Overall critical current of CC tapes and devices when local critical currents fluctuate along the tape length

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Introduction

fluctuation of critical current along the tape length is a common feature of CC tapes

what is the value of „critical current“ that should be used in the design of a device?
Problems discussed

1) Is there a long-length equivalent of the short sample’s critical current?

2) Could this long-length critical current be predicted from the parameters of statistical distribution of local \( I_c \) ‘s?

3) Knowing the \( I_c \) fluctuation property of a single tape, what are the consequences to operation of cables and coils?

4) Conclusions (answers)
Overall critical current

the value of current at which the same electrical field is registered on the whole length as in the short sample testing

\[ E_i = E_c \left( \frac{I}{I_{ci}} \right)^n \]

in case of non-fluctuating \( n \)

\[ I_{c,ovrI} = \left[ \frac{N}{\sum_{i=1}^{N} \left( \frac{1}{I_{c,i}} \right)^n} \right]^{\frac{1}{n}} \]

Statistical description of $I_c(x)$ data

mean value

$$I_{c,\text{mean}} = \frac{\sum_{i=1}^{N} I_{c,i}}{N}$$

variance

$$\text{var}_{I_c} = \frac{\sum_{i=1}^{N} (I_{c,\text{mean}} - I_{c,i})^2}{N}$$

coefficient of variation

$$c_{\text{var}} = \frac{\sqrt{\text{var}_{I_c}}}{I_{c,\text{mean}}}$$

overall critical current is always lower than the mean

deterioration because of $I_c$ fluctuations

$$\delta_{\text{fluct}} = \frac{I_{c,\text{mean}} - I_{c,\text{ovrl}}}{I_{c,\text{mean}}}$$
Statistical description of $I_c(x)$ data

**Gauss**

**Probability density:**

$$f_G(I_c) = \frac{1}{\sqrt{2\pi \text{var}_{I_c}}} e^{-\frac{(I_c - I_{c,\text{mean}})^2}{2\text{var}_{I_c}}}$$

**Mean**

$$I_{c,\text{mean}} = \frac{1}{\Gamma(1+\frac{1}{s_{Ic}})}$$

**Standard deviation:**

$$\sigma_{Ic} = \sqrt{\text{var}_{Ic}}$$

**Cumulative probability:**

$$F_G(I_c) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{I_c - I_{c,\text{mean}}}{\sigma_{Ic}\sqrt{2}} \right) \right]$$

**Weibull**

**Probability density:**

$$f_W(I_c) = e^{-\left(\frac{I_c}{I_{c,\text{scale}}}\right)^{s_{Ic}}} \frac{s_{Ic}}{I_{c,\text{scale}}} \left(\frac{I_c}{I_{c,\text{scale}}}\right)^{s_{Ic} - 1}$$

**Scale**

$$I_{c,\text{mean}} = I_{c,\text{scale}}^{1+\frac{1}{s_{Ic}}}$$

**Shape:**

$$c_{var}^2 = \left[ \Gamma \left( 1 + \frac{2}{s_{Ic}} \right) - \left( \Gamma \left( 1 + \frac{1}{s_{Ic}} \right) \right)^2 \right]$$

**Cumulative probability:**

$$F_W(I_c) = 1 - e^{-\left(\frac{I_c}{I_{c,\text{scale}}}\right)^{s_{Ic}}}$$

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Relation between the overall critical current and the mean value

computational exercise – many sets of **artificially generated data with Gaussian distribution**

\[ \delta_{\text{fluct}} = \frac{I_{c,\text{mean}} - I_{c,\text{ovrl}}}{I_{c,\text{mean}}} \]

all the results can be fitted by general formula:

\[ \delta_{\text{fluct,G}} = 1.03n c_{\text{var}}^{2.2} \]

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Relation between the overall critical current and the mean value

\[ \delta_{fluct} = \frac{I_{c,mean} - I_{c,ovrl}}{I_{c,mean}} \]

\[ I_{c,ovrl} = \left[ \frac{N}{\sum_{i=1}^{N} \left( \frac{1}{I_{c,i}} \right)} \right]^{\frac{1}{n}} \]

fit Gaussian:
\[ \delta_{fluct,G} = 1.03nc_{var}^{2.2} \]

points – exact formula

\[ c_{var} = \sigma_{Ic}/I_{c,mean} \]

\[ \delta_{fluct} = (I_{c,mean} - I_{c,ovrl})/I_{c,mean} \]

fit Gaussian, n = 35
- Tape S, n = 35
- Tape T, n = 35

- fit Gaussian, n = 25
- Tape S, n = 25
- Tape T, n = 25

reduction of overall critical current slightly underestimated
Assumptions:
• zero resistance at the terminations
• magnetic field produced by transported current is equivalent for all the tapes

\[
\delta_{\text{fluct}} = \frac{I_{c,\text{mean}} - I_{c,\text{ovrl}}}{I_{c,\text{mean}}}
\]
Estimation of the overall critical current for cables

\[ \delta_{f,\text{fluct}} = \frac{I_{c,\text{mean}} - I_{c,\text{ovrl}}}{I_{c,\text{mean}}} \]

\[ I_{c,\text{ovrl},ns} = \sum_{j=1}^{M} I_{c,\text{ovrl},j} \]

a) no electrical contact between tapes

for each of the tapes:

\[ I_{c,\text{ovrl},j} = \left[ \frac{N}{\sum_{i=1}^{N} \left( \frac{1}{I_{c,ij}} \right)^n} \right]^{1/n} \]

overall critical current of the cable is reduced the same way as the tape':

\[ I_{c,\text{ovrl},ns} = M(1 - \delta_{f,\text{fluct}})I_{c,\text{mean}} \]
Estimation of the overall critical current for cables

\[
\delta_{\text{fluct}} = \frac{I_{c,\text{mean}} - I_{c,\text{ovrl}}}{I_{c,\text{mean}}}
\]

- a) no electrical contact between tapes

Considering a non-equivalent self-field:

\[
I_{c,\text{cable}} = \sum_{j=1}^{M} \alpha_j I_{c,\text{tape}}
\]

\[
I_{c,\text{ovrl,ns}} = \sum_{j=1}^{M} \alpha_j (1 - \delta_{\text{fluct}}) I_{c,\text{mean}} = (1 - \delta_{\text{fluct}}) \sum_{j=1}^{M} \alpha_j I_{c,\text{mean}} = (1 - \delta_{\text{fluct}}) I_{c,\text{cable}}
\]

Self-field has no influence on the degradation due to \( I_c \) fluctuations
Estimation of the overall critical current for cables

\[ \delta_{fluct} = \frac{I_{c,mean} - I_{c,ovrl}}{I_{c,mean}} \]

\( x_{i+1} = (i-1) \Delta x \)

\( x_i = i \Delta x \)

b) perfect electrical contact between tapes

\[ E(i \Delta x) = E_c \left( \frac{I_{cable}}{I_{c,i}} \right)^n \]

\( \bar{I}_{c,i} = \sum_{j=1}^{M} I_{c_{ij}} \)

for \( M >> 1 \):

\( \bar{I}_{c,i} \approx M I_{c,mean} \)

overall critical current of the cable is not reduced because of \( I_c \) fluctuations

\( I_{c,ovrl,sh} = M I_{c,mean} \)
Estimation of the overall critical current for cables

\[ \delta_{fluct} = \frac{I_{c,\text{mean}} - I_{c,\text{ovrl}}}{I_{c,\text{mean}}} \]

b) perfect electrical contact between tapes

considering a non-equivalent self-field:

\[ \bar{I}_{c,i} = \sum_{j=1}^{M} \alpha_j I_{cij,\text{self}} \quad \bar{I}_{c,i} \approx \sum_{j=1}^{M} \alpha_j I_{c,\text{mean}} \quad I_{c,\text{ovrl,sh}} = I_{c,\text{mean}} \sum_{j=1}^{M} \alpha_j \]

current sharing prevents the reduction of overall critical current that would be caused by \( I_c \) fluctuations

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Estimation of the overall critical current for coils

**single insulated tape** (pancake coil)

Turn 1,2,............$M = 10$

\[
\delta_{fluct} = \frac{I_{c,\text{mean}} - I_{c,\text{ovrl}}}{I_{c,\text{mean}}}
\]

Step 1: computation neglecting $I_c(x)$

\[
U_{\text{coil}} = E_c \sum_{j=1}^{M} 2\pi R_{\text{turn},j} \left( \frac{I_{\text{coil}}}{I_{c,\text{turn},j}} \right)^n
\]

Step 2: modification by $\delta_{fluct}$

\[
U_{\text{coil,f}} = E_c \sum_{j=1}^{M} 2\pi R_{\text{turn},j} \left( \frac{I_{\text{coil}}}{(1 - \delta_{fluct})I_{c,\text{turn},j}} \right)^n = (1 - \delta_{fluct})U_{\text{coil}}
\]

Overall critical current of the coil is reduced the same way as for the tape.
Estimation of the overall critical current for coils

cabled conductor from parallel tapes

\[ \delta_{\text{fluct}} = \frac{I_{c,\text{mean}} - I_{c,\text{ovrl}}}{I_{c,\text{mean}}} \]

\( I_{c,\text{ovrl},ns} = \sum_{j=1}^{M} (1 - \delta_{\text{fluct}}) \alpha_j I_{c,\text{mean}} = (1 - \delta_{\text{fluct}}) I_{c,\text{coil}} \)

Overall critical current is reduced the same way as for the single tape

a) insulated tapes:

a) non-insulated tapes, current sharing possible:

\[ I_{c,\text{ovrl},sh} = \sum_{j=1}^{M} \alpha_j I_{c,\text{mean}} = I_{c,\text{coil}} \]

No reduction caused by \( I_c \) fluctuation expected

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Conclusions

- Fluctuation of critical current in CC tapes causes that the “overall critical current”, $I_{c,ovrl}$, measured on long tape, is lower than the mean value, $I_{c,mean}$, of the $I_c(x)$ data.

- Basic statistical characterization (mean, standard deviation) of $I_c(x)$ data allows to estimate the minimal expected reduction of $I_{c,ovrl}$ in regard to $I_{c,mean}$.

- Cables and pancake coils from insulated tapes would suffer from $I_c(x)$ fluctuation in the same way as the single tape.

- Sharing of current between parallel non-insulated tapes could result in the overall critical current equal to $I_{c,mean}$.
Statistical description of $I_c(x)$ data

practical method: check of the cumulative probability

<table>
<thead>
<tr>
<th>Sample</th>
<th>RMSE for Gaussian</th>
<th>RMSE for Weibull</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tape S</td>
<td>0.02411</td>
<td>0.0211</td>
</tr>
<tr>
<td>Tape T</td>
<td>0.01344</td>
<td>0.0335</td>
</tr>
</tbody>
</table>

Is such deviation from ideal statistical models significant?
### Statistical description of $I_c(x)$ data

#### Mean value

$$I_{c,\text{mean}} = \frac{\sum_{i=1}^{N} I_{c,i}}{N}$$

#### Variance

$$\text{var}_{I_c} = \frac{\sum_{i=1}^{N} (I_{c,\text{mean}} - I_{c,i})^2}{N}$$

#### Coefficient of variation

$$c_{\text{var}} = \frac{\text{var}_{I_c}}{I_{c,\text{mean}}}$$

<table>
<thead>
<tr>
<th>Sample</th>
<th>Width [mm]</th>
<th>Length [m]</th>
<th>$\Delta x$ [mm]</th>
<th>$I_{c,\text{mean}}$ [A]</th>
<th>$\text{var}_{I_c}$ [A$^2$]</th>
<th>$c_{\text{var}}$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tape S</td>
<td>4</td>
<td>50</td>
<td>5</td>
<td>187.16</td>
<td>12.18</td>
<td>1.87</td>
</tr>
<tr>
<td>Tape T</td>
<td>12</td>
<td>10</td>
<td>1</td>
<td>650.97</td>
<td>552.72</td>
<td>3.61</td>
</tr>
</tbody>
</table>
Introduction

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$E_i = E_c \left( \frac{I}{I_{ci}} \right)^n$