Magnetic Vector Potential-Based Formulations for Electromagnetic Modeling of Superconductors: an Alternative to the $H$ Formulation

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$H$ formulation: the de facto standard for simulating electromagnetic behavior of HTS

At least 45 research groups have used it since 2006!
Complex HTS applications have been successfully simulated with the $H$ formulation.

Roebel cables

Large magnets

Zermeno et al. 2013 SuST 26 052001

Xia et. al 2019 SuST 32 095005
Why look for alternatives?

• Efficiently simulate HTS tapes in electrical machines
  → $T$-$A$ formulation

• Use different constitutive law from power-law
  → Quasi critical state model
T-A formulation
So far used to simulate thin superconductors

- Model proposed by Zhang et al. 2017 SuST 30 024005

- Magnetic vector potential $A \rightarrow$ magnetic field (everywhere)

- Current vector potential $T \rightarrow J$ in infinitely thin superconductors

- Very easy to apply the desired current
Here extended to **thick** superconductors

Bi-2223 or MgB$_2$ tapes

Stacks of coated conductors

Sumitomo, Ansaldo Superconductors

Uglietti et al. 2014 IEEE TAS 24 4800704
How do we impose the desired current?

\[ I = \iint_S \mathbf{J} \, dS = \iint_S \nabla \times \mathbf{T} \, dS = \oint_L \mathbf{T} \, dl \]

\[ T_x = 0 \]

\[ T_y = 0 \quad T_y = 0 \]

\[ T_x = I/a \quad b \]

\[ T_y = I/b \]

\[ T_x = 0 \]
Application to HTS coils

Insulated turns = 1D approach

Coupled turns = 2D approach
Application to HTS machines

Magnetized stack of tapes

Stator coil/ magnetizing coil

Rotor

A. Patel et al. 2015 IEEE TAS 25 (3) 5203405
We can now simulate superconducting turns in HTS machines!

Superconducting motor SUTOR: Oswald et al. 2012 Physics Procedia 36 765
Everything done in the same model

Superconducting motor SUTOR: Oswald et al. 2012 Physics Procedia 36 765
The coupling scenario strongly influences the AC losses of the stator’s coils.
Quasi critical state model
Parametric solution of magnetostatic problem with time as parameter

\[ \nabla \times (\nabla \times \mathbf{A}) = \mu_0 \mathbf{J} \]

\[ J = J_c \operatorname{erf} \left( \frac{E}{E_c} \right) \]

\[ E = -\frac{\partial A}{\partial t} - \nabla \phi \]

\[ \frac{\partial A}{\partial t} \approx \frac{A_{t+\Delta t} - A_t}{\Delta t} \]

Error function \( \operatorname{erf}(x) \)

Campbell 2007 SuST 20 292
Gömöry & Inanir 2012 IEEE TAS 22 4704704
Gömöry & Shen 2017 SuST 30 064005
A “quasi” critical state model

• AC magnetization of an elliptical conductor
• Magnetization loops change with frequency
• Due to smooth $E-J$ relationship
Can a single static calculation predict the cyclic AC losses of an HTS tape?

Yes, if we know the field distribution at the peak* $Q = 4J_c \int_{\Omega} |A_p| d\Omega$

*Claassen 2006 APL 88 122512
*Campbell 2007 SuST 20 292
*Pardo et al. 2007 SuST 20 351
The static model satisfactorily predicts the total losses of an HTS coated conductor

- In-phase AC current and AC field
- Various field orientations
- Perpendicular field component dominates

About one second per simulation!

Ogawa et al. 2005 Cryogenics 45 23
Conclusions

T-A formulation
• Efficient tool for modeling HTS in electrical machines
• Different coupling scenarios considered

Quasi critical state model
• Departure from CSM behavior at low frequency
• Promising for quick one-step calculation of AC losses (single tapes)
Thank you very much for your attention!