^+g$	569	*1 800
$ZZZZ$	$u\bar{u} \to ZZZZ$	408	*1 070
$WWWW$	$u\bar{u} \to W^+W^-W^+W^-$	496	*1 350
$t\bar{t}b\bar{b}\,(m_b \neq 0)$	$d\bar{d} \to t\bar{t}b\bar{b}$	275	178
	$gg \to t\bar{t}b\bar{b}$	1 530	5 685
$t\bar{t}+2j$	$gg \to t\bar{t}gg$	4 700	13 827
$Zb\bar{b}+1j\,(m_b \neq 0)$	$dug \to ue^+e^-b\bar{b}$	708	*1 070
$Wb\bar{b}+1j\,(m_b \neq 0)$	$u\bar{d} \to e^+\nu_e b\bar{b}g$	312	67
$Wb\bar{b}+2j\,(m_b \neq 0)$	$u\bar{d} \to e^+\nu_e b\bar{b}s\bar{s}$	648	181
	$u\bar{d} \to e^+\nu_e b\bar{b}d\bar{d}$	1 220	895
	$u\bar{d} \to e^+\nu_e b\bar{b}gg$	3 923	5387
$WWb\bar{b}\,(m_b \neq 0)$	$d\bar{d} \to \nu_e e^+\bar{\nu}_\mu\mu^- b\bar{b}$	292	115
	$gg \to \nu_e e^+\bar{\nu}_\mu\mu^- b\bar{b}$	1 068	*5 300
$WWb\bar{b}+1j\,(m_b = 0)$	$u\bar{u} \to \nu_e e^+\bar{\nu}_\mu\mu^- b\bar{b}g$	3 612	*2 000
$H+3j$ in GF	$gg \to Hggg$	9 325	8 961
$t\bar{t}Z+1j$	$u\bar{u} \to t\bar{t}e^+e^-g$	1408	1 220
	$gg \to t\bar{t}e^+e^-g$	4230	19 560
$t\bar{t}H+1j$	$gg \to t\bar{t}Hg$	1 517	1 505
$H+3j$ in VBF	$u\bar{u} \to Hgu\bar{u}$	432	101
$H+4j$ in VBF	$u\bar{u} \to Hggu\bar{u}$	1 176	669
$H+5j$ in VBF	$u\bar{u} \to Hgggu\bar{u}$	15 036	29 200

How easy is NLO these days?

pp⟶tt+j

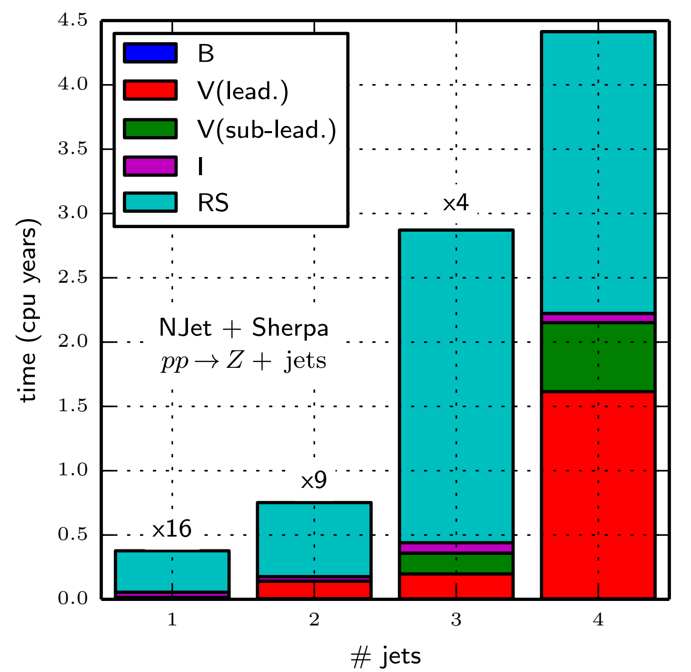
```
import model loop_sm-no_b_mass
define p = g u u~ c c~ d d~ s s~ b b~
define j = g u u~ c c~ d d~ s s~ b b~
generate p p > t~ t j [QCD]
output my_pp_ttj
calculate_xs NLO
```

e.g. MadGraph5_aMC@NLO v2.1.1
[Alwall et al. 1405.0301]

generation time ~ 5 mins
total cross section ~ 30 mins (20 cores)

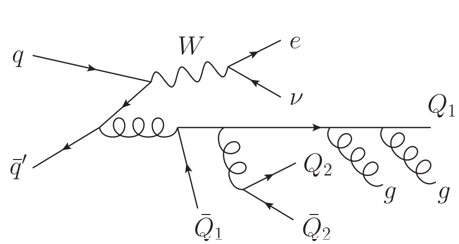
$$\sigma^{\mathrm{NLO}}_{pp\to\bar{t}tj}(\mu_R = \mu_F = m_t) = 687(7)^{+23}_{-58}\,\mathrm{pb}$$

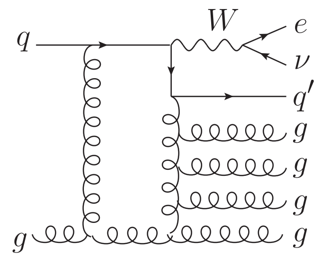
Bottleneck "only" in CPU time



An example : W+5 jets !!

BlackHat Collaboration, Z.Bern et al

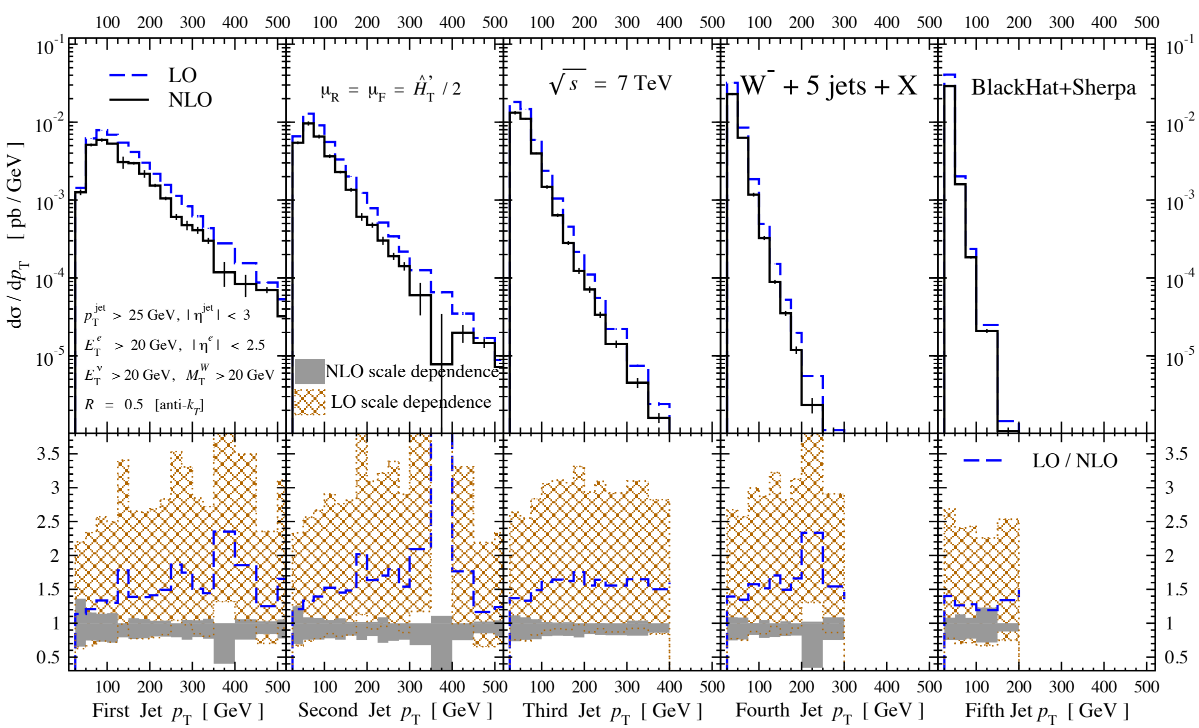




Real $2 \to 8$ SHERPA

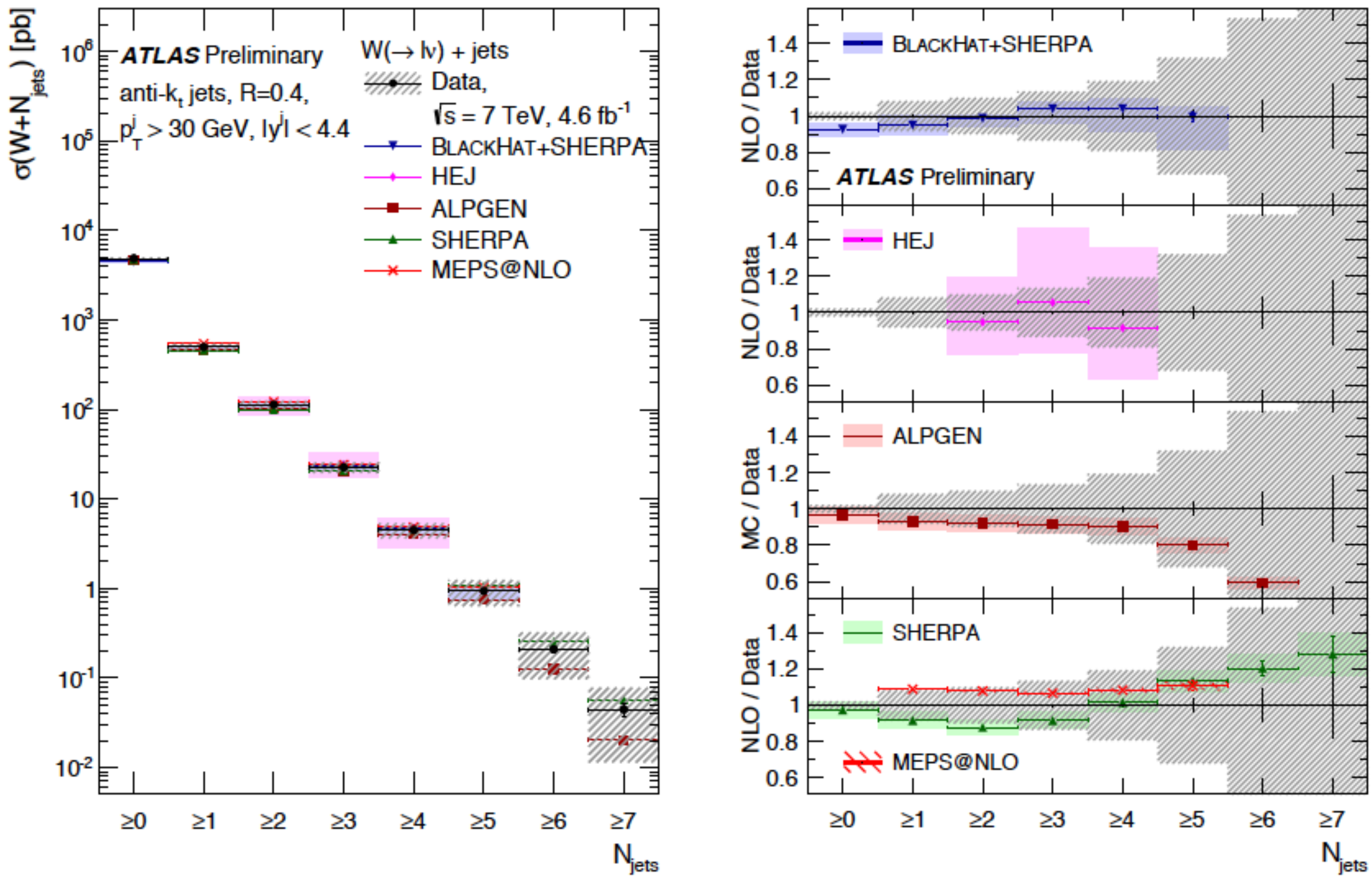
Virtual $2 \to 7$ BlackHat

Dynamical Scale choice $\mu_R = \mu_F = \frac{\hat{H}'_T}{2} \equiv \frac{1}{2}\sum_m p_T^m + E_T^W$



- Dramatic reduction in scale dependence (~20%)
- Up to 50% correction (non-trivial in shape)

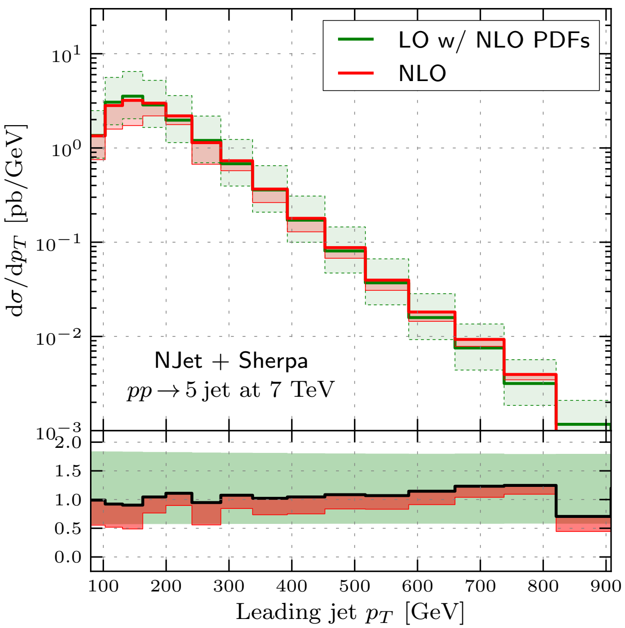
There are (already) measurements up to 7 jets !

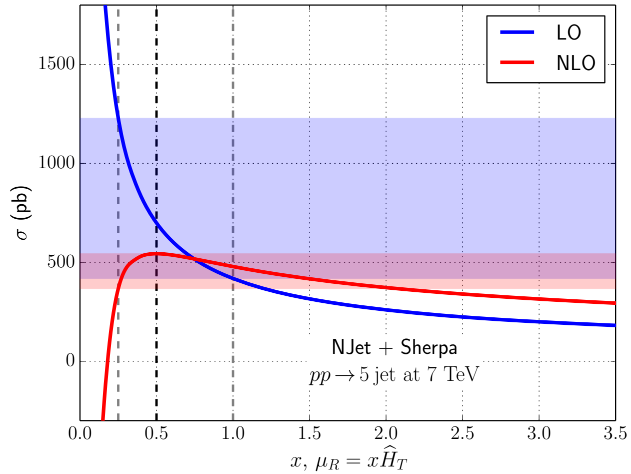


Multi-jet production

$pp \to 5\,\text{jets at NLO}$

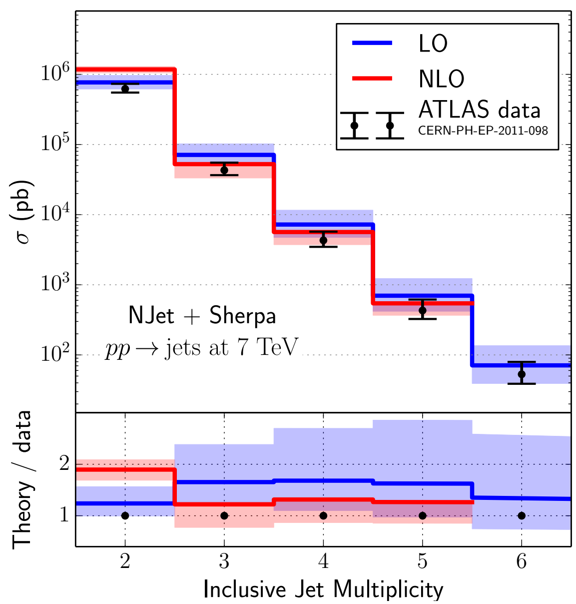
Njet+Sherpa (Badger, Biedermann, Uwer, Yundin





$$\widehat{H}_T = \sum_{i=1}^{N_{\text{parton}}} p_{T,i}^{\text{parton}}$$

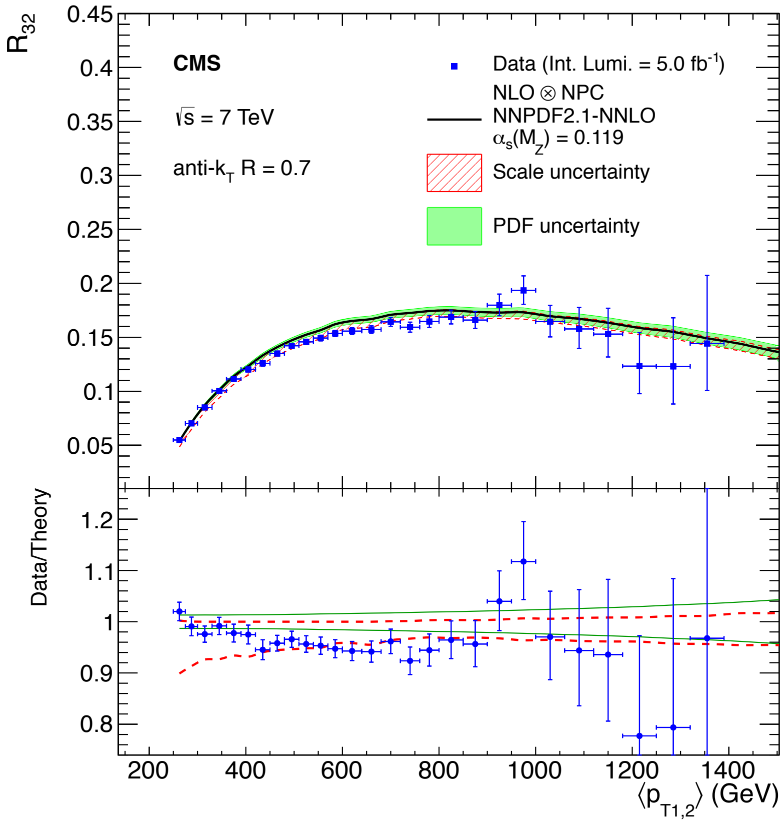
- Better stability
- NLO in very good agreement with data!

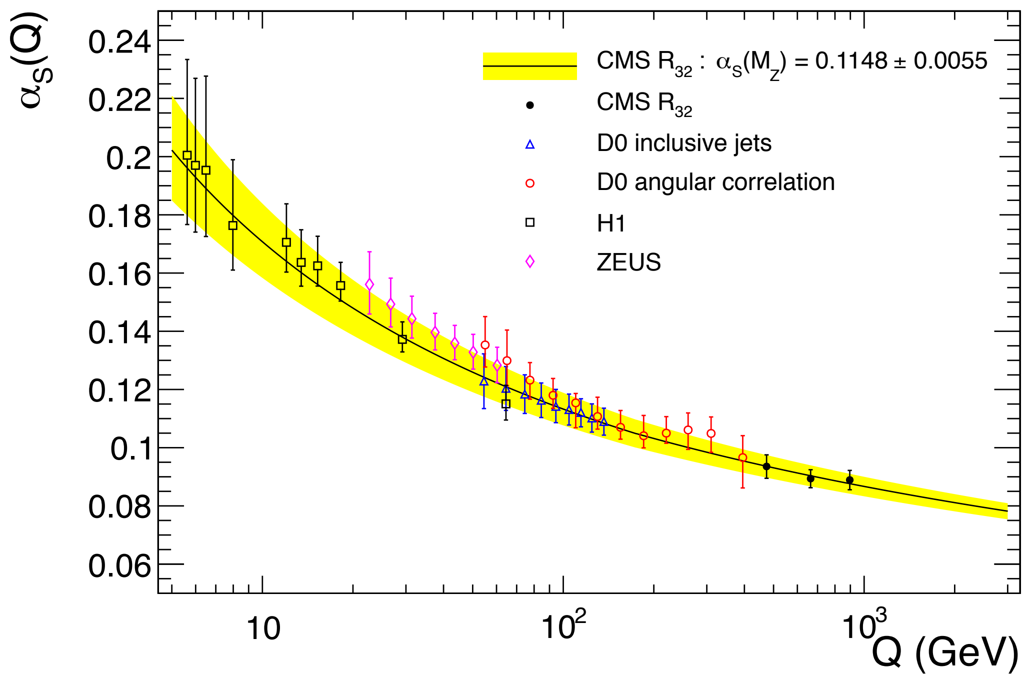


4jets in agreement with previous calculation by BlackHat (Z.Bern et al)

Multi-jet production

Use ratios 3-jets/2-jets to extract coupling constant





$$Q = \langle p_{T1,2} \rangle = \frac{p_{T1} + p_{T2}}{2}$$

- Coupling extraction in agreement with HERA/Tevatron

$$\alpha_S(M_Z) = 0.1148 \pm 0.0014\,(\text{exp.}) \pm 0.0018\,(\text{PDF}) \pm 0.0050\,(\text{theory})$$

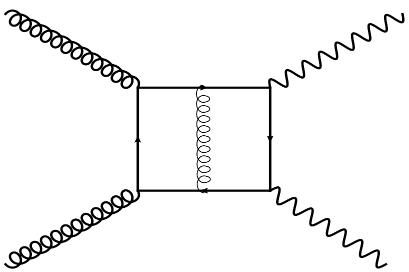
- Not everything solved at NLO yet… but constant progress

- Parton Showers @NLO

- Automated EW corrections   MADGRAPH5_AMC@NLO   Sherpa+Recola

  - QCD dominant (except very large pT)
  - Coupling hierarchy ~ respected
  - Large cancellations in EW contributions

- Loop induced Processes $gg \to VV$



- Enhanced by gluon luminosity
- Corrections for gg channel usually large (color, logs)

F. Caola, et al (2015-2016)
J. Campbell, K. Ellis, M. Czakon, S. Kirchner (2015)

- BSM (arbitrary, higher dimensional operators, etc)

~Automated!   BSM@NLO+aMC@NLO
MadGolem

Recap of third lecture

- "New" methods allow to compute amplitudes in a more efficient way : helicity, color, recursions

- Many tools available for LO : qualitative for colliders...

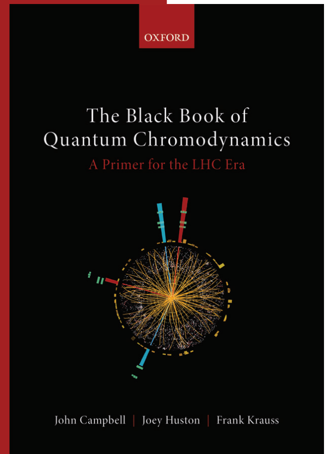
- Higher order calculations needed: scale dependence and uncertainties estimates, large higher order corrections, precision. more realistic (more partons), new channels with large luminosities, etc

- How to do NLO: subtraction method for "real" plus new techniques for numerical computation of virtual amplitudes

- Automation for NLO : very simple to compute, input card, definitions and run!

- Many high multiplicities observables computed for LHC : multi-jet

- NLO might not be enough for some processes...



The Black Book of Quantum Chromodynamics

A Primer for the LHC Era

John Campbell, Fermi National Accelerator Laboratory
Joey Huston, Michigan State University
Frank Krauss, Durham University

- Landmark textbook on modern Quantum Chromodynamics
- Pedagogical style, based on lectures, written by practitioners in the field
- Detailed calculations and discussions of all aspects relevant for physics at hadron colliders such as the LHC
- Full of illustrative data
- Clear layout, detailed index, exhaustive references

The Black Book of Quantum Chromodynamics is an in-depth introduction to the particle physics of current and future experiments at particle accelerators. The book offers the reader an overview of practically all aspects of the strong interaction necessary to understand and appreciate modern particle phenomenology at the energy frontier. Aimed at graduate students and researchers, it also serves as a comprehensive reference for LHC experimenters and theorists.

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