Quasi-3D Thermal Simulation of Quench Propagation in Superconducting Magnets

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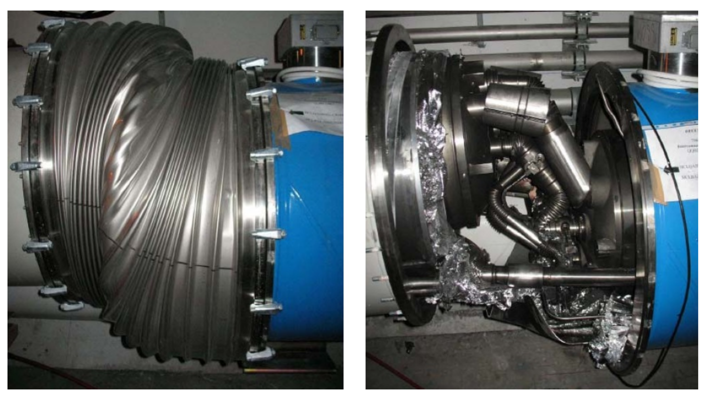
Motivation: Improvement of quench detection

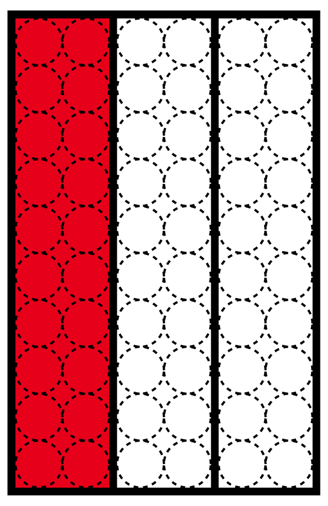
Summary report on the analysis of the 19th September 2008 incident at the LHC:



"Within the first second, an electrical arc developed and punctured the helium enclosure, leading to release of helium into the insulation vacuum of the cryostat [...]. [The resulting large pressure] forces displaced dipoles [...] and knocked the short straight section cryostats housing the quadrupoles and vacuum barriers from their external support jacks [...], in some locations breaking their anchors in the concrete floor of the tunnel."

- At the Large Hadron Collider (LHC) at CERN, superconducting accelerator magnets are used to achieve high magnetic fields.
- Above the critical temperature θ_{crit}, a sudden transition from superconducting to normal conducting state happens: a **quench**.
- **Worst case**: The whole stored energy of 1.1 GJ – enough to melt 1.5 tons of copper – concentrates in a tiny volume.
- **Goal**: Get more reliable and accurate thresholds for quench detection by magneto-thermal simulations.
- **Problem**: Magnet's cross-section has a diameter of 570 mm and is over 10 m long ⇒ Conventional 3D simulations are too expensive.
- **Idea**: Combine a 2D finite-element method (FEM) in the cross-section with a 1D spectral-element method (SEM) in longitudinal direction into a **quasi-3D (Q3D) FE-SE method**.

Benchmark: Quench propagation in Rutherford cables



- **Model**: Three Rutherford cables of length $\ell_z = 1\,\text{m}$ wrapped with glass fibre insulation.
- Each cable contains wires made of superconducting Nb_3Sn filaments embedded in a copper matrix.
- **Bulk model**: The cables are considered to be solid and the material properties are homogenized.
- Solve the transient heat conduction equation

$$-\nabla\cdot(\lambda\nabla\theta(\vec{r},t)) + C_V\partial_t\theta(\vec{r},t) = q(\vec{r},t) \qquad \text{(PDE)}$$

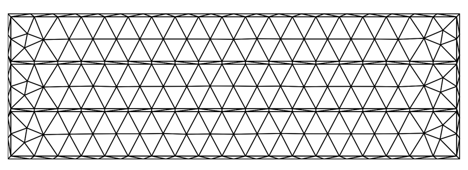
with constant material properties and **boundary conditions** (BCs),

$$\text{adiabatic BCs,} \quad -\lambda\,\partial_n\theta(x,y,z) = 0|_{\Gamma_{\text{hull}}},$$
$$\text{isothermal BCs,} \quad \theta(x,y,0) = \theta(x,y,\ell_z) = \theta_{\text{Dir}}.$$

- **Scenario**: The left cable (marked in red) quenches at $z = z_q$.

Finite Element Method & Spectral Element Method

2D Finite Element Method

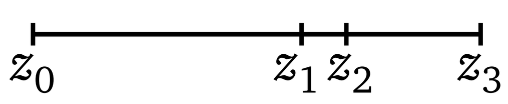


- A triangular mesh in the xy-cross-section can resolve geometrical details.
- Approximation:

$$\theta(x,y;t) \approx \sum_{j=1}^{J} u_j(t) N_j(x,y)$$

with linear nodal shape functions N_j with local support around the j-th node.
- The coefficients u_j live in the physical space.

1D Spectral Element Method



- Non-uniform line elements in z-direction can resolve steep quench fronts.
- Approximation:

$$\theta(z;t) \approx \sum_{k=1}^{K}\sum_{q=1}^{N+1} \tilde{u}_q^{(k)}(t)\,\phi_q^{(k)}(z)$$

with modal orthogonal polynomials $\phi_q^{(k)}$ of order q with local support in the k-th element.
- The coefficients $\tilde{u}_q^{(k)}$ live in the frequency space.

Connecting the dimensions: Quasi-3D FE-SE formulation

- **Galerkin method**: Multiply (PDE) with test functions $N_i(x,y)\phi_p^{(k)}(z)$ and integrate over the 3D volume V.
- **Approximation** as triple sum:

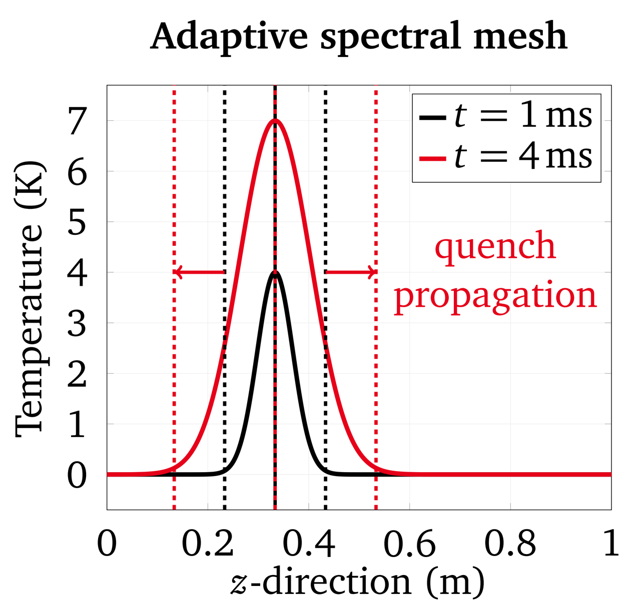
$$\theta(x,y,z;t) \approx \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{q=1}^{N+1} \tilde{u}_{jq}^{(k)}(t)\,N_j(x,y)\,\phi_q^{(k)}(z).$$

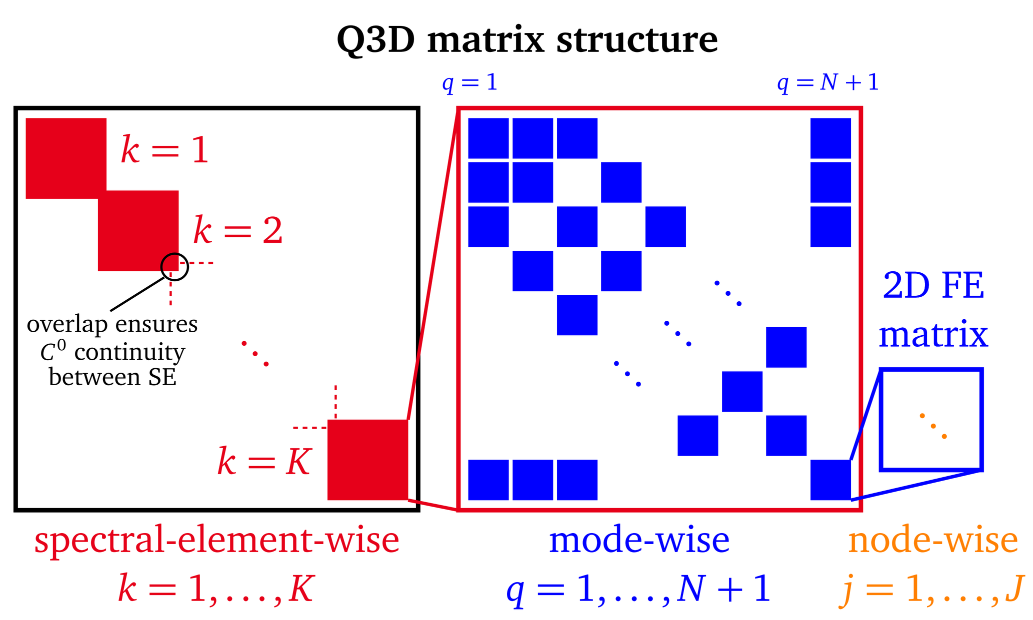
- **System of equations**: $\mathbf{K}_\lambda^{\text{Q3D}}\tilde{\mathbf{u}}(t) + \mathbf{M}_{C_V}^{\text{Q3D}}\partial_t\tilde{\mathbf{u}}(t) = \mathbf{q}^{\text{Q3D}}(t)$

with dimensions $(J(KN+1)\times J(KN+1))$. All Q3D matrices and vectors can be constructed out of 2D FEM and 1D SEM matrices and vectors by Kronecker tensor products,

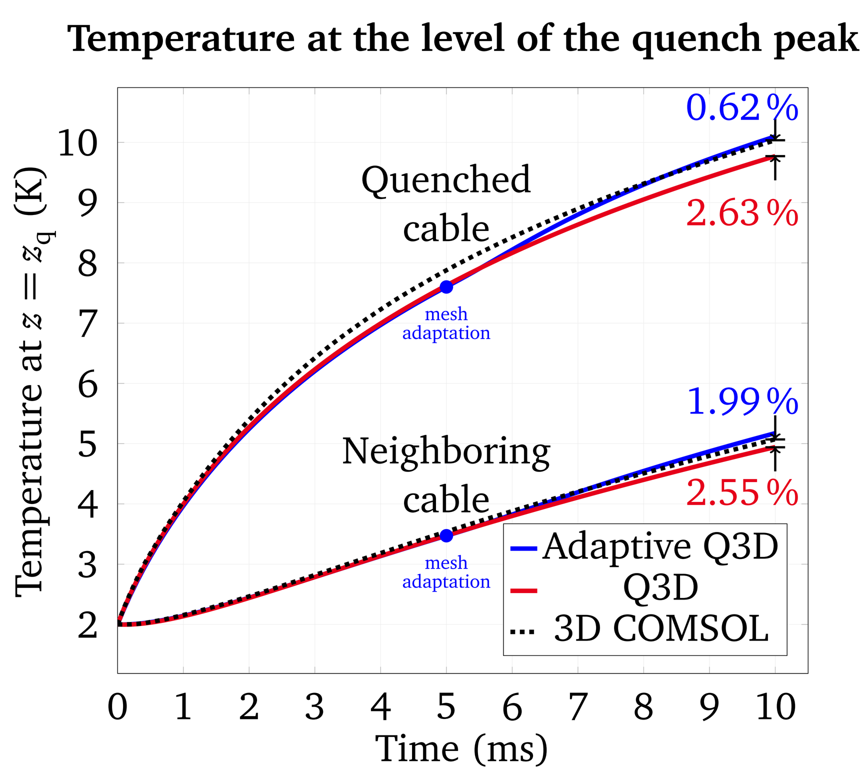
$$\mathbf{K}_\lambda^{\text{Q3D}} = \mathbf{M}^{\text{SE}}\otimes\mathbf{K}_\lambda^{\text{FE}} + \mathbf{K}^{\text{SE}}\otimes\mathbf{M}_\lambda^{\text{FE}} \qquad \text{Q3D stiffness matrix,}$$
$$\mathbf{M}_{C_V}^{\text{Q3D}} = \mathbf{M}^{\text{SE}}\otimes\mathbf{M}_{C_V}^{\text{FE}} \qquad \text{Q3D mass matrix,}$$
$$\mathbf{q}^{\text{Q3D}}(t) = \mathbf{q}^{\text{SE}}(t)\otimes\mathbf{q}^{\text{FE}}(t) \qquad \text{Q3D load vector.}$$

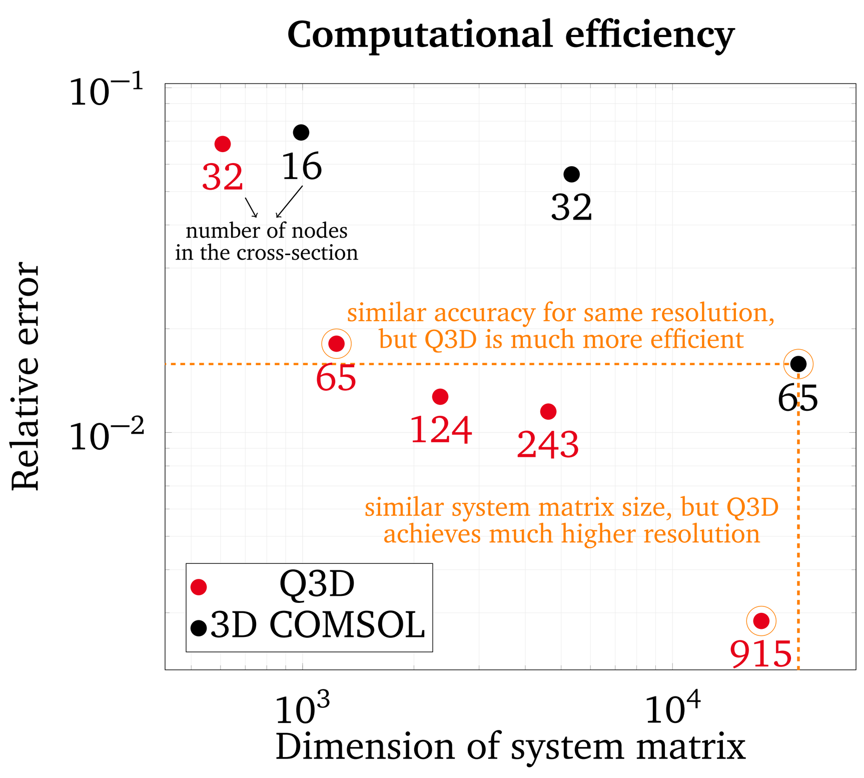
- Discretize in time with the implicit Euler method.
- Impose the BCs and solve the system with a standard solver.
- Obtain the physical solution by a backward transform of the frequency solution $\tilde{\mathbf{u}}$ at every FE node.





Simulation results: Comparison with 3D COMSOL





⇒ The Q3D method delivers accurate results (even better with spectral mesh adaptation) and needs much less computational effort than the conventional 3D FEM to do so.

Future steps

→ Develop an appropriate adaptive spectral mesh strategy.

→ Consider nonlinear material properties.

→ Do a magnetic and magneto-thermal simulation.

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