Exercises for the Lecture on Experimental Facilities

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The problems are not self-contained. You may need to look up some formulae in a textbook or on line. A good reference is: A. De Angelis and M. Pimenta: "Introduction to Particle and Astroparticle Physics". You may also want to check the Particle Data Group "Review of Particle Properties".

1. The radiation length of electrons in air is $X_r \simeq 37$ g/cm^2. Considering that the atmospheric pressure at sea level is $p_0 \simeq 1 \times 10^5$ N/m^2, show that the vertical thickness of the atmosphere is about 30 radiation lengths.

2. (a) Determine the Cerenkov radiation emission angle θ_C in the atmosphere at sea level for a charged particle moving with $\beta \sim 1$. The refractive index of the atmosphere at sea level is $n_{at} \simeq 1.00029$.
   (b) Same question but when the medium is liquid water.
   (c) Determine the threshold (minimum) energy for an electron to emit Cerenkov light in air.
   (d) Same question but for a muon.

3. Considering that the average galactic magnetic field is a few microGauss, show that cosmic rays (protons) with energies below 10^{18} eV should be of galactic origin while those with energies 10^{19} eV or above should be extragalactic.

4. What components of Galactic Cosmic Rays can have a larger energy, protons or heavier nuclei?

5. Consider observatories of cosmic rays and gammas based on IACT (imaging atmospheric Cerenkov telescopes) and SDA (surface particle detector arrays). Explain the meaning of the following concepts:
   (a) Field of View
   (b) Energy threshold
   (c) Sensitivity
   (d) Duty cycle
   (e) Effective area of detection of a ground array. Explain why it depends on the energy of the CR or gamma ray.

6. (a) Why are IACT's (Cerenkov Telescopes) in general more sensitive to lower energies than Surface Arrays?
   (b) Why IACT's have a much shorter duty cycle than Surface Arrays?
   (c) Which one of the detector techniques, IACT's or Surface Arrays, have a larger Field of View? Why is that?
   (d) How can a Surface Array measure the energy of the primary CR or Gamma Ray? How is that done with IACT's?

7. In an extensive air shower (EAS), a primary proton produces pions as it collides with the atmospheric nuclei. As it develops, the shower contains many muons and also many electromagnetic cascades as well. Explain why it contains muons and additional e.m. cascades.

8. In an EAS, a muon is produced at an altitude of 10 km. If the muon is able to reach sea level before decaying, determine the minimum energy it should have. Will this muon produce Cerenkov light in the atmosphere?

9. From the graph of the CR spectrum, in the region 10 GeV$< E <$ 1 PeV, the spectral flux (particles per unit area, time, solid angle and energy) can be fitted to a function:

$$j(E) = A \left(\frac{\text{eV}}{E}\right)^{\alpha}, \quad \text{with} \quad \alpha \simeq 2.7$$

   (a) By fitting the graph (choosing an adecuate point), find the value of A in $[1/(\text{m}^2 \cdot \text{s} \cdot \text{sr} \cdot \text{eV})]$. (sr = steradian)
   (b) Consider a satellite with a flat detector face of 1 m^2. Determine the rate of primary cosmic rays with energies above 1 TeV that hit the detector (do not forget that the incidence is from the outside only, and do the solid angle integration correctly).

(c) What about energies above 100 TeV? Express the result in [particles/($m^2\cdot$ day)].

10. Consider an electromagnetic shower caused by a vertical cosmic gamma ray of 1 TeV. Assume that the shower begins at 20 km a.s.l. ($x = 0$) and as it develops, all particles in the shower (electrons, positrons and photons) at a given height have the same energy.

    (a) Determine the number of particles in the shower as a function of depth x. You will need the radiation length X_r from (1).

    (b) The shower reaches its maximum when the energy of the particles reaches the critical value $E_C \simeq 80\ MeV$. Determine x_{Max}.

    (c) E_C occurs when the energy loss in electrons and positrons start to be dominated by ionization instead of bremsstrahlung. Find in the literature the expression for the energy loss by ionization and show that E_C for electrons is indeed around 80 MeV.

11. Consider the Sun, a star 1.5×10^{11} m away from Earth. The luminous solar radiation intensity on Earth is about 1.5 kW/m^2.

    (a) Determine the total power emitted by the Sun.

    (b) The net reaction where that power is produced is $p + p \rightarrow {}^4_2He + 2e^+ + 2\nu$, releasing about 26 MeV per reaction. Most of this energy is eventually emitted as photons. However the neutrinos escape, carrying in average about 0.4 MeV each (these are the so-called "*pp* neutrinos"). There are other reactions in the Sun that produce neutrinos, but these are the most abundant. Determine the flux of neutrinos on Earth, in 1/cm^2 s.

    (c) The solar neutrinos that are detectable by Super KamiokaNDE (SuperK) are not *pp* neutrinos. They have energies above 5 MeV and they come from the decay of the isotope ${}^8_5B$ (boron-8), and they constitute are a very small fraction of the total flux that arrive to Earth, about 5×10^6 1/cm^2 s. Try to estimate roughly the number of Solar neutrinos detected by SuperK per day. (You need to look up the size of SuperK and the ν-e cross section at those energies).

12. Let us try to recreate a Cerenkov emission from the e.m. shower produced by a 1 TeV primary photon. Assume that the shower is vertical and initiates at 12 km a.s.l. Also assume for simplicity that the radiation length is a fixed value of 400 m, independent of altitude. An e.m. shower is quite collimated, with a spread not more than 0.5^o, while the Cerenkov angle in air is about 1.5^o. So clearly the size of the light pool on the ground will be determined mainly by the Cerenkov spread.

    Now consider that, along the shower, each electron or positron as it travels, emits about 10 Cerenkov photons per meter travelled. Consider this emission only up to X_{Max}, which is where the shower has maximal multiplicity (particles reach the critical energy $E_C \simeq 80$ MeV).

    (a) Determine the diameter of the Cerenkov light pool on the ground, which is at an altitude of 2000 m a.s.l.

    (b) Determine the number of Cerenkov photons in the light pool (here we are disregarding absorption in the lower atmosphere, which is not quite realistic).

    (c) Assuming that the Cerenkov photons are uniformly spread inside the light pool, determine how many photons will be caught by a 12 m diameter IACT that lies completely inside the light pool.

13. How can one learn about the acceleration regions of Cosmic Rays by measuring Gamma Rays?

14. Find out about the mechanism called *Fermi Acceleration*.

15. Why is it important to measure the VHE Gamma Rays that come from the Galactic Center?

16. How can one test Lorentz invariance by measuring cosmic Gamma Rays?

17. Why Gamma Rays are better to identify the sources of Cosmic Rays than the Cosmic Rays themselves?

18. What are "Fermi bubbles"?

19. Concerning the search for the particles that constitute the Dark Matter of the Universe, explain what is called "direct searches" and "indirect searches".

20. The GZK cutoff is the upper limit for the energy of CR that can reach the Earth from very far extragalactic distances, due to the energy loss of a CR particle (e.g. a proton) when it collides with a CMB photon and produces a pion at the resonance of the Δ baryon, e.g.:

$$p + \gamma_{CMB} \rightarrow \Delta^{+} \rightarrow p + \pi^{0}$$

Determine the energy of the GZK cutoff.

Some references and reading

1. A. De Angelis and M. Pimenta, *Introduction to Particle and Astroparticle Physics*, second ed., Springer, 2018.
2. M. Tanabashi et al. (Particle Data Group), *Review of Particle Physics*, Phys. Rev. D98, 010001 (2018).
3. HAWC website: https://www.hawc-observatory.org
4. SGSO: A. Albert et al., *Science Case for a Wide FoV VHE Gamma-Ray Observatory in the Southern Hemisphere*, arXiv:1902.08429 [astro-ph.HE]
5. CTA website: https://www.cta-observatory.org
6. CTA Consortium (B.S. Acharya et al.), *Science with the Cerenkov Telescope Array*, arXiv:1709.07997 [astro-ph.IM]