Motivation: Improvement of quench detection

Summary report on the analysis of the 19th September 2008 incident at the LHC:

“Within the first second, an electrical arc developed and punctured the helium enclosure, leading to release of helium into the insulation vacuum of the cryostat [-]. The resulting large pressure forces displaced dipoles [-]..."}

- At the Large Hadron Collider (LHC) at CERN, superconducting accelerator magnets are used to achieve high magnetic fields.
- Above the critical temperature $T_{\text{cr}}$, a sudden transition from superconducting to normal conducting state happens: a quench.
- Worst case: The whole stored energy of 1.1 GJ – enough to melt 1.5 tons of copper – concentrates in a tiny volume.
- Goal: Get more reliable and accurate thresholds for quench detection by magneto-thermal simulations.
- Problem: Magnet’s cross-section has a diameter of 570 mm and is over 10 m long ⇒ Conventional 3D simulations are too expensive.
- Idea: Combine a 2D finite-element method (FEM) in the cross-section with a 1D spectral-element method (SEM) in longitudinal direction into a quasi-3D (Q3D) SEM method.

Benchmark: Quench propagation in Rutherford cables

- Model: Three Rutherford cables of length $\ell = 1$ m wrapped with glass fibre insulation.
- Each cable contains wires made of superconducting NbSn filaments embedded in a copper matrix.
- Bulk model: The cables are considered to be solid and the material properties are homogenized.
- Solve the transient heat conduction equation
  \[
  -\nabla \cdot (\lambda \nabla \theta(t)) + c_v \frac{\partial \theta(t)}{\partial t} = q(t)
  \]
  with constant material properties and boundary conditions (BCs), adiabatic BCs, \( \lambda \frac{\partial \theta(x,y,z)}{\partial n} = 0 \) at walls, isothermal BCs, \( \theta(x,y,0) = \theta(x,y,\ell) = \theta_{\text{init}} \).
- Scenario: The left cable (marked in red) quenches at $z \approx z_i$.

Finite Element Method & Spectral Element Method

2D Finite Element Method

- A triangular mesh in the $xy$-cross-section can resolve geometrical details.
- Approximation:
  \[
  \theta(x,y,t) \approx \sum_{j=1}^{J} u_j(t)N_j(x,y)
  \]
  with linear nodal shape functions $N_j$ with local support around the $j$-th node.
- The coefficients $u_j$ live in the physical space.

1D Spectral Element Method

- Non-uniform line elements in $z$-direction can resolve steep quench fronts.
- Approximation:
  \[
  \theta(z,t) \approx \sum_{k=1}^{K} \sum_{q=1}^{Q_k} \tilde{u}_q(t) \phi_q^{(k)}(z)
  \]
  with modal orthogonal polynomials $\phi_q^{(k)}$ of order $q$ with local support in the $k$-th element.
- The coefficients $\tilde{u}_q^{(k)}$ live in the frequency space.

Connecting the dimensions: Quasi-3D FE-SE formulation

- Galerkin method: Multiply (PDE) with test functions $N_j(x,y) \phi^{(k)}(z)$ and integrate over the 3D volume $V$.
- Approximation as triple sum:
  \[
  \theta(x,y,z;t) \approx \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{q=1}^{Q_k} \hat{u}_q^{(k)}(t) N_j(x,y) \phi^{(k)}(z).
  \]
- System of equations: $K^{\text{Q3D}} \tilde{u}(t) + M^{\text{Q3D}} \dot{\tilde{u}}(t) = q^{\text{Q3D}}(t)$
  with dimensions $(J(KN+1) \times J(KN+1))$. All Q3D matrices and vectors can be constructed out of 2D FEM and 1D SEM matrices and vectors by Kronecker tensor products,
  \[
  K^{\text{Q3D}} = M^{\text{SE}} \otimes K^{\text{FE}} + K^{\text{SE}} \otimes M^{\text{FE}} \]
  $M^{\text{Q3D}} = M^{\text{SE}} \otimes M^{\text{FE}}$,
  $q^{\text{Q3D}}(t) = q^{\text{SE}}(t) \otimes q^{\text{FE}}(t)$.
- Discretize in time with the implicit Euler method.
- Impose the BCs and solve the system with a standard solver.
- Obtain the physical solution by a backward transform of the frequency solution $\tilde{u}$ at every FE node.

Simulation results: Comparison with 3D COMSOL

- The Q3D method delivers accurate results (even better with spectral mesh adaptation) and needs much less computational effort than the conventional 3D FEM to do so.

Future steps

- Develop an appropriate adaptive spectral mesh strategy.
- Consider nonlinear material properties.
- Do a magnetic and magneto-thermal simulation.

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