A. Pich, arXiv:1805.08597

Flavour Physics & $\not{C}\not{P}$

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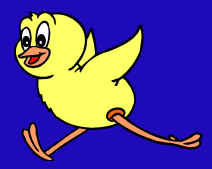
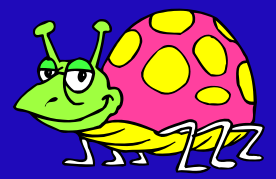
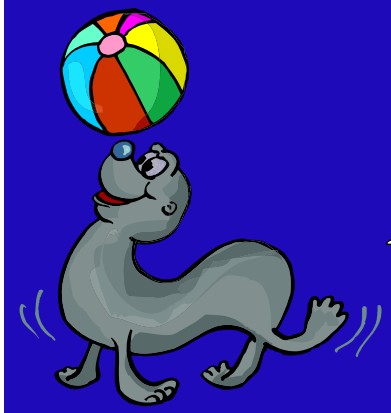
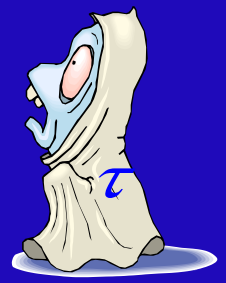
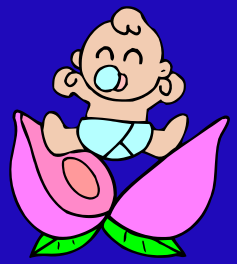
Flavour Physics & $\not{C}\not{P}$

1. Quark Mixing
2. P^0-$\bar{P}^0$ Mixing & CP Violation
3. Searching for New Physics

Quarks		Leptons		Bosons
 up	 down	 electron	 neutrino e	 photon
 charm	 strange	 muon	 neutrino μ	 gluon
 top	 beauty	 tau	 neutrino τ	 Z^0 $W^{\pm}$
				 Higgs

Flavour Structure of the Standard Model

$$\begin{pmatrix} u & \nu_e \\ d & e^- \end{pmatrix}, \begin{pmatrix} c & \nu_\mu \\ s & \mu^- \end{pmatrix}, \begin{pmatrix} t & \nu_\tau \\ b & \tau^- \end{pmatrix}$$

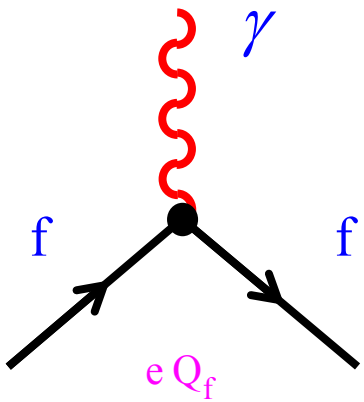
- **Pattern of masses**
- **Flavour Mixing**
- $\not{CP}$

Related to SSB

Scalar Sector (Higgs)

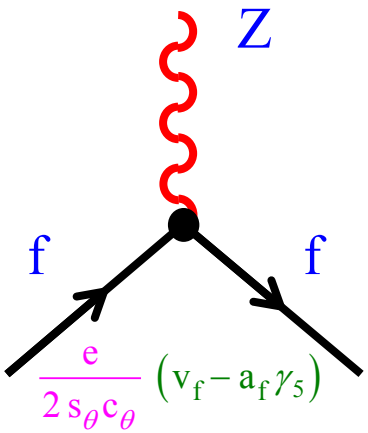
- **Kaon Factories :** u , d , s
- **τcF :** c , τ
- **BF:** b , c , τ
- **LHC :** t , b , c
- **LC :** t , b , c
- **νF :** ν_e , ν_μ , ν_τ

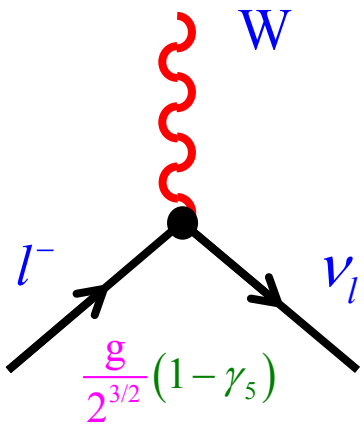
Universality: Family–Independent Couplings



NEUTRAL CURRENTS

Flavour Conserving

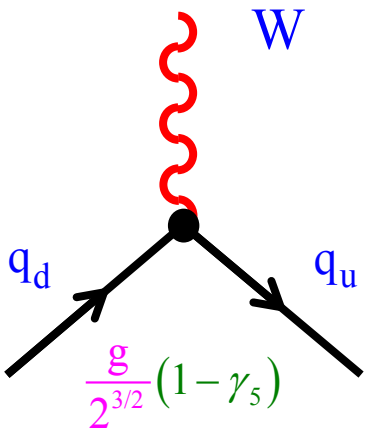


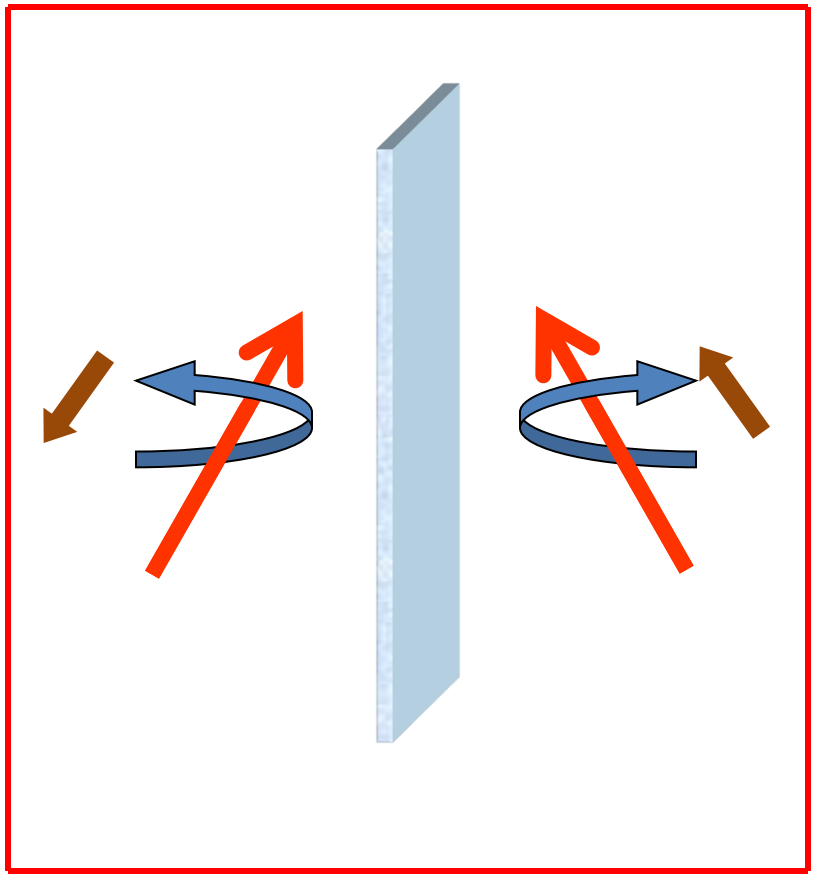


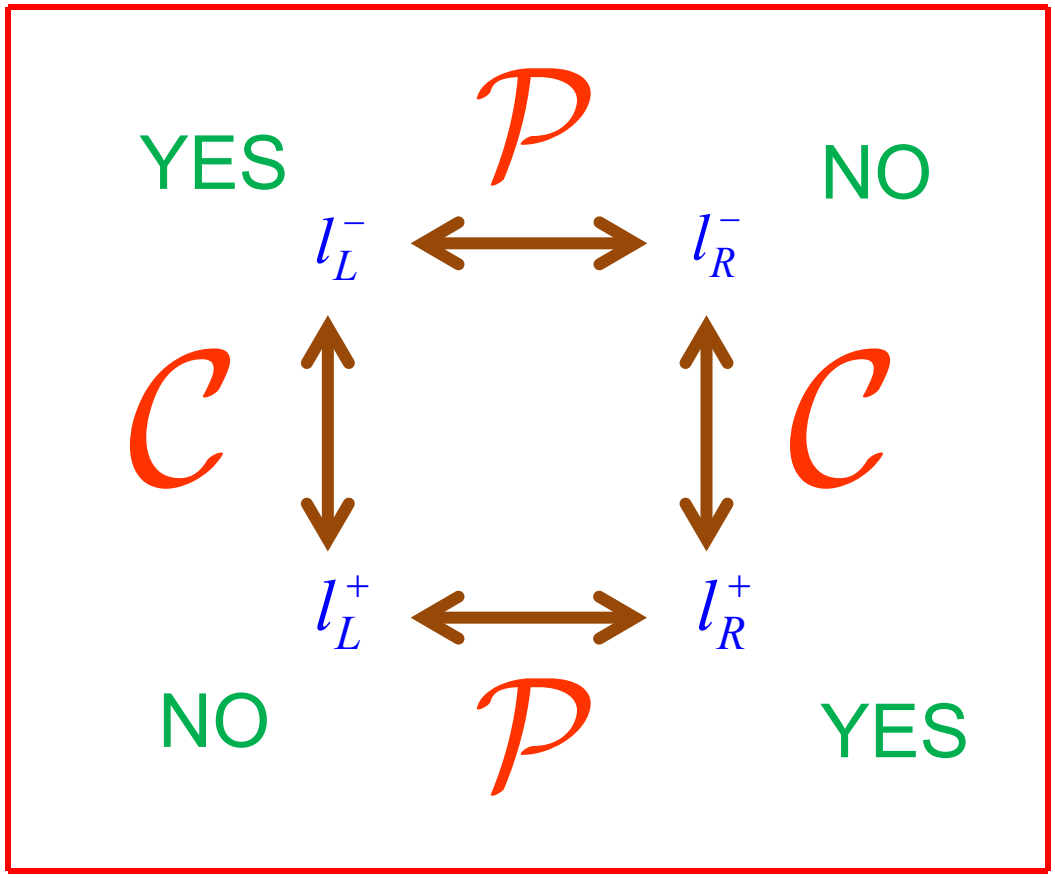
CHARGED CURRENTS

Flavour Changing

Left Handed







$\not{\mathcal{P}}$ **and** $\not{\mathcal{C}}$ **in Weak Interactions**

$\mathcal{CP}$ **still a good symmetry** **(1 family)**

FERMION MASSES

Scalar – Fermion Couplings allowed by Gauge Symmetry

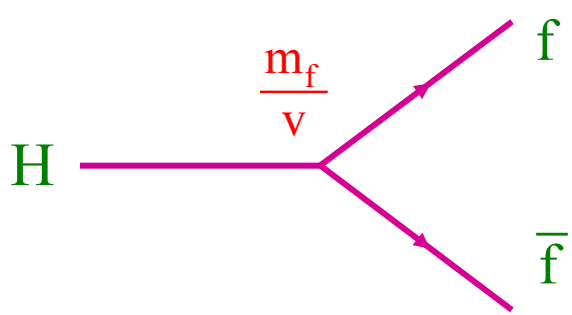
$$\mathcal{L}_Y = -(\bar{q}_u, \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right] - (\bar{\nu}_l, \bar{l})_L \, c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R + \text{h.c.}$$

SSB

$$\mathcal{L}_Y = -\left(1 + \frac{H}{\text{v}}\right) \left\{ m_{q_d} \, \bar{q}_d \, q_d + m_{q_u} \, \bar{q}_u \, q_u + m_l \, \bar{l} \, l \right\}$$

Fermion Masses are New Free Parameters

$$\left[m_{q_d}, \, m_{q_u}, \, m_l \right] = \left[c^{(d)}, \, c^{(u)}, \, c^{(l)} \right] \frac{\text{v}}{\sqrt{2}}$$



Couplings Fixed: $g_{Hf\bar{f}} = \frac{m_f}{v}$

FERMION GENERATIONS

$\mathbf{N_G = 3}$ **Identical Copies** **Masses are the only difference**

$$Q = 0 \qquad Q = -1 \qquad \begin{pmatrix} \nu'_j & u'_j \\ l'_j & d'_j \end{pmatrix} \qquad Q = +2/3 \qquad Q = -1/3 \qquad (j = 1, \cdots, \mathrm{N_G})$$

WHY ?

$$\mathcal{L}_Y = -\sum_{jk} \left\{ \left(\overline{u}'_j, \overline{d}'_j\right)_L \left[c^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c^{(u)}_{jk} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] - \left(\overline{\nu}'_j, \overline{l}'_j\right)_L c^{(l)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} + \text{h.c.}$$

SSB

$$\mathcal{L}_Y = -\left(1 + \frac{H}{\mathrm{v}}\right) \left\{ \overline{d}'_L \cdot \mathbf{M}'_d \cdot d'_R + \overline{u}'_L \cdot \mathbf{M}'_u \cdot u'_R + \overline{l}'_L \cdot \mathbf{M}'_l \cdot l'_R + \text{h.c.} \right\}$$

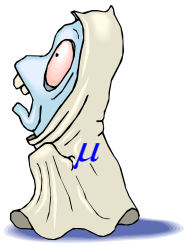
Arbitrary Non-Diagonal Complex Mass Matrices

$$\left[\mathbf{M}'_d \,,\, \mathbf{M}'_u \,,\, \mathbf{M}'_l \right]_{jk} = \left[c^{(d)}_{jk} \,,\, c^{(u)}_{jk} \,,\, c^{(l)}_{jk} \right] \frac{\mathrm{v}}{\sqrt{2}}$$

DIAGONALIZATION OF MASS MATRICES

$$\mathbf{M}'_d = \mathbf{H}_d \cdot \mathbf{U}_d = \mathbf{S}_d^\dagger \cdot \boldsymbol{\mathcal{M}}_d \cdot \mathbf{S}_d \cdot \mathbf{U}_d$$
$$\mathbf{M}'_u = \mathbf{H}_u \cdot \mathbf{U}_u = \mathbf{S}_u^\dagger \cdot \boldsymbol{\mathcal{M}}_u \cdot \mathrm{S}_u \cdot \mathbf{U}_u$$
$$\mathbf{M}'_l = \mathbf{H}_l \cdot \mathbf{U}_l = \mathbf{S}_l^\dagger \cdot \boldsymbol{\mathcal{M}}_l \cdot \mathbf{S}_l \cdot \mathbf{U}_l$$

$$\mathbf{H}_\mathrm{f} = \mathbf{H}_\mathrm{f}^\dagger$$
$$\mathbf{U}_\mathrm{f} \cdot \mathbf{U}_\mathrm{f}^\dagger = \mathbf{U}_\mathrm{f}^\dagger \cdot \mathbf{U}_\mathrm{f} = 1$$
$$\mathbf{S}_\mathrm{f} \cdot \mathbf{S}_\mathrm{f}^\dagger = \mathbf{S}_\mathrm{f}^\dagger \cdot \mathbf{S}_\mathrm{f} = 1$$



$$\mathcal{L}_Y = -\left(1+\frac{H}{\mathrm{v}}\right)\left\{\bar{\mathrm{d}}\cdot\boldsymbol{\mathcal{M}}_d\cdot\mathrm{d} + \bar{\mathrm{u}}\cdot\boldsymbol{\mathcal{M}}_u\cdot\mathrm{u} + \bar{l}\cdot\boldsymbol{\mathcal{M}}_l\cdot l\right\}$$

$$\boldsymbol{\mathcal{M}}_u = \mathrm{diag}\,(m_u, m_c, m_t) \quad ; \quad \boldsymbol{\mathcal{M}}_d = \mathrm{diag}\,(m_d, m_s, m_b) \quad ; \quad \boldsymbol{\mathcal{M}}_l = \mathrm{diag}\,(m_e, m_\mu, m_\tau)$$

$$\mathrm{d}_L \equiv \mathbf{S}_d \cdot \mathrm{d}'_L \quad ; \quad \mathrm{u}_L \equiv \mathbf{S}_u \cdot \mathrm{u}'_L \quad ; \quad l_L \equiv \mathbf{S}_l \cdot l'_L$$
$$\mathrm{d}_R \equiv \mathbf{S}_d \cdot \mathbf{U}_d \cdot \mathrm{d}'_R \quad ; \quad \mathrm{u}_R \equiv \mathbf{S}_u \cdot \mathbf{U}_u \cdot \mathrm{u}'_R \quad ; \quad l_R \equiv \mathbf{S}_l \cdot \mathbf{U}_l \cdot l'_R$$

Mass Eigenstates
$\neq$
Weak Eigenstates

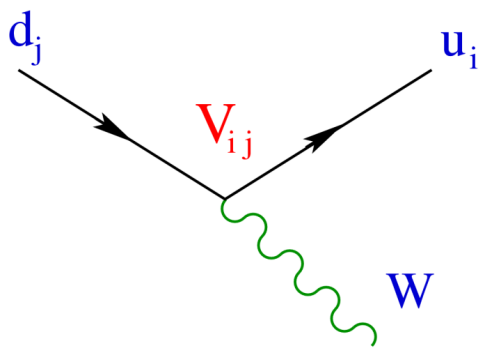
$$\bar{\mathrm{f}}'_L\,\mathrm{f}'_L = \bar{\mathrm{f}}_L\,\mathrm{f}_L \quad ; \quad \bar{\mathrm{f}}'_R\,\mathrm{f}'_R = \bar{\mathrm{f}}_R\,\mathrm{f}_R \quad \Rightarrow \quad \mathcal{L}'_\mathrm{NC} = \mathcal{L}_\mathrm{NC}$$

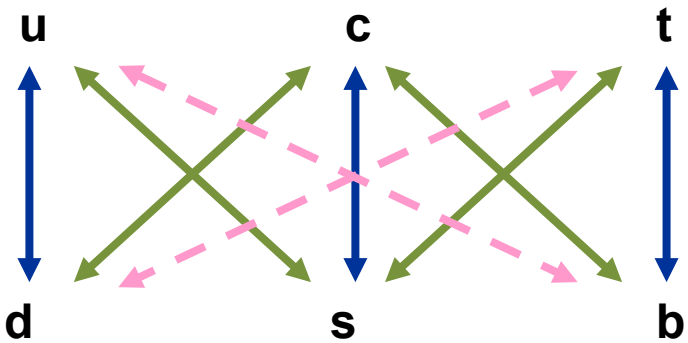
$$\bar{\mathrm{u}}'_L\,\mathrm{d}'_L = \bar{\mathrm{u}}_L \cdot \mathbf{V} \cdot \mathrm{d}_L \quad ; \quad \mathbf{V} \equiv \mathbf{S}_u \cdot \mathbf{S}_d^\dagger \quad \Rightarrow \quad \mathcal{L}'_\mathrm{CC} \neq \mathcal{L}_\mathrm{CC}$$

QUARK MIXING

Flavour Changing Charged Currents

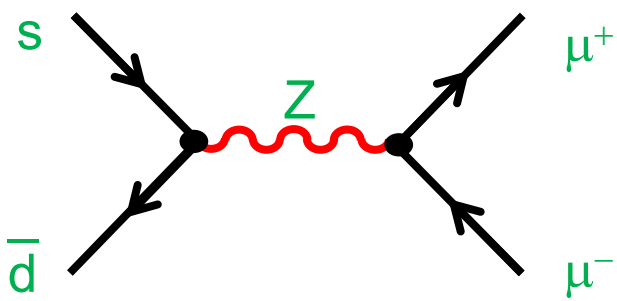
$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_{ij} \bar{u}_i \gamma^\mu (1-\gamma_5) V_{ij} d_j + \sum_l \bar{\nu}_l \gamma^\mu (1-\gamma_5) l \right] + \text{h.c.}$$





Flavour Conserving Neutral Currents (GIM)

NO

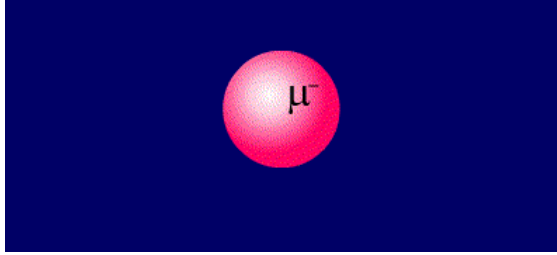


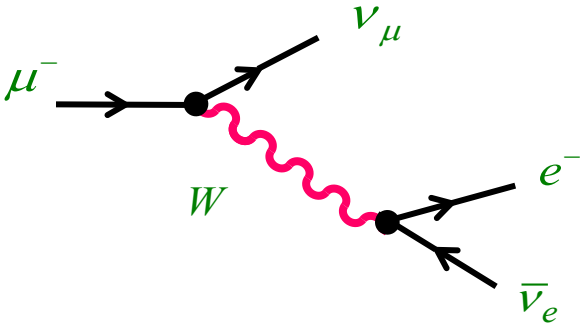
LHCb, 1706.00758

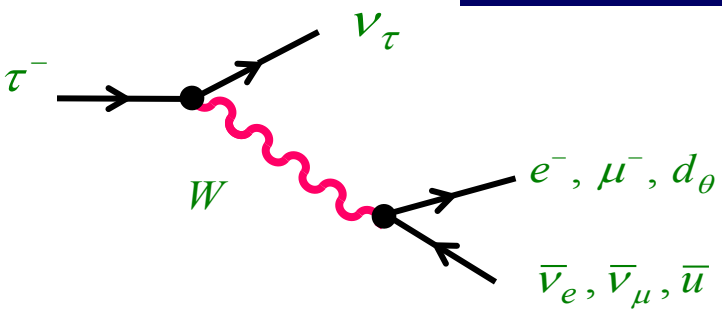
$$\text{Br}(K_S \to \mu^+\mu^-) < 1.0 \times 10^{-9}$$

(95% CL)

Weak Decays





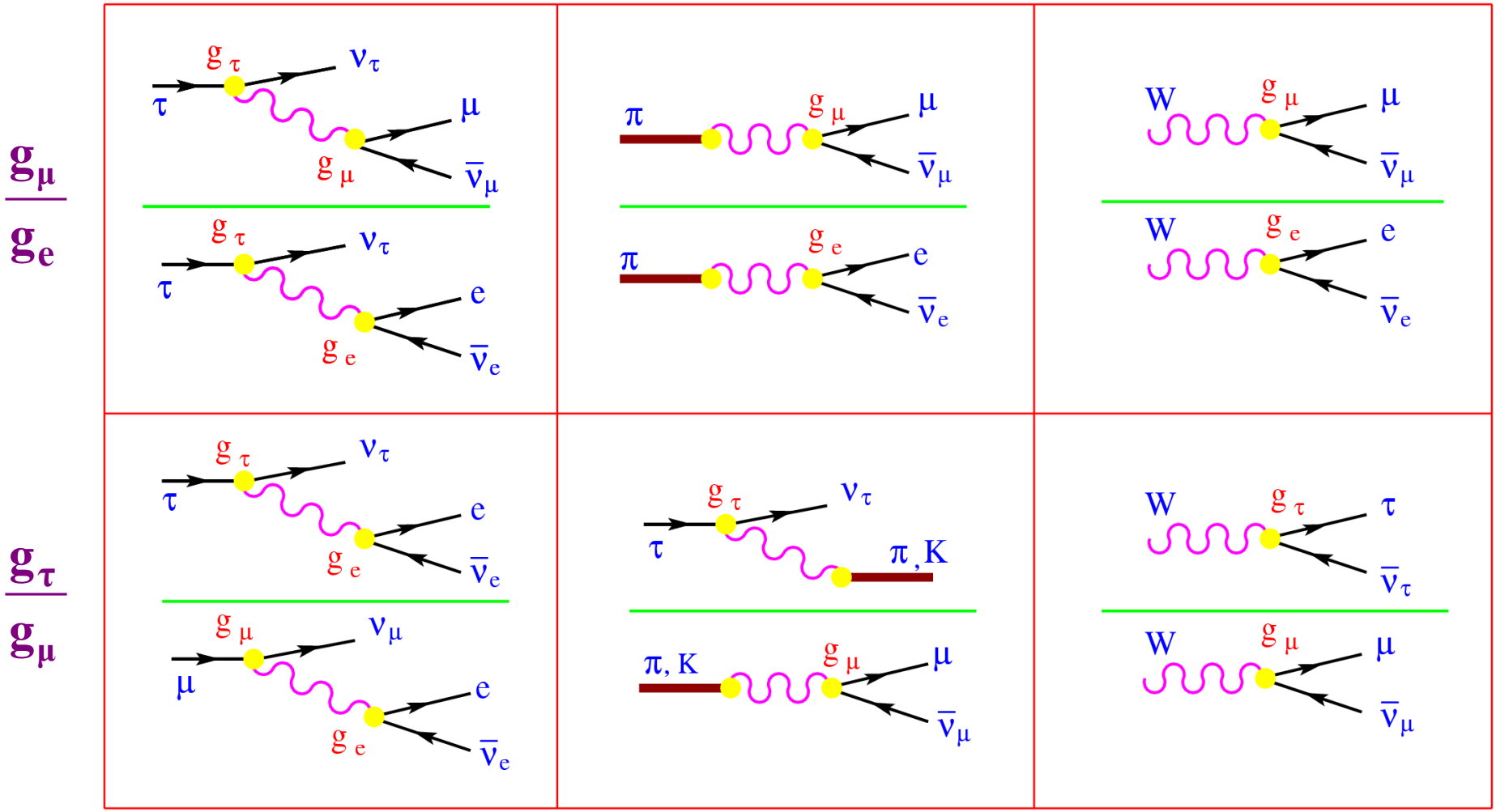


$$T(l \to \nu_l\, l' \bar{\nu}_{l'}) \sim \frac{g^2}{M_W^2 - q^2} \xrightarrow{q^2 << M_W^2} \frac{g^2}{M_W^2} = 4\sqrt{2}\, G_F$$

$$\frac{1}{\tau_\mu} = \frac{G_F^2\, m_\mu^5}{192\,\pi^3}\, f(m_e^2/m_\mu^2)\, r_{EW} \quad \Rightarrow \quad G_F = (1.166\,378\,7 \pm 0.000\,000\,6) \times 10^{-5}\ \mathrm{GeV}^{-2}$$

$$r_{EW} = \left[1 + \frac{\alpha(m_\mu)}{2\pi}\left(\frac{25}{4} - \pi^2\right) + C_2 \frac{\alpha(m_\mu)^2}{\pi^2}\right] = 0.9958 \quad ; \quad f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x$$

LEPTON UNIVERSALITY



CHARGED CURRENT UNIVERSALITY

A. Pich, arXiv:1310.7922

$|g_\mu / g_e|$

$B_{\tau\to\mu}/B_{\tau\to e}$	1.0018 ± 0.0014
$B_{\pi\to\mu}/B_{\pi\to e}$	1.0003 ± 0.0012
$B_{K\to\mu}/B_{K\to e}$	0.9978 ± 0.0020
$B_{K\to\pi\mu}/B_{K\to\pi e}$	1.0010 ± 0.0025
$B_{W\to\mu}/B_{W\to e}$	0.996 ± 0.010

PIENU 1506.05845

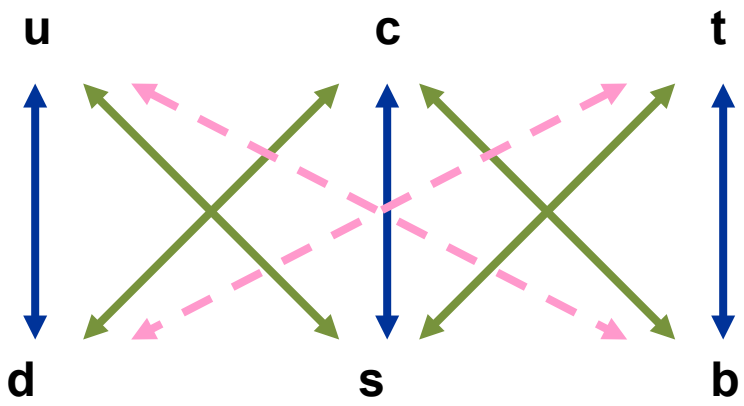
$|g_\tau / g_\mu|$

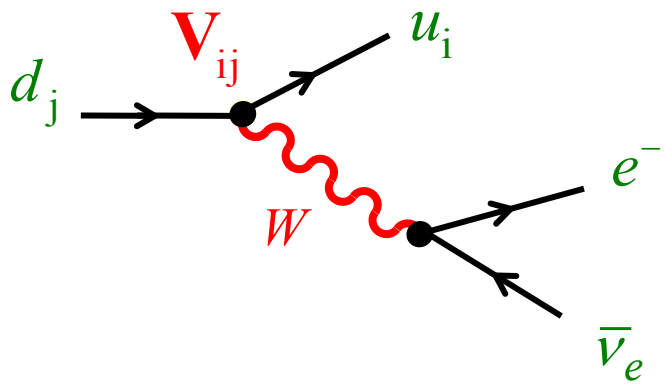
$B_{\tau\to e}\ \tau_\mu/\tau_\tau$	1.0011 ± 0.0015
$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	0.9962 ± 0.0027
$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	0.9858 ± 0.0070
$B_{W\to\tau}/B_{W\to\mu}$	1.034 ± 0.013

$|g_\tau / g_e|$

$B_{\tau\to\mu}\ \tau_\mu/\tau_\tau$	1.0030 ± 0.0015
$B_{W\to\tau}/B_{W\to e}$	1.031 ± 0.013

Flavour Changing Charged Currents





$$\Gamma(d_j \to u_i\, e^-\, \bar{\nu}_e) \propto \left|V_{ij}\right|^2$$

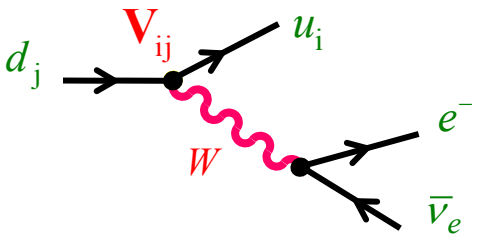
We measure decays of hadrons **(no free quarks)**

Important QCD Uncertainties

V_{ij} Determination

$(0^- \to 0^-)$

$K \to \pi \ell \nu$, $D \to K \ell \nu$...



$$\langle P'(k')|\bar{u}_i \gamma^\mu d_j|P(k)\rangle = C_{PP'}\left\{(k+k')^\mu f_+(q^2) + (k-k')^\mu f_-(q^2)\right\}$$

$$\Gamma(P \to P' l \nu) = \frac{G_F^2 M_P^5}{192\pi^3} |V_{ij}|^2 C_{PP'}^2 |f_+(0)|^2 \mathbf{I} (1+\delta_{RC})$$

$$\mathbf{I} \approx \int_0^{(M_P - M_{P'})^2} \frac{dq^2}{M_P^8} \lambda^{3/2}(q^2, M_P^2, M_{P'}^2) \left|\frac{f_+(q^2)}{f_+(0)}\right|^2$$

$f_-(q^2)$ suppressed

$(k-k')^\mu \bar{l}\gamma_\mu(1-\gamma_5)\nu_l \sim m_l$

- Measure the q^2 distribution ➡ $\mathbf{I}$
- Measure Γ ➡ $f_+(0)\ |V_{ij}|$
- Get a theoretical prediction for $f_+(0)$ ➡ $|V_{ij}|$

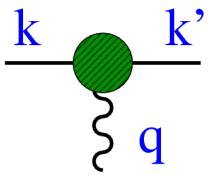
Theory is always needed: Symmetries

Symmetry (CVC):

$$\partial_\mu V^\mu_{ij} \equiv \partial_\mu \left(\bar{u}_i \gamma^\mu d_j\right) \sim m_{u_i} - m_{d_j} = 0$$

$$V^\mu_{ij}(x) = e^{iP\cdot x}\, V^\mu_{ij}(0)\, e^{-iP\cdot x}$$

$$\left\langle P_i'(k')\middle| V^\mu_{ij}(x) \middle| P_j(k)\right\rangle = e^{iq\cdot x}\, C_{PP'}\, (k+k')^\mu\, f_+^{ij}(q^2)$$



Clebsh-Gordan: $C_{PP'} = 1/\sqrt{2}$ $(P' = \pi^0)$, 1 (otherwise)

$$\partial_\mu V^\mu_{ij} = 0 \quad \Rightarrow \quad N_{ij} = \int d^3x\, V^0_{ij}(x) = \int d^3x\, u_i^\dagger(x)\, d_j(x)$$

$$C_{PP'}\, \Delta_{\vec{k}\vec{k}'} = \left\langle P_i'(k')\middle| N_{ij} \middle| P_j(k)\right\rangle = \left\langle P_i'(k')\middle| \int d^3x\, V^0_{ij}(x) \middle| P_j(k)\right\rangle$$

$$= C_{PP'}\, (2\pi)^3\, \delta^3(\vec{q})\, 2k^0\, f_+^{ij}(0) = C_{PP'}\, \Delta_{\vec{k}\vec{k}'}\, f_+^{ij}(0)$$

$$\Rightarrow \quad \boxed{f_+^{ij}(0) = 1}$$

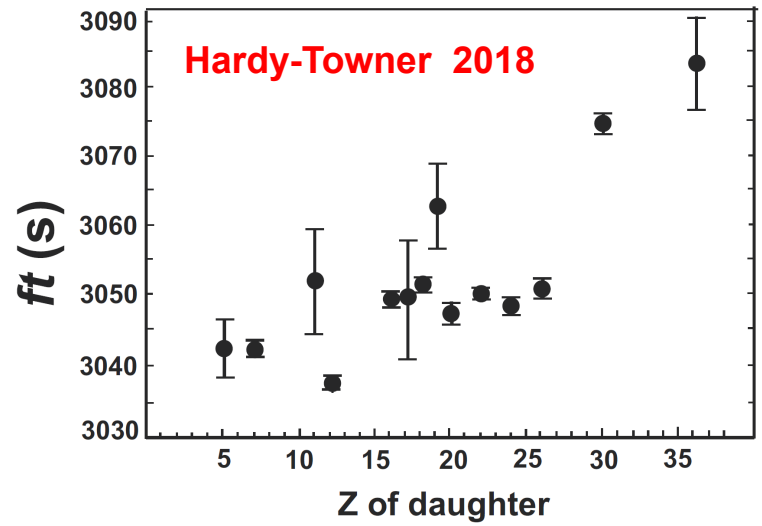
$|V_{ud}|$

$$f_+(0) = 1 + O[(m_u - m_d)^2]$$

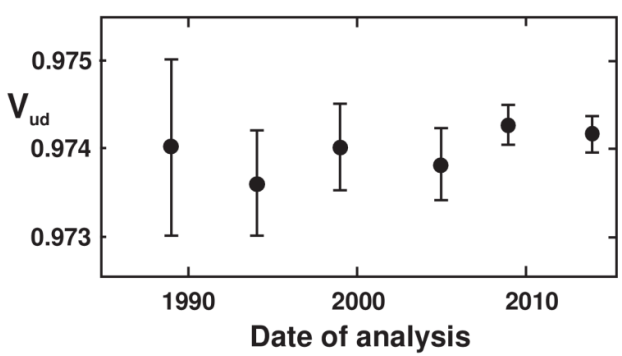
Superallowed Nuclear β Transitions $(0^+ \to 0^+)$

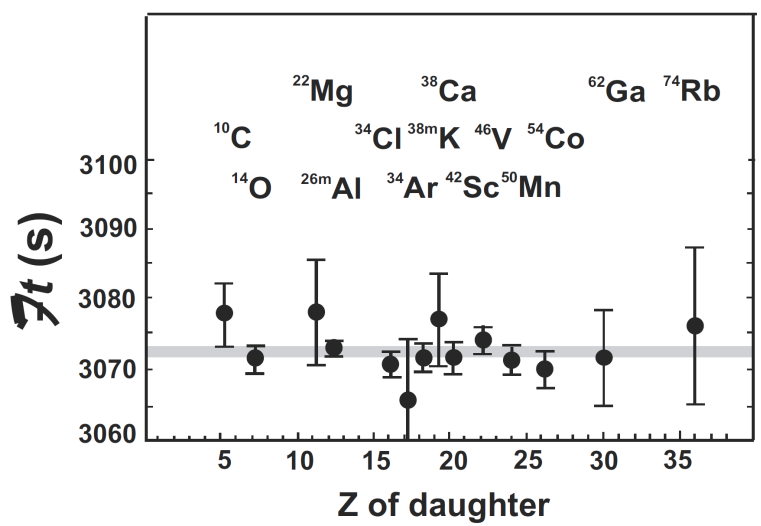
$$|V_{ud}|^2 = \frac{\pi^3 \ln 2}{ft\, G_F^2 m_e^5 (1+\delta_{RC})} = \frac{(2984.48 \pm 0.05)\ \mathrm{s}}{ft\,(1+\delta_{RC})}$$

(Marciano – Sirlin)



$|V_{ud}| = 0.97420 \pm 0.00021$





Superallowed Nuclear β Transitions $(0^+ \to 0^+)$

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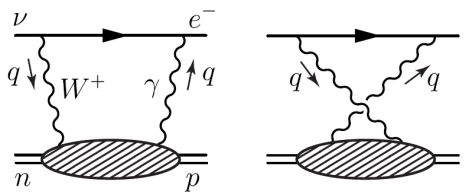
$$\delta_{RC} = \Delta_R^V + \Delta_{Nucl} \quad , \quad \mathcal{F}t = ft\,(1+\Delta_{Nucl}) = 3072.27\,(72)\ \mathrm{s}$$

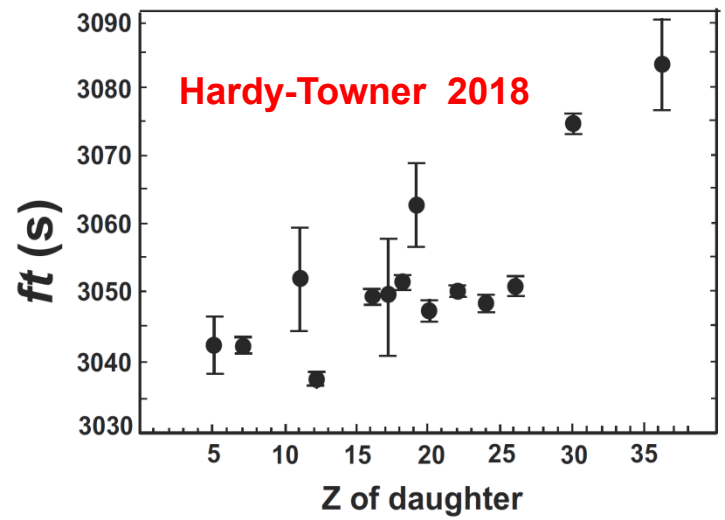


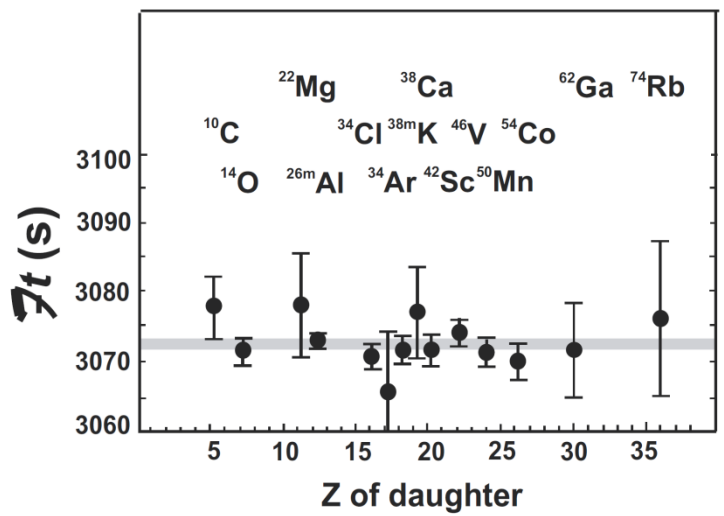
Nucleus-independent radiative correction

$$\Delta_R^V = \begin{cases} 0.02361\,(38) & \text{Marciano-Sirlin, 2006} \\ 0.02467\,(22) & \text{Seng et al, 1807.10197} \end{cases}$$

$$|V_{ud}| = \begin{cases} \mathbf{0.97420\ (21)} & \text{Marciano-Sirlin} \\ \mathbf{0.97366\ (15)} & \text{Seng et al} \end{cases}$$

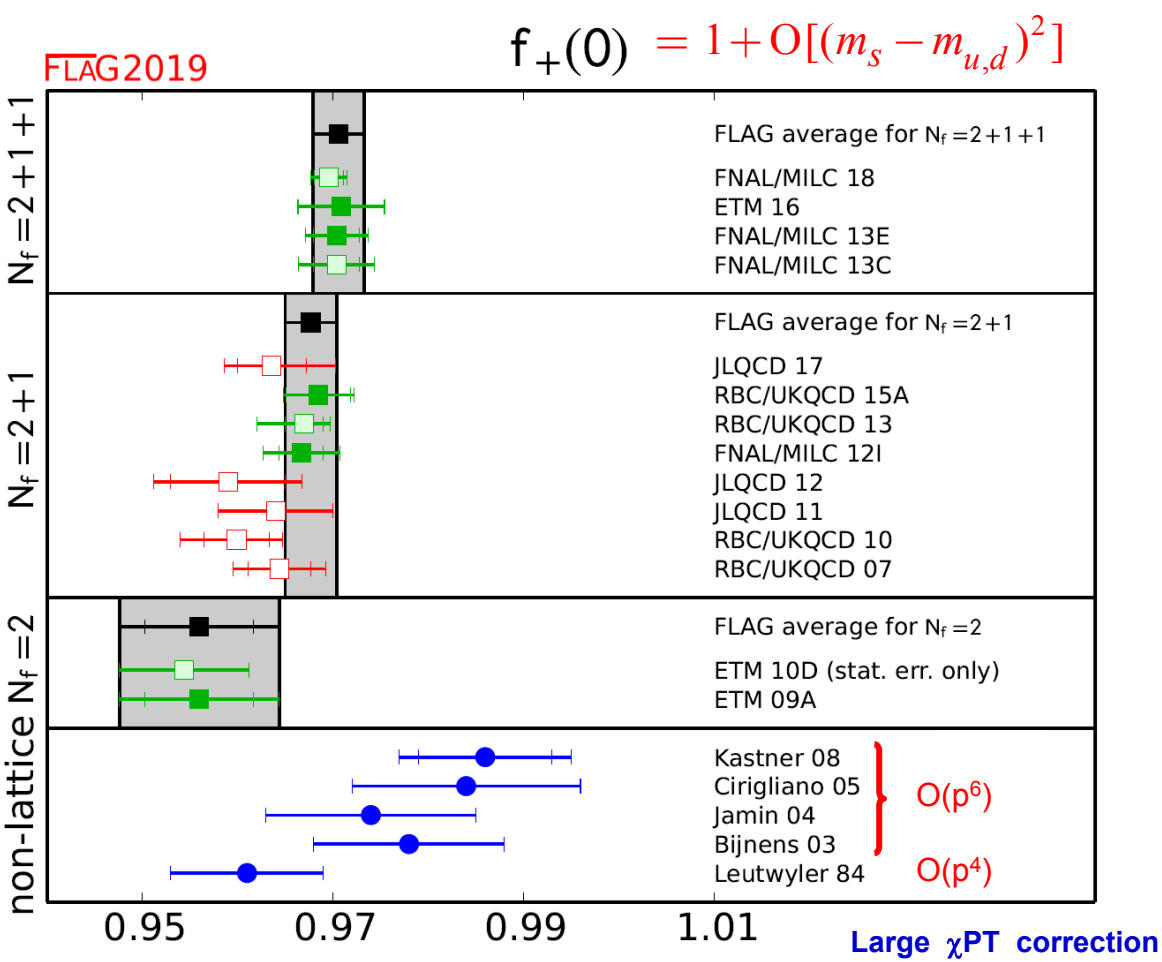
Independent check needed!

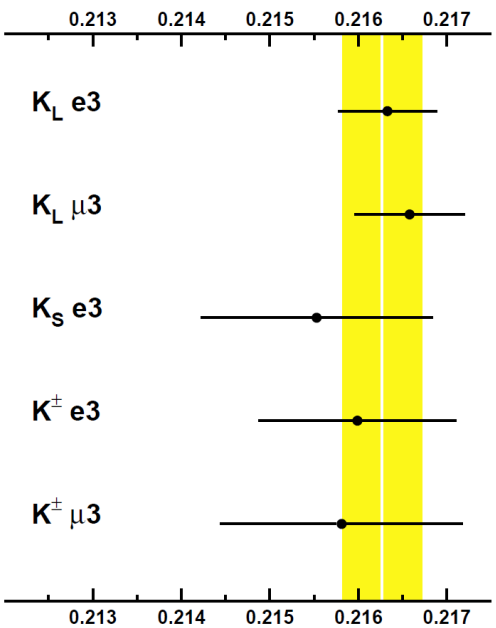




$K \to \pi \ell \nu$ Decays

Flavianet, arXiv:1005.2323 [hep-ph]
Moulson, arXiv:1411.5252 [hep-ph]





$$|f_+(0)\, V_{us}| = 0.2165 \pm 0.0004$$

$$f_+(0) = 0.9706 \pm 0.0027 \quad \Rightarrow \quad |V_{us}| = 0.2231 \pm 0.0007$$

Heavy Quark Symmetry

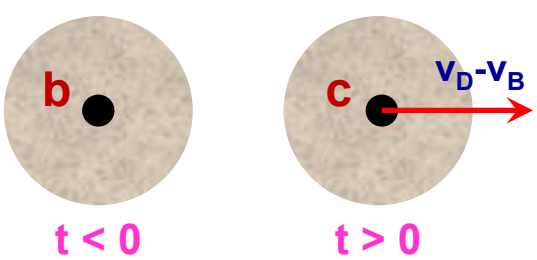
- **Atomic Physics:** $\mu \equiv m_e M_N/(m_e+M_N) \simeq m_e \ll M_N$, S_N/M_N
  - **Flavour Symmetry:** Same chemical properties for different isotopes $(Z = Z' , M_N \neq M_{N'})$
  - **Spin Symmetry:** Atoms with nuclear spin J are (2J+1) degenerate

- **Heavy-Light Mesons Qq:** $M_Q \gg m_q, \Lambda$; $\delta P_Q \sim \Lambda$; $\delta v_Q \sim \Lambda/M_Q \ll 1$
  - Q is practically at rest and acts as a static source of gluons $(\lambda_Q \sim 1/M_Q \ll R_{had} \sim 1/\Lambda)$
  - The interaction is M_Q and J_Q independent ➡ **Flavour and Spin Symmetries**

    $B \leftrightarrow D$ $B \leftrightarrow B^*$

- **$B \to D \ell \nu$:** $P_Q^\mu \equiv M_Q v^\mu + k^\mu$, $v^2 = 1$, $k \sim \Lambda$, $Q(x) \approx e^{-iM_Q v\cdot x}\, h_v^{(Q)}(x)$, $|M(P)\rangle \equiv \sqrt{M_P}\, |\tilde{M}(v)\rangle$

$$\langle D|\bar{c}\gamma^\mu b|B\rangle \quad \Rightarrow \quad \left\langle \tilde{D}(v_D)\middle|\bar{h}_{v_D}^{(c)}\gamma^\mu h_{v_B}^{(b)}\middle|\tilde{B}(v_B)\right\rangle = \xi(v_D\cdot v_B)\,(v_D+v_B)^\mu$$

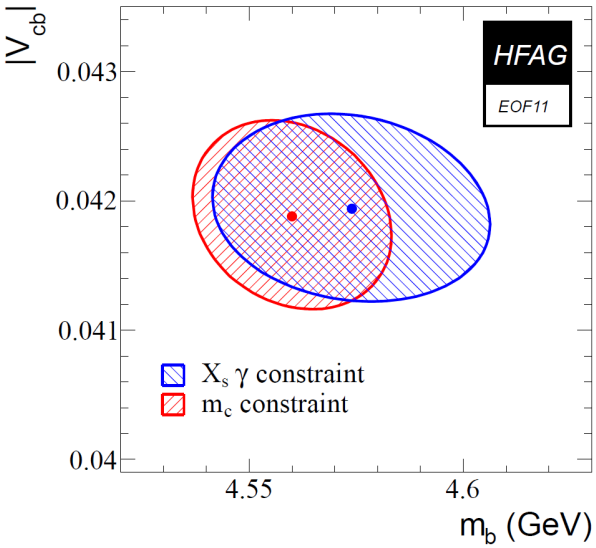


Nothing changes at zero recoil: $v_D = v_B$ $[(q^2)_{max} = (M_B-M_D)^2]$

➡ $\boxed{\xi(1) = 1}$

Inclusive B Decays (OPE, HQET)

$$\Gamma(\bar{B} \to X_c \ell \bar{\nu}) = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} \left\{ f(\rho) + k(\rho)\frac{\mu_\pi^2}{2m_b^2} + g(\rho)\frac{\mu_G^2}{2m_b^2} \right\}$$



Fits to lepton energy, hadronic invariant mass and photon energy moments

HFAG 2016: $|V_{cb}|_{incl} = \begin{cases} (42.19 \pm 0.78)\cdot 10^{-3} & \text{Kinetic mass} \\ (41.98 \pm 0.45)\cdot 10^{-3} & \text{1S mass} \end{cases}$

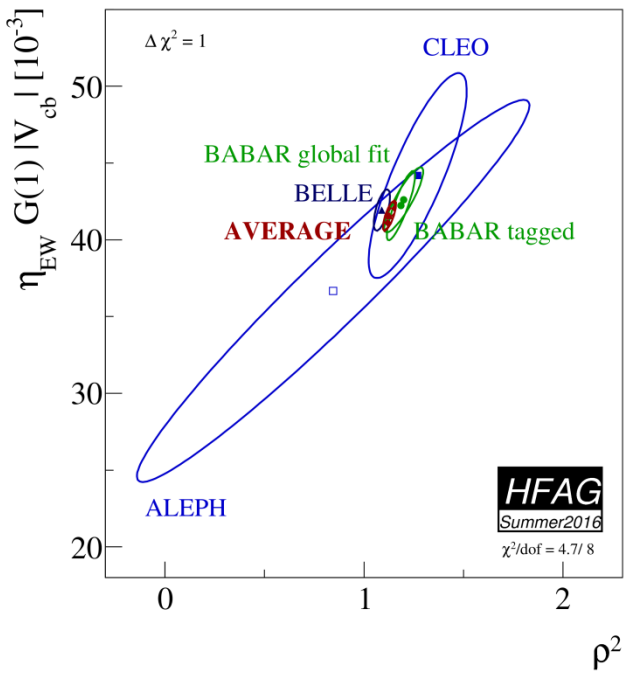
PDG 2018: $|V_{cb}|_{incl} = (42.2 \pm 0.8)\cdot 10^{-3}$

Gambino- Healey-Turczyk, 1606.06174

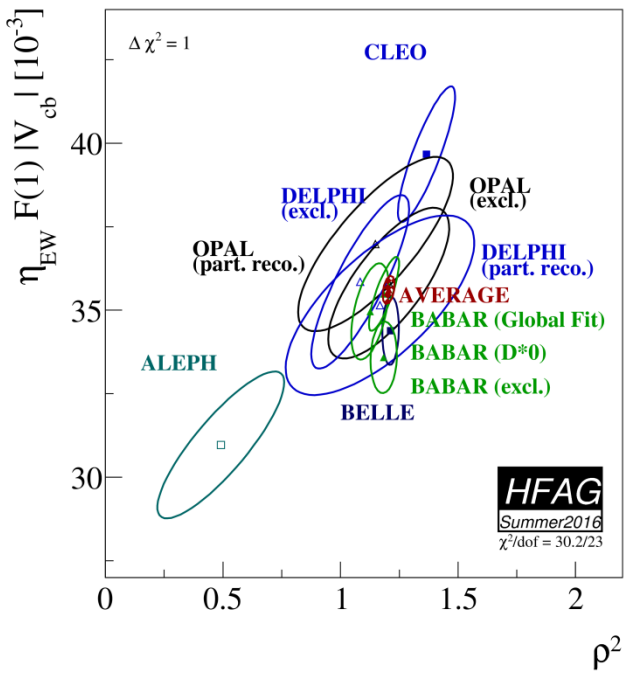
Higher Power Corrections

$$|V_{cb}| = (42.00 \pm 0.63) \times 10^{-3}$$

$B \to D \ell \nu$



$B \to D^* \ell \nu$



QCD Symmetries at $1/M_Q \to 0$

HQET

Caprini-Lellouch-Neubert parametrization

$$\eta_{EW}\, G(1)\, |V_{cb}| = (41.57 \pm 1.00) \cdot 10^{-3}$$

$$\eta_{EW}\, F(1)\, |V_{cb}| = (35.61 \pm 0.43) \cdot 10^{-3}$$

FNAL / MILC :

$$\eta_{EW}\, G(1) = 1.061 \pm 0.010 \quad \Rightarrow \quad |V_{cb}| = (39.18 \pm 0.94_{exp} \pm 0.36_{th}) \cdot 10^{-3}$$

$$\eta_{EW}\, F(1) = 0.912 \pm 0.014 \quad \Rightarrow \quad |V_{cb}| = (39.05 \pm 0.47_{exp} \pm 0.58_{th}) \cdot 10^{-3}$$

$$\Rightarrow \quad \boxed{|V_{cb}|_{excl} = (39.10 \pm 0.60) \cdot 10^{-3}}$$

3.3 σ discrepancy with inclusive measurement

V_{cb}

HFLAV 2016

$$|V_{cb}|_{incl} = (42.10 \pm 0.78) \cdot 10^{-3}$$

3.3 σ discrepancy

$$|V_{cb}|_{excl} = (39.10 \pm 0.60) \cdot 10^{-3}$$

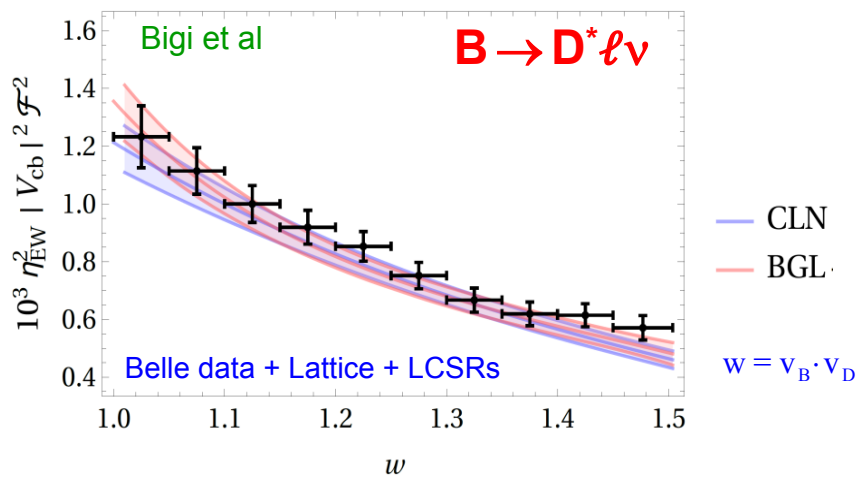
(CLN parametrization)

$|V_{cb}|_{excl}$ Parametrization Dependence:

BGL agrees with inclusive result

Bigi-Gambino-Schacht, 1703.06124, 1707.09509;
Grinstein-Kobach, 1703.08170;
Bernlochner et al, 1703.05330, 1708.07134

- Caprini-Lellouch-Neubert (CLN)
  (HQET relations valid within 2%)
- Boyd-Grinstein-Lebed (BGL)



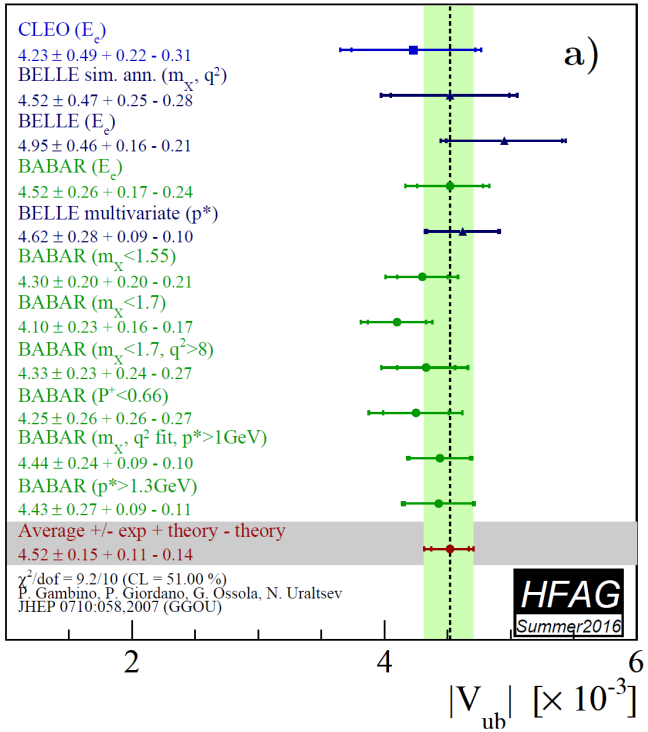
Belle 1809.03290

$$|V_{cb}| = \begin{cases} (38.4 \pm 0.2 \pm 0.6 \pm 0.6) \cdot 10^{-3} \\ (42.5 \pm 0.3 \pm 0.7 \pm 0.6) \cdot 10^{-3} \end{cases}$$

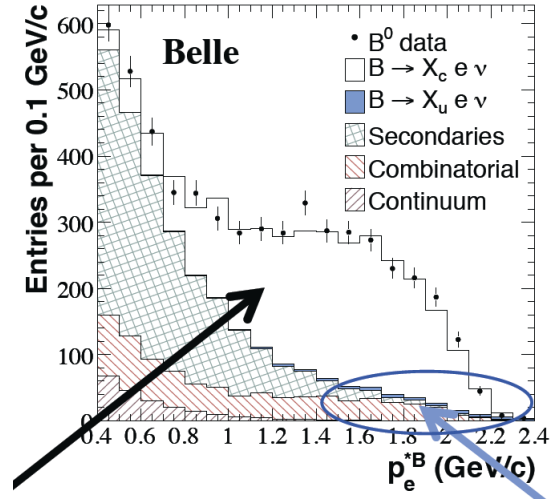
$B \to D \ell \nu$ ➡ $|V_{cb}| = (40.49 \pm 0.97) \cdot 10^{-3}$

Bigi-Gambino-Schacht, 1606.08030

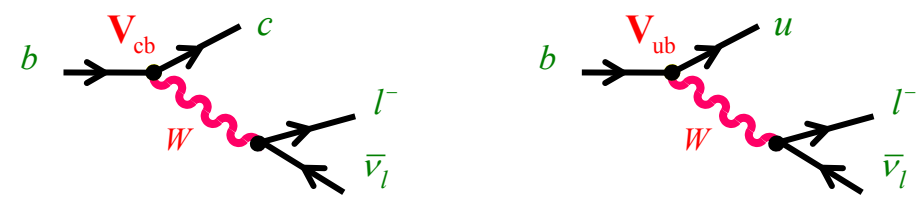
$B \to X_u \ell \nu$



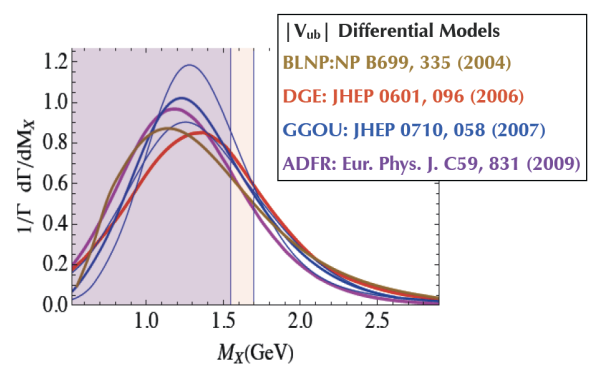
Framework	$\lvert V_{ub}\rvert[10^{-3}]$
BLNP	$4.44 \pm 0.15^{+0.21}_{-0.22}$
DGE	$4.52 \pm 0.16^{+0.15}_{-0.16}$
GGOU	$4.52 \pm 0.15^{+0.11}_{-0.14}$
ADFR	$4.08 \pm 0.13^{+0.18}_{-0.12}$
BLL (m_X/q^2 only)	$4.62 \pm 0.20 \pm 0.29$
LLR (*BABAR*) [531]	$4.43 \pm 0.45 \pm 0.29$
LLR (*BABAR*) [532]	$4.28 \pm 0.29 \pm 0.29 \pm 0.26 \pm 0.28$
LNP (*BABAR*) [532]	$4.40 \pm 0.30 \pm 0.41 \pm 0.23$



$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 \approx \frac{1}{50}$$

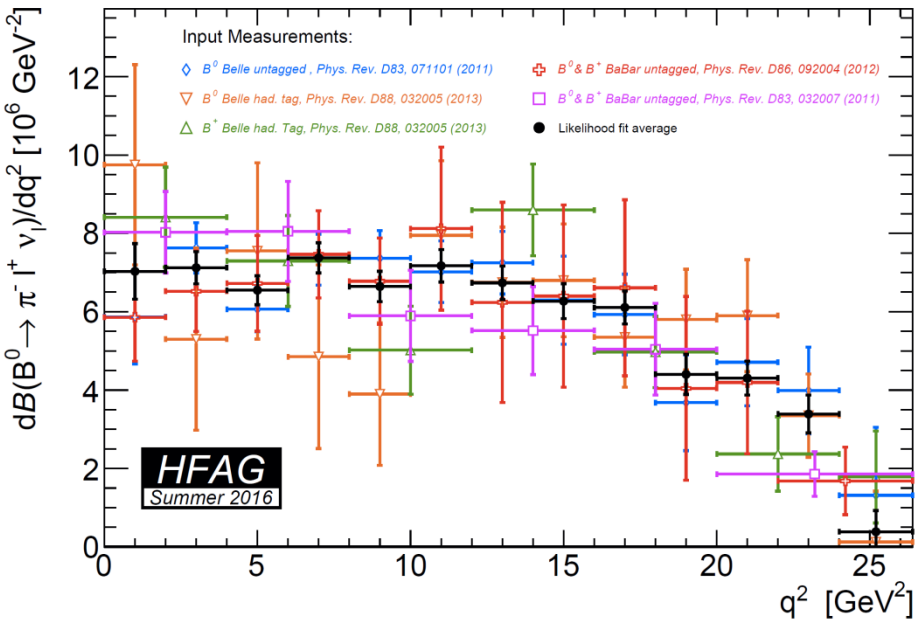


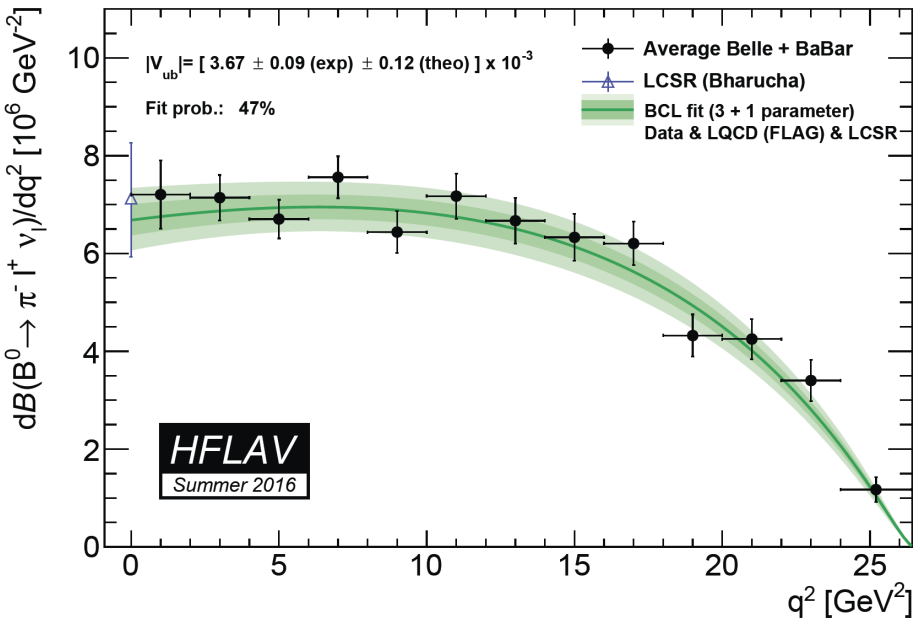
- **Large backgrounds from $B \to X_c l \nu$**
- **Strong experimental cuts**
- **Large theoretical uncertainties**



HFAG 2016: $$\left|V_{ub}\right|_{incl} = \left(4.52 \pm 0.15\,^{+0.11}_{-0.14}\right)\cdot 10^{-3}$$

$B \to \pi \ell \nu$





HFAG 2016:

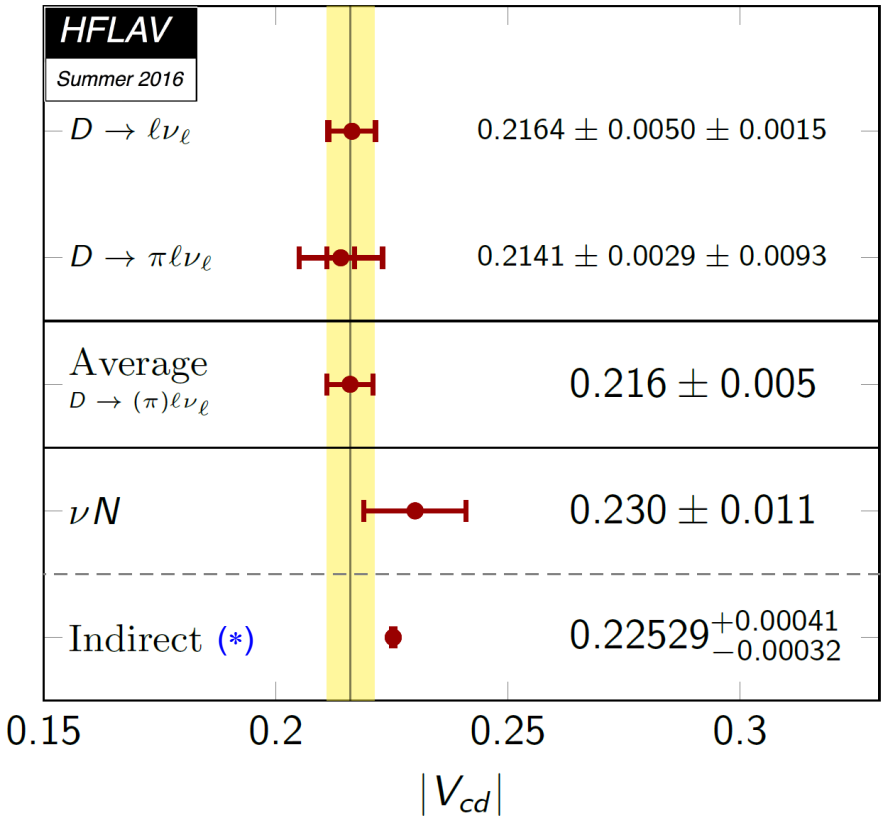
$$\left|V_{ub}\right|_{excl} = \left(3.67 \pm 0.09_{exp} \pm 0.12_{th}\right) \cdot 10^{-3}$$

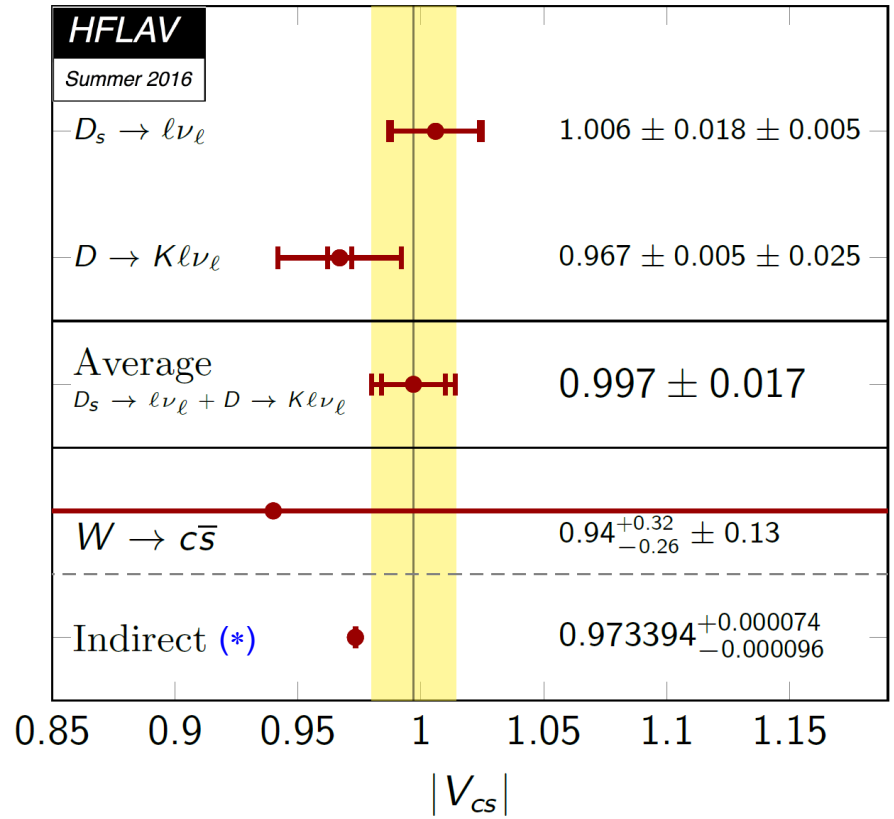
3.4 σ discrepancy with inclusive measurement

$$\left|V_{ub}\right| = (3.94 \pm 0.36) \times 10^{-3}$$

PDG 2018

$|V_{cd}|$ & $|V_{cs}|$





(*) Global CKM fit (unitarity assumed)

V_{ij} DETERMINATION

CKM entry	Value	Source
$\lvert V_{ud}\rvert$	$\mathbf{0.97420 \pm 0.00021}$ 0.9763 ± 0.0016 0.9749 ± 0.0026	Nuclear β decay $n \to p\, e^- \bar{\nu}_e$ $\pi^+ \to \pi^0 e^+ \nu_e$
$\lvert V_{us}\rvert$	$\mathbf{0.2231 \pm 0.0007}$ 0.2253 ± 0.0005 0.2213 ± 0.0023	$K \to \pi\, e^- \bar{\nu}_e$ $K/\pi \to \mu\nu$, Lattice, V_{ud} τ decays
$\lvert V_{cd}\rvert$	0.230 ± 0.011 $\mathbf{0.216 \pm 0.005}$	$\nu d \to c X$ $D \to (\pi) l \nu$, Lattice
$\lvert V_{cs}\rvert$	$\mathbf{0.997 \pm 0.017}$	$D \to K l \nu$, $D_s \to l \nu$, Lattice
$\lvert V_{cb}\rvert$	0.0405 ± 0.0010 $\mathbf{0.0420 \pm 0.0006}$	$B \to D^* l \bar{\nu}_l$, $D l \bar{\nu}_l$ $b \to c\, l \bar{\nu}_l$
$\lvert V_{ub}\rvert$	0.00367 ± 0.00015 0.00449 ± 0.00029 $\mathbf{0.00394 \pm 0.00036}$	$B \to \pi\, l \bar{\nu}_l$ $b \to u\, l \bar{\nu}_l$
$\lvert V_{tb}\rvert / \sqrt{\sum_q \lvert V_{tq}\rvert^2}$ $\lvert V_{tb}\rvert$	> 0.975 (95% CL) $\mathbf{1.019 \pm 0.025}$	$t \to bW / t \to qW$ $p\bar{p} \to tb + X$

$$\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2 + \lvert V_{ub}\rvert^2 = \mathbf{0.9989 \pm 0.0005}$$

$$\lvert V_{cd}\rvert^2 + \lvert V_{cs}\rvert^2 + \lvert V_{cb}\rvert^2 = \mathbf{1.042 \pm 0.034}$$

$$\lvert V_{ub}\rvert^2 + \lvert V_{cb}\rvert^2 + \lvert V_{tb}\rvert^2 = \mathbf{1.040 \pm 0.051}$$

$$\sum_j \left(\lvert V_{uj}\rvert^2 + \lvert V_{cj}\rvert^2 \right) = \mathbf{2.002 \pm 0.027} \quad \text{(LEP)}$$

V_{ij} DETERMINATION

CKM entry	Value	Source
$\lvert V_{ud}\rvert$	0.97366 ± 0.00015	Nuclear β decay
Seng et al	0.9763 ± 0.0016	$n \to p\, e^- \bar{\nu}_e$
	0.9749 ± 0.0026	$\pi^+ \to \pi^0 e^+ \nu_e$
$\lvert V_{us}\rvert$	0.2231 ± 0.0007	$K \to \pi\, e^- \bar{\nu}_e$
	0.2253 ± 0.0005	$K/\pi \to \mu\nu$, Lattice, V_{ud}
	0.2213 ± 0.0023	τ decays
$\lvert V_{cd}\rvert$	0.230 ± 0.011	$\nu d \to c X$
	0.216 ± 0.005	$D \to (\pi) l \nu$, Lattice
$\lvert V_{cs}\rvert$	0.997 ± 0.017	$D \to K l \nu$, $D_s \to l \nu$, Lattice
$\lvert V_{cb}\rvert$	0.0405 ± 0.0010	$B \to D^* l \bar{\nu}_l$, $D l \bar{\nu}_l$
	0.0420 ± 0.0006	$b \to c\, l \bar{\nu}_l$
$\lvert V_{ub}\rvert$	0.00367 ± 0.00015	$B \to \pi\, l \bar{\nu}_l$
	0.00449 ± 0.00029	$b \to u\, l \bar{\nu}_l$
	0.00394 ± 0.00036	
$\lvert V_{tb}\rvert / \sqrt{\sum_q \lvert V_{tq}\rvert^2}$	> 0.975 (95% CL)	$t \to bW / t \to qW$
$\lvert V_{tb}\rvert$	1.019 ± 0.025	$p\bar{p} \to tb + X$

$$\lvert V_{ud}\rvert^2 + \lvert V_{us}\rvert^2 + \lvert V_{ub}\rvert^2 = 0.9978 \pm 0.0004$$

$$\lvert V_{cd}\rvert^2 + \lvert V_{cs}\rvert^2 + \lvert V_{cb}\rvert^2 = 1.042 \pm 0.034$$

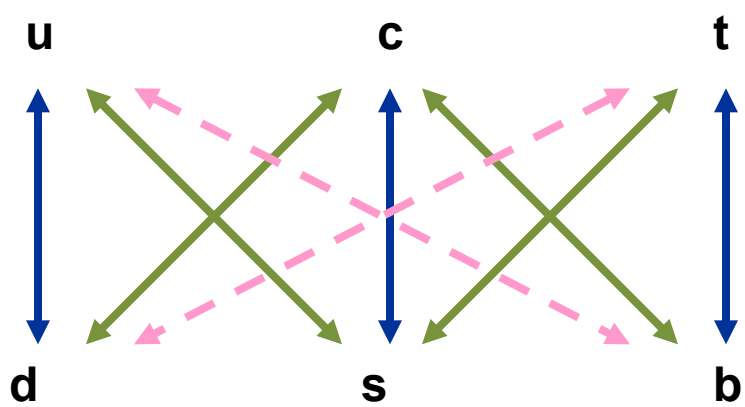
$$\lvert V_{ub}\rvert^2 + \lvert V_{cb}\rvert^2 + \lvert V_{tb}\rvert^2 = 1.040 \pm 0.051$$

$$\sum_j \left(\lvert V_{uj}\rvert^2 + \lvert V_{cj}\rvert^2 \right) = 2.002 \pm 0.027 \quad \text{(LEP)}$$

Hierarchical Structure

$$\mathbf{V} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$$\lambda \approx \sin\theta_C \approx 0.223 \quad ; \quad A \approx 0.84 \quad ; \quad \sqrt{\rho^2+\eta^2} \approx 0.4$$



QUARK MIXING MATRIX

- **Unitary** $N_G \times N_G$ **Matrix:** N_G^2 **parameters**

$$\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1}$$

$\frac{1}{2} N_G (N_G - 1)$ moduli, $\frac{1}{2} N_G (N_G + 1)$ phases

- $2 N_G - 1$ **arbitrary phases:** $\bar{u}_i \mathbf{V}_{ij} d_j$

$$u_i \to e^{i\phi_i} u_i \quad ; \quad d_j \to e^{i\theta_j} d_j \quad \Rightarrow \quad \mathbf{V}_{ij} \to e^{i(\theta_j - \phi_i)} \mathbf{V}_{ij}$$

$\mathbf{V}_{ij}$ **Physical Parameters:**

$\frac{1}{2} N_G (N_G - 1)$ **moduli** ; $\frac{1}{2} (N_G - 1)(N_G - 2)$ **phases**

- **$N_f = 2$: 1 angle, 0 phases** **(Cabibbo)**

$$\mathbf{V} = \begin{bmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{bmatrix} \quad \Rightarrow \quad \text{No } \not{CP}$$

- **$N_f = 3$: 3 angles, 1 phase** **(CKM)** $c_{ij} \equiv \cos\theta_{ij}$; $s_{ij} \equiv \sin\theta_{ij}$

$$\mathbf{V} = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$\lambda \approx \sin\theta_C \approx 0.223$; $A \approx 0.84$; $\sqrt{\rho^2+\eta^2} \approx 0.4$ $\quad \delta_{13} \neq 0 \quad (\eta \neq 0) \Rightarrow \not{CP}$

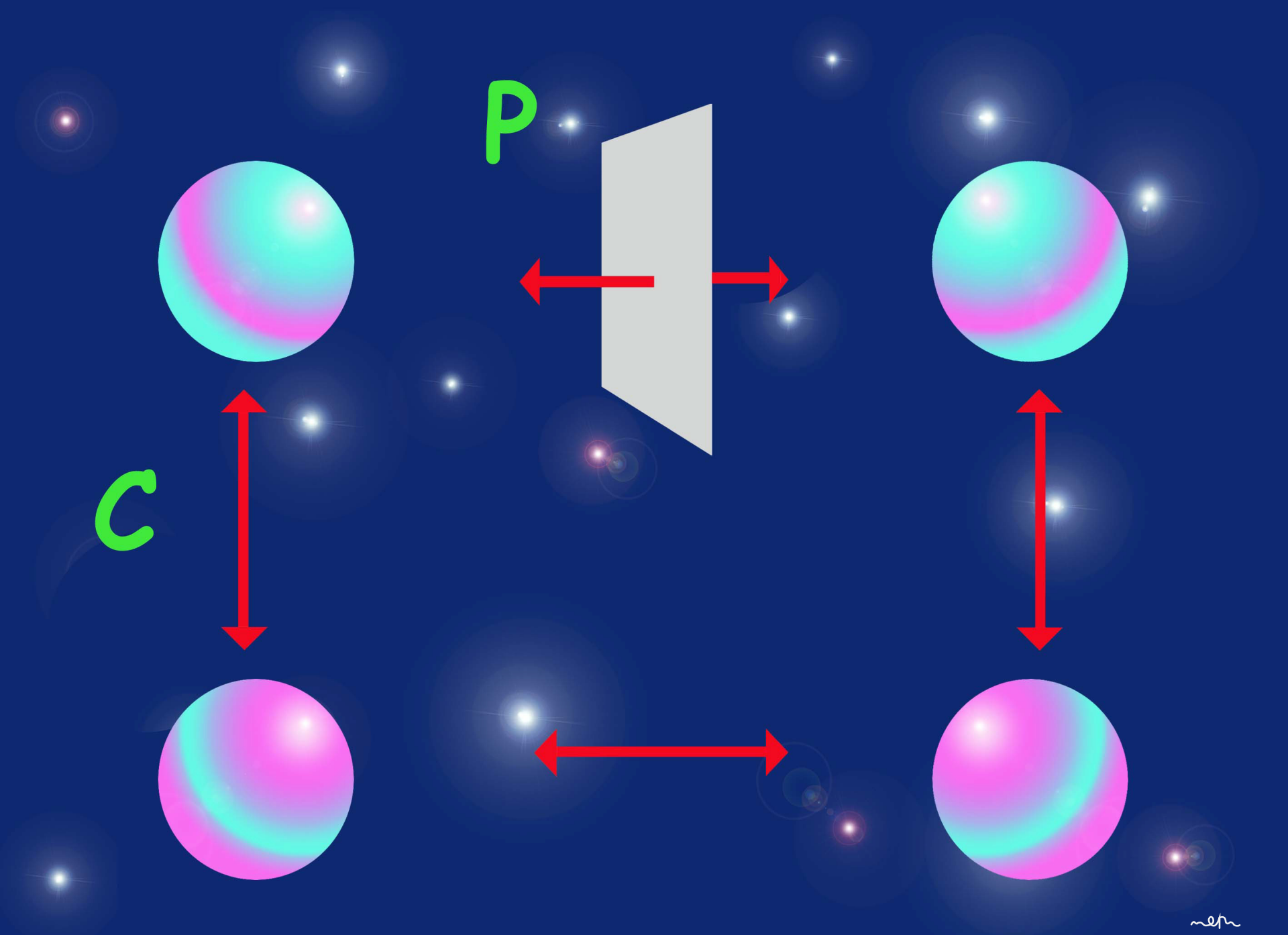
PDG parametrization of the CKM matrix

$$\mathbf{V} = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}\, e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}\, e^{i\delta_{13}} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Wolfenstein: $s_{12} \equiv \lambda \quad , \quad s_{23} \equiv A\lambda^2 \quad , \quad s_{13}\, e^{-i\delta_{13}} \equiv A\lambda^3(\rho - i\eta)$

$$\Rightarrow \quad \mathbf{V} \approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3\,(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3\,(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$


P
C

Backup

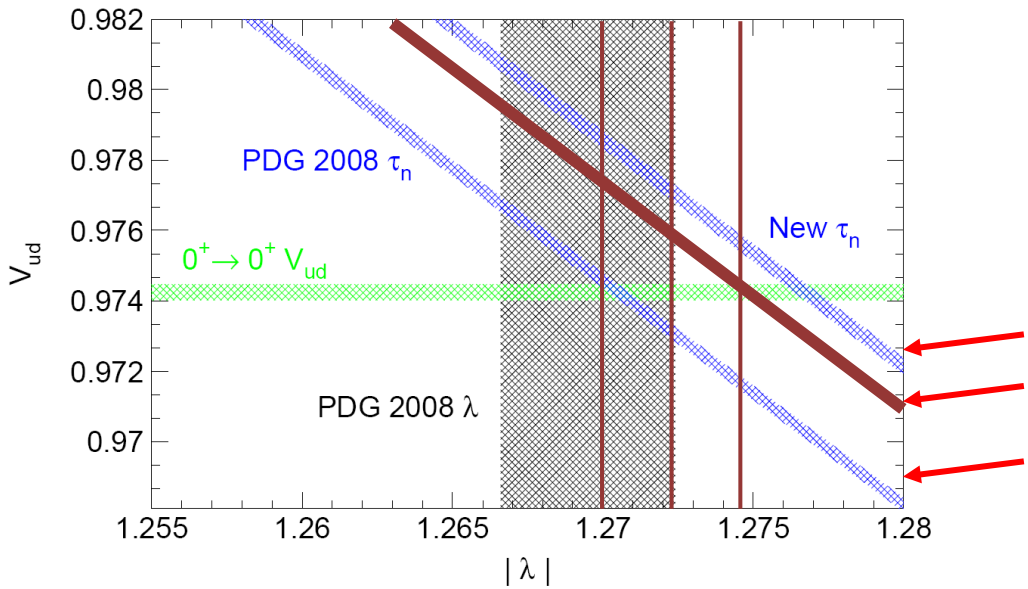
• Neutron Decay:

$$|V_{ud}|^2 = \frac{(4908.7 \pm 1.9)\ \mathrm{s}}{\tau_n (1 + 3\lambda^2)}$$

(Czarnecki – Marciano – Sirlin)

PDG10: $\tau_n = (885.7 \pm 0.8)$ s , $\lambda \equiv g_A / g_V = -1.2694 \pm 0.0028$

PDG18: $\tau_n = (879.3 \pm 0.9)$ s , $\lambda \equiv g_A / g_V = -1.2724 \pm 0.0023$



$|V_{ud}| = 0.9763 \pm 0.0016$

$\tau_n = (878.5 \pm 0.7 \pm 0.3)$ s

(Serebrov et al, 2005)

PDG18

PDG10

• Pion Decay:

$$\mathrm{Br}(\pi^+ \to \pi^0 e^+ \nu_e) = (1.036 \pm 0.006) \times 10^{-8}$$

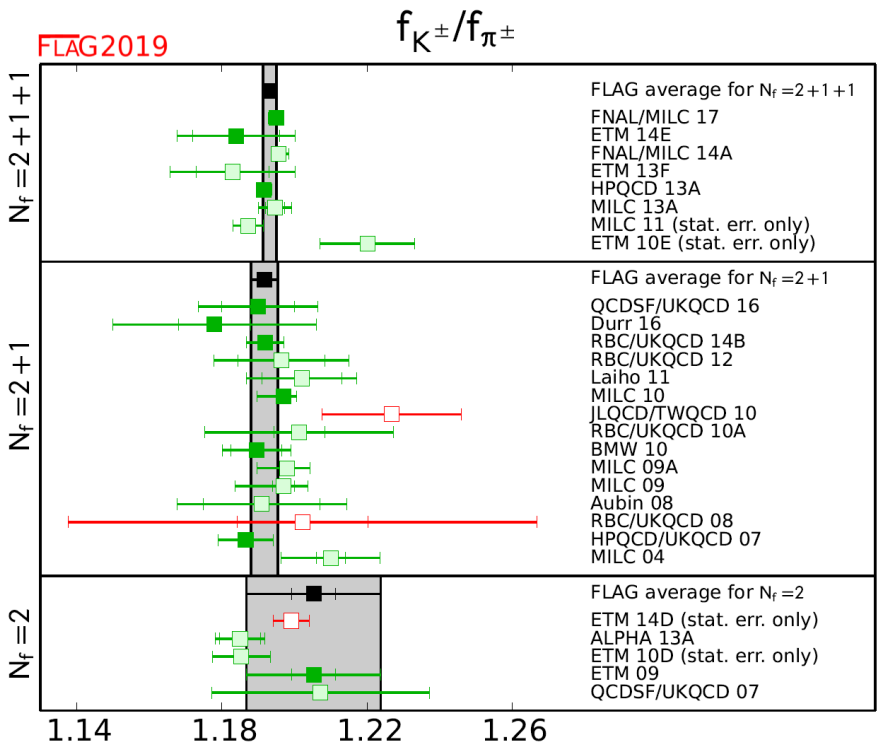
(PIBETA)

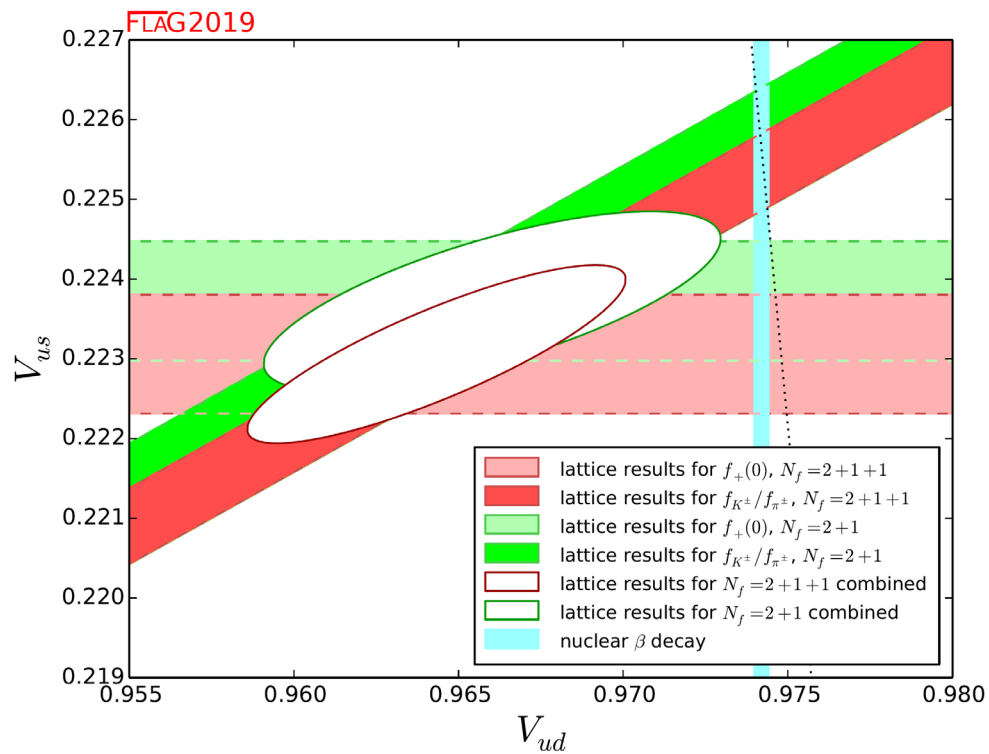
$|V_{ud}| = 0.9749 \pm 0.0026$

$\Gamma(K^+ \to \mu^+ \nu_\mu) / \Gamma(\pi^+ \to \mu^+ \nu_\mu)$

$$\frac{f_K}{f_\pi}\frac{|V_{us}|}{|V_{ud}|} = 0.2760 \pm 0.0004 \quad \Rightarrow \quad \frac{|V_{us}|}{|V_{ud}|} = 0.2313 \pm 0.005$$

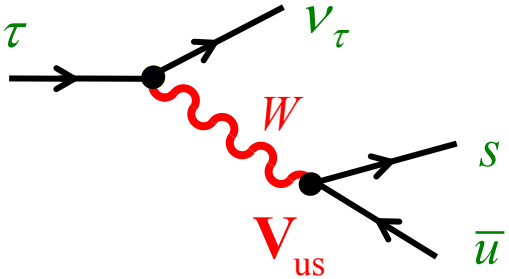
$$\langle 0 | \bar{d}_i \gamma^\mu \gamma_5 u_j | P(k) \rangle = i\, f_P\, k^\mu$$





$$f_K / f_\pi = 1.1932 \pm 0.0019$$

$$R_{\tau,us} = \frac{\Gamma(\tau^- \to \nu_\tau X_s^-)}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)}$$



Gámiz-Jamin-Pich-Prades-Schwab

$$\delta R_\tau \equiv \frac{R_{\tau,ud}}{|\mathbf{V_{ud}}|^2} - \frac{R_{\tau,us}}{|\mathbf{V_{us}}|^2} \approx 24\,\frac{m_s^2(m_\tau^2)}{m_\tau^2}\,\Delta(\alpha_s) = 0.240 \pm 0.032$$

$$\Rightarrow \quad |\mathbf{V_{us}}|^2 = \frac{R_{\tau,S}}{\dfrac{R_{\tau,ud}}{|\mathbf{V_{ud}}|^2} - \delta R_\tau^{\text{th}}}$$

HFAG 2016: $R_{\tau,ud} = 3.4718 \pm 0.0072$; $R_{\tau,S} = 0.1633 \pm 0.0027$

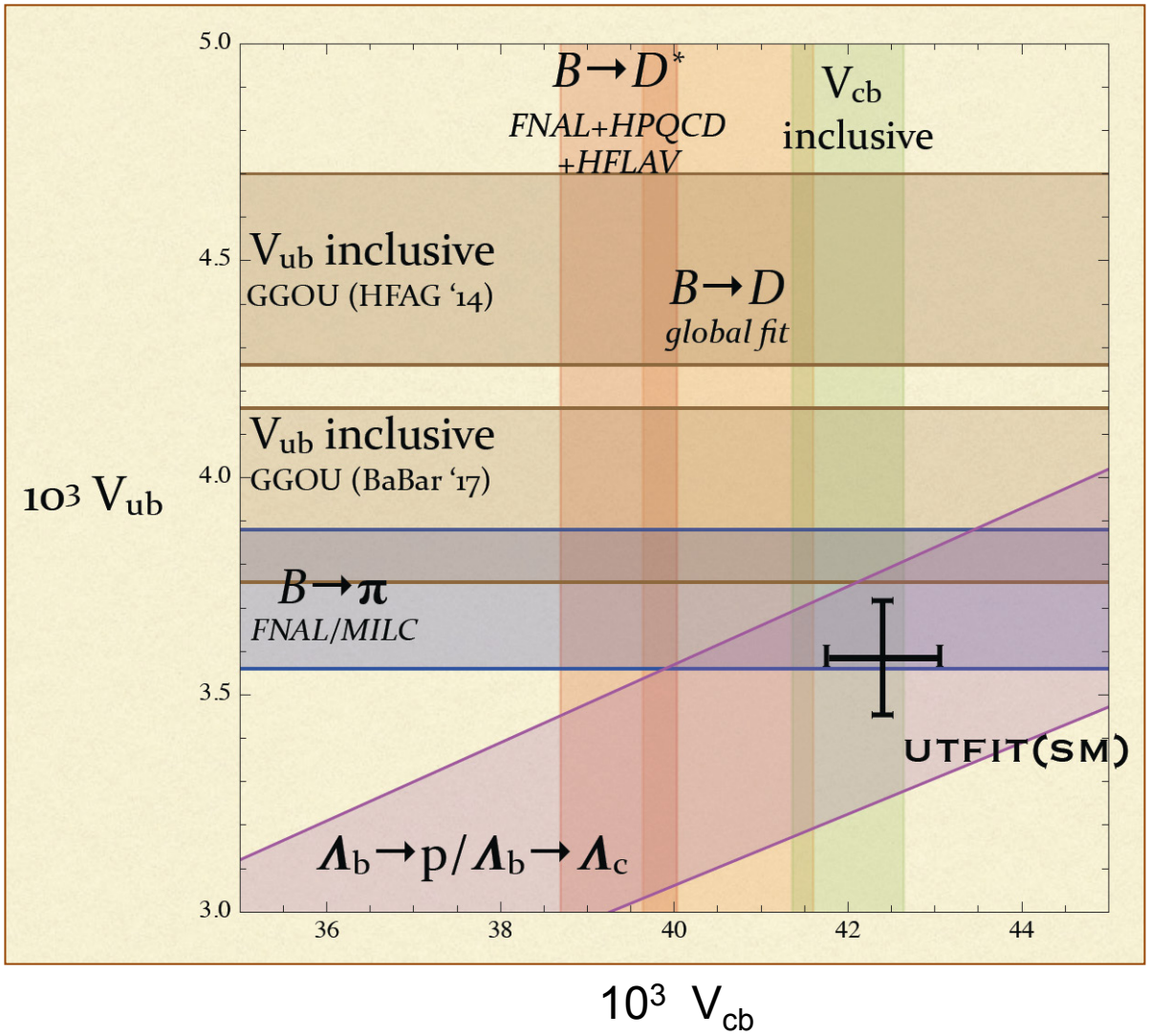
$$\Rightarrow \quad |\mathbf{V_{us}}| = 0.2186 \pm 0.0018_{\text{exp}} \pm 0.0010_{\text{th}}$$

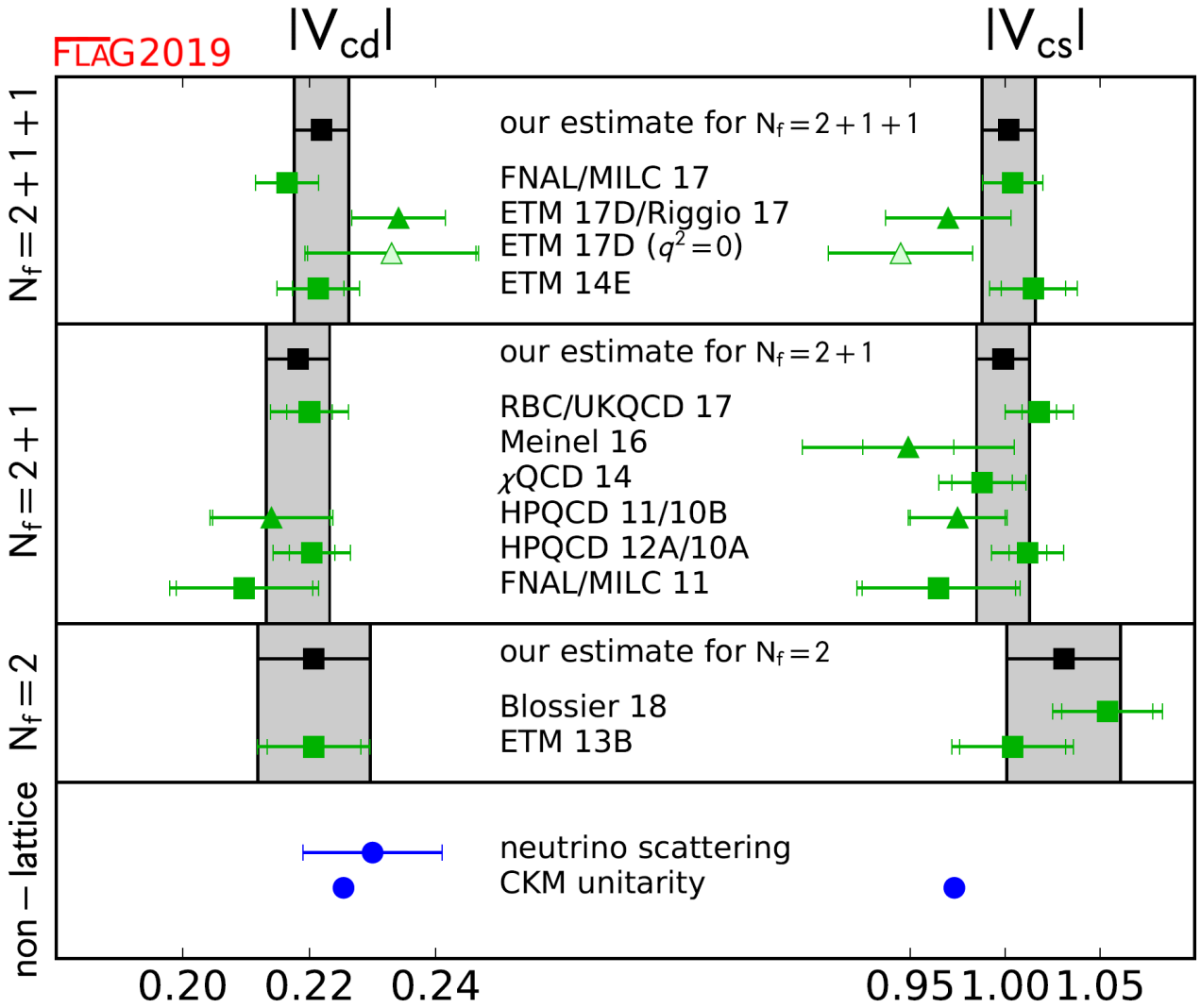
Replacing $\tau \to \nu K(\pi)$ by $K \to \nu\mu(\pi)$ data: **$|V_{us}| = 0.2213 \pm 0.0023$**

With better data, could give a very precise V_{us} determination

V_{ub} discrepancy remains, but…

P. Gambino, CKM 2018





$$|V_{cd}| = 0.2219 \pm 0.0043 \qquad |V_{cs}| = 1.002 \pm 0.014$$