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Accurate calculation of field expansion coefficients in FEM magnetostatic simulations

V. Marusov

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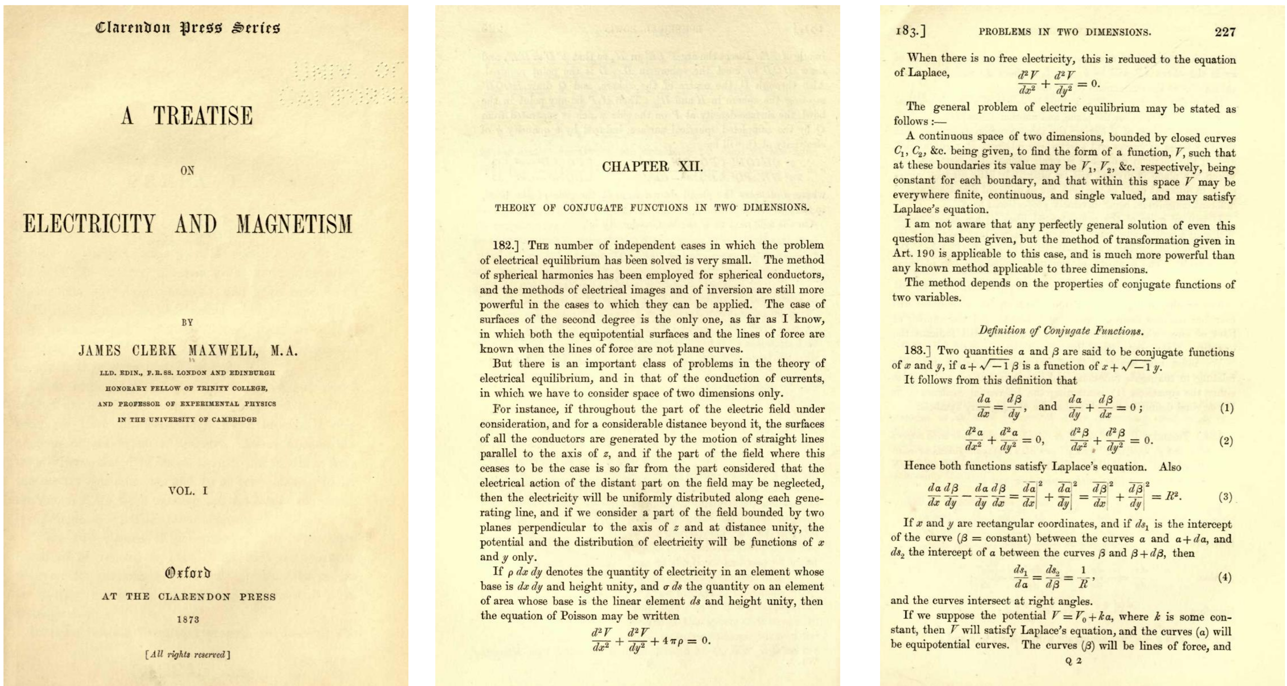


James Clerk Maxwell, 1873: *Potential must be an analytic function; thus, methods of the theory of functions of a complex variable are very powerful tools for solving 2D problems of electro- and magnetostatics.*

An analytic function of the complex variable $\boldsymbol{z} = x + \boldsymbol{i}y$ is representable by its Taylor series:

$$\boldsymbol{B}(\boldsymbol{z}) = B_y + \boldsymbol{i}B_x = \sum_{n=0}^{\infty} (\boldsymbol{z} - \boldsymbol{z}_0)^n \frac{1}{n!} \frac{d^n \boldsymbol{B}}{d\boldsymbol{z}^n}\bigg|_{\boldsymbol{z}=\boldsymbol{z}_0}$$

The multipole field expansion, a particular form of the Taylor series:

$$\boldsymbol{B}(\boldsymbol{z}) = \sum_{n=1}^{\infty} \boldsymbol{C}_n \left(\frac{\boldsymbol{z}}{R_{ref}}\right)^{n-1}, \text{ here } \boldsymbol{C}_n = \frac{R_{ref}^{n-1}}{(n-1)!} \frac{d^{n-1}\boldsymbol{B}}{d\boldsymbol{z}^{n-1}}\bigg|_{\boldsymbol{z}=0}$$

Multipoles due to the current density $\boldsymbol{j}$ and the magnetization density $\boldsymbol{m} = m_x + \boldsymbol{i}m_y$ in an area

$$\boldsymbol{C}_n = -\frac{\mu_0 R_{ref}^{n-1}}{2\pi} \iint \frac{\boldsymbol{j}(x,y)}{(x+\boldsymbol{i}y)^n} dx\,dy$$

$$\boldsymbol{C}_n = \boldsymbol{i}n \frac{\mu_0 R_{ref}^{n-1}}{2\pi} \iint \frac{\boldsymbol{m}(x,y)}{(x+\boldsymbol{i}y)^{n+1}} dx\,dy$$

Integrals for finite elements with certain shape functions can be calculated exactly

~*HRMCALC* - **ANSYS** COMMAND TO CALCULATE COMPLEX FIELD EXPANSION COEFFICIENTS.

INPUT PARAMETERS

External **ANSYS** command ~*HRMCALC* calculates the complex field expansion coefficients for 2-D magnetic or magneto-structural problems, accounting separately the contribution of elements with the current density defined and elements with non-zero magnetisation. It does not accept any command line arguments, but is controlled by several variables, which may be set to desired values before issuing the command.

Table 1 - ~*HRMCALC* input variables

Variable	Meaning	Default value
REF_RAD	Reference radius(1)	1.0
MU_ZERO	Free space permeability(1)	$2{\cdot}\pi{\cdot}10^{-7}$
HARM_OUT	Number of harmonics to calculate(1)	15
X_C_OUT, Y_C_OUT	Expansion centre(1)	0.0, 0.0
X_SYMM, Y_SYMM	Symmetry keys(1)	0, 0
OFF_STAT	Statistics printout key(1)	0
OFF_DISP	"Ignore displacements" key(1)	0

REF_RAD of the dimension *[length]* is the reference radius as defined in formula (1).

[1] See detailed description below

MU_ZERO of the dimension *[inductance]·[length]*$^{-1}$ is the ratio between flux density and field in free space. In the SI(MKS) unit system it is equal to $2{\cdot}\pi{\cdot}10^{-7}$ H/m, in CGS(Gauss' system) it is equal to 1. If MU_ZERO is not set by user, MKS system assumed.

X_C_OUT, Y_C_OUT of the dimension *[length]* define the origin of the Cartesian coordinate system (assumed to be parallel to the global the Cartesian coordinate system) where harmonics will be calculated. If X_C_OUT, Y_C_OUT are not set by user, harmonics will be calculated at the global Cartesian coordinate system origin.

OFF_STAT controls the printout of the ~*HRMCALC* command. If OFF_STAT is not set or set to a value ≤ 0, then an additional information (number of elements with defined current density and magnetisation, status of control variables) will be reported. If OFF_STAT 0, printout is suppressed.

OFF_DISP : if this variable is set to a positive value, then ~*HRMCALC* calculates harmonics, ignoring displacements (caused by mechanical loads) of elements. Note, that switch can lead to insensible results, if there are elements with non-zero magnetisation, because the magnetisation will be anyway calculated for displaced elements.

X_SYMM, Y_SYMM define the symmetry of the problem relatively lines x = 0 and y = 0 (in the global Cartesian system). With the default values 0,0 no symmetries are assumed. That is, in such a case ~*HRMCALC* calculates harmonics so as the model represents the whole magnet. If X_SYMM ≠ 0, then it is assumed that every element has a mirror image relatively the line x = 0. If X_SYMM > 0, then the mirrored current has the same sign and if X_SYMM < 0, it has the opposite sign. Similar conventions for the symmetry relatively the line y = 0 are accepted if Y_SYMM ≠ 0. In terms of flux all possible situations are listed below:

- X_SYMM = 0 or not set : no conditions of flux are set on the line x= 0;
- X_SYMM > 0 : flux is normal to the line x= 0;
- X_SYMM < 0 : flux is parallel to the line x= 0;
- Y_SYMM = 0 or not set : no conditions of flux are set on the line y= 0;
- Y_SYMM > 0 : flux is normal to the line y= 0;
- Y_SYMM < 0 : flux is parallel to the line y= 0;

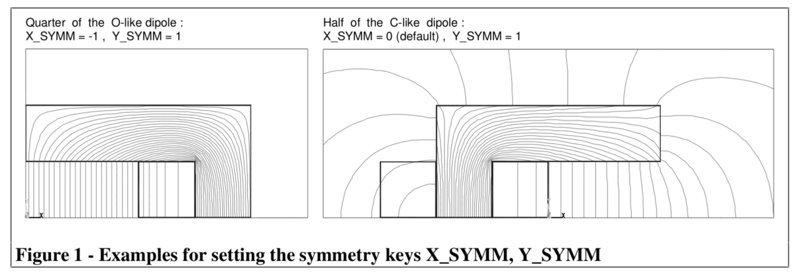


Figure 1 - Examples for setting the symmetry keys X_SYMM, Y_SYMM

OUTPUT PARAMETERS

~*HRMCALC* places results of the harmonic calculation in two arrays : CURR_HRM (harmonics of the field induced by elements with non-zero current density) and MAGN_HRM (harmonics of the field induced by elements with non-zero magnetisation. Both arrays have the dimensions (HARM_OUT, 2). First index of the arrays is the harmonic number (1 - dipole, 2 - quadrupole, 3 - sextupole, etc.), second index meaning is as follows : 1 - real ("normal") term of the harmonic, 2 - imaginary ("skew") one. For instance, the element CURR_HRM(2,2) gives the value of skew quadrupole for the field induced by currents, and the sum CURR_HRM(3,1) + MAGN_HRM(3,1) yields the value of normal sextupole for the whole field. Note, there is *no need to create these array* before to issue the ~*HRMCALC* command. To learn more about arrays in **ANSYS**, read the **ANSYS** help on the ***DIM** command.

RESTRICTIONS AND TIPS

Present version of ~*HRMCALC* cannot treat properly models which have symmetries relatively lines other then x = 0, y = 0. In particular, does not include special switches for multipole symmetries. This option will be implemented in future. In order to keep the compatibility with future version it is strongly recommended to use values ±1 if X_SYMM, Y_SYMM are desired to be non-zero.

However, present version can be also used to simulate high-order magnets in an efficient way, if user will follow certain rules:

- make the ANSYS input for the smallest non-symmetrical part of the magnet, that is, 45° sector for a quadrupole, the 30° sector for a sextupole, 22.5° for an octupole, etc. The sector must be located at the global coordinate system origin and one of the borders must coincide with the line x = 0 or y = 0. Use for the meshing PLANE13 elements, KEYOPT(1) = 4, if you have a coupled, magneto-structural problem and PLANE53 if the problem is purely magnetic.
- set magnetic boundary conditions on azimuthal borders (remember, the absence of constrains it equivalent to the flux-normal condition). Set corresponding value of X_SYMM or Y_SYMM depending on which border line lies on the coordinate system axis. For instance, if the is a border on the line y = 0 and the flux is parallel to the border, then Y_SYMM = -1 must be set. Do not define X_C_OUT, Y_C_OUT. In order to improve the solution precision is recommended not to set a constrain on the flux on the outer radial border, but to mesh it with the infinite boundary elements INFIN9. The input in such a case must include the outermost air area.
- if mechanical DOFs are included, constrain UX, UY at the sector center and outer arc. On the radial border lines constrain the displacement, normal to the lines. Set Maxwell surfaces on the borders of ferromagnetic areas.
- after the problem solution run ~*HRMCALC*.

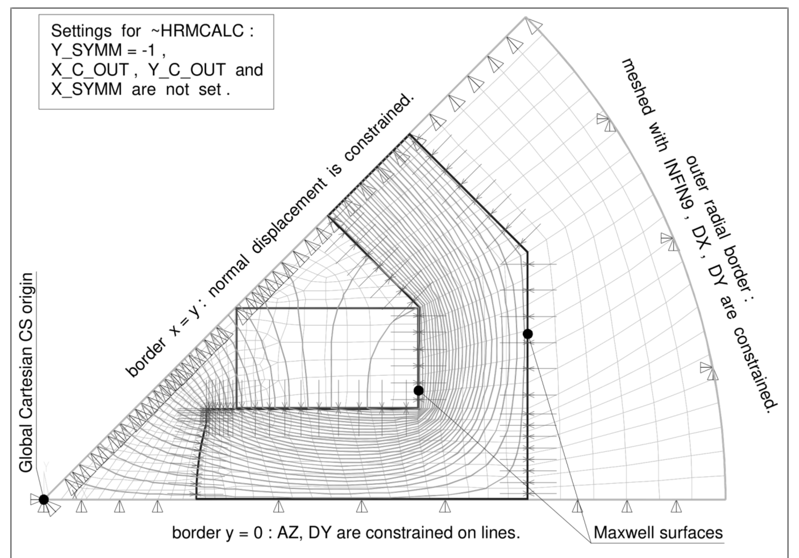


Figure 2 - Quadrupole input example. The magneto-structural element PLANE13 is used.

~*HRMCALC* will calculate harmonics of the field, which would be induced just by "doubled" input (completed with the mirror image relatively the line where X_SYMM or Y_SYMM is set). In other words, it will be harmonics created by $1/(2{\cdot}N)$ part of whole magnet. In general all elements of the arrays CURR_HRM and MAGN_HRM may be filled with non-zero values. However, in the whole magnet with 2*N*-pole symmetry only harmonics $(2{\cdot}k+1){\cdot}N$, here the *k* is a non-zero integer, can exist. For example in a dipole magnet ($N = 1$), only odd harmonics are allowed, for a quadrupole magnet ($N = 2$) allowed numbers of harmonics are: 2, 6, 10, 14, … , $2{\cdot}(2{\cdot}k+1)$.

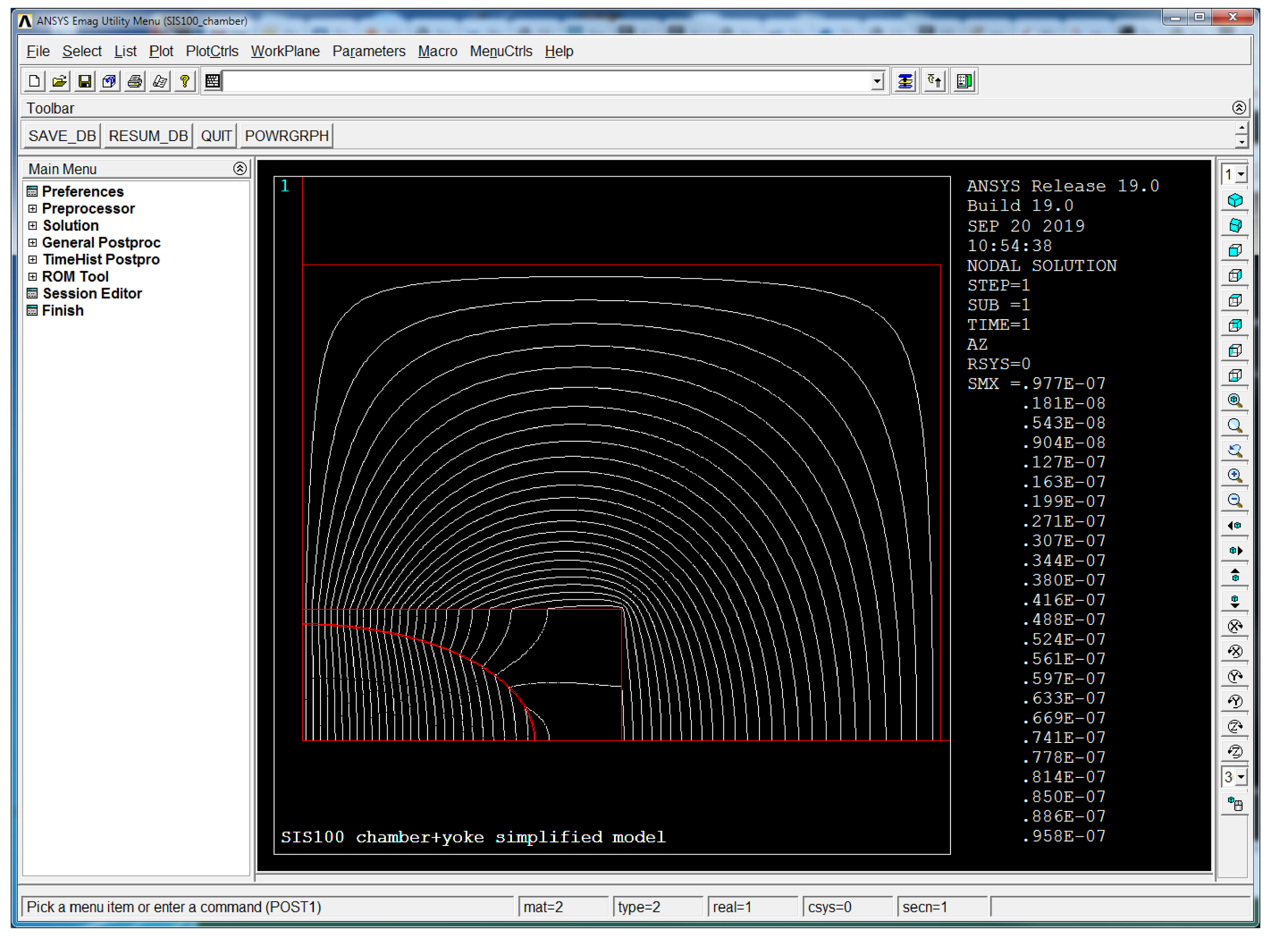
In order to obtain harmonics for the whole 2*N*-pole magnet, allowed elements of the arrays CURR_HRM and MAGN_HRM must be multiplied by 2*N* (= 4 for a quadrupole, = 6 for a sextupole, etc.) and non-allowed should be assigned to zero.

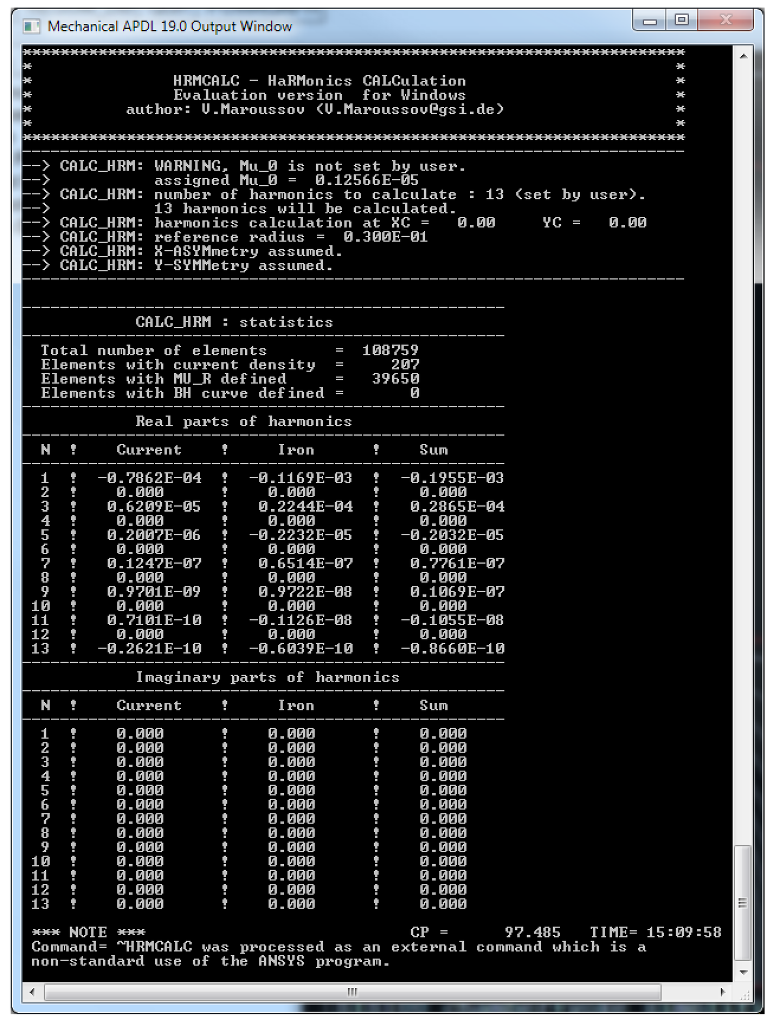
GENERAL REMARKS

~*HRMCALC* works especially well, providing a high precision, when the iron yoke is located relatively far from the magnet coil and the field is mostly created by conductors (that is typical for high-field superconductor magnets). If the field is mostly shaped by the yoke, check carefully the result, solving your problem with various mesh configurations.

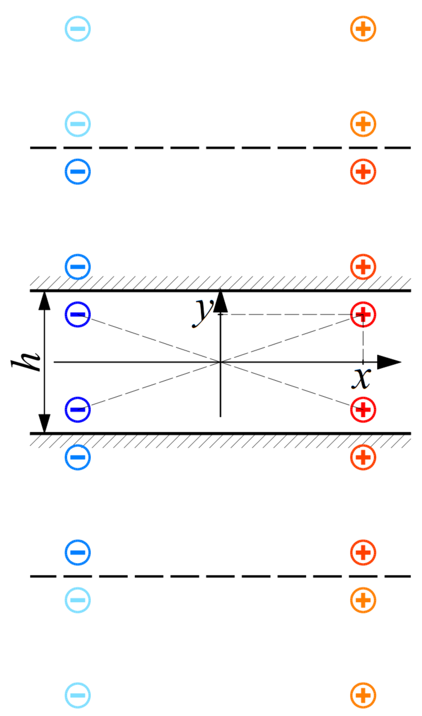
With this approach, only the magnetization density is used from the FEM solution. Effectively, the Biot-Savart option is enabled for both conductors and the yoke.

Case study: multipoles due to eddy currents in the vacuum chamber of SIS100 dipole magnet





Target values calculation: method of images +integration over contour



Results for various mesh densities

Target values: multipoles by the method of images, [*T*] @ R_{ref} = 3 cm						
B_1	B_3	B_5	B_7	B_9	B_{11}	B_{13}
-1.955E-04	2.867E-05	-2.014E-06	8.054E-08	1.127E-08	-1.011E-09	-3.777E-11

Number of elements			(Calculated in ANSYS) ⁄ (Target values)						
Total	With current	With μ_r >1	B_1	B_3	B_5	B_7	B_9	B_{11}	B_{13}
155728	248	57183	0.9998	0.9994	1.0090	0.9642	0.9438	1.0242	1.7713
108759	207	39650	0.9998	0.9994	1.0090	0.9636	0.9482	1.0440	2.2929
69592	168	25480	0.9998	0.9994	1.0090	0.9636	0.9491	1.0499	2.4147
39186	127	14430	0.9998	0.9994	1.0090	0.9667	0.9340	0.9842	1.1779
17568	87	6500	0.9998	0.9990	1.0090	0.9685	0.9225	1.0331	-0.9452

Results are stable over ×10 mesh density range