Overall critical current of CC tapes and devices when local critical currents fluctuate along the tape length

F. Gömöry, M. Adámek, J. Šouc
Institute of Electrical Engineering, Slovak Academy of Sciences, Bratislava, Slovakia

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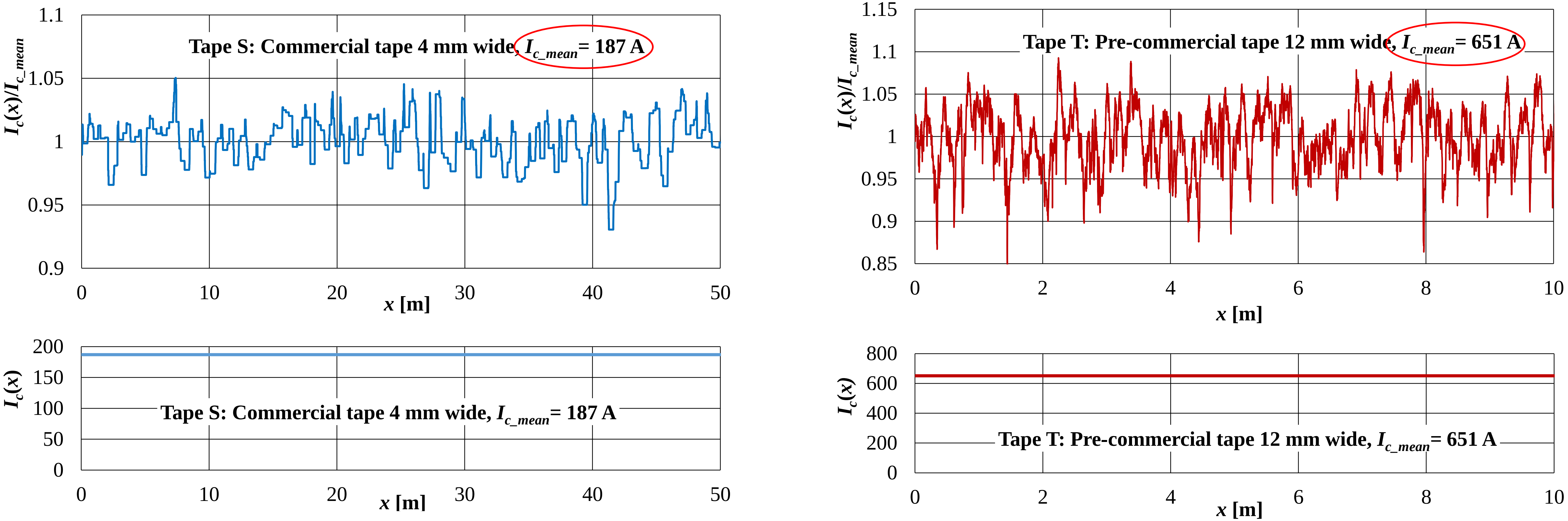
Introduction

fluctuation of critical current along the tape length is a common feature of CC tapes



what is the value of „critical current“ that should be used in the design of a device?

Problems discussed

1) Is there a long-length equivalent of the short sample's critical current ?

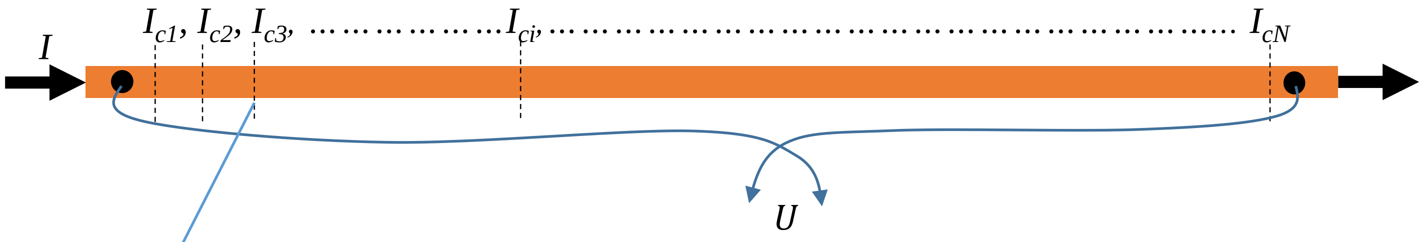
2) Could this long-length critical current be predicted from the parameters of statistical distribution of local I_c 's ?

3) Knowing the I_c fluctuation property of a single tape, what are the consequences to operation of cables and coils ?

4) Conclusions (answers)

Overall critical current

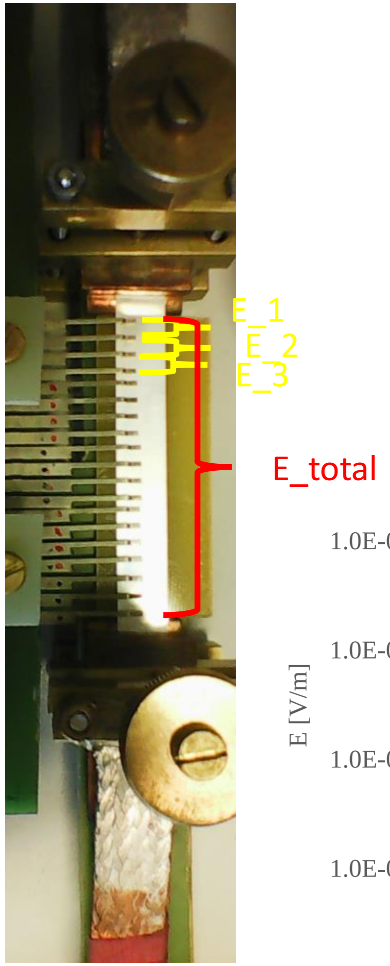
the value of current at which the same electrical field is registered on the whole length as in the short sample testing

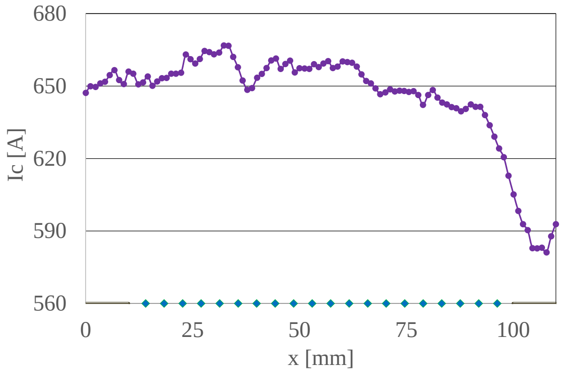


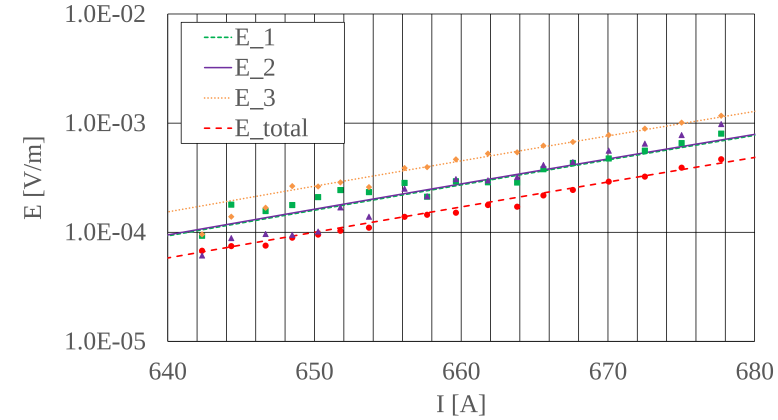
$$E_i = E_c \left(\frac{I}{I_{ci}} \right)^n$$

in case of non-fluctuating *n*

$$I_{c,ovrl} = \left[\frac{N}{\sum_{i=1}^{N} \left(\frac{1}{I_{c,i}} \right)^n} \right]^{\frac{1}{n}}$$







Fee M, Fleshler S, Otto A, Malozemoff A P 2001 *IEEE. Trans. Appl. Supercond* **11** 3337-340
Wang Y, Xiao L, Lin L, Xu X, Lu Y, Teng Y 2003 *Cryogenics* **43** 71-77

Statistical description of $I_c(x)$ data

mean value

$$I_{c,mean} = \frac{\sum_{i=1}^{N} I_{c,i}}{N}$$

variance

$$var_{Ic} = \frac{\sum_{i=1}^{N} \left(I_{c,mean} - I_{c,i}\right)^2}{N}$$

coefficient of variation

$$c_{var} = \frac{\sqrt{var_{Ic}}}{I_{c,mean}}$$

overall critical current is always lower than the mean

deterioration because of I_c fluctuations

$$\delta_{fluct} = \frac{I_{c,mean} - I_{c,ovrl}}{I_{c,mean}}$$

Statistical description of $I_c(x)$ data

Gauss

Probability density:

$$f_G(I_c) = \frac{1}{\sqrt{2\pi var_{Ic}}} e^{-\frac{(I_c - I_{c,mean})^2}{2var_{Ic}}}$$

mean

standard deviation: $\sigma_{Ic} = \sqrt{var_{Ic}}$

Cumulative probability:

$$F_G(I_c) = \frac{1}{2}\left[1 + erf\left(\frac{(I_c - I_{c,mean})}{\sigma_{I_c}\sqrt{2}}\right)\right]$$

Weibull

$$f_W(I_c) = e^{-\left(\frac{I_c}{I_{c,scale}}\right)^{s_{Ic}}} \frac{s_{Ic}}{I_{c,scale}} \left(\frac{I_c}{I_{c,scale}}\right)^{s_{Ic}-1}$$

scale $I_{c,mean} = I_{c,scale}\Gamma\left(1 + \frac{1}{s_{Ic}}\right)$

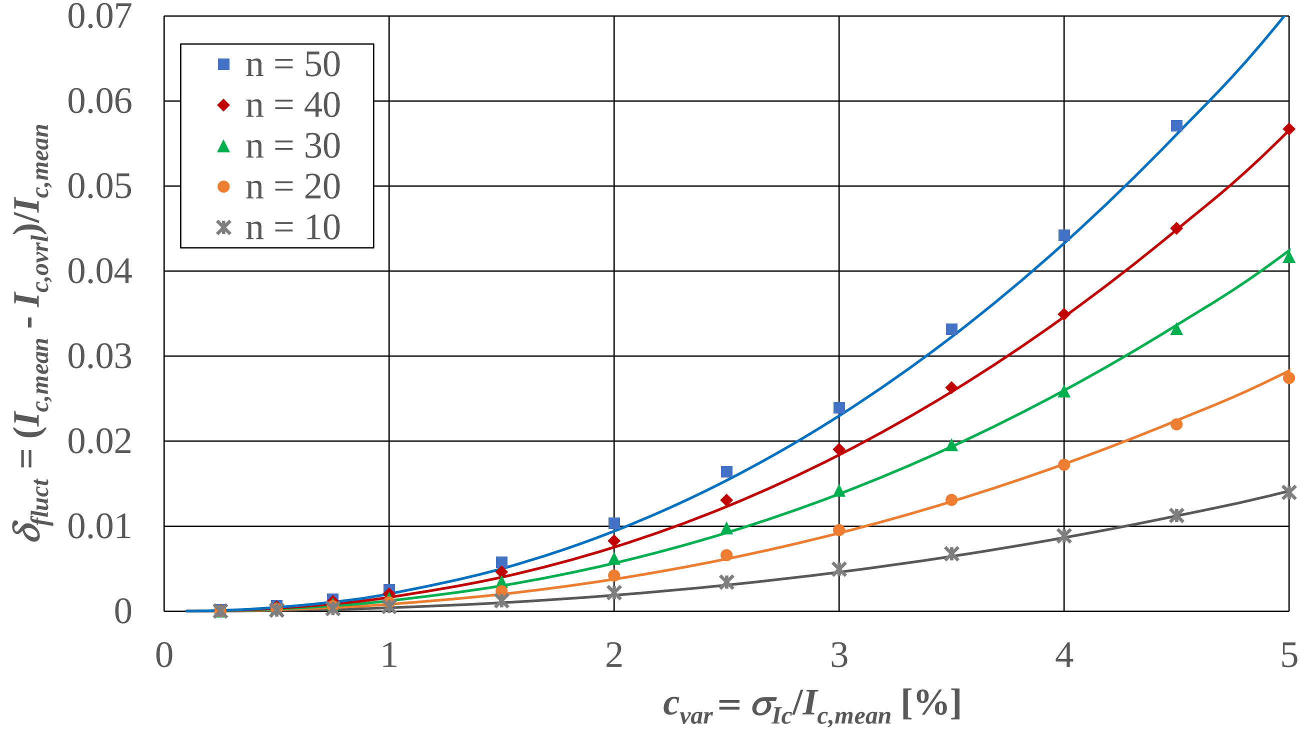
shape: $c_{var}^2 = \left[\Gamma\left(1 + \frac{2}{s_{Ic}}\right) - \left(\Gamma\left(1 + \frac{1}{s_{Ic}}\right)\right)^2\right]$

$$F_W(I_c) = 1 - e^{-\left(\frac{I_c}{I_{c,scale}}\right)^{s_{Ic}}}$$

Relation between the overall critical current and the mean value

$$\delta_{fluct} = \frac{I_{c,mean} - I_{c,ovrl}}{I_{c,mean}}$$

computational exercise – many sets of **artificially generated data with Gaussian distribution**



points – exact formula

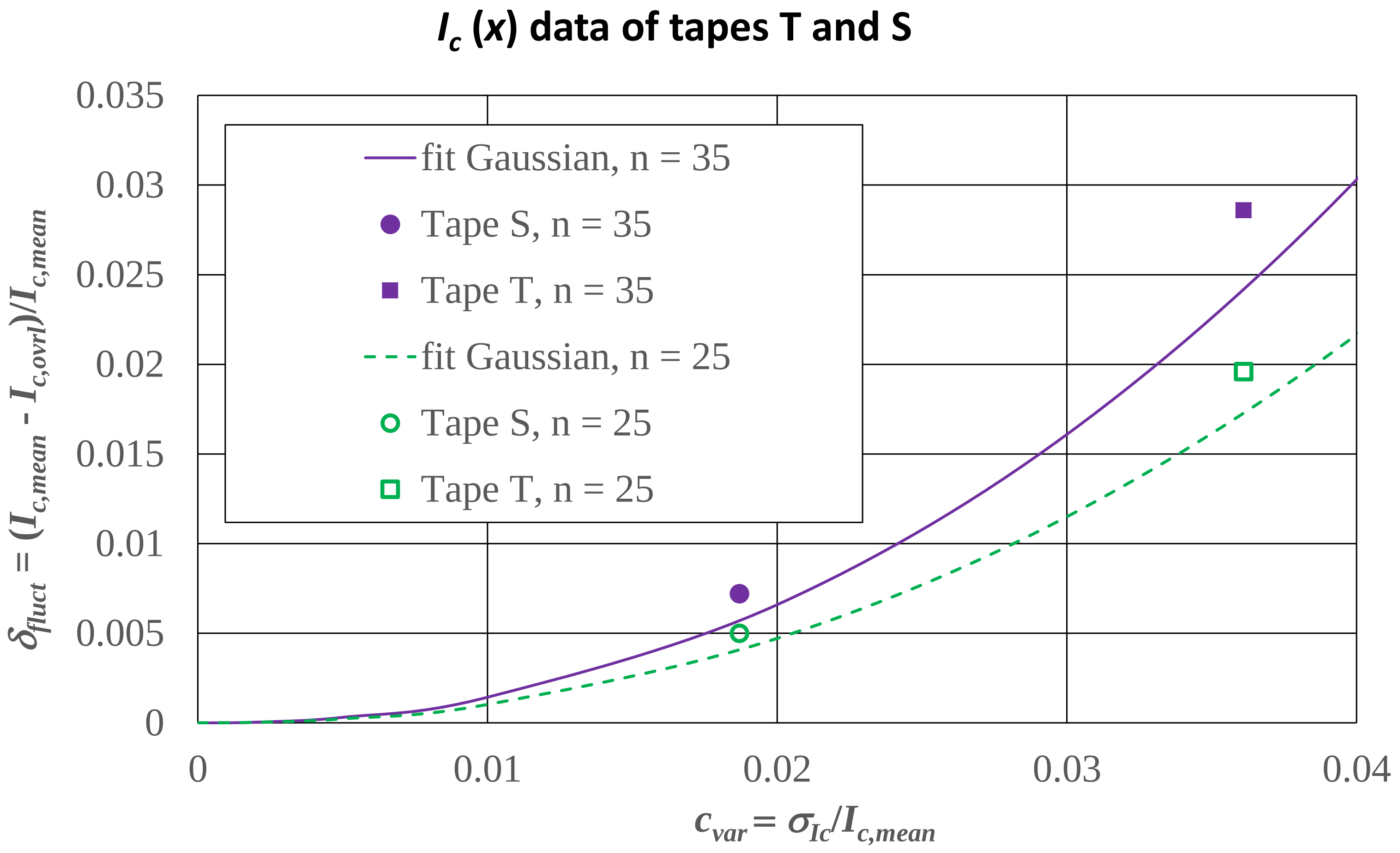
$$I_{c,ovrl} = \left[\frac{N}{\sum_{i=1}^{N}\left(\frac{1}{I_{c,i}}\right)^{n}}\right]^{\frac{1}{n}}$$

bigger impact of fluctuations at higher *n*

all the results can be fitted by general formula: $\delta_{fluct,G} = 1.03 n c_{var}^{2.2}$

Relation between the overall critical current and the mean value

$$\delta_{fluct} = \frac{I_{c,mean} - I_{c,ovrl}}{I_{c,mean}}$$



points – exact formula

$$I_{c,ovrl} = \left[\frac{N}{\sum_{i=1}^{N}\left(\frac{1}{I_{c,i}}\right)^{n}}\right]^{\frac{1}{n}}$$

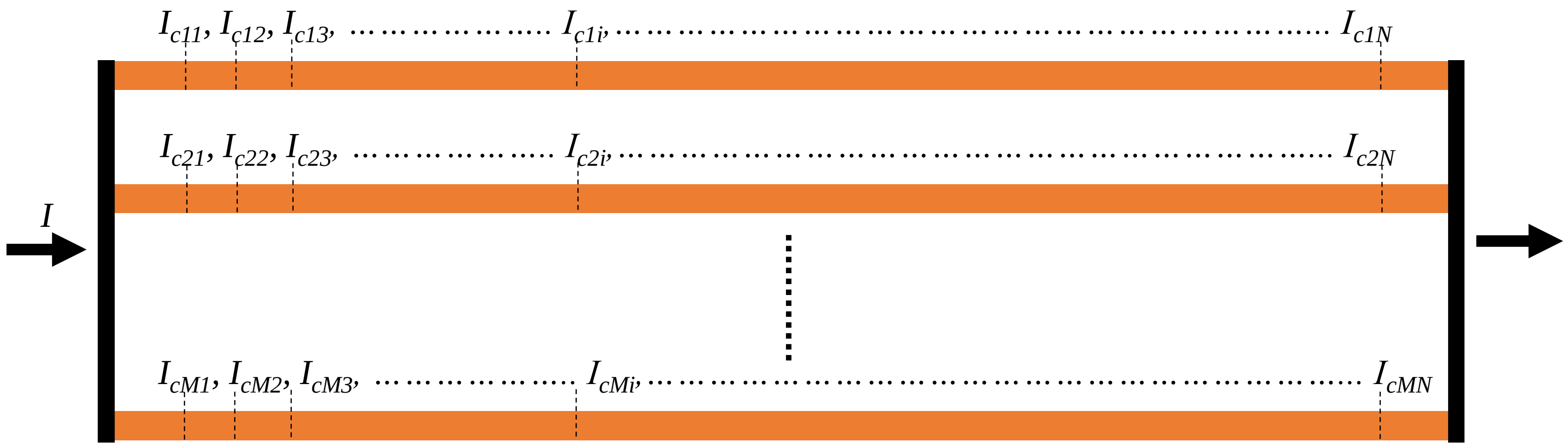
fit Gaussian:

$$\delta_{fluct,G} = 1.03nc_{var}^{2.2}$$

reduction of overall critical current slightly underestimated

Estimation of the overall critical current for cables

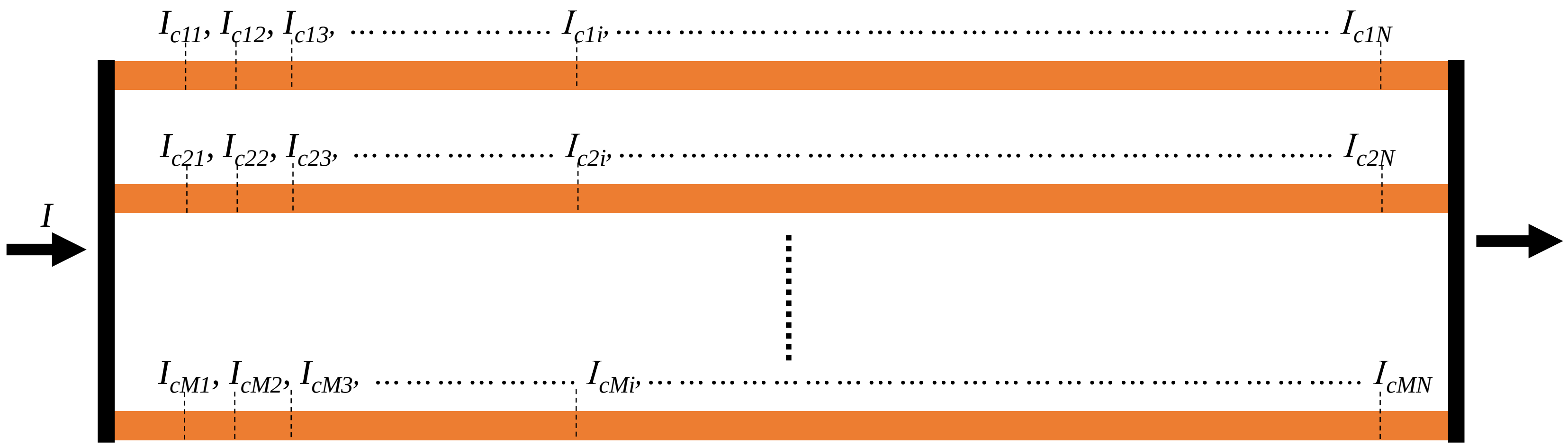
$$\delta_{fluct} = \frac{I_{c,mean} - I_{c,ovrl}}{I_{c,mean}}$$



Assumptions:
- zero resistance at the terminations
- magnetic field produced by transported current is equivalent for all the tapes

Estimation of the overall critical current for cables

$$\delta_{fluct} = \frac{I_{c,mean} - I_{c,ovrl}}{I_{c,mean}}$$



a) no electrical contact between tapes

for each of the tapes:

$$I_{c,ovrl,j} = \left[\frac{N}{\sum_{i=1}^{N}\left(\frac{1}{I_{cij}}\right)^{n}}\right]^{\frac{1}{n}}$$

cable current is the sum of tape currents:

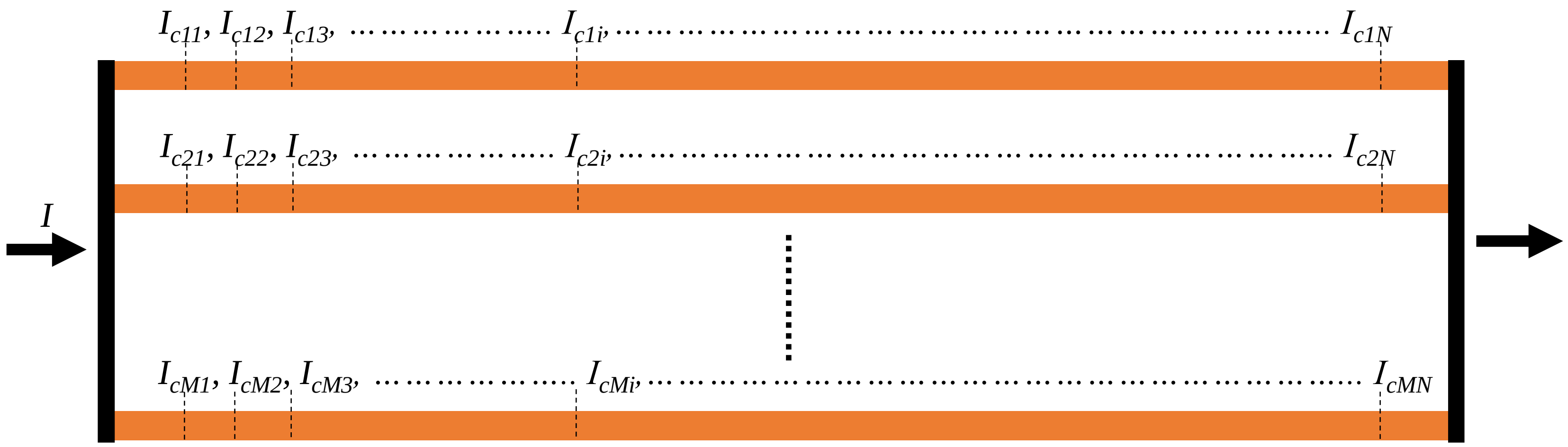
$$I_{c,ovrl,ns} = \sum_{j=1}^{M} I_{c,ovrl,j}$$

overall critical current of the cable is reduced the same way as the tape':

$$I_{c,ovrl,ns} = M\left(1 - \delta_{fluct}\right)I_{c,mean}$$

Estimation of the overall critical current for cables

$$\delta_{fluct} = \frac{I_{c,mean} - I_{c,ovrl}}{I_{c,mean}}$$



a) no electrical contact between tapes

factor reflecting the I_c suppression due to self-field

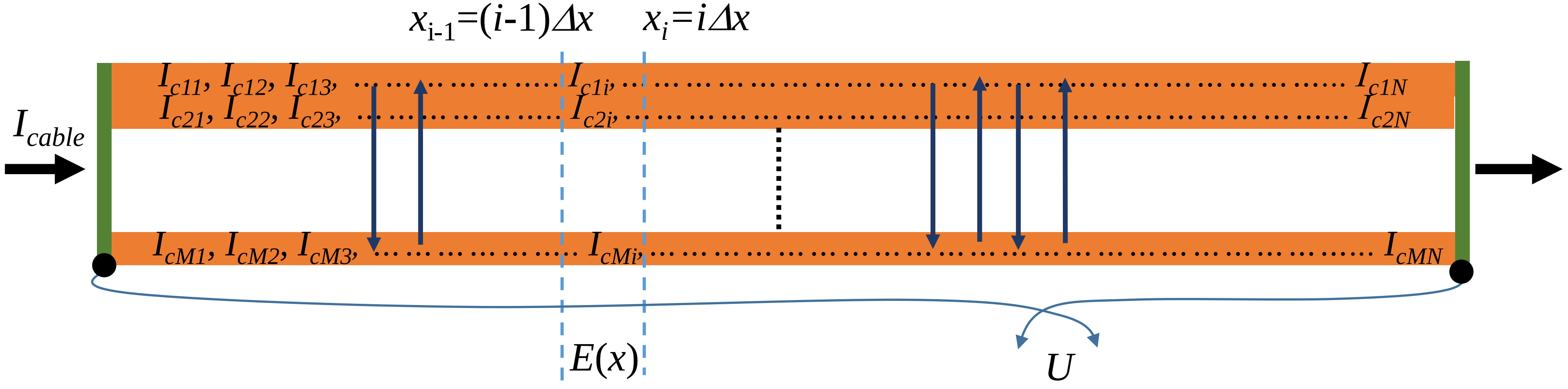
considering a non-equivalent self-field: $$I_{c,\mathrm{cable}} = \sum_{j=1}^{M} \alpha_j I_{c,tape}$$

$$I_{c,ovrl,ns} = \sum_{j=1}^{M} \alpha_j (1 - \delta_{fluct}) I_{c,mean} = (1 - \delta_{fluct}) \sum_{j=1}^{M} \alpha_j I_{c,mean} = (1 - \delta_{fluct}) I_{c,cable}$$

self-field has no influence on the degradation due to I_c fluctuations

Estimation of the overall critical current for cables

$$\delta_{fluct} = \frac{I_{c,mean} - I_{c,ovrl}}{I_{c,mean}}$$



b) perfect electrical contact between tapes

$$E(i\Delta x) = E_c \left(\frac{I_{cable}}{\bar{I}_{c,i}}\right)^n \qquad \bar{I}_{c,i} = \sum_{j=1}^{M} I_{cij}$$

for $M >> 1$:

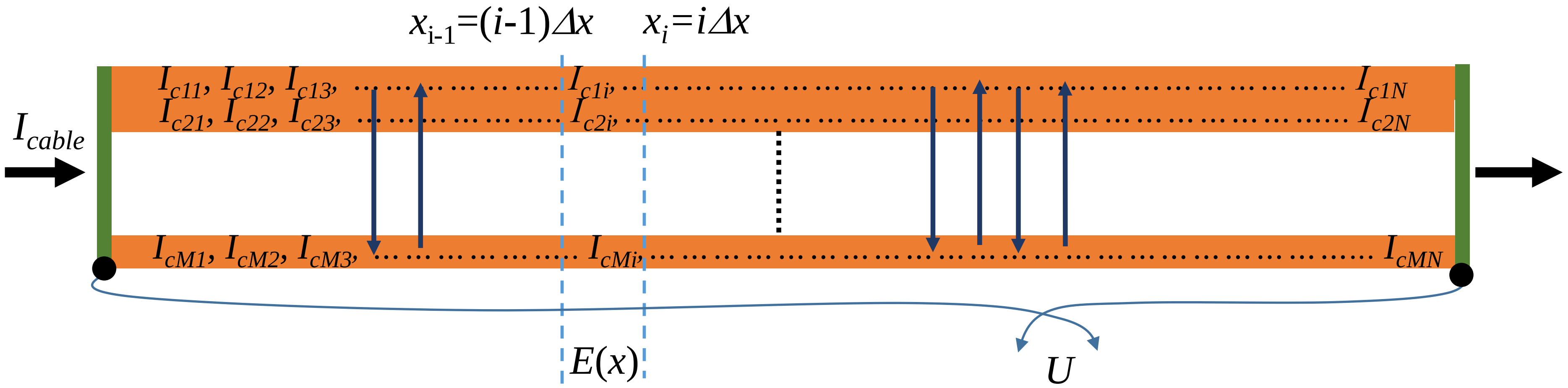
$$\bar{I}_{c,i} \approx M I_{c,mean}$$

overall critical current of the cable is not reduced because of I_c fluctuations

$$I_{c,ovrl,sh} = M I_{c,mean}$$

Estimation of the overall critical current for cables

$$\delta_{fluct} = \frac{I_{c,mean} - I_{c,ovrl}}{I_{c,mean}}$$



b) perfect electrical contact between tapes

considering a non-equivalent self-field:

$$\bar{I}_{c,i} = \sum_{j=1}^{M} \alpha_j I_{cij,self} \qquad \bar{I}_{c,i} \approx \sum_{j=1}^{M} \alpha_j I_{c,mean} \qquad I_{c,ovrl,sh} = I_{c,mean} \sum_{j=1}^{M} \alpha_j$$

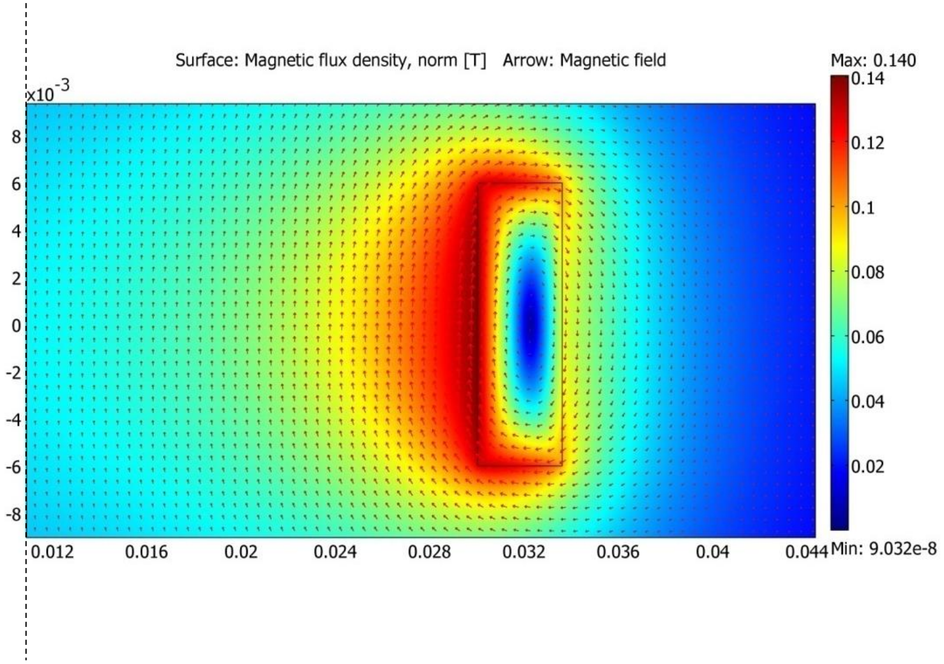
current sharing prevents the reduction of overall critical current that would be caused by I_c fluctuations

Estimation of the overall critical current for coils

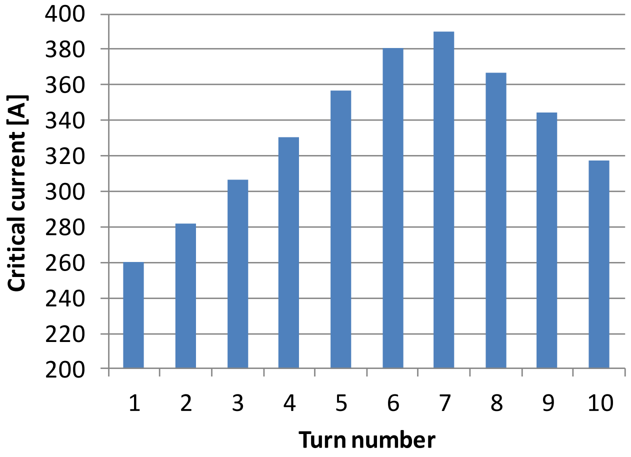
$$\delta_{fluct} = \frac{I_{c,mean} - I_{c,ovrl}}{I_{c,mean}}$$

single insulated tape (pancake coil)

Turn 1,2,...........M = 10



Step 1: computation neglecting $I_c(x)$



$$U_{coil} = E_c \sum_{j=1}^{M} 2\pi R_{turn,j} \left(\frac{I_{coil}}{I_{c,turn,j}} \right)^n$$

Step 2: modification by δ_{fluct}

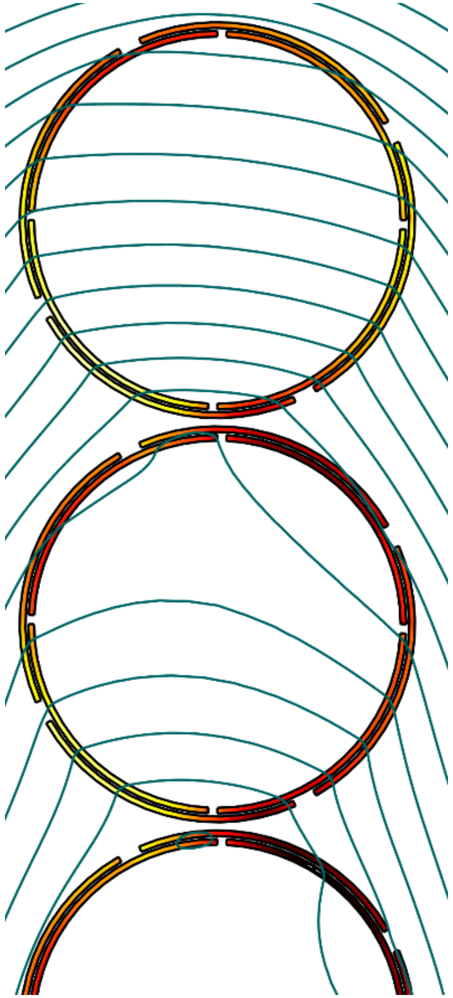
$$U_{coil,f} = E_c \sum_{j=1}^{M} 2\pi R_{turn,j} \left(\frac{I_{coil}}{(1-\delta_{fluct}) I_{c,turn,j}} \right)^n = (1-\delta_{fluct}) U_{coil}$$

overall critical current of the coil is reduced the same way as for the tape

Estimation of the overall critical current for coils

$$\delta_{fluct} = \frac{I_{c,mean} - I_{c,ovrl}}{I_{c,mean}}$$

cabled conductor from parallel tapes



a) insulated tapes:

$$I_{c,ovrl,ns} = \sum_{j=1}^{M} (1 - \delta_{fluct}) \alpha_j I_{c,mean} = (1 - \delta_{fluct}) I_{c,coil}$$

overall critical current is reduced the same way as for the single tape

a) non-insulated tapes, current sharing possible:

$$I_{c,ovrl,sh} = \sum_{j=1}^{M} \alpha_j I_{c,mean} = I_{c,coil}$$

no reduction caused by I_c fluctuation expected

Conclusions

- Fluctuation of critical current in CC tapes causes that the “overall critical current”, $I_{c,ovrl}$, measured on long tape, is lower than the mean value, $I_{c,mean}$, of the $I_c(x)$ data

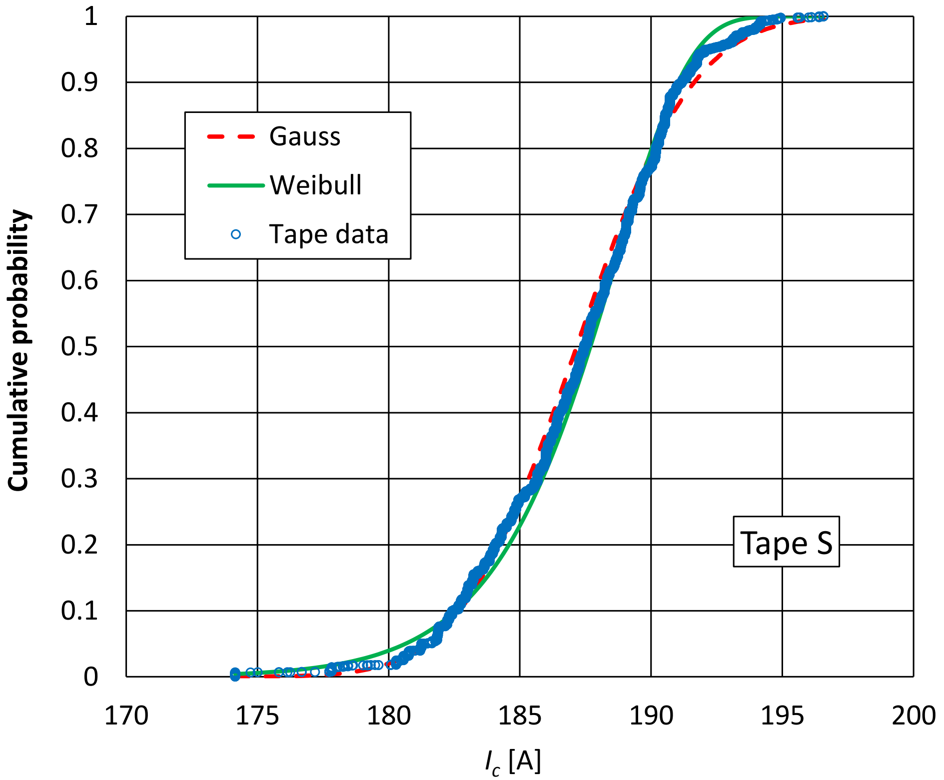
- Basic statistical characterization (mean, standard deviation) of $I_c(x)$ data allows to estimate the minimal expected reduction of $I_{c,ovrl}$ in regard to $I_{c,mean}$

- Cables and pancake coils from insulated tapes would suffer from $I_c(x)$ fluctuation in the same way as the single tape

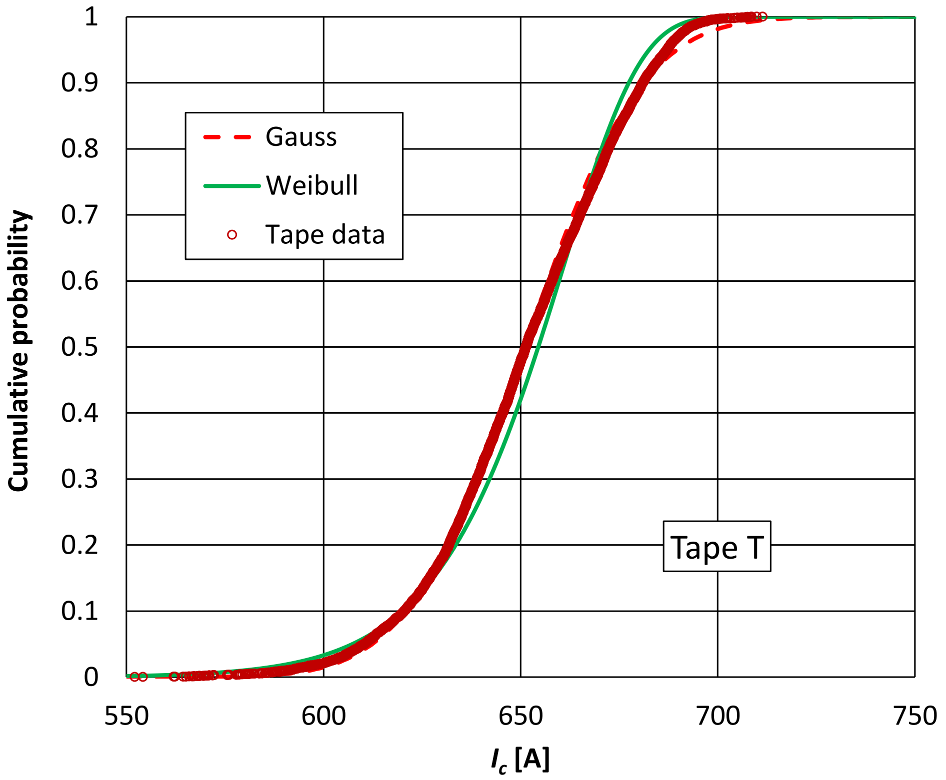
- Sharing of current between parallel non-insulated tapes could result in the overall critical current equal to $I_{c,mean}$

Statistical description of $I_c(x)$ data

practical method: check of the cumulative probability



Sample	RMSE for Gaussian	RMSE for Weibull
Tape S	0.02411	0.0211
Tape T	0.01344	0.0335



Is such deviation from ideal statistical models significant?

Statistical description of $I_c(x)$ data

mean value

$$I_{c,mean} = \frac{\sum_{i=1}^{N} I_{c,i}}{N}$$

variance

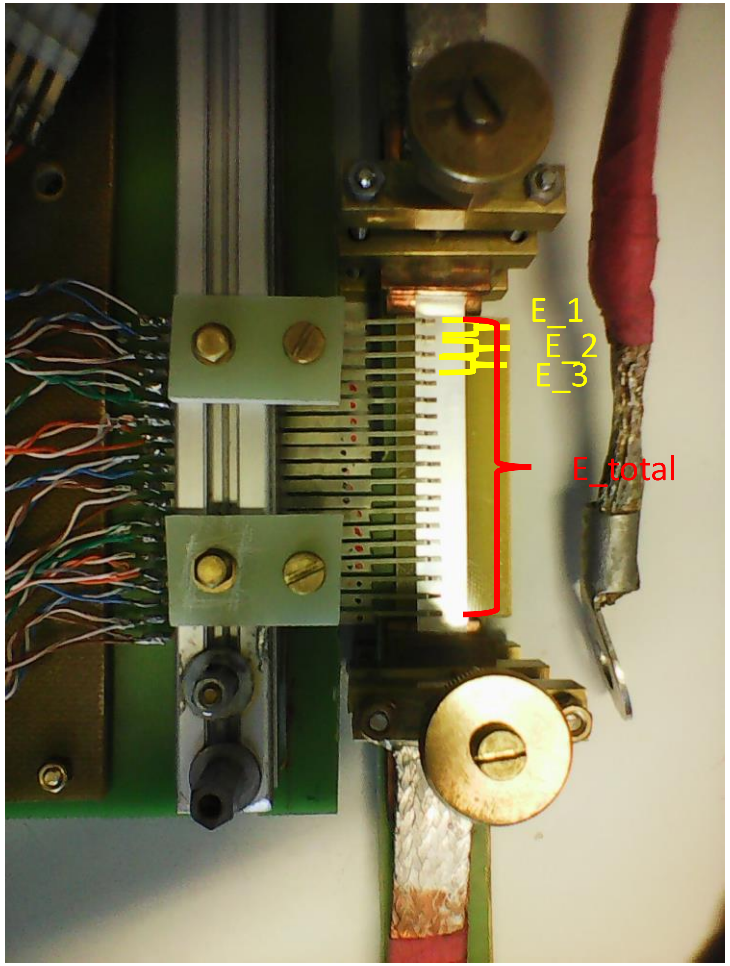
$$var_{Ic} = \frac{\sum_{i=1}^{N}\left(I_{c,mean} - I_{c,i}\right)^2}{N}$$

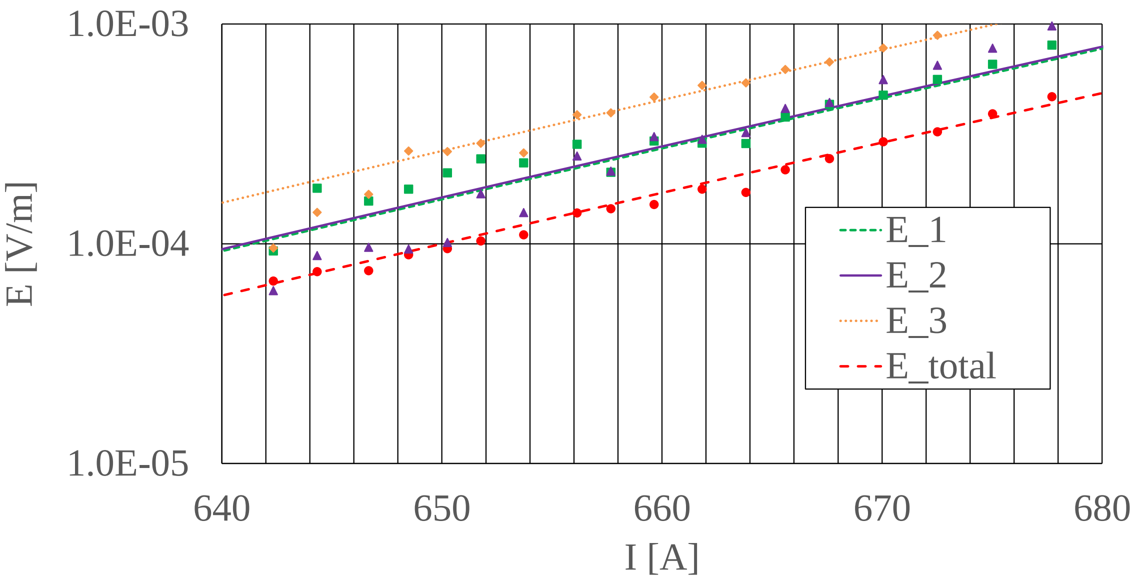
coefficient of variation

$$c_{var} = \frac{\sqrt{var_{Ic}}}{I_{c,mean}}$$

Sample	Width [mm]	Length [m]	Δx [mm]	$I_{c,mean}$ [A]	var_{Ic} [A^2]	c_{var} [%]
Tape S	4	50	5	187.16	12.18	1.87
Tape T	12	10	1	650.97	552.72	3.61

Introduction





$$E_i = E_c \left(\frac{I}{I_{ci}} \right)^n$$