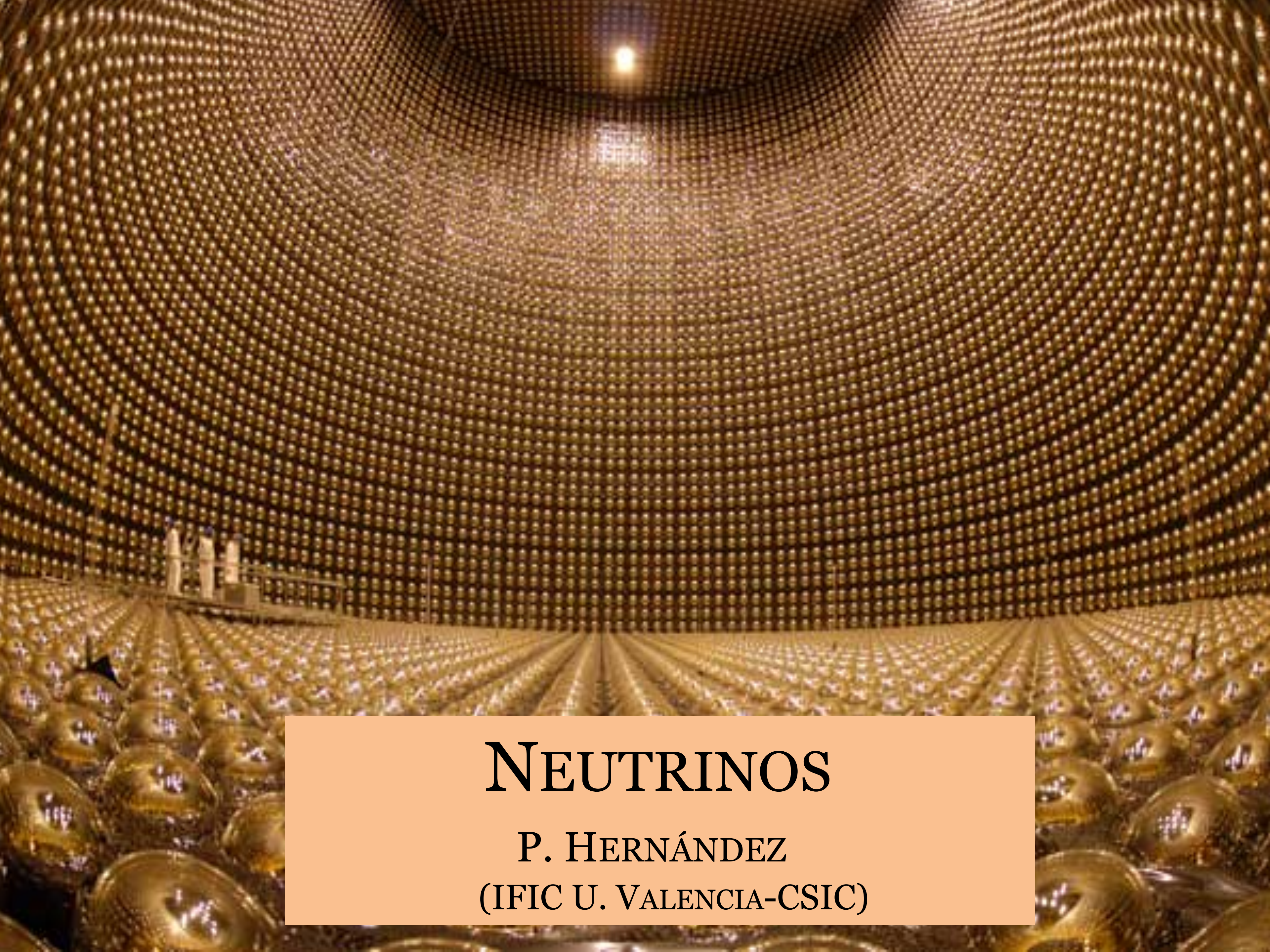


NEUTRINOS

P. HERNÁNDEZ
(IFIC U. VALENCIA-CSIC)

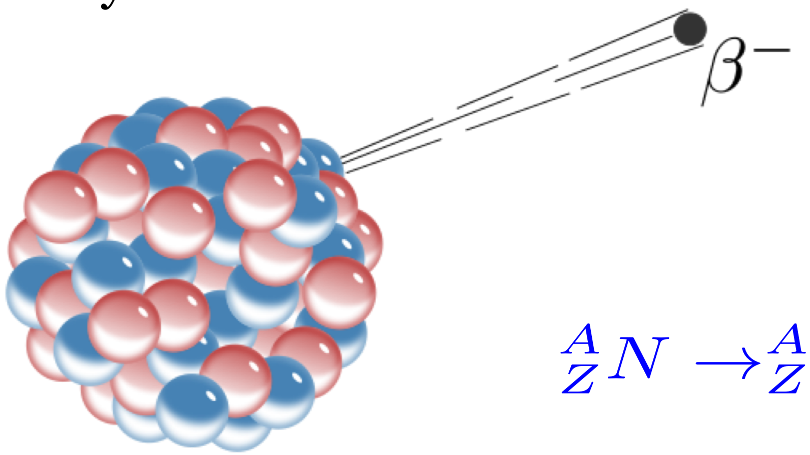
LECTURE I

- Introduction: Neutrinos in the Standard Model
- Neutrino masses and mixing : Majorana versus Dirac
- Neutrino oscillations in vacuum and in matter
- Experimental evidence for neutrino masses & mixings

Neutrino: the dark particle

1900 Radioactivity: Becquerel, M & P Curie, Rutherford....

β decay

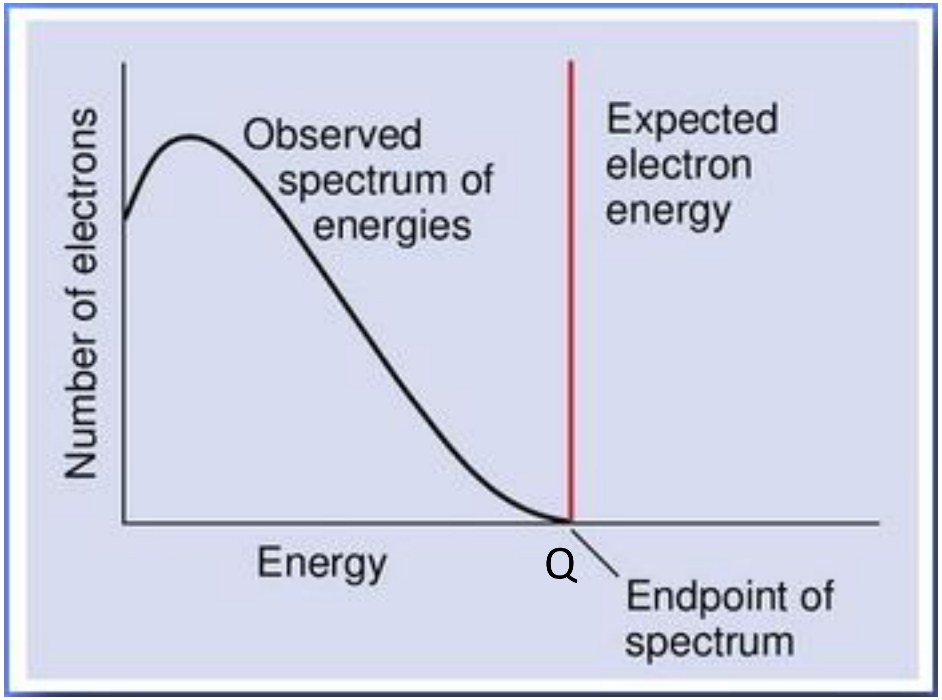


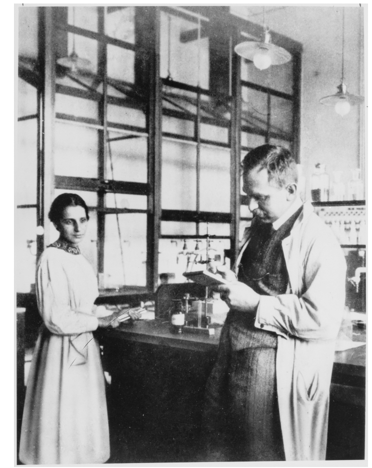
$$^A_Z N \rightarrow ^A_{Z+1} N' + e^-$$

Energy conservation: $E_{\text{electron}} \simeq (M_N - M_{N'})c^2 = Q = \text{constante}$

1911/1914

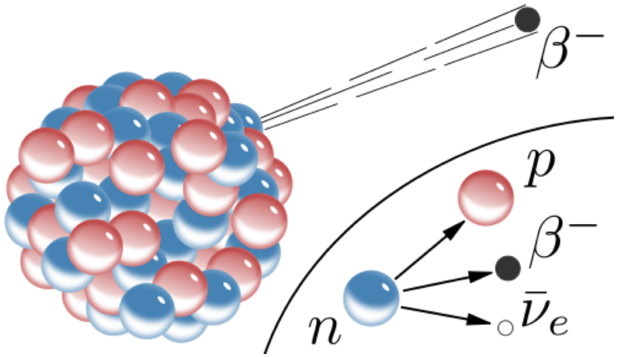
Electron spectrum:





Meitner, Hahn
(Nobel 1944 only him!)

Chadwick (Nobel 1935)



1930

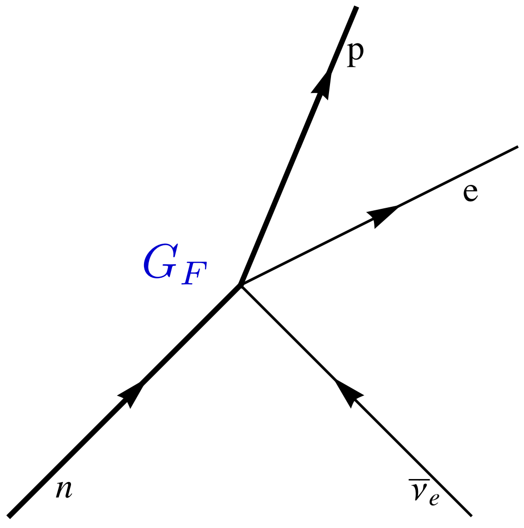
Pauli (Nobel 1945)

Dear Radioactive Ladies and Gentlemen,

As the bearer of these lines, to whom I graciously ask you to listen, will explain to you in more detail, how because of the "wrong" statistics of the N and Li^6 *nuclei and the continuous beta spectrum, I have hit upon a desperate remedy to save the "exchange theorem" of statistics and the law of conservation of energy. Namely, the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call neutrons, which have spin 1/2 and obey the exclusion principle, and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the neutrons should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a neutron is emitted in addition to the electron such that the sum of the energies of the neutron and the electron is constant...*

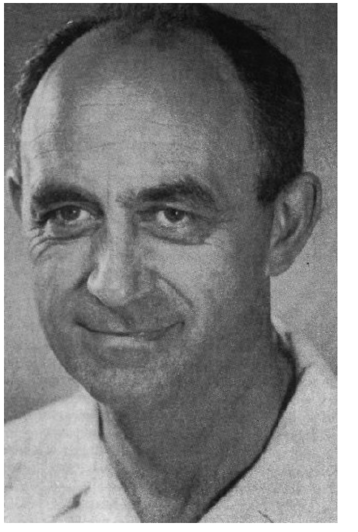
Unfortunately, I cannot personally appear in Tübingen since I am indispensable here in Zürich because of a ball on the night from December 6 to 7….

1934: Theory of beta decay



$$n + \nu \to p + e^-$$

$$p + \bar{\nu} \to n + e^+$$



E. Fermi
(Nobel 1938)

Nature did not publish his article: "contained speculations too remote from reality to be of interest to the reader…"

Bethe-Peierls (1934): compute the neutrino cross section using this theory

$$\sigma \simeq 10^{-44} cm^2, \; E(\bar{\nu}) = 2 \text{ MeV}$$

"there is not practically possible way of detecting a neutrino"

How to detect them ?

$$
\begin{aligned}
\lambda &\simeq \frac{1}{n\sigma} \\
\lambda|_{@\mathrm{water}} &\simeq 1.5\times 10^{21}\ cm \simeq 1600\ \text{Light Years} \\
\lambda|_{@\mathrm{interstelar}} &\simeq 10^{44}\ cm \simeq 10^{26}\ \text{Light Years}
\end{aligned}
$$

“I have done a terrible thing. I have postulated a particle that cannot be detected”

W. Pauli

Pauli’s worst insult to a theory: “Not even wrong”

Revealing Pauli’s dark matter was just a question of time and ingenuity…

In a 1000kg detector, a $10^{11}\ \nu/\mathrm{cm}^2/\mathrm{s}$ a few events per day

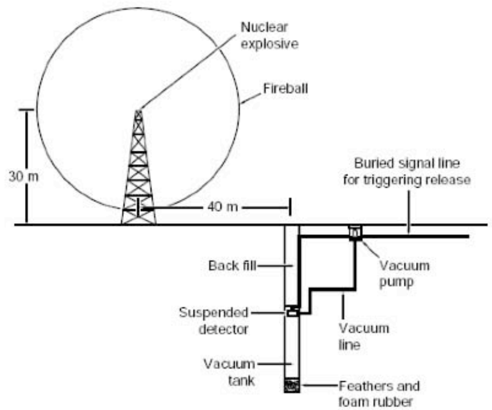
Reactors: ~ 10^{20}/second!

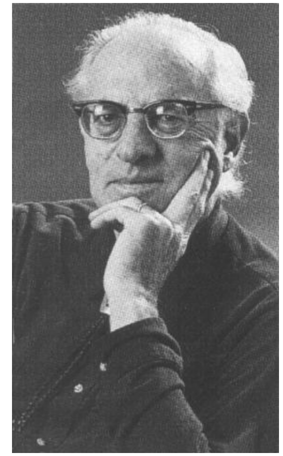
(10^{11}/s@100 meters)

1956 anti-neutrino detection

Poltergeist project

First idea: put the detector close to a nuclear explosion !

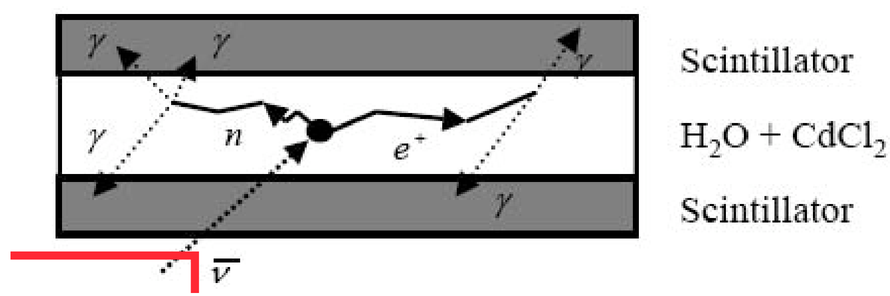




Reines Nobel 95 Cowan (died 74)

Finally used the reactor Savannah River to discover the anti-neutrino

Golden signal



Modern versions of Reines&Cowan experiment: Chooz, Dchooz, Daya Bay, RENO… still making discoveries today

The flavour of neutrinos

1937 μ discovered in cosmic rays

Is a heavy version of the electron and not the nuclear agent (pion)

$$\pi \rightarrow \mu \ \bar{\nu}_\mu$$

Pontecorvo

The neutrino that accompanies the μ is different to that in beta decay

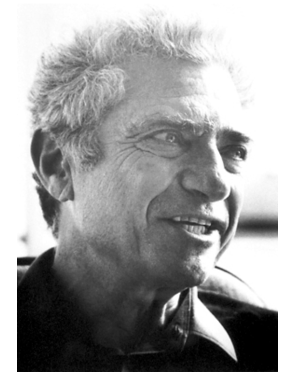
Neutrino cross section in Fermi theory grows with energy, it should be easier to observe: the first experiment with an accelerator neutrino beam !

Neutrino Flavour

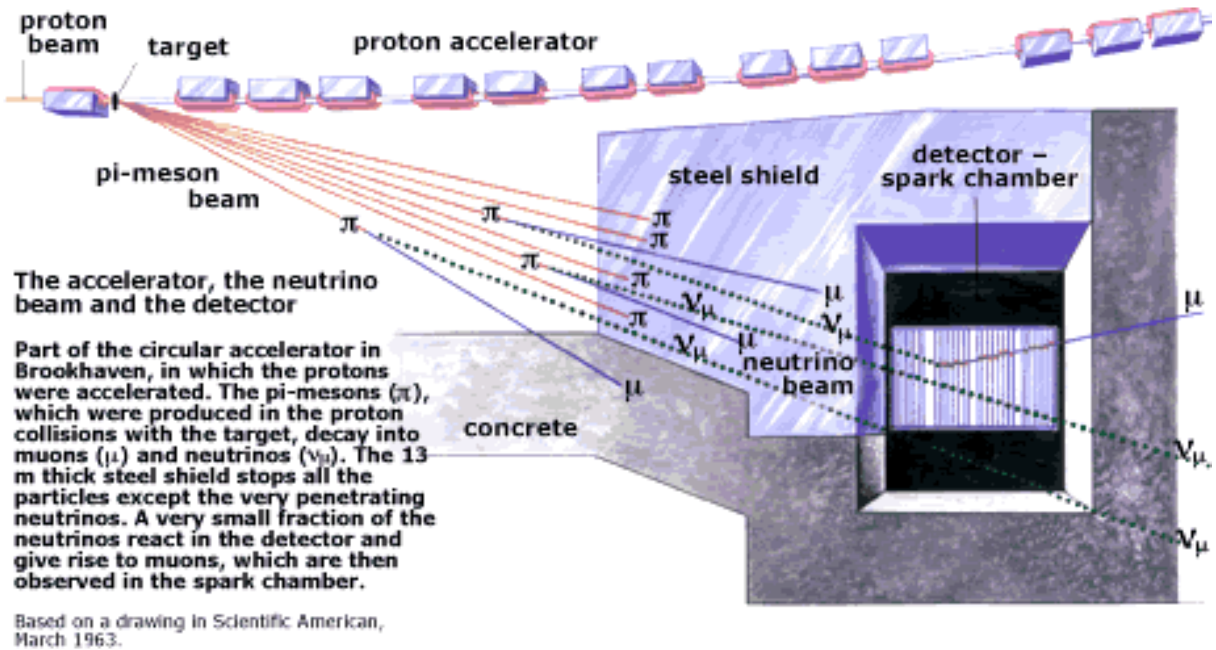
$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$


Lederman


Schwartz


Steinberger

Nobel 1988

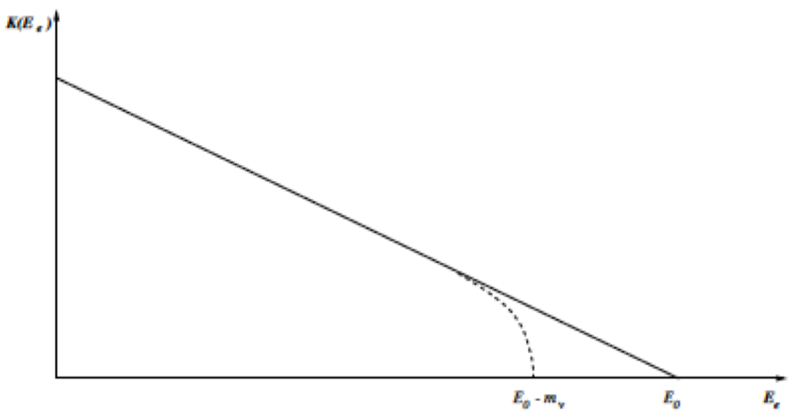


Modern versions of Lederman, Schwartz, Steinberger experiment are accelerator neutrino experiments: MINOS, OPERA, T2K, NoVA,...

Kinematical effects of neutrino mass

Most stringent from Tritium beta-decay

$$H^3 \rightarrow^3 He + e^- + \bar{\nu}_e$$



$m_{\nu_e} < 2.2\text{eV}$ (Mainz-Troitsk)

$m_{\nu_\mu} < 170\text{keV}$ (PSI: $\pi^+ \rightarrow \mu^+ \nu_\mu$)

$m_{\nu_\tau} < 18.2\text{MeV}$ (LEP: $\tau^- \rightarrow 5\pi\nu_\tau$)

Standard Model neutrinos assumed massless

Next generation of tritium beta decay experiment: Katrin

Goal: $m_{\nu e} < 0.2$ eV

Neutrinos in the Standard Model

$$SU(3) \times SU(2) \times U(1)_Y$$

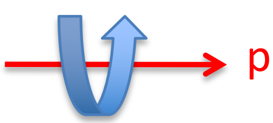
$(1,2)_{-\frac{1}{2}}$	$(3,2)_{-\frac{1}{6}}$	$(1,1)_{-1}$	$(3,1)_{-\frac{2}{3}}$	$(3,1)_{-\frac{1}{3}}$
$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L$	$\begin{pmatrix}u^i \\ d^i\end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L$	$\begin{pmatrix}c^i \\ s^i\end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L$	$\begin{pmatrix}t^i \\ b^i\end{pmatrix}_L$	τ_R	t^i_R	b^i_R

$$\Psi_{L/R} \equiv P_{L/R}\Psi$$

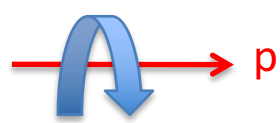
$$P_{L/R} \equiv \frac{1 \mp \gamma_5}{2}$$

$$\underbrace{P_{L,R}}_{\text{chiral projector}} \simeq_{v\to c} \underbrace{P_{\mp}}_{\text{helicity projector}}$$

Left-handed



Right-handed

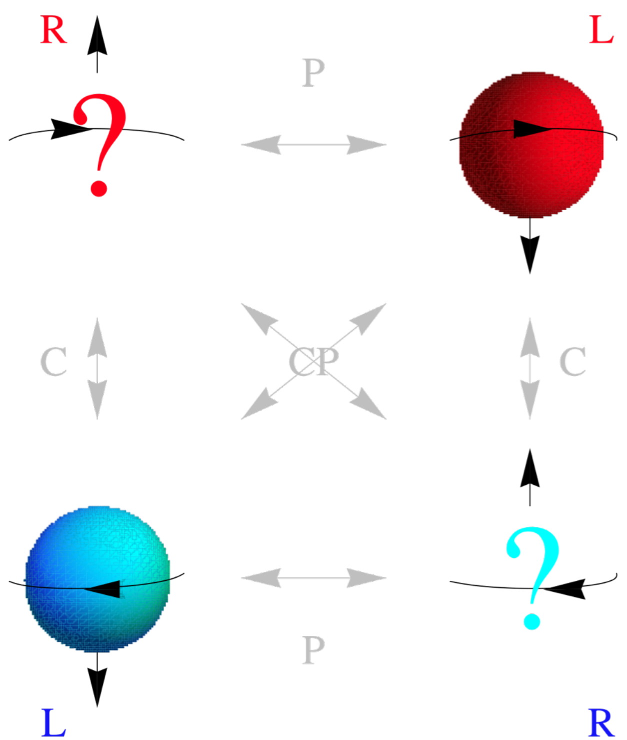


Neutrinos are Weyl fermions: two component spinor describing a massless fermion with negative helicity + antifermion with positive helicity

Breaking of C and P

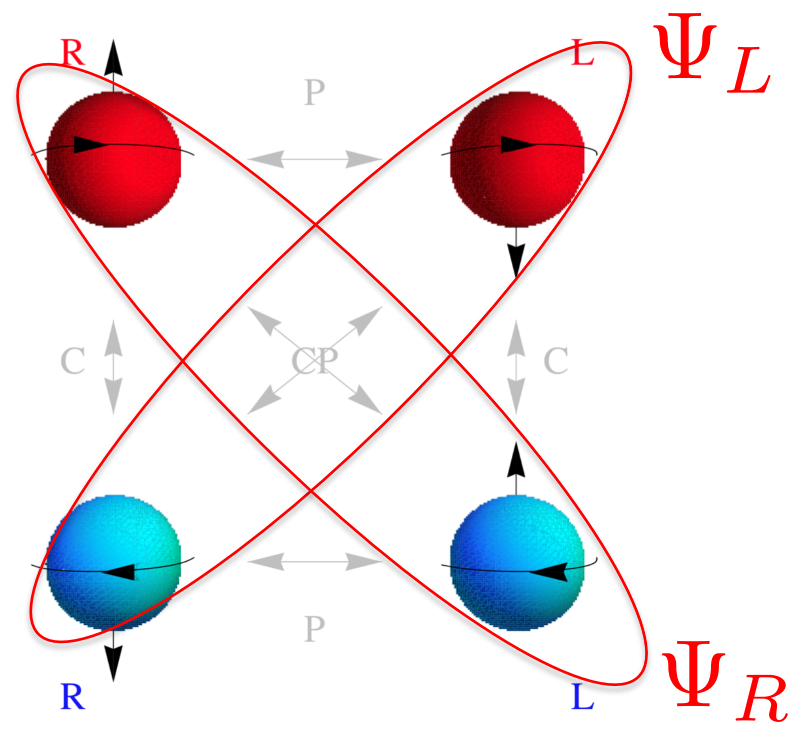
Weyl fermion= 2-component spinor

(Minimal spin ½)

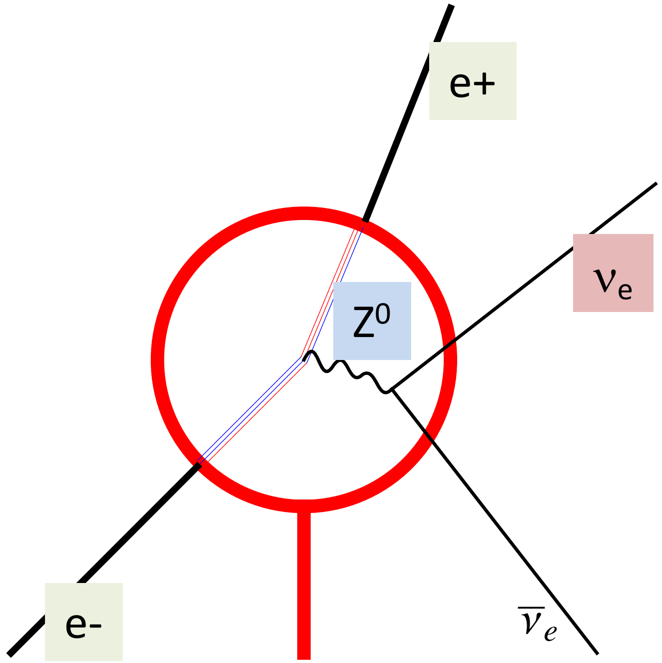


Dirac fermion= 4-component spinor

(Minimal spin ½ + Parity)



Neutrinos in the Standard Model



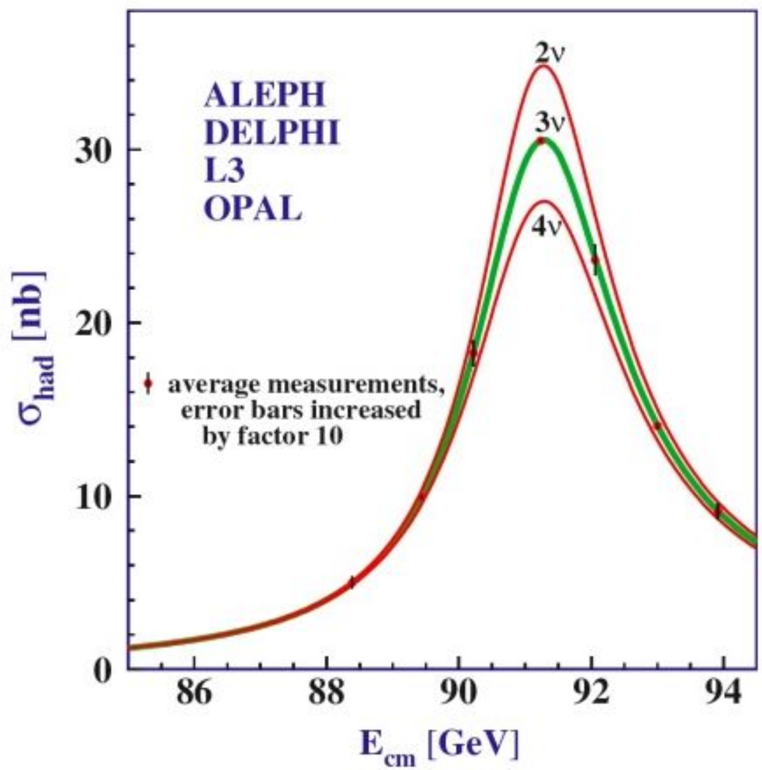
At LEP:

$$e^+e^- \rightarrow Z^0 \rightarrow f\bar{f}$$

Only three neutrinos -> three SM families

Neutral currents: NC

$$N_\nu = \frac{\Gamma_{\rm inv}}{\Gamma_{\nu\bar{\nu}}} = 2.984 \pm 0.008$$



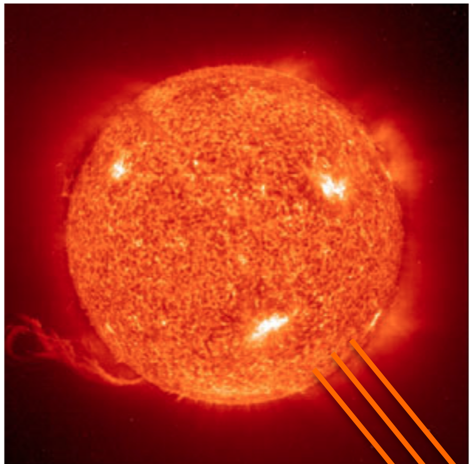
The most elusives particles have been key in the discovery of the weak interactions and in establishing the two most intringuing features of the SM:

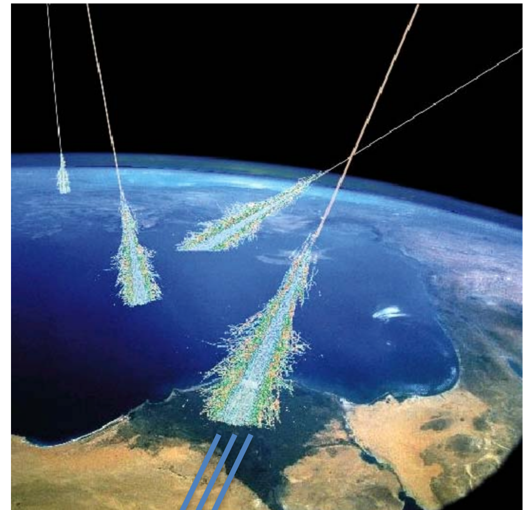
3-fold repetition of family structures

chiral nature of the weak interactions

Ubiquitous Neutrinos

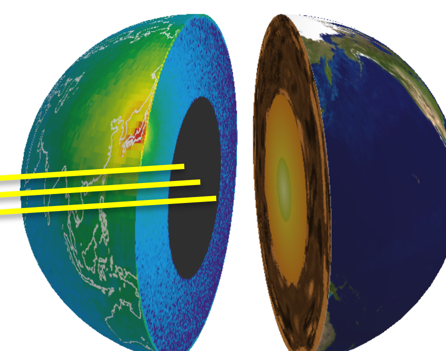
They are everywhere…





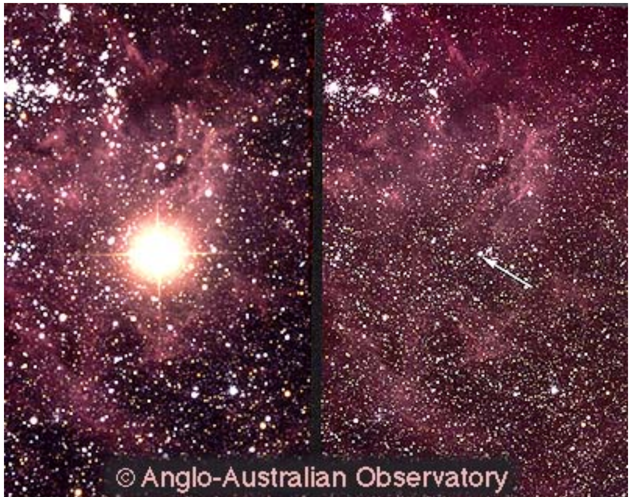
Atmosphere: ~20/second

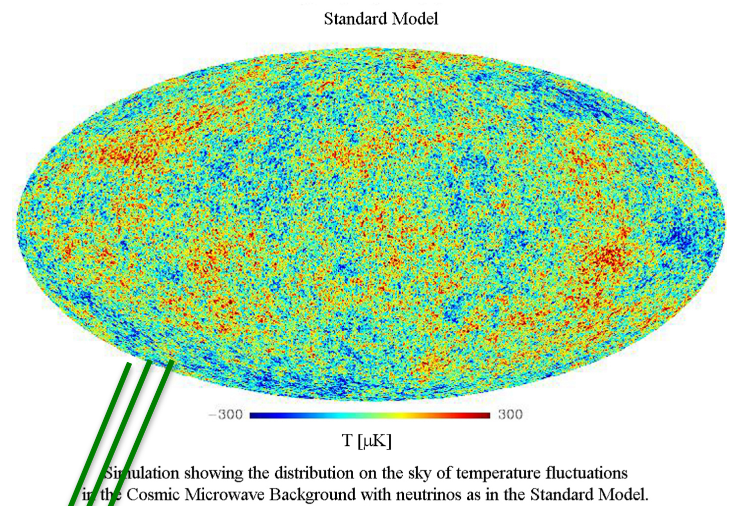
Sun: 5 x 10^{12}/second



Earth: ~10^{9}/second

Ubiquitous Neutrinos





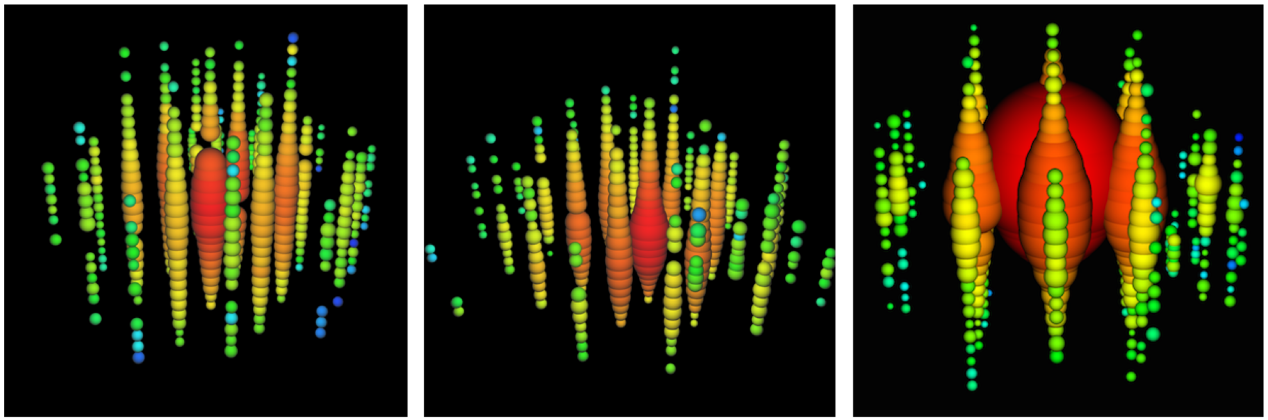
Big Bang: ~2 x 10^{12}/second

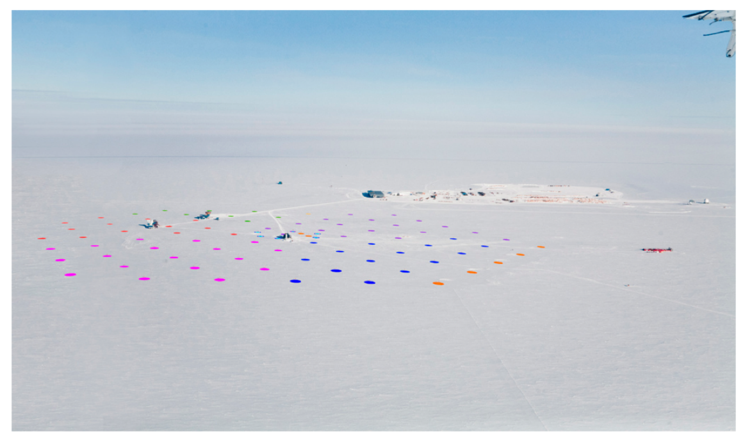
Supernova 1987: ~10^{12}/second

@168000 Light years!
10^8 farther from Earth

Ubiquitous Neutrinos

PeV neutrinos from still unknown sources...

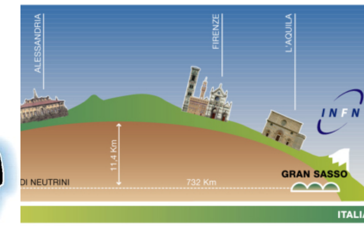




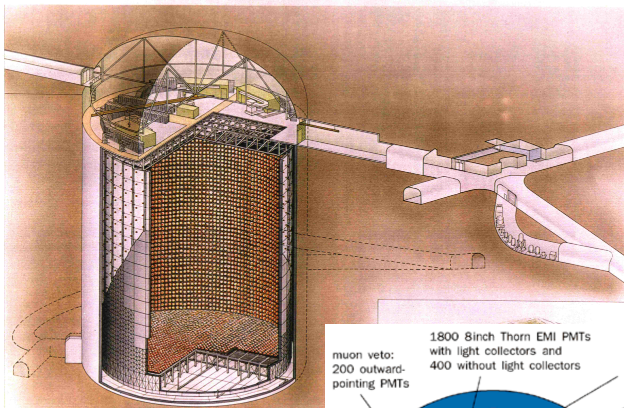
Icecube

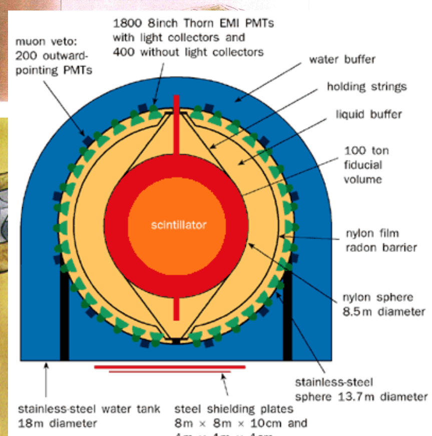
Using many of these sources, and others man-made, two decades of revolutionary neutrino experiments have demonstrated that neutrinos are not quite standard, because they have a tiny mass & massive neutrinos require to extend the SM!

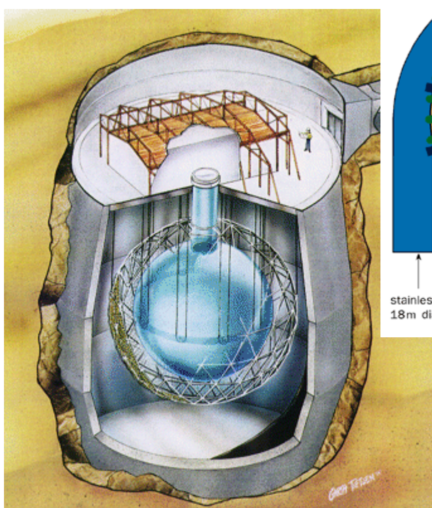
MINOS, Opera



SuperKamiokande





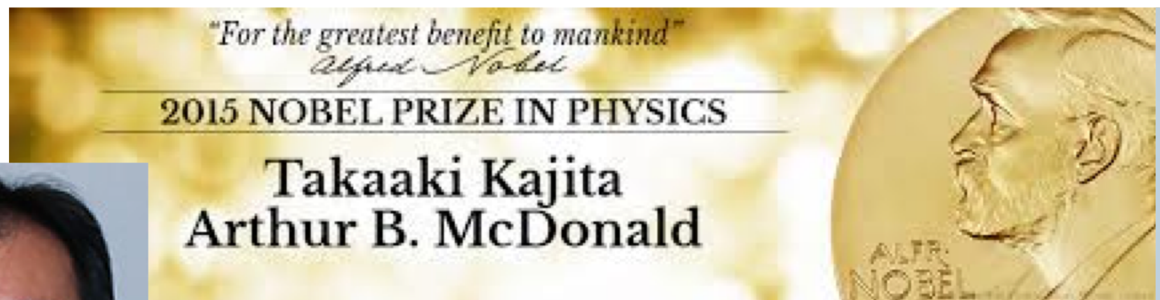


Borexino

SNO

…and more

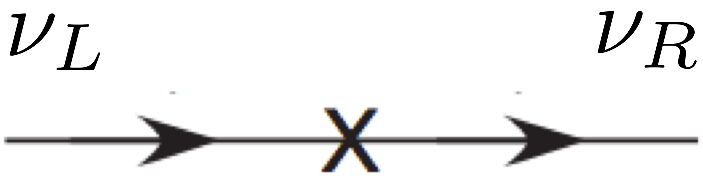
"For the discovery of neutrino oscillations,
which shows that neutrinos have mass"



Massive (free) fermions

Dirac fermion of mass m:

$$-\mathcal{L}_m^{\mathrm{Dirac}} = m\bar{\psi}\psi = m(\overline{\psi_L + \psi_R})(\psi_L + \psi_R) = m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L)$$



A massive particle must have both helicities... $\nu_{\mathrm{D}} = \nu_L + \nu_R$

Massive (free) fermions

Majorana fermion of mass m (Weyl representation)

$$-\mathcal{L}_m^{Majorana} = \frac{m}{2}\overline{\psi^c}\psi + \frac{m}{2}\overline{\psi}\psi^c \equiv \frac{m}{2}\psi^T C\psi + \frac{m}{2}\bar{\psi}C\bar{\psi}^T,$$

$$\psi^c \equiv C\bar{\psi}^T = C\gamma_0\psi^* \qquad C = i\gamma_2\gamma_0$$

$$\nu_L \qquad \nu_L^c = C\overline{\nu_L}^T$$

Massive field is both particle and antiparticle $\quad \nu_{\mathrm{M}} = \nu_L + \nu_L^c$

Massive fermions & Weak Interactions ?

Dirac fermion of mass m:

$$-\mathcal{L}_m^{\mathrm{Dirac}} = m\bar{\psi}\psi = m(\overline{\psi_L + \psi_R})(\psi_L + \psi_R) = m(\overline{\psi_L}\psi_R + \overline{\psi_R}\psi_L)$$

Breaks SU(2)xU(1) gauge invariance!

Majorana fermion of mass m (Weyl representation)

$$-\mathcal{L}_m^{Majorana} = \frac{m}{2}\overline{\psi^c}\psi + \frac{m}{2}\bar{\psi}\psi^c \equiv \frac{m}{2}\psi^T C\psi + \frac{m}{2}\bar{\psi}C\bar{\psi}^T,$$

No gauge/global symmetry of ψ possible!

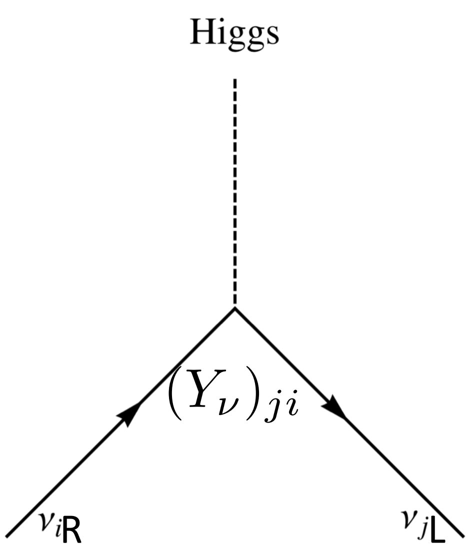
Spontaneous symmetry breaking can induce Dirac masses for all fermions but Majorana masses only for neutrinos !

Massive Dirac neutrinos & SSB ?

$$\widetilde{\phi} \equiv \sigma_2 \phi^*, \qquad \widetilde{\phi} : (1, 2, -\frac{1}{2}), \quad \langle \widetilde{\phi} \rangle = \begin{pmatrix} \frac{v}{2} \\ 0 \end{pmatrix}$$

Massive Dirac neutrino via Yukawa coupling: SM + ν_R

$$-\mathcal{L}_m^{\mathrm{Dirac}} = Y_\nu \underbrace{\bar{L}\ \widetilde{\phi}}_{(1,1,0)} \ \underbrace{\nu_R}_{(1,1,0)} + h.c \rightarrow SSB \ \rightarrow Y_\nu \bar{\nu}_L \frac{v}{\sqrt{2}} \nu_R + h.c.$$



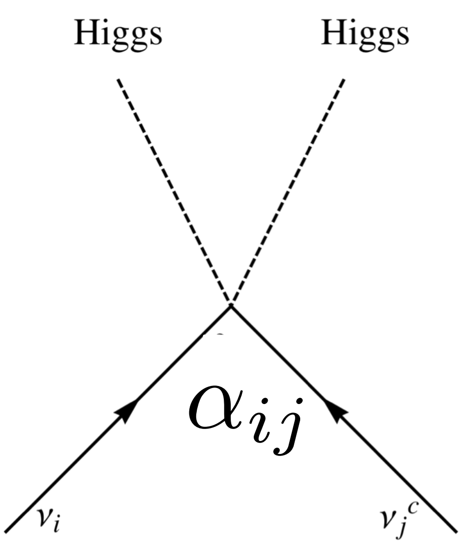
$$m_\nu = Y_\nu \frac{v}{\sqrt{2}}$$

Massive Majorana neutrinos & SSB ?

$$\widetilde{\phi} \equiv \sigma_2 \phi^*, \qquad \widetilde{\phi} : (1, 2, -\frac{1}{2}), \;\; \langle \widetilde{\phi} \rangle = \begin{pmatrix} \frac{v}{2} \\ 0 \end{pmatrix}$$

Massive Majorana neutrino via Weinberg's coupling

$$-\mathcal{L}^{\mathrm{Majorana}} = \alpha \bar{L}\; \widetilde{\phi}\; C \widetilde{\phi}^T \bar{L}^T + h.c. \rightarrow SSB \rightarrow \alpha \frac{v^2}{2} \bar{\nu}_L C \bar{\nu}_L^T + h.c.$$



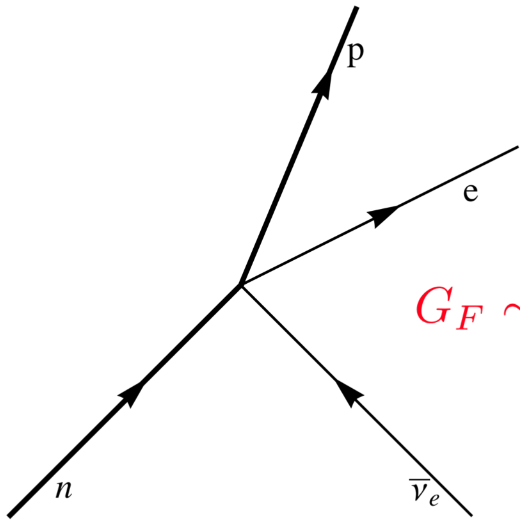
$$m_\nu = \alpha \frac{v^2}{2}$$

$$[\alpha] = -1$$

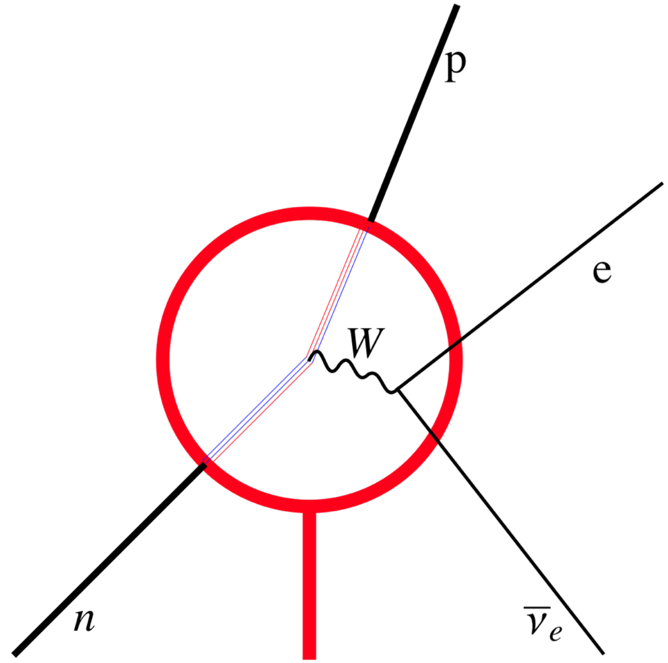
$$\alpha \equiv \frac{\lambda}{\Lambda}$$

Implies the existence of a new physics scale unrelated to v !

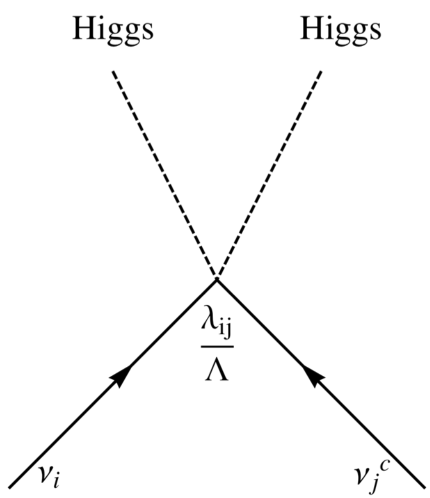
Neutrinos have tiny masses -> a new physics scale, what ?



$$G_F \sim \frac{1}{M_W^2}$$



SM



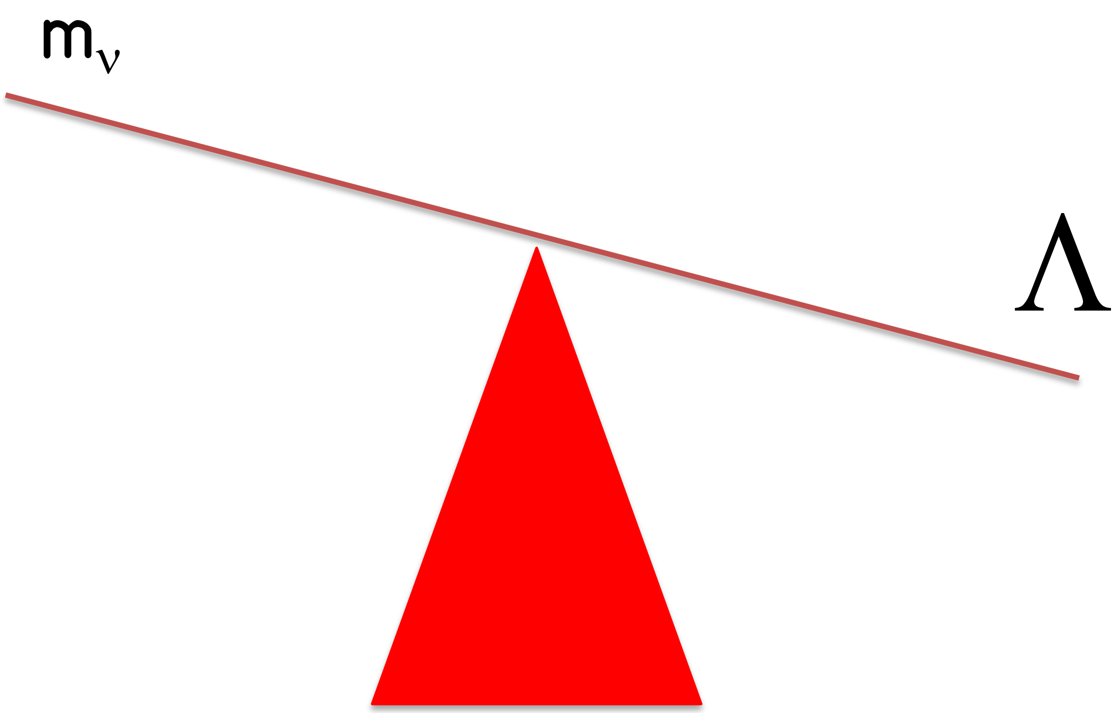
νSM ?

$$m_\nu = \lambda \frac{v^2}{\Lambda}$$

Scale at which new particles will show up

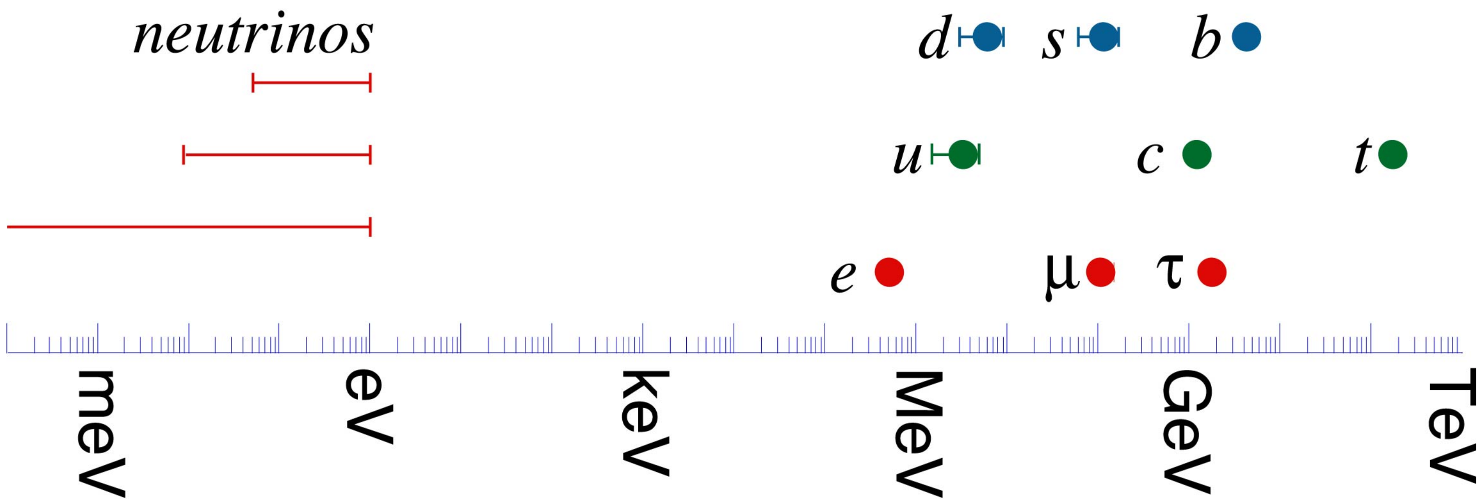
Seesaw mechanism:

Minkowski
Gell-Mann, Ramond Slansky
Yanagida, Glashow
Mohapatra, Senjanovic



Massive Majorana neutrinos & SSB ?

If $\Lambda >> v$ natural explanation for the smallness of neutrino mass



$$m_f(charged) \sim Yv, \quad m_\nu \sim Y\frac{v^2}{\Lambda} \sim m_f\frac{v}{\Lambda}$$

Neutrino masses & lepton mixing (Dirac)

Yukawa couplings are generic complex matrices in flavour space

$$(M_f)_{ij} = Y_{ij} \frac{v}{\sqrt{2}}$$

$$-\mathcal{L}_m^{lepton} = \bar{\nu}_{Li} \underbrace{(M_\nu)_{ij}}_{3\times n_R} \nu_{Rj} + \bar{l}_{Li} \underbrace{(M_l)_{ij}}_{3\times 3} l_{Rj} + h.c.$$

$$M_\nu = U_\nu^\dagger \ \mathrm{Diag}(m_1, m_2, m_3) \ V_\nu, \ \ M_l = U_l^\dagger \ \mathrm{Diag}(m_e, m_\mu, m_\tau) \ V_l$$

In the mass eigenbasis

$$\mathcal{L}_{\mathrm{gauge-lepton}} \supset -\frac{g}{\sqrt{2}} \bar{l}'_{Li} \underbrace{(U_l^\dagger U_\nu)_{ij}}_{U_{PMNS}} \gamma_\mu W_\mu^- \nu'_{Lj} + h.c.$$

Pontecorvo-Maki-Nakagawa-Sakata

$U_{\mathrm{PMNS}}(\theta_{12}, \theta_{13}, \theta_{23}, \delta)$ unitary matrix analogous to CKM

Neutrino masses & lepton mixing (Majorana)

Are generic complex matrices in flavour space

$$-\mathcal{L}_m^{lepton} = \frac{1}{2}\bar{\nu}_{Li}(M_\nu)_{ij}\nu^c_{Lj} + \bar{l}_{Li}(M_l)_{ij}l_{Rj} + h.c.$$

$$M_\nu^T = M_\nu \rightarrow M_\nu = U_\nu^T \mathrm{Diag}(m_1, m_2, m_3) U_\nu$$

In the mass eigenbasis

$$\mathcal{L}_{\mathrm{gauge-lepton}} \supset -\frac{g}{\sqrt{2}}\bar{l}'_{Li}\underbrace{(U_l^\dagger U_\nu)_{ij}}_{U_{PMNS}}\gamma_\mu W_\mu^- \nu'_{Lj} + h.c.$$

$U_{\mathrm{PMNS}}(\theta_{12}, \theta_{13}, \theta_{23}, \delta, \alpha_1, \alpha_2)$ depends on three CP phases

Exercise: make sure you agree

Neutrino Mixing

flavour eigenstates (in combination with e, μ, τ)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta, \ldots) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{\text{PMNS}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1} & 0 \\ 0 & 0 & e^{i\alpha_2} \end{pmatrix}}_{\text{Majorana phases}}$$

$$c_{ij} \equiv \cos\theta_{ij} \;\; s_{ij} \equiv \sin\theta_{ij}$$

Total lepton number

Massive neutrinos imply that family number is not conserved

Dirac neutrinos conserve total lepton number: $\sum_{\alpha=e,\mu,\tau} \mathcal{L}(\alpha)$

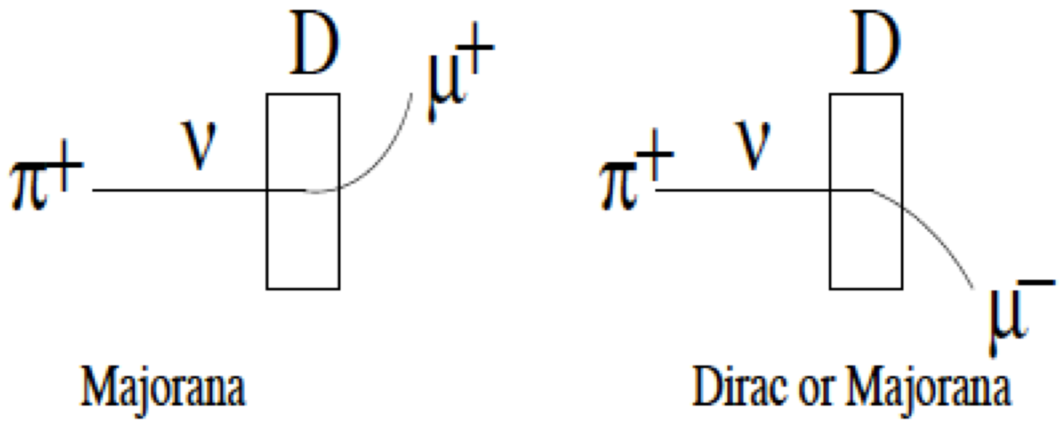
$$L_\alpha \to e^{i\theta} L_\alpha, l_{R\alpha} \to e^{i\theta} l_{R\alpha}, \nu_{R\alpha} \to e^{i\theta} \nu_{R\alpha}$$

Majorana neutrinos violate also this global symmetry

-> a new mechanism to explain the matter/antimatter asymmetry emerges

Majorana versus Dirac

In principle clear experimental signatures

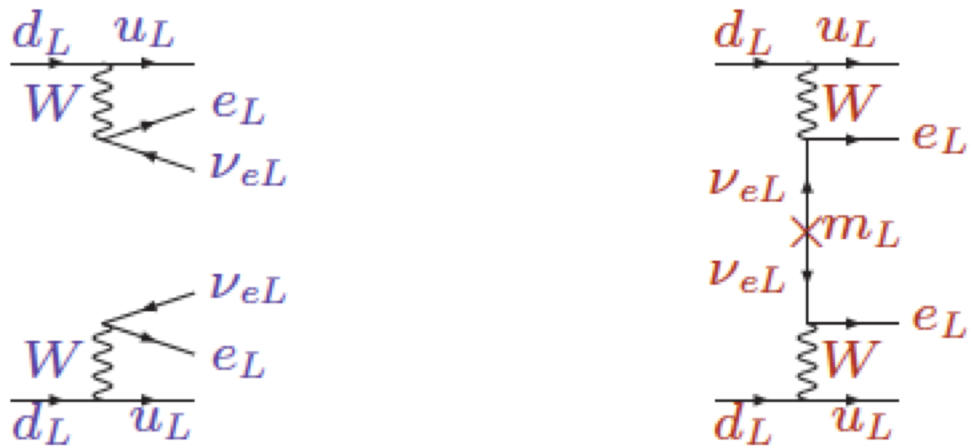


In practice theses processes are extremely rare:

$$\mathrm{Rate}(+) = \mathrm{Rate}(-)\left(\frac{m_\nu}{E}\right)^2$$

Neutrinoless double-β decay

Best hope is neutrinoless double-β decay



$T_{2\beta 2\nu} \sim 10^{18} - 10^{21}$ years

$T^{-1}_{2\beta 0\nu} \sim \left(\frac{m_\nu}{E}\right)^2 10^9\, T^{-1}_{2\beta 2\nu}$

If neutrinos are Majorana this process must be there at some level

Neutrinoless double-β decay

$$T_{2\beta0\nu}^{-1} \simeq \underbrace{G^{0\nu}}_{\text{Phase}} \underbrace{\left|M^{0\nu}\right|^2}_{\text{Nuclear M.E.}} \underbrace{\left|\sum_i \left(V_{MNS}^{ei}\right)^2 m_i\right|^2}_{|m_{ee}|^2}$$

Present bounds:

Sarazin 2012

Isotope	$T_{1/2}^{2\nu}$ (yr)	Experiment	$T_{1/2}^{0\nu}$ (yr) (90% C.L.)	Experiment	$\langle m_{ee}\rangle$ (eV) Min.	Max.	
^{48}Ca	$4.2^{+2.1}_{-1.0}\ 10^{19}$	NEMO-3	$5.8\ 10^{22}$	CANDLES [111]	3.55	9.91	
^{76}Ge	$1.5 \pm 0.1\ 10^{21}$	HDM	**$1.9\ 10^{25}$**	HDM [46]	0.2	0.4	GERDA '13
^{82}Se	$9.0 \pm 0.7\ 10^{19}$	NEMO-3	$3.2\ 10^{23}$	NEMO-3 [40]	0.85	2.08	
^{96}Zr	$2.0 \pm 0.3\ 10^{19}$	NEMO-3	$9.2\ 10^{21}$	NEMO-3 [35]	3.97	14.39	
^{100}Mo	$7.1 \pm 0.4\ 10^{18}$	NEMO-3	**$1.0\ 10^{24}$**	NEMO-3 [40]	**0.31**	**0.79**	
^{116}Cd	$3.0 \pm 0.2\ 10^{19}$	NEMO-3	$1.7\ 10^{23}$	SOLOTVINO [81]	1.22	2.30	
^{130}Te	$0.7 \pm 0.1\ 10^{21}$	NEMO-3	**$2.8\ 10^{24}$**	CUORICINO [65]	**0.27**	**0.57**	
^{136}Xe	$2.38 \pm 0.14\ 10^{21}$	Kamland	**$5.7\ 10^{24}$**	Kamland-Zen [93]	**0.25**	**0.6**	
^{150}Nd	$7.8 \pm 0.7\ 10^{18}$	NEMO-3	$1.8\ 10^{22}$	NEMO-3 [37]	[illegible]	[illegible]	

^{136}Xe	EXO-Kamland '12	0.12	0.25
	Kamland '16	0.061	0.165

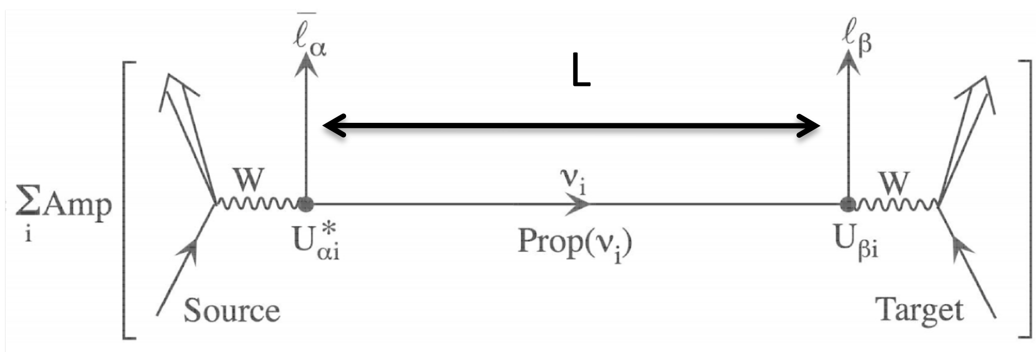
Neutrino oscillations

1968 Pontecorvo

If neutrinos are massive

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta, \ldots) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

A neutrino experiment is an interferometer in flavour space, because neutrinos are so weakly interacting that can keep coherence over very long distances !



ν_i pick up different phases when travelling in vacuum

Neutrino oscillations

$$P(\nu_\alpha \to \nu_\beta) = \sum_{ij} U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} e^{-i\frac{(m_i^2 - m_j^2)L}{2E}}$$

Many ways to derive the oscillation probability master formula

Quantum mechanics with neutrinos as plane waves
Quantum mechanics with neutrinos as wave packets
Quantum Field Theory <-> neutrinos as intermediate states

The basic ingredients:

✓ Uncertainty in momentum at production & detection (they must be better localized than baseline)

✓ Coherence of mass eigenstates over macroscopic distances

Neutrino oscillations in QM (plane waves)

$$|\nu_\alpha(t_0)\rangle = \sum_i U^*_{\alpha i}|\nu_i(\mathbf{p})\rangle, \qquad \hat{H}|\nu_i(\mathbf{p})\rangle = E_i(\mathbf{p})|\nu_i(\mathbf{p})\rangle, \quad \mathbf{p}^2 + m_i^2 = E_i^2(\mathbf{p})$$

$\downarrow$ time evolution

$$|\nu_\alpha(t)\rangle = \sum_i U^*_{\alpha i} e^{-iE_i(\mathbf{p})(t-t_0)}|\nu_i(\mathbf{p})\rangle$$

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta)(t) &= |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = |\sum_i U_{\beta i}U^*_{\alpha i} e^{-iE_i(t-t_0)}|^2 \\ &= \sum_{i,j} e^{-i(E_i-E_j)(t-t_0)} U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j} \end{aligned}$$

$$E_i(\mathbf{p}) - E_j(\mathbf{p}) \simeq \frac{1}{2}\frac{m_i^2 - m_j^2}{|\mathbf{p}|} + \mathcal{O}(m^4) \qquad L \simeq t - t_0, v_i \simeq c$$

$$\boxed{P(\nu_\alpha \to \nu_\beta)(L) \simeq \sum_{i,j} e^{i\frac{\Delta m^2_{ji} L}{2E}} U_{\beta i}U^*_{\alpha i}U^*_{\beta j}U_{\alpha j}}$$

Neutrino oscillations in QM (plane waves)

Well founded criticism to this derivation

Why same p for the i-th states ?

Why plane waves if the neutrino source is localized ?

Why t <-> L conversion ?

Neutrino oscillations in QM (wavepackets)

B. Kayser '81,... many more authors...

Wave packet created at source @ $(t_0, \mathbf{x}_0) = (0, \mathbf{0})$

$$|\nu_\alpha(t, \mathbf{x})\rangle = \sum_i U^*_{\alpha i} \int_{\mathbf{p}} \underbrace{f_i^S(\mathbf{p} - \mathbf{Q}_i)}_{\text{Wave packet at source}} \quad e^{-iE_i(\mathbf{p})t} \; e^{i\mathbf{p}\cdot\mathbf{x}} \; |\nu_i\rangle$$

$$E_i(\mathbf{p}) \equiv \sqrt{\mathbf{p}^2 + m_i^2}$$

For example: $f_i^S(\mathbf{p} - \mathbf{Q}_i) \simeq e^{-(\mathbf{p}-\mathbf{Q}_i)^2/2\sigma_S^2}$

$\sigma_S \quad \leftrightarrow \quad$ momentum uncertainty

$\mathbf{Q}_i \quad \leftrightarrow \quad$ average momentum of i – th wavepacket

Wave packet created at detector @ $(t_0, \mathbf{x}_0) = (t, \mathbf{L})$

$$|\nu_\beta(t, \mathbf{x})\rangle = \sum_j U^*_{\beta j} \int_{\mathbf{p}} f_j^D(\mathbf{p} - \mathbf{Q}'_j) \; e^{-iE_j(\mathbf{p})(t-T)} \; e^{i\mathbf{p}(\mathbf{x}-\mathbf{L})} \; |\nu_j\rangle$$

Neutrino oscillations in QM (wavepackets)

$$\mathcal{A}(\nu_\alpha \to \nu_\beta) = \int_{\mathbf{x}} \langle \nu_\beta(t,\mathbf{x}) | \nu_\alpha(t,\mathbf{x}) \rangle$$
$$= \sum_i U^*_{\alpha i} U_{\beta i} \int_{\mathbf{p}} e^{iE_i(\mathbf{p})T} \; e^{-i\mathbf{p}\mathbf{L}} \; \underbrace{f_i^{D*}(\mathbf{p}-\mathbf{Q}'_i) f_i^S(\mathbf{p}-\mathbf{Q}_i)}_{overlap}$$

For Gaussian wave packets overlap is also gaussian:

$$f_i^{D*} f_i^S = f_i^{ov}(\mathbf{p} - \langle \mathbf{Q} \rangle_i) \; e^{-(\mathbf{Q}_i - \mathbf{Q}'_i)^2/4/(\sigma_S^2 + \sigma_D^2)}$$

$$\langle \mathbf{Q} \rangle_i \equiv \left(\frac{\mathbf{Q}_i}{\sigma_S^2} + \frac{\mathbf{Q}'_i}{\sigma_D^2} \right) \sigma_{ov}^2$$
$$\sigma_{ov}^2 \equiv \frac{1}{1/\sigma_S^2 + 1/\sigma_D^2}$$

group velocity

$$E_i(\mathbf{p}) \simeq E_i(\langle \mathbf{Q} \rangle_i) + \overbrace{\left.\frac{\partial E}{\partial p_k}\right|_{\langle \mathbf{Q} \rangle_i}}^{\mathbf{v}_i} (p_k - \langle Q_k \rangle_i) + \mathcal{O}(p_k - \langle Q_k \rangle_i)^2$$

$$\mathcal{A}(\nu_\alpha \to \nu_\beta) \propto \sum_i U^*_{\alpha i} U_{\beta i} \; e^{iE_i(\langle \mathbf{Q} \rangle_i)T} \; e^{-i\langle \mathbf{Q} \rangle_i \mathbf{L}} \; e^{-(\mathbf{Q}_i - \mathbf{Q}'_i)^2/4/(\sigma_S^2+\sigma_D^2)} \; e^{-(\mathbf{L}-\mathbf{v}_i T)^2 \sigma_{ov}^2/2}$$

Neutrino oscillations in QM (wavepackets)

$$\langle \mathbf{Q} \rangle_i \simeq \langle \mathbf{Q}' \rangle_i, \quad \mathbf{L} || \langle \mathbf{Q} \rangle_i$$

$$L^{-1}_{coh}(i,j) \sim \sigma_{ov} \frac{|\mathbf{v}_i - \mathbf{v}_j|}{\sqrt{\mathbf{v}_i^2 + \mathbf{v}_j^2}} \simeq \frac{|m_j^2 - m_i^2|}{2\langle Q \rangle} \frac{\sigma_{ov}}{\langle Q \rangle}$$

$$L_{coh} \sim L_{\rm osc} \frac{\langle Q \rangle}{\sigma_{ov}}$$

$$P(\nu_\alpha \to \nu_\beta) \propto \int_{-\infty}^{\infty} dT \, |\mathcal{A}(\nu_\alpha \to \nu_\beta)|^2$$

$$\propto \sum_{i,j} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} \, e^{i \frac{m_j^2 - m_i^2}{2E} L} \times e^{-L^2 / L_{coh}(i,j)^2} \times e^{-\left(\frac{\Delta_{ij} E(\langle Q \rangle)}{2\sigma_{ov} \langle v \rangle} \right)^2},$$

$L > L_{coh}$ coherence is lost

There must be sufficient uncertainty in production & detection so that wave packets include all mass eigenstates: $\Delta E \ll \sigma$

Problems: normalization is arbitrary, needs to be imposed a posteriori

$$\sum_\beta P(\nu_\alpha \to \nu_\beta) = 1$$

Can be cured in QFT...

Neutrino Oscillation

$$\boxed{P(\nu_\alpha \to \nu_\beta) = \sum_{ij} U_{\alpha i} U^*_{\beta i} U^*_{\alpha j} U_{\beta j} e^{-i\frac{(m_i^2 - m_j^2)L}{2E}}}$$

α ≠ β appearance probability
α = β disappearance or survival probability

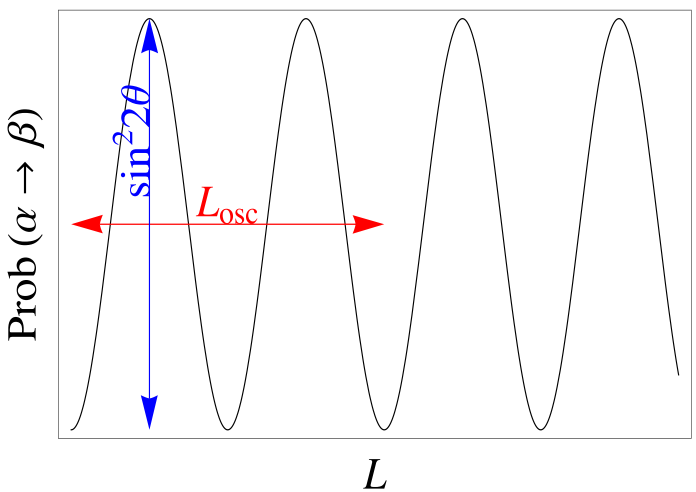
$$L_{osc} \sim \frac{E}{m_i^2 - m_j^2}$$

$$\begin{aligned}
P(\nu_\alpha \to \nu_\beta) \quad &= \quad \underbrace{2\sum_{i<j} Re[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}] + \sum_{i=j} |U_{\alpha i}|^2 |U_{\beta i}|^2}_{\delta_{\alpha\beta}} \\
\text{CP-even} \quad &- \quad 4\sum_{i<j} \mathrm{Re}[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}] \sin^2\left[\frac{\Delta m^2_{ji} L}{4E}\right] \\
\text{CP-odd} \quad &- \quad 2\sum_{i<j} \mathrm{Im}[U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}] \sin\left[\frac{\Delta m^2_{ji} L}{2E}\right]
\end{aligned}$$

Neutrino Oscillation: 2ν

Only one oscillation frequency, $U = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2(eV^2)L(km)}{E(GeV)}\right)$$



$$P(\nu_\alpha \to \nu_\alpha) = 1 - P(\nu_\alpha \to \nu_\beta)$$

$$L_{osc}(km) = \frac{\pi}{1.27}\frac{E(GeV)}{\Delta m^2(eV^2)}$$

Neutrino Oscillation: 2ν

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$$P(\nu_\alpha \to \nu_\beta) = \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m^2(eV^2)L(km)}{E(GeV)}\right)$$

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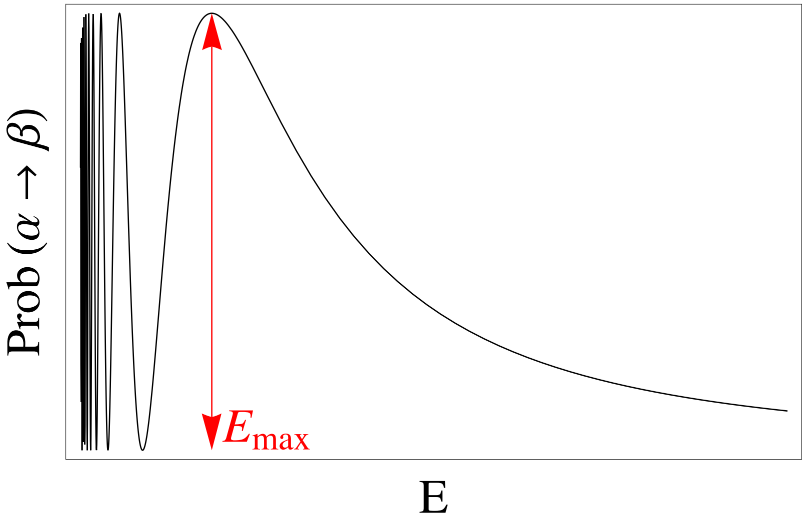


$$E_{max}(GeV) = 1.27\frac{\Delta m^2(eV^2)L(km)}{\pi/2}$$

L, E dependence give Δm^2 amplitude of oscillation gives θ

Optimal experiment: $\frac{E}{L} \sim \Delta m^2$

$\frac{E}{L} \gg \Delta m^2$ Oscillation suppressed

$$P(\nu_\alpha \to \nu_\beta) \propto \sin^2 2\theta \left(\Delta m^2\right)^2$$

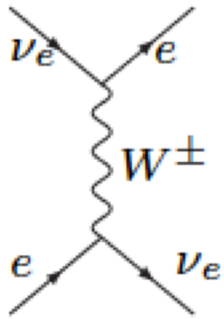
$\frac{E}{L} \ll \Delta m^2$ Fast oscillation regime

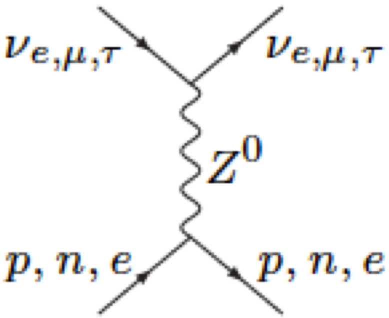
$$P(\nu_\alpha \to \nu_\beta) \simeq \sin^2 2\theta \, \langle \sin^2 \frac{\Delta m^2 L}{4E} \rangle \simeq \frac{1}{2} \sin^2 2\theta = |U^*_{\alpha 1} U_{\beta 1}|^2 + |U^*_{\alpha 2} U_{\beta 2}|^2$$

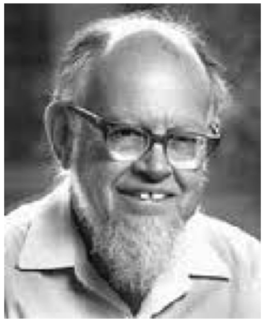
Equivalent to incoherent propagation: sensitivity to mass splitting is lost

Neutrino Oscillations in matter

Many neutrino oscillation experiments involve neutrinos propagating in matter (Earth for atmospheric neutrinos or accelerator experiments, Sun for solar neutrinos)







Wolfenstein

Index of refraction (coherent forward scattering) can strongly affect the oscillation probability

$$\mathcal{H}_{CC} = \frac{G_F}{\sqrt{2}} [\bar{e}\gamma_\mu(1-\gamma_5)\nu_e][\bar{\nu}_e\gamma^\mu(1-\gamma_5)e] = \frac{G_F}{\sqrt{2}} [\bar{e}\gamma_\mu(1-\gamma_5)e][\bar{\nu}_e\gamma^\mu(1-\gamma_5)\nu_e]$$

$$\langle \bar{e}\gamma_\mu P_L e\rangle_{\text{unpol.medium}} = \delta_{\mu 0}\frac{N_e}{2}$$

$$\mathcal{L} \simeq \bar{\nu}\left(i\partial\!\!\!/ - M_\nu - \gamma_0 V_m\right)\nu + \ldots$$

Neutrino oscillations in constant matter

$$E^2 - \mathbf{p}^2 \quad = \quad \pm 2\, V_m E + M_\nu^2$$

+: neutrinos, -: antineutrinos

Effective mixing angles and masses depend on energy

$$\begin{pmatrix} \tilde{m}_1^2 & 0 & 0 \\ 0 & \tilde{m}_2^2 & 0 \\ 0 & 0 & \tilde{m}_3^2 \end{pmatrix} = \tilde{U}^\dagger_{\mathrm{PMNS}} \left(M_\nu^2 \pm 2E \begin{pmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau \end{pmatrix} \right) \tilde{U}_{\mathrm{PMNS}}$$

For two families (- neutrinos, + antineutrinos):

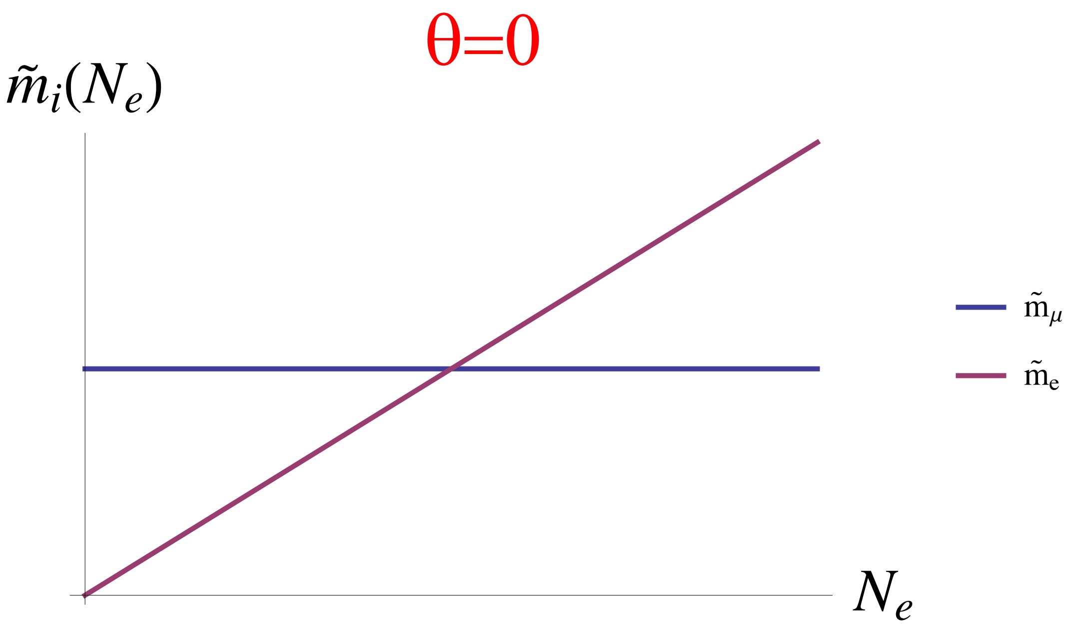
$$\sin^2 2\tilde{\theta} = \frac{\left(\Delta m^2 \sin 2\theta\right)^2}{\left(\Delta m^2 \cos 2\theta \pm 2\sqrt{2}\, G_F E\, N_e\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}$$

$$\Delta \tilde{m}^2 = \sqrt{\left(\Delta m^2 \cos 2\theta \pm 2\sqrt{2} E\, G_F\, N_e\right)^2 + \left(\Delta m^2 \sin 2\theta\right)^2}$$

$$\Delta m^2 \cos 2\theta \pm 2\sqrt{2}\, G_F E\, N_e = 0 \qquad \sin^2 2\tilde{\theta} = 1, \quad \Delta \tilde{m}^2 = \Delta m^2 \sin 2\theta$$

MSW resonance

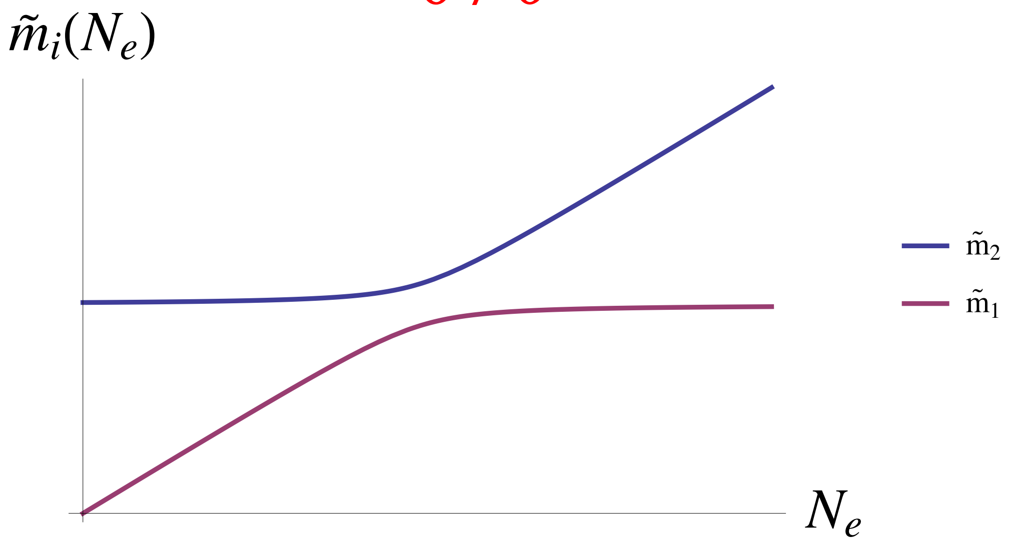
Mikheyev, Smirnov ‘85



MSW resonance

Mikheyev, Smirnov '85

$\theta \neq 0$



$$\Delta m^2 \cos 2\theta \pm 2\sqrt{2}\, G_F E\, N_e = 0$$

MSW Resonance:

-Only for ν or $\bar{\nu}$, not both

-Only for one sign of $\Delta m^2 \cos 2\theta$

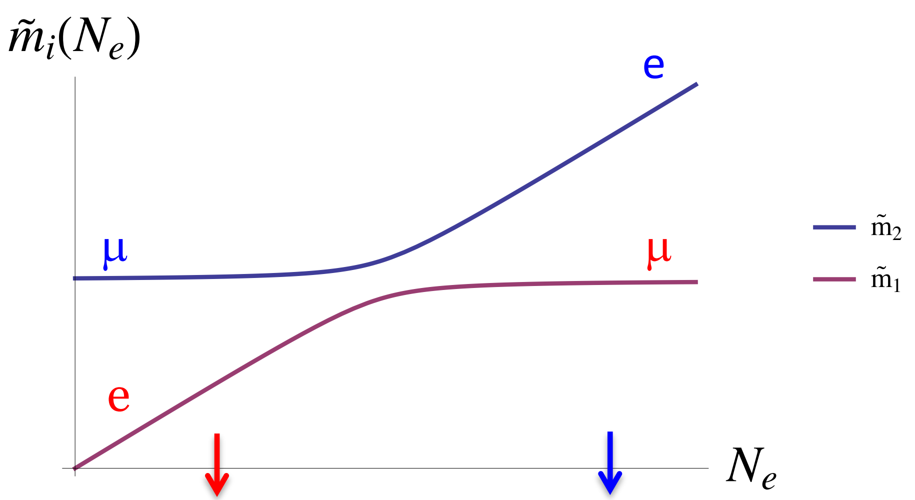
Neutrinos in variable matter

Solar neutrinos propagate in variable matter:

$$N_e(r) \propto N_e(0) e^{-r/R}$$

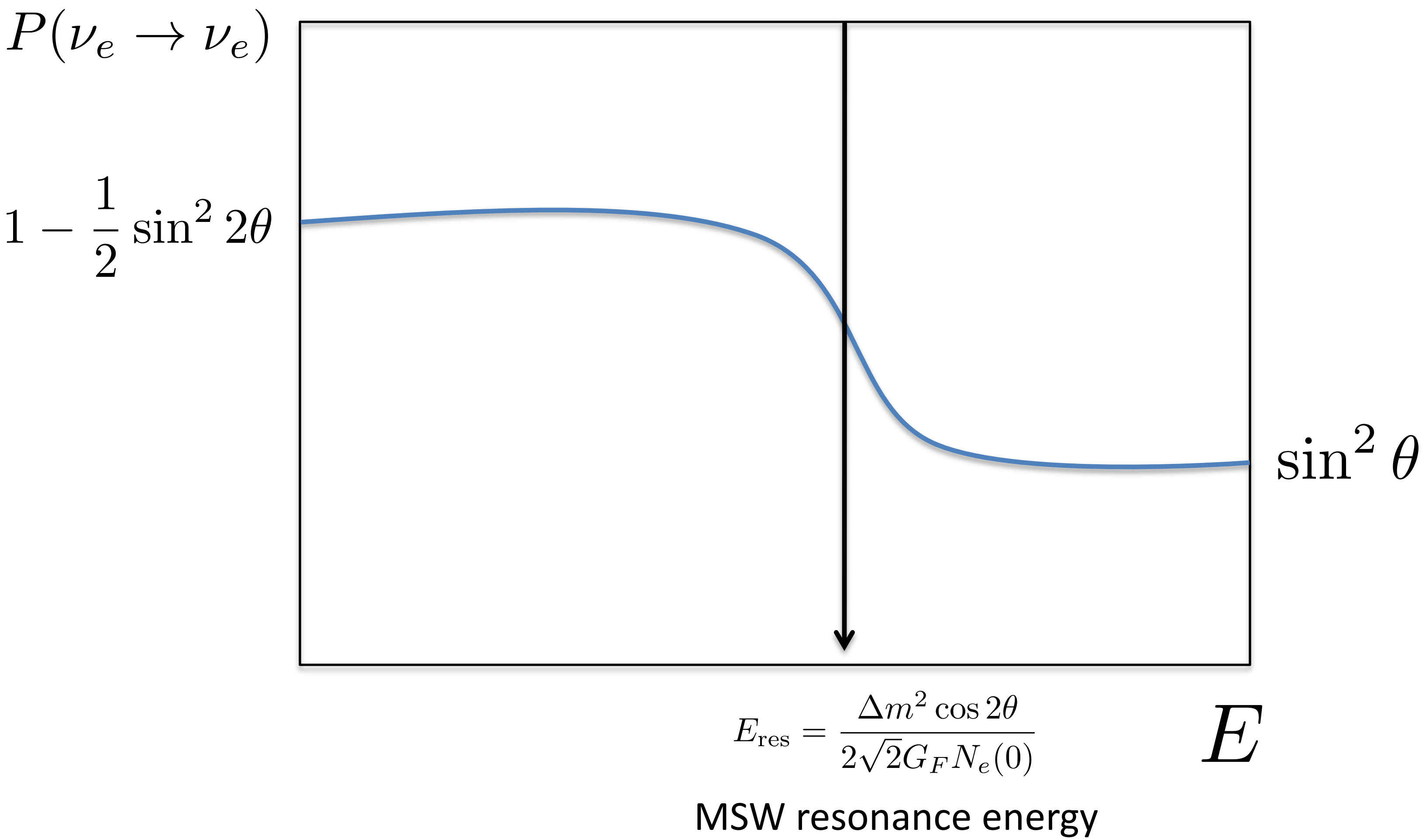
If the variation is slow enough: adiabatic approximation (if a state is at r=0 in an eigenstate $\tilde{m}_i^2(0)$ it remains in the i-th eigenstate until it exits the sun)

$$P(\nu_e \to \nu_e) = \sum_i |\langle \nu_e | \tilde{\nu}_i(\infty) \rangle|^2 |\langle \tilde{\nu}_i(0) | \nu_e \rangle|^2$$



$P(\nu_e \to \nu_e) \simeq 1 - \frac{1}{2}\sin^2 2\theta$ $\quad$ $P(\nu_e \to \nu_e) \simeq \sin^2 \theta$

Solar neutrinos

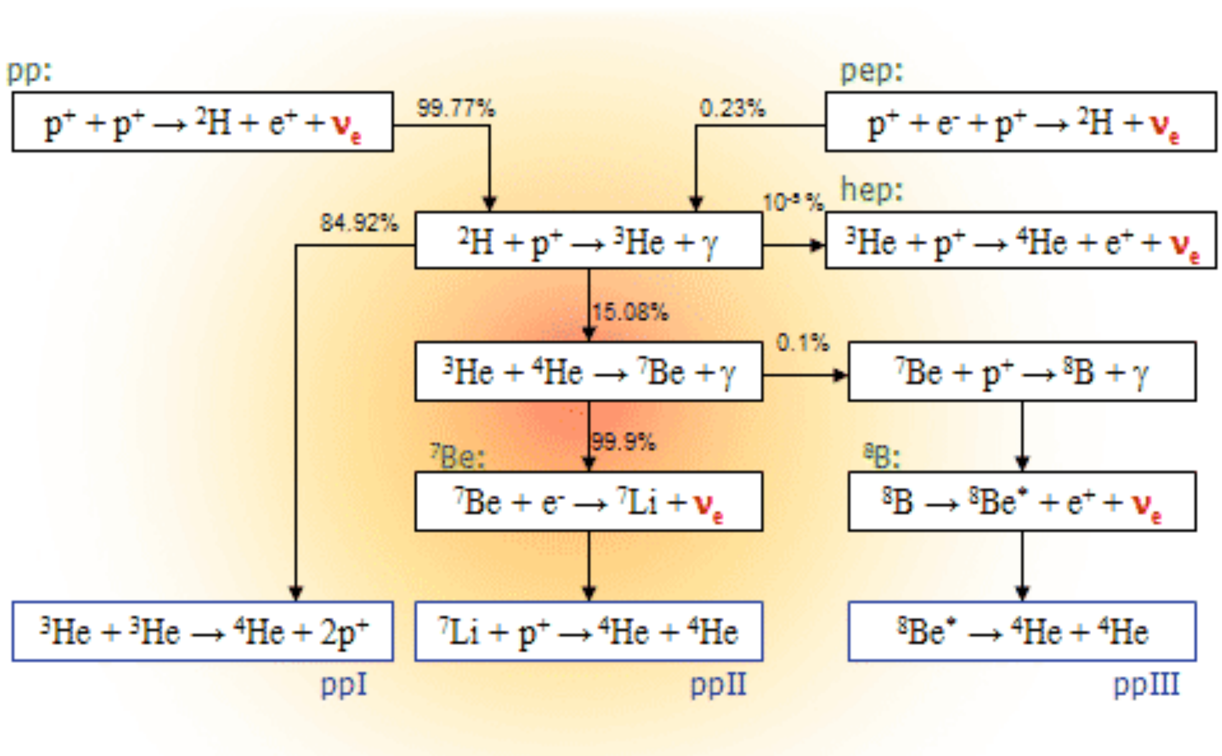


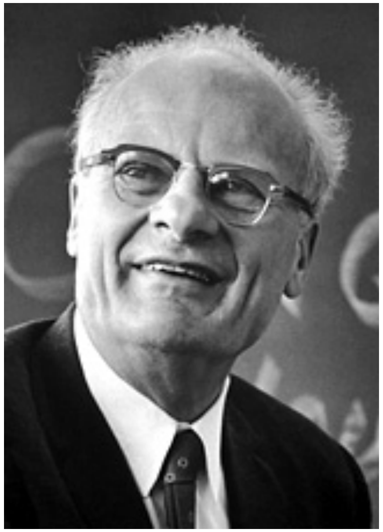
In most physical situations: piece-wise constant matter or adiabatic approx. good enough

Stars shine neutrinos

1939 Bethe

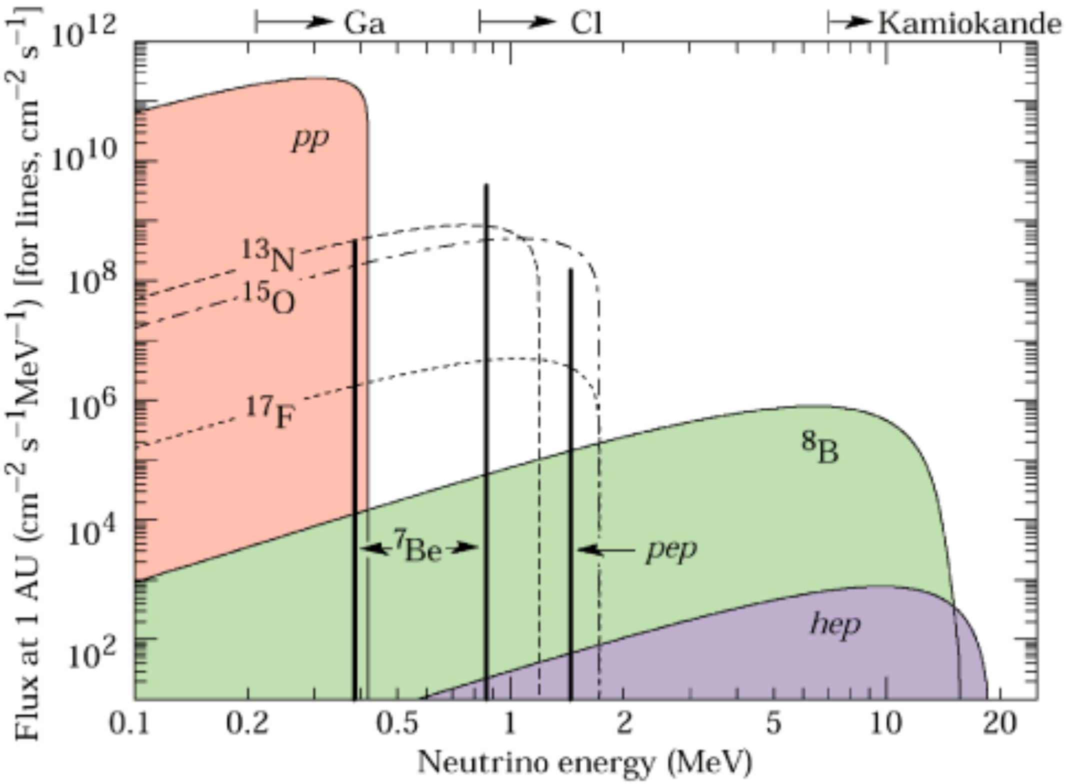
Stablishes the theory of stelar nucleosynthesis





Nobel 1967

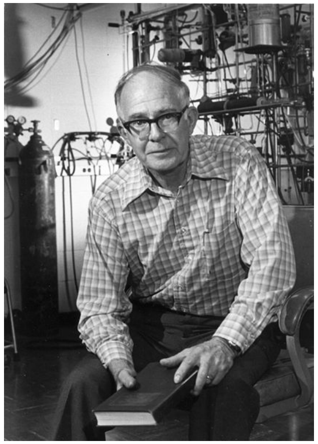
¿How many neutrinos from the Sun ?



Bahcall

The hero of the caves

1966 detects for the first time
solar neutrinos in a tank of
400000 liters 1280m underground
(Homestake mine)



R. Davis
Nobel 2002

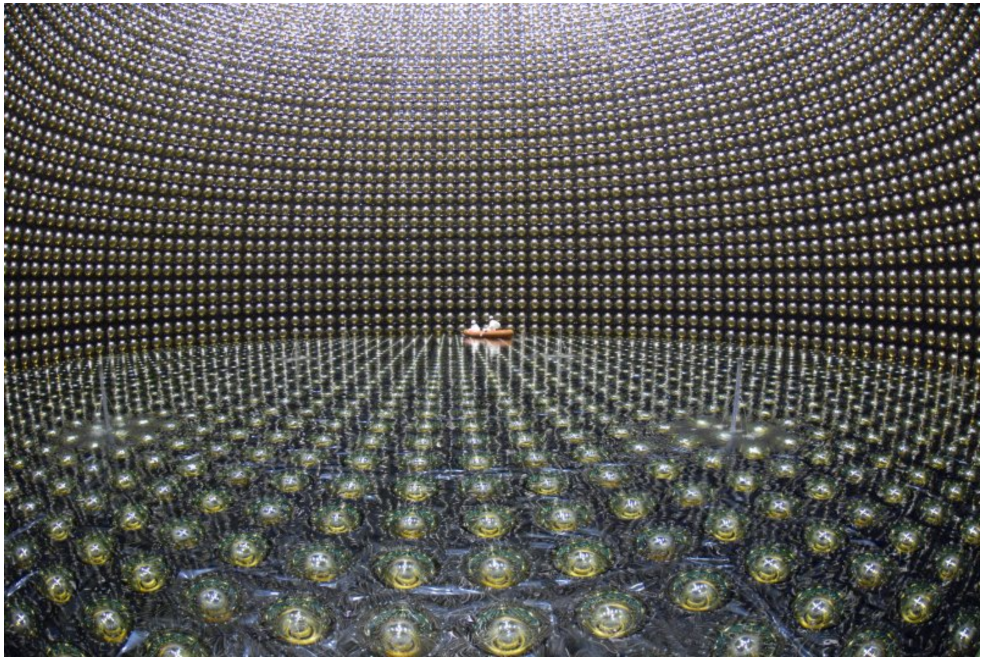
$$^{37}\mathrm{Cl} + \nu_e \to ^{37}\mathrm{Ar} + e^-$$

Did not convince because he saw 0.4 of the expected....

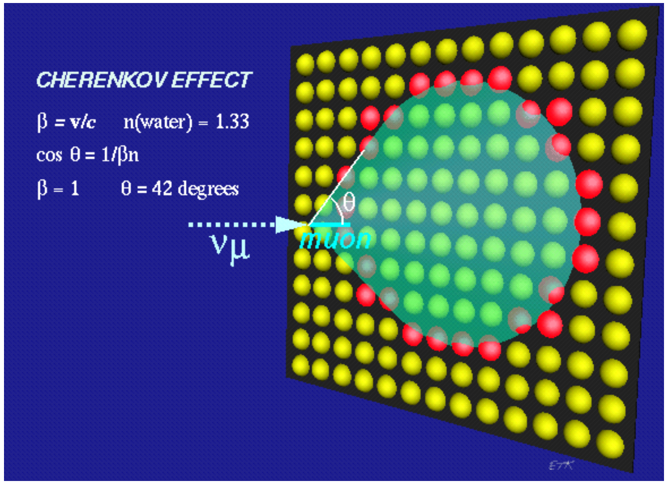
Problem in detector ? In solar model ? In neutrinos ?

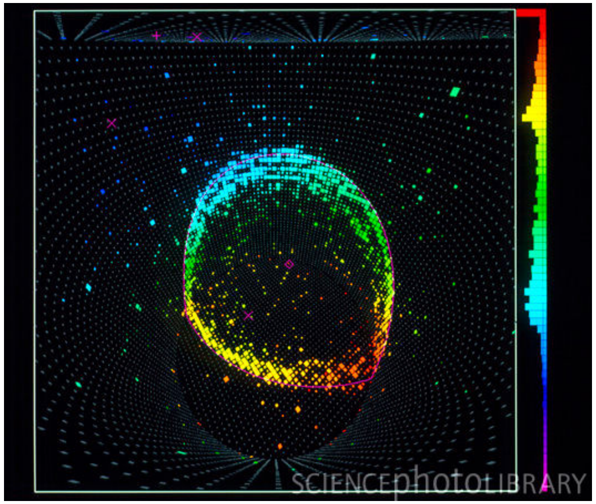
Other radiochemical experiments: Gallium with lower-threshold confirmed

Underground cathedrals of light



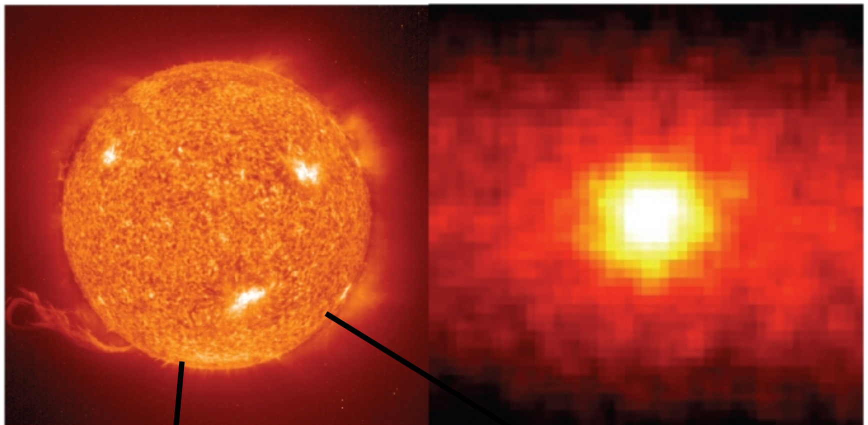
Koshiba (Nobel 2002)





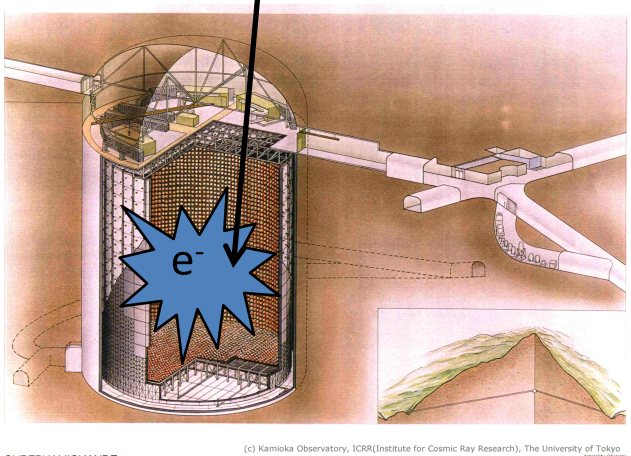
Allows to reconstruct velocity and direction, e/μ particle identification

Solar Neutrinos



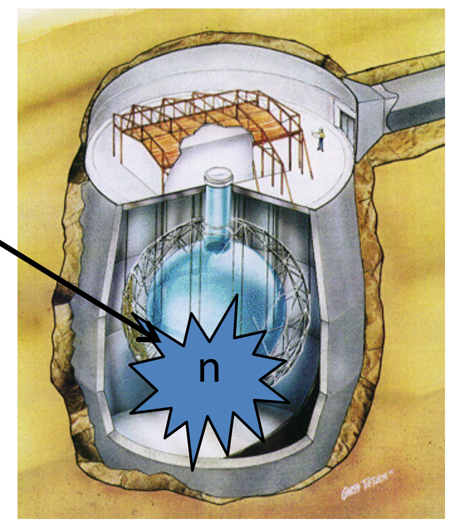
Neutrinography of the sun

SuperKamiokande (22.5 kton)



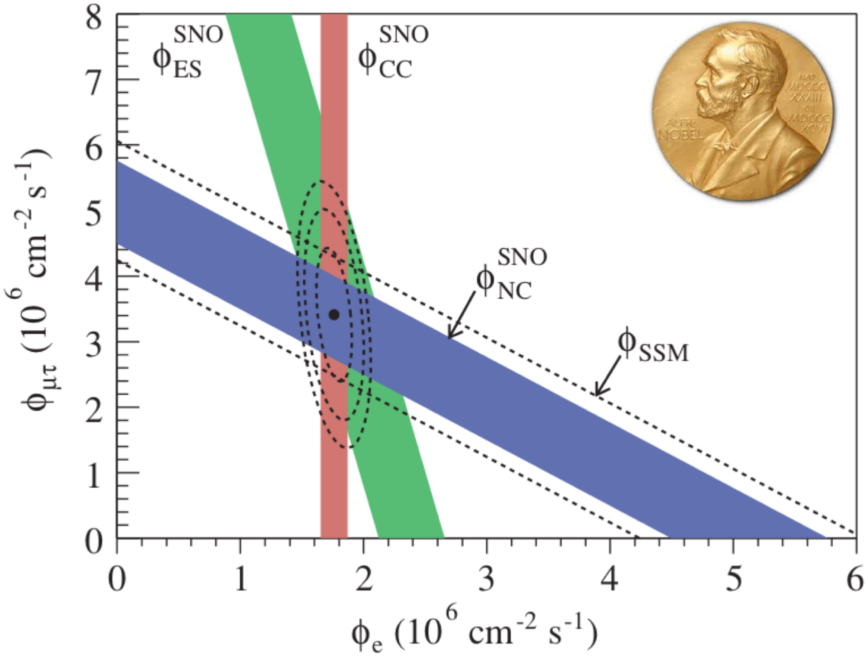
$$\nu_e + e^- \rightarrow \nu_e + e^-$$

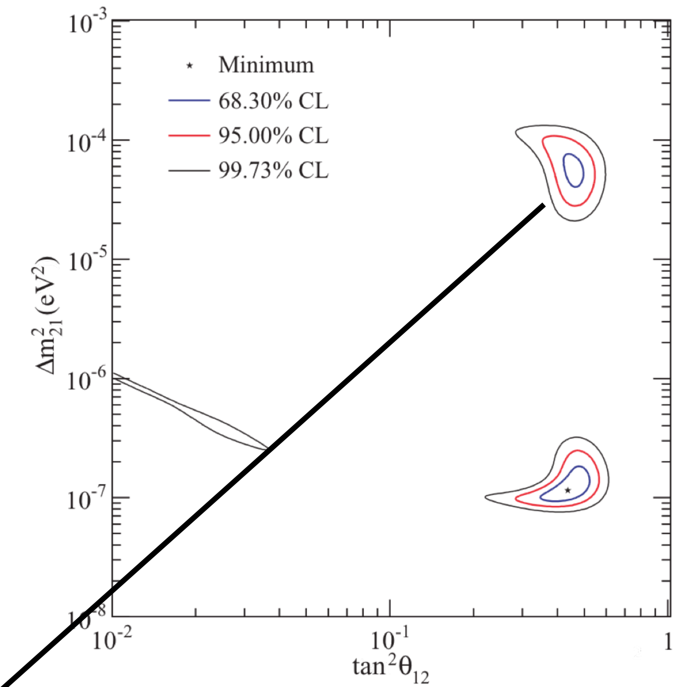
SNO



$$NC: \quad \nu_i + d \rightarrow p + n + \nu_i$$
$$CC: \quad \nu_e + d \rightarrow p + p + e^-$$

Flavour of solar neutrinos





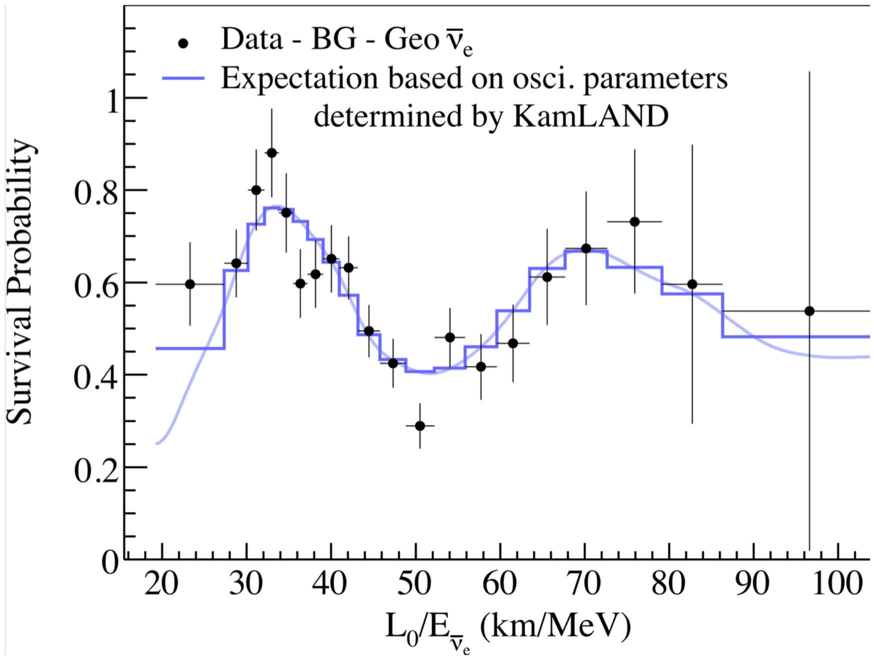
$$|\Delta m^2|^{-1} \sim \frac{O(100Km)}{O(MeV)}$$

Can be tested in the Earth with Reines&Cowen experiment !

KamLAND: solar oscillation

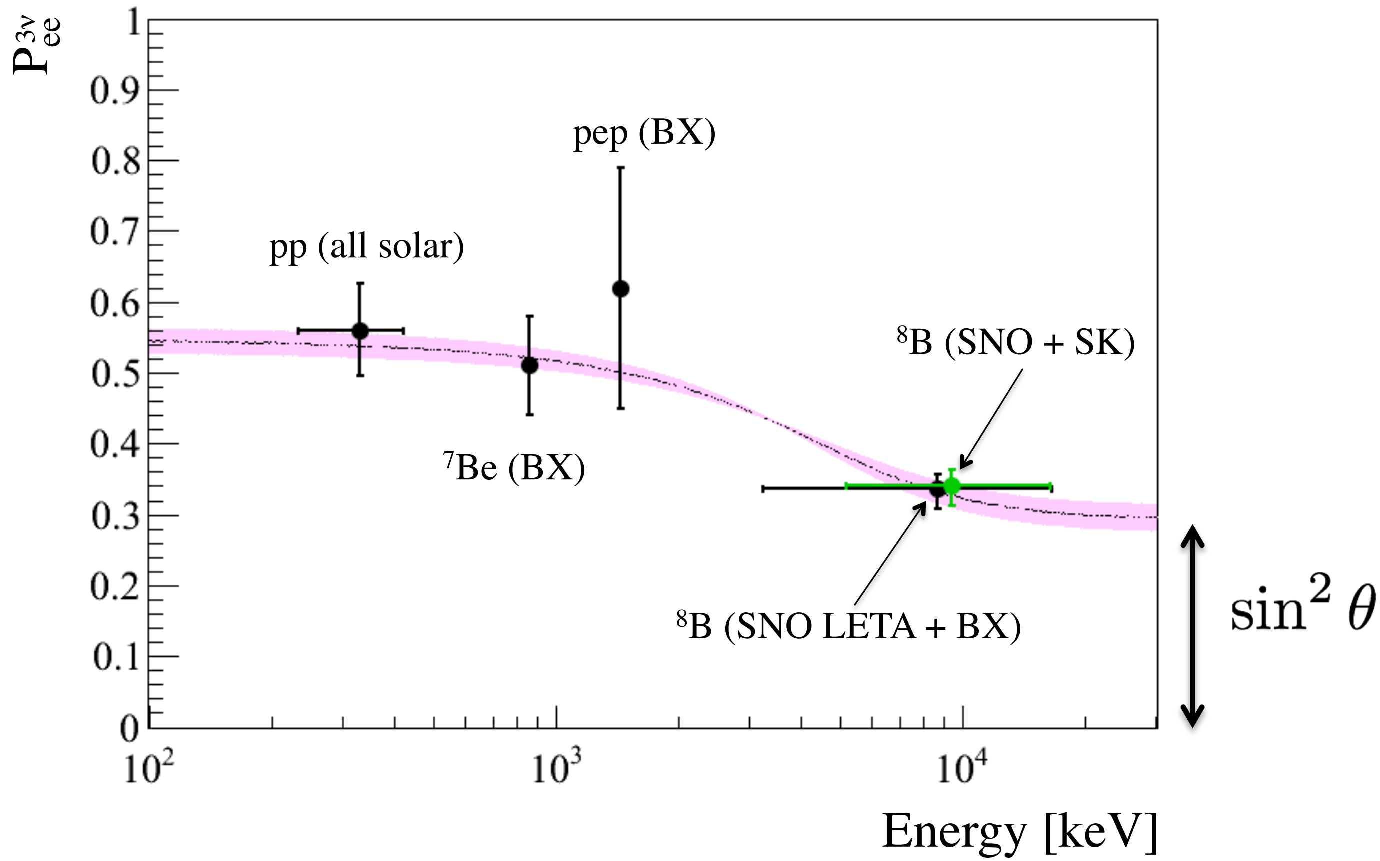
$$\overline{\nu}_e \to \overline{\nu}_e$$

Reines&Cowan experiment ½ century later
at 170 km from Japenese reactors ...



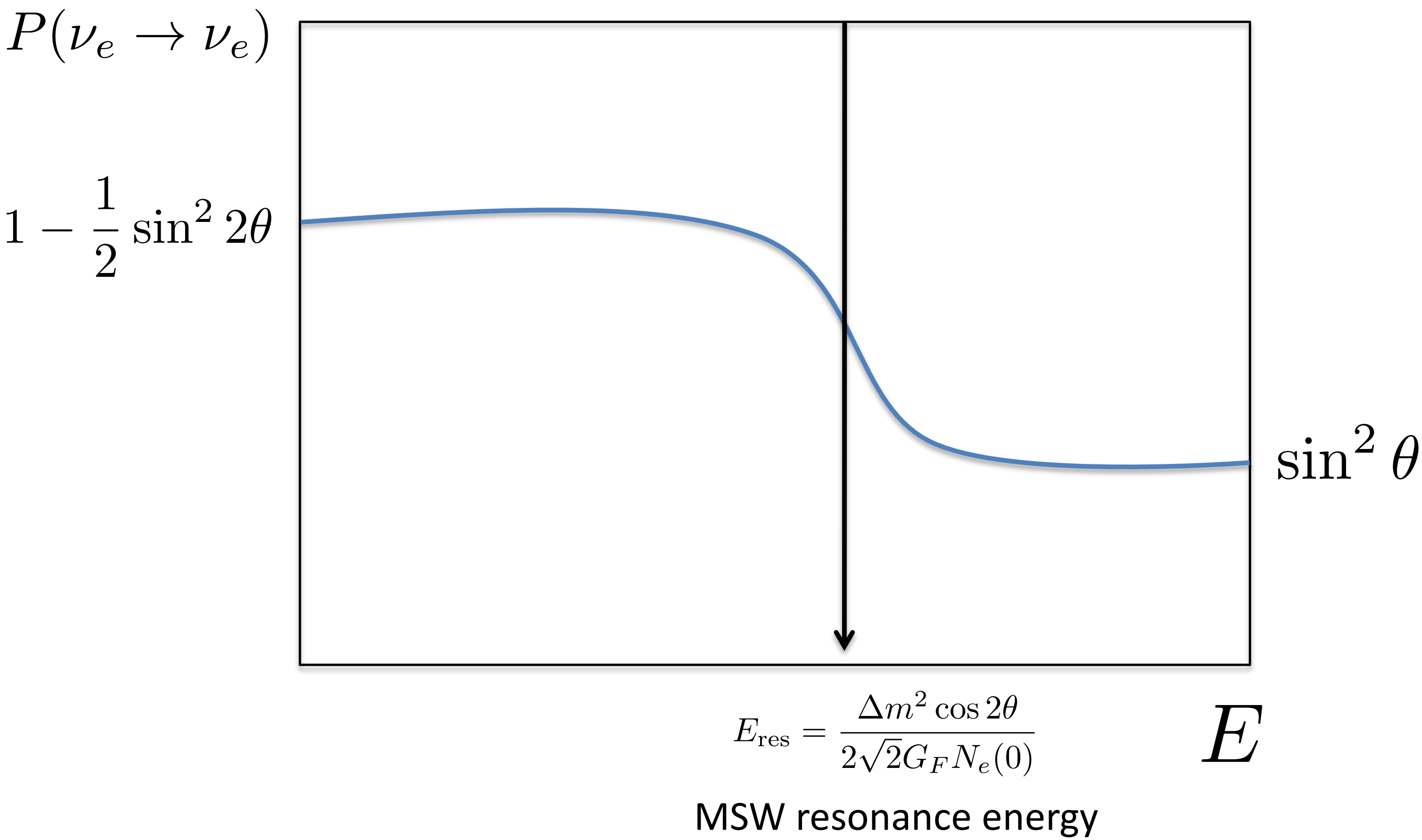
$$\boxed{\Delta m^2_{\text{solar}} \simeq 8 \times 10^{-5}\ eV^2}$$

Solar neutrinos and MSW



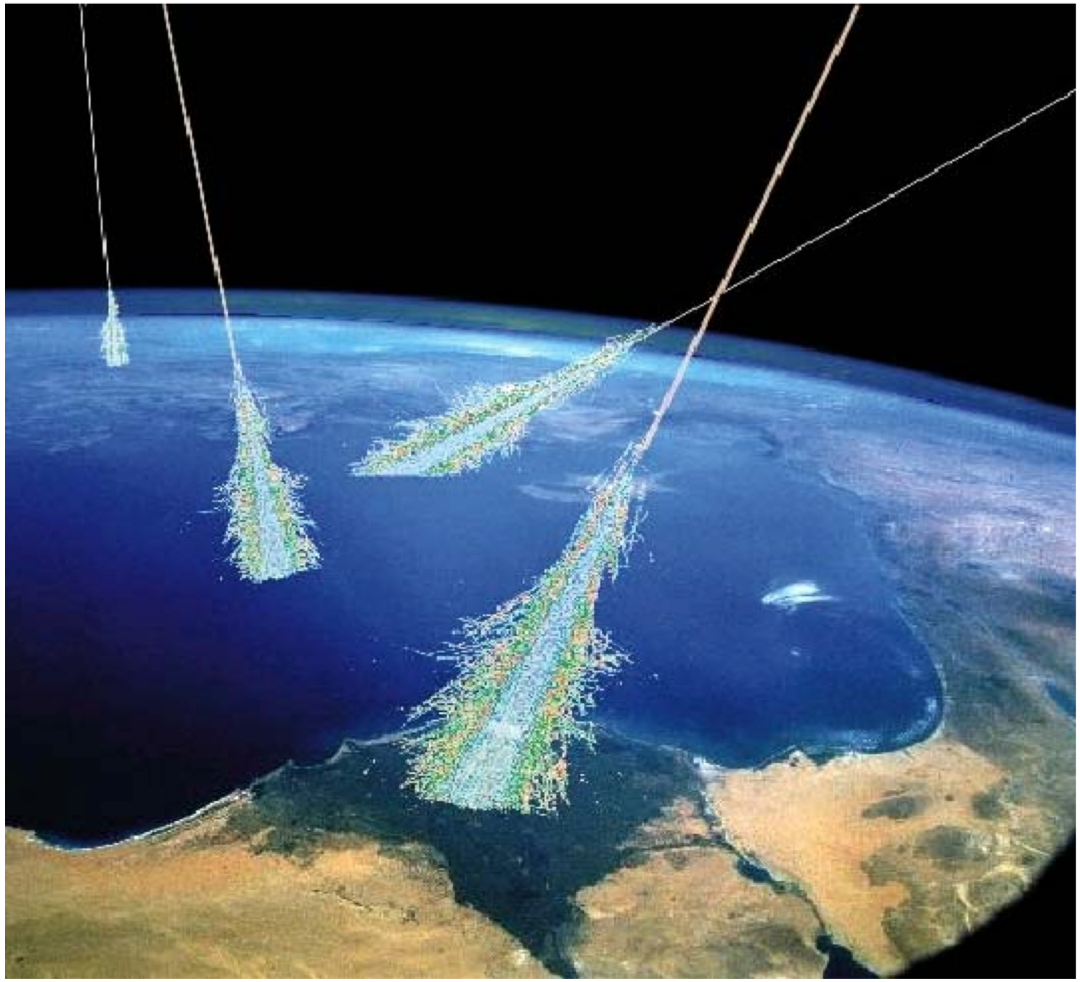
Borexino

Solar neutrinos



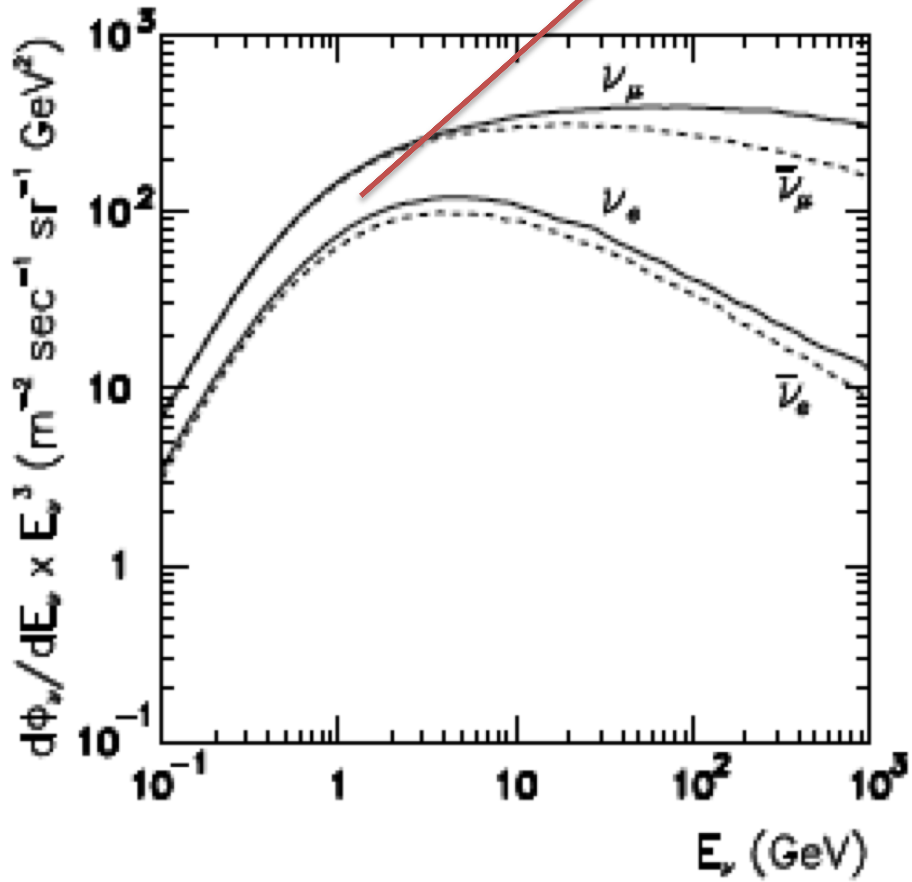
In most physical situations: piece-wise constant matter or adiabatic approx. good enough

Atmospheric Neutrinos



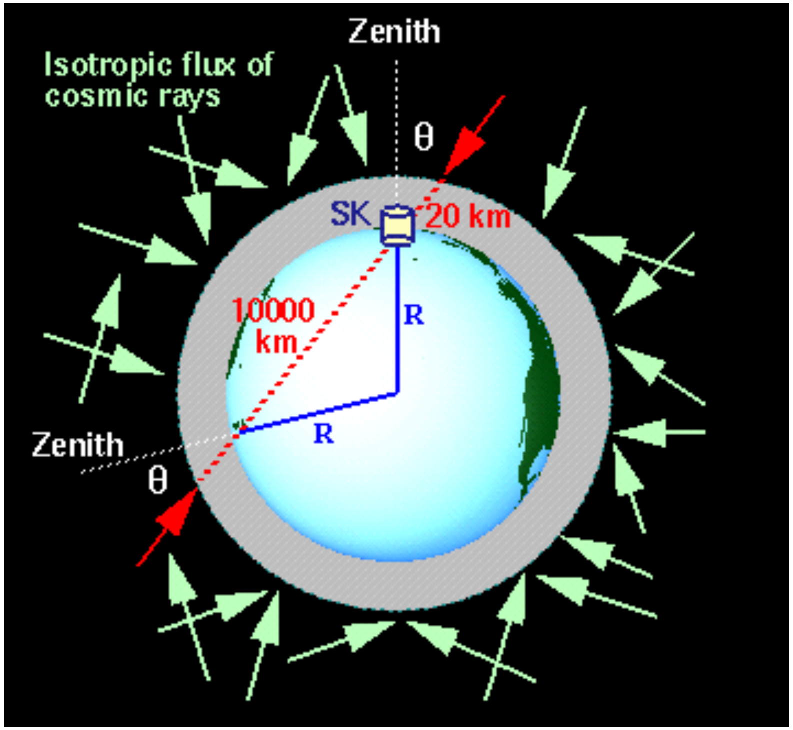
Atmospheric Neutrinos

$\nu_\mu/\nu_e \sim 2$

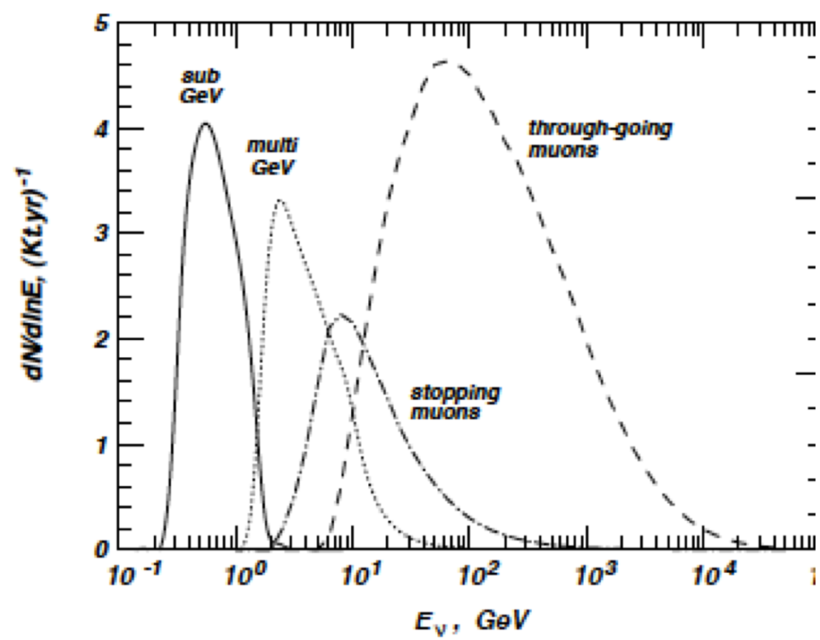


Produced in the atmosphere when primary cosmic rays collide with it, producing π, K

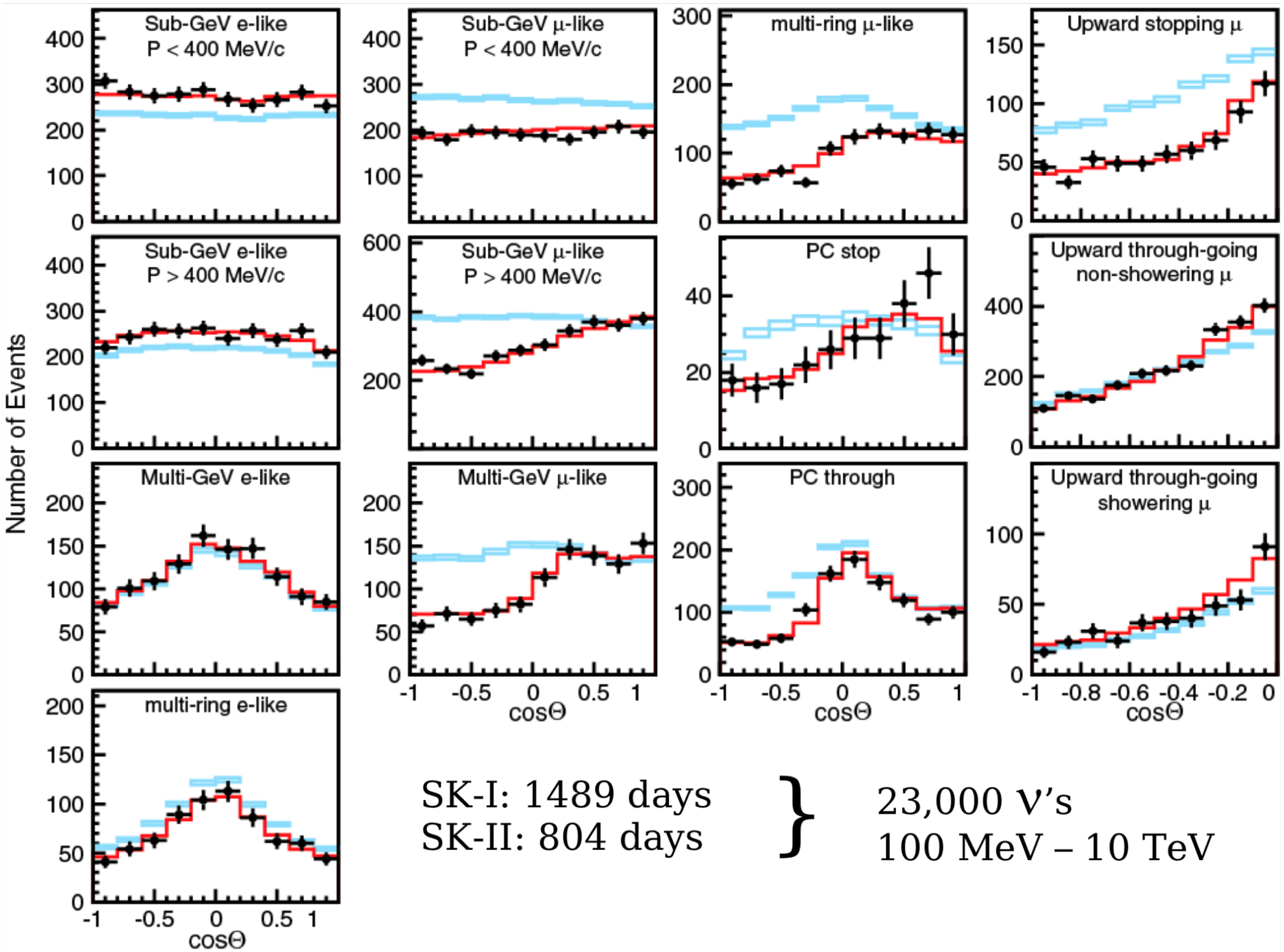
Atmospheric Neutrinos



$$L = 10 - 10^4 \ \mathrm{Km}$$



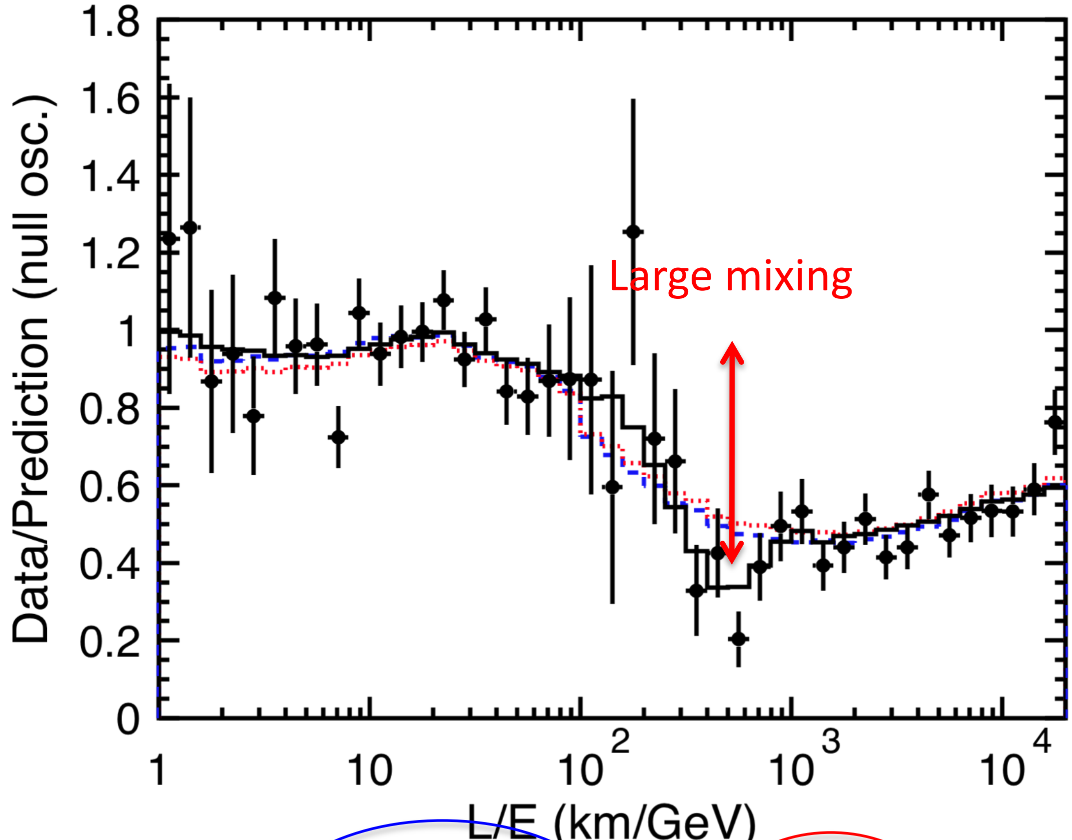
Atmospheric Neutrinos



SK-I: 1489 days
SK-II: 804 days

23,000 ν's
100 MeV – 10 TeV

Atmospheric Oscillation



$$\Delta m^2_{\rm atm} = 2.5 \times 10^{-3} eV^2$$

$$|\Delta m^2|^{-1} \sim \frac{O(1000Km)}{O(GeV)} \sim \frac{O(1km)}{O(MeV)}$$

Reines&Cowan experiment at 1km!

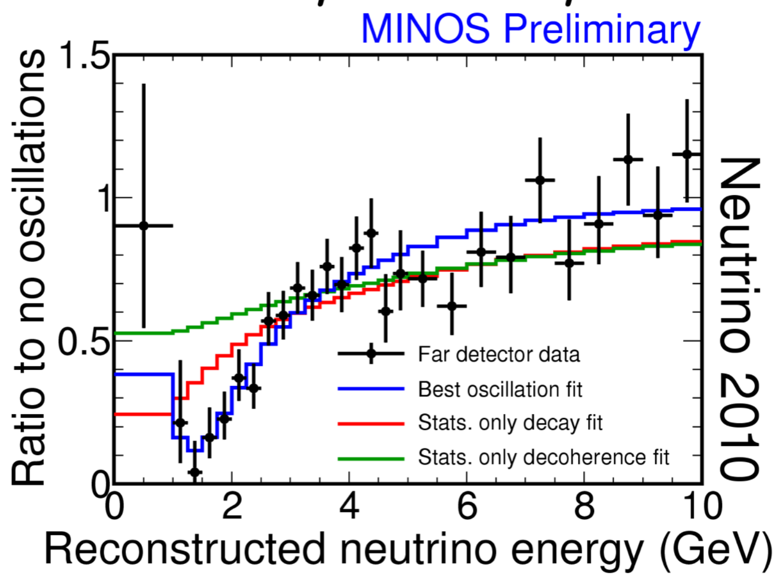
Lederman&co experiment at 1000km!

Lederman&co neutrinos oscillate with the atmospheric wave length

Pulsed neutrino beams to 700 km baselines

MINOS

$$\nu_\mu \to \nu_\mu$$

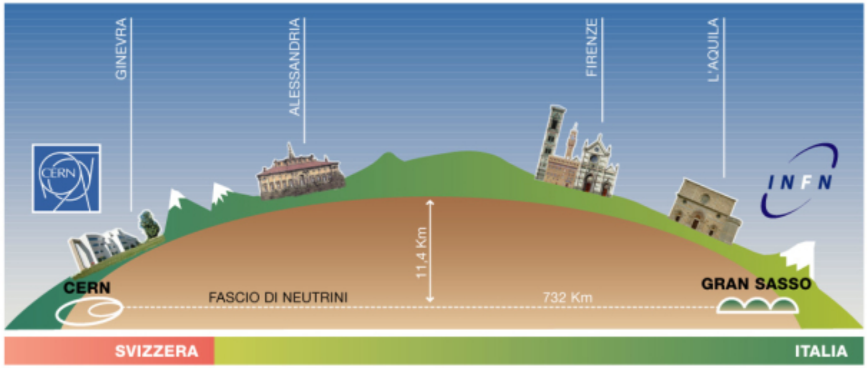


$$|\Delta m^2_{\rm atmos}| \simeq 2.5 \times 10^{-3}\ eV^2$$

$$\sin^2 2\theta_{\rm atmos} \simeq 1$$

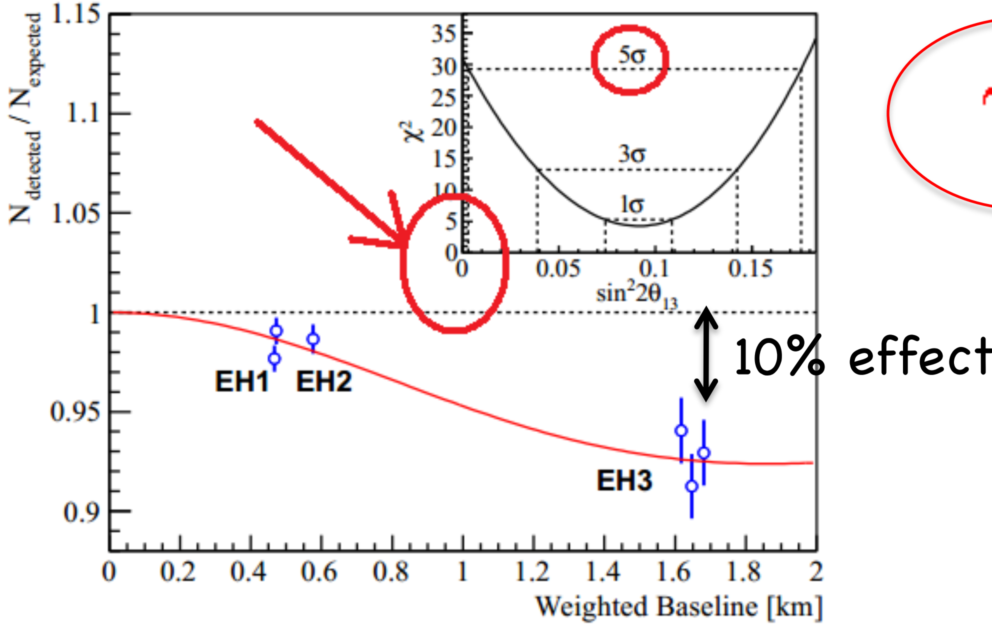
$$\nu_\mu \to \nu_\tau$$

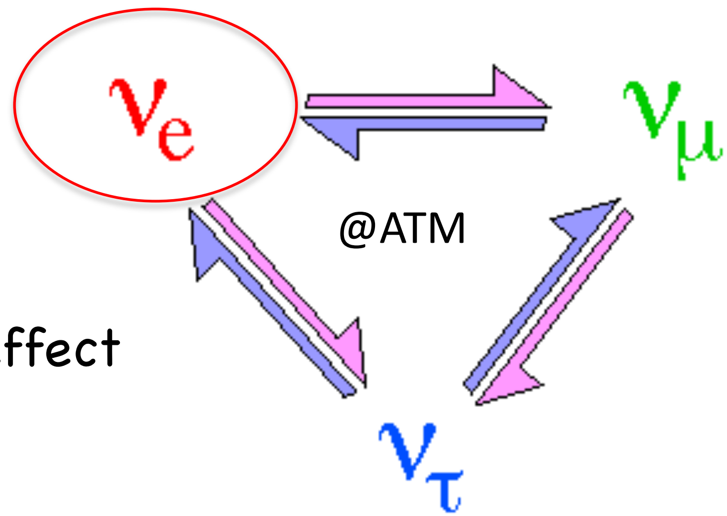
OPERA



Reines&Cowan (reactor) neutrinos oscillate with atmospheric wave length

Double Chooz, Daya Bay, RENO



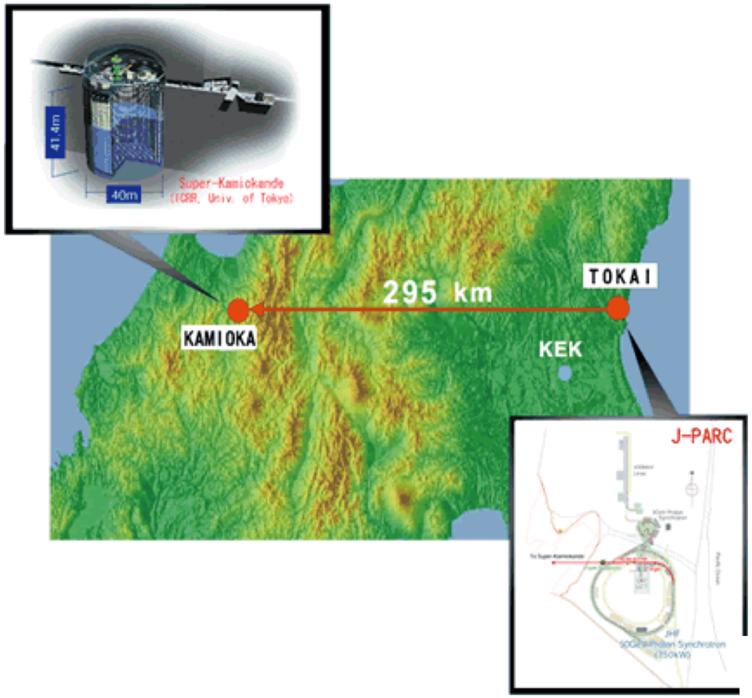


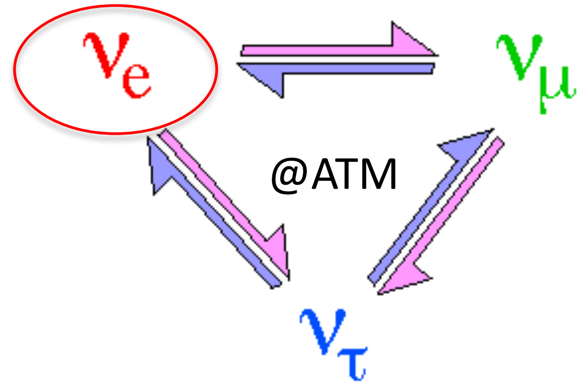
$$\bar{\nu}_e \to \bar{\nu}_e$$

Two different wave lengths

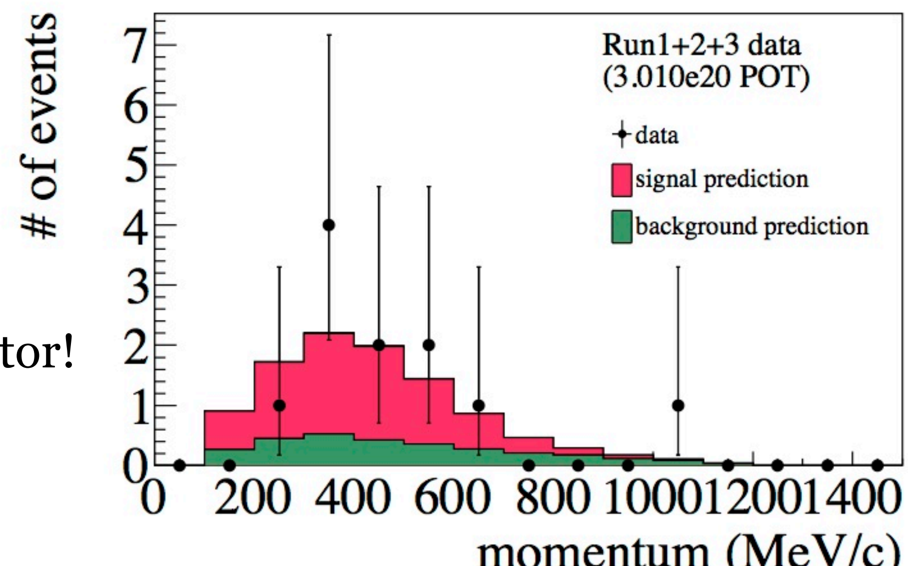
Modern copies of the influential experiment Chooz that barely missed the effect and set a limit

T2K



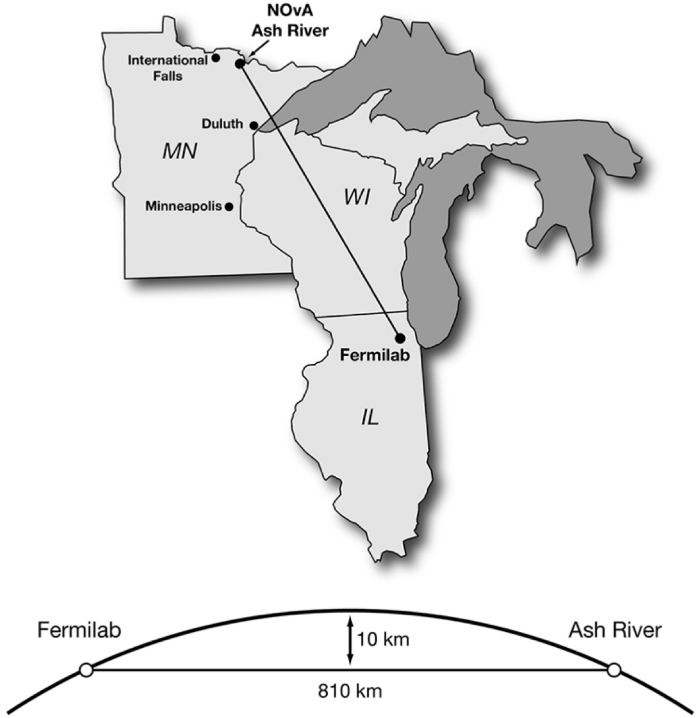


$$\nu_\mu \rightarrow \nu_e$$



Using the SuperKamiokande detector!

NOνA



L=810km

$$\nu_\mu \rightarrow \nu_e$$

3ν scenario

$$\Delta m^2_{23} = m^2_3 - m^2_2 \equiv \Delta m^2_{atm}$$

$$\Delta m^2_{12} = m^2_2 - m^2_1 \equiv \Delta m^2_{sol}$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{23}(\theta_{23})U_{13}(\theta_{13},\delta)U_{12}(\theta_{12}) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Solar and atmospheric osc. decouple as 2x2 mixing phenomena:

- hierarchy $\dfrac{|\Delta m^2_{atm}|}{|\Delta m^2_{sol}|} > 10$
- small θ_{13}

$$E_\nu / L \sim \Delta m^2_{23} \gg \Delta m^2_{12}$$

$$\begin{aligned}
P(\nu_e \to \nu_\mu) &= s^2_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{23}}{4E} L\right) \\
P(\nu_e \to \nu_\tau) &= c^2_{23} \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{23}}{4E} L\right) \\
P(\nu_\mu \to \nu_\tau) &= c^4_{13} \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{23}}{4E} L\right) \\
P(\bar{\nu}_e \to \bar{\nu}_e) &= 1 - \sin^2 2\theta_{13} \sin^2\left(\frac{\Delta m^2_{23}}{4E} L\right)
\end{aligned}$$

$$E_\nu/L \sim \Delta m^2_{23} \gg \Delta m^2_{12} \qquad \theta_{13} \to 0$$

$$\begin{aligned}
P(\nu_e \to \nu_\mu) &= 0 \\
P(\nu_e \to \nu_\tau) &= 0 \\
P(\nu_\mu \to \nu_\tau) &= \sin^2 2\theta_{23} \sin^2\left(\frac{\Delta m^2_{23}}{4E} L\right) \\
P(\bar{\nu}_e \to \bar{\nu}_e) &= 1
\end{aligned}$$

Experiments in the atmospheric range are described approximately by 2x2 mixing with

$$(\Delta m^2_{23}, \theta_{23}) = (\Delta m^2_{atm}, \theta_{atm})$$

$$E_\nu/L \sim \Delta m^2_{12} \ll \Delta m^2_{23}$$

$$P(\nu_e \to \nu_e) = P(\bar{\nu}_e \to \bar{\nu}_e) \simeq c^4_{13} \left(1 - \sin^2 2\theta_{12} \, \sin^2 \left(\frac{\Delta m^2_{12}}{4E} \, L \right) \right) + s^4_{13}$$

$$E_\nu/L \sim \Delta m^2_{12} \ll \Delta m^2_{23} \qquad \theta_{13} \to 0$$

$$P(\nu_e \to \nu_e) = P(\bar{\nu}_e \to \bar{\nu}_e) \simeq \qquad 1 - \sin^2 2\theta_{12} \sin^2\left(\frac{\Delta m^2_{12}}{4E} L\right)$$

Experiments in the solar range are described approximately by 2x2 mixing with

$$(\Delta m^2_{12}, \theta_{12}) = (\Delta m^2_{\rm sol}, \theta_{\rm sol})$$

The measurement of $\theta_{13} \sim 9^\circ$ implies that corrections to these approximations are sizeable and need to be included in all analyses

Standard 3ν scenario

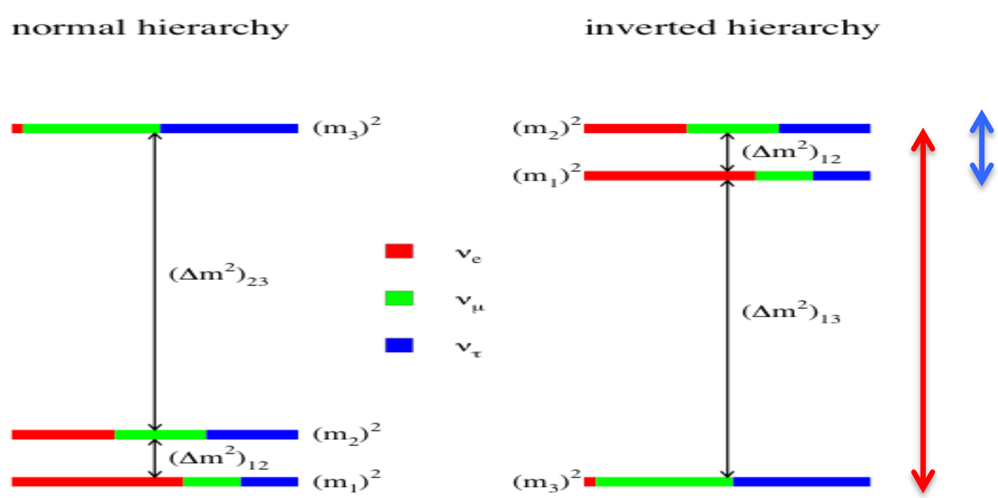
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS}(\theta_{12}, \theta_{23}, \theta_{13}, \delta, \ldots) \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$\theta_{12} \sim 34^\circ$

$\theta_{23} \sim 42^\circ$ o 48°

$\theta_{13} \sim 8.5^\circ$

$\delta \sim \ ?$



$7.5 \cdot 10^{-5}\mathrm{eV}^2$

$2.5 \cdot 10^{-3}\mathrm{eV}^2$