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Experimental Verification and Analytical Study of Influence of Rotor Eccentricity on Electromagnetic Characteristics of Permanent Magnet Machine

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I. ABSTRACT

In this study, we present an analytical model for the permanent magnet synchronous machines (PMSM) in which rotor eccentricity is applied. We simplified the analysis model through several assumptions and modeled the magnetization. Based on electromagnetic field theory and the perturbation theory, the governing equations were derived in the air-gap region. Further, the relationship between the magnetic vector potential in each region and the appropriate boundary conditions is used to obtain the undefined coefficients to derive the magnetic flux density characteristics in each region. The validity of the analytical results was verified by comparing them to the results of the two- dimensional finite element analysis (FEM) and experiments.

II .ANALYSIS MODEL

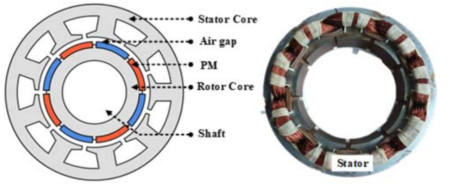


Fig. 1. Permanent magnet machine: FE Analysis and prototype model

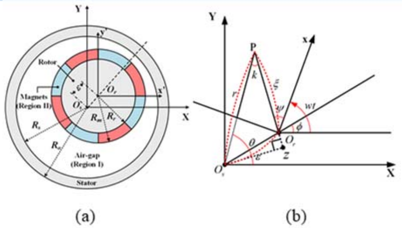
- Fig. 1 shows FE analysis and prototype model.
- Table 1 is specification of the analysis model.

Table 1. Specification of the analysis model

Parameter	Value (unit)	Parameter	Value (unit)
Pole number	8	Slot number	9
R_s	47(mm)	R_m	43 (mm)
R_r	38 (mm)	B_r	1.28 (T)
μ_r (magnet)	1	Turn	60
L_{stk}	30 (mm)	w_r	1000 (rpm)
	0.9	ec	0.25

- The FE analysis model is simplified through several assumptions to apply the analytical method.

III. ANALYSIS OF THE ROTOR ECCENTRICITY



(a) (b)

Fig. 2. (a) Simplified analytical model, (b) The stator and rotor coordinate system.

❖ Relationship of the rotor and stator coordinate system

$$\xi = r - \varepsilon\cos(\theta-\phi) + O(\varepsilon^2) \tag{1}$$

$$\psi = (\theta - wt) + \frac{\varepsilon}{r}\sin(\theta-\phi) + O(\varepsilon^2), \qquad \varepsilon = ec \times g \tag{2}$$

- The simplified analytical model and analysis area can be defined as shown in Fig. 2(a) for the mathematical modeling.
- It consists of an air-gap region (Region I) and a PM region (Region II).
- The stator and rotor coordinate system is shown in Fig. (b) to explain the coordinate transformation.
- The relationship between each coordinate system can be expressed as follows equation (1) and (2).

❖ Magnetization modeling

$$\mathbf{M} = \sum_{n=1}^{\infty} \mathrm{M}_{rn}\cos(np\theta)\cdot\mathbf{i_r} + \mathrm{M}_{\theta n}\sin(np\theta)\cdot\mathbf{i_\theta} \tag{3}$$

❖ Perturbation theory

$$A_{z1}(r,\theta,\varepsilon) = A_{z1}^{(0)}(r,\theta) + \varepsilon A_{z1}^{(1)}(r,\theta) + \cdots$$
$$A_{z2}(r,\theta,\varepsilon) = A_{z2}^{(0)}(r,\theta) + \varepsilon A_{z2}^{(1)}(r,\theta) + \cdots \tag{4}$$

❖ Governing equation

$$\frac{\partial^2 A_{z1}^{(0)}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{z1}^{(0)}}{\partial r} - \frac{q^2}{r^2}A_{z1}^{(0)} = 0$$
$$\frac{\partial^2 A_{z2}^{(0)}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{z2}^{(0)}}{\partial r} - \frac{q^2}{r^2}A_{z2}^{(0)} = -\frac{\mu_0 q M_n \sin(q\theta)}{r}$$
$$\frac{\partial^2 A_{z1}^{(1)}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{z1}^{(1)}}{\partial r} - \frac{q^2}{r^2}A_{z1}^{(1)} = 0 \tag{5}$$
$$\frac{\partial^2 A_{z2}^{(1)}}{\partial r^2} + \frac{1}{r}\frac{\partial A_{z2}^{(1)}}{\partial r} - \frac{q^2}{r^2}A_{z2}^{(1)} = 0$$

❖ Boundary condition

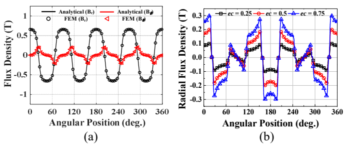
$$\mathbf{n}_{R_r} \times \mathbf{H}_2 = 0$$
$$\mathbf{n}_{R_m} \times (\mathbf{H}_1 - \mathbf{H}_2) = 0 \tag{6}$$
$$\mathbf{n}_{R_m} \cdot (\mathbf{B}_1 - \mathbf{B}_2) = 0$$

❖ Magnetic field density in air-gap region

$$B_{r1} = \sum_{n=1,3,5\cdots}^{\infty} q[A_1 r^{q-1} + B_1 r^{-q-1}]\cos(q\theta)$$
$$+\varepsilon(q-1)\left(W_1 r^{q-2} + X_1 r^{-q}\right)\cos\left[(q-1)\theta+\phi\right]$$
$$+\varepsilon(q+1)\left(Y_1 r^{q} + Z_1 r^{-q-2}\right)\cos\left[(q+1)\theta-\phi\right] \tag{7}$$
$$B_{\theta 1} = \sum_{n=1,3,5\cdots}^{\infty} -q[A_1 r^{q-1} - B_1 r^{-q-1}]\sin(q\theta)$$
$$-\varepsilon(q-1)\left(W_1 r^{q-2} - X_1 r^{-q}\right)\sin\left[(q-1)\theta+\phi\right]$$
$$-\varepsilon(q+1)\left(Y_1 r^{q} - Z_1 r^{-q-2}\right)\sin\left[(q+1)\theta-\phi\right]$$

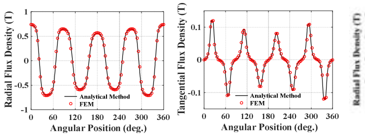
- The rotor eccentricity can be treated as one kind of perturbation phenomena.
- The governing equations applied with perturbation theory in each region are expressed by the equation (4).
- Substituting equation (4) and $\nabla \times \mathbf{A} = \mathbf{B}$ into equation (6) can be derive the boundary conditions in each region.
- The general solutions are obtained according to the governing equation (5) and boundary conditions.
- The magnetic flux density in air gap region is expressed by equation (7).

IV. RESULTS AND DISCUSSION



(a) (b)

Fig. 3. (a) Flux density in air gap without the rotor eccentricity, (b) Additional flux density according to the rotor eccentricity ratio.



(a) (b)

Fig. 4. Flux density in air gap region for 25% eccentricity: (a) Radial direction, (b) tangential direction.

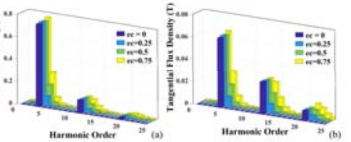


Fig. 5. The harmonics of the air-gap flux density according to the eccentricity ratio: (a) Radial direction, (b) Tangential direction.

- Fig. 3(a) and (b) show the flux density without the rotor eccentricity and additional flux density according to the rotor eccentricity ratio.
- The magnetic flux density due to the eccentricity can be derived by adding the magnetic flux density and the change in magnetic flux density according to the eccentricity when the rotor is at the center.
- Fig. 4 is the flux density in air gap region for 25% eccentricity.
- It can be confirmed that the analytical results good agreement with FEM results in Fig. 4.
- Fig.5 indicates the harmonics of flux density in air gap region according to the eccentricity ratio.
- It can be confirmed that the 5th and 13th harmonics are increased significantly as the eccentricity ratio growing.

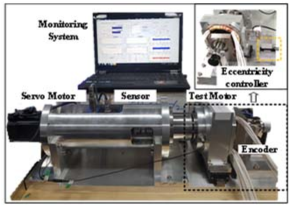


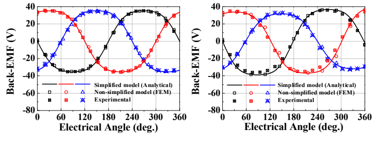
Fig. 6. Experimental set

❖ Back-EMF

$$\Phi = \int \mathbf{B}\cdot d\mathbf{S}, \quad E(t) = -N_{ph}\frac{d\Phi}{dt} \tag{8}$$

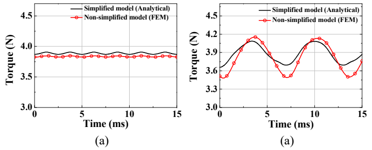
❖ Electro magnetic torque

$$T_e = \frac{e_a i_a + e_b i_b + e_c i_c}{w_r} \tag{9}$$



(a) (b)

Fig. 7. Analysis results for back-EMF: (a) *ec* = 0, (b) *ec* = 0.25.



(a) (a)

Fig. 8. Analysis results for Torque: (c) *ec* = 0, (d) *ec* = 0.25.

- In order to find back-EMF, it is necessary to know the magnetic flux linking the coil using equation (8).
- The back-EMF can be found using Faraday' law.
- The back-EMF in other coils of the same phase are not necessary similar because of the eccentricity. They should be calculated individually by a proper shift.
- The electromagnetic torque can be expressed as equation (9).
- Fig. 7(a) and (b) shows the back-EMF at *ec* = 0 and 0.25.
- Fig. 7(c) and (d) indicate the torque when 7 A_{peak} is applied at *ec* = 0 and 0.25.
- It can be confirmed that electro magnetic torque increases significantly at *ec* = 0.25.
- The analytical results are compared with the FEM and experimental.
- It can be seen that the results are in good agreement with FEM and experimental.

V. CONCLUSION

In this paper, the magnetic field characteristics the electromagnetic variation of the PMSM are analyzed according to the rotor eccentricity. In order to apply the analytical method, a simplified analytical model is presented, and the governing equations and general solutions are obtained using the magnetic vector potential and the polar coordinate system. According to the rotor eccentricity, the relationship between each coordinate system is summarized using the coordinate system about the center of the stator and the rotor. The variation of magnetic field due to eccentricity is considered through perturbation theory, and the undefined coefficient is derived through appropriate boundary conditions. We confirmed the additional harmonics in flux density, back-EMF, and electromagnetic torque according to the rotor eccentricity. Such analytical solution may be helpful for analyzing the electromagnetic influence of the variation of magnetic field due to rotor eccentricity. The results obtained through the analytical method were verified by the FEM and experiment.