

## 1. Introduction

In this paper, an optimized method to design superconducting magnets for 14T actively shielded MRI is proposed. At first, the feasible current-carrying region assumed as possible coil zone is subdivided into two-dimensional array grids, in which each grid represents one loop of a current-carrying conductor. Then the current distribution is obtained using linear programming by a detailed consideration of the superconductor consumption, central field strength, imaging region homogeneity and stray field leakage range. Afterwards the initial rectangular section of the magnet is determined by reshaping the current-carrying region and then the method of non-linear programming is adopted to refine the final configuration of each coil with the limitation of each coil position and section size. In addition, the spherical harmonics elimination of central magnetic fields is proposed as the optimization strategies used in the non-linear programming method to get the least level of inhomogeneity over the imaging region. Besides all the current centers are chosen based on the location of actual superconducting wires of each conductor. Finally this design method has obtained high field homogeneity in the central zone by four sets of coils with total length around 3.5m and inner diameter nearly 1m. The detailed analysis and optimization approach will be presented.

## 2. Optimization method

### 2.1 Linear Programming

The two-dimensional array source grids shown in Fig. 1 are the candidate domain for the main and shielded coils in which each grid represents one current loop of the actual conductor coaxial with the z-axis. The basic structure of the conductor shown in Fig. 2 is comprising of Nb<sub>3</sub>Sn Rutherford cable (each strand shown as a yellow circle) and cooper stabilizer channel (orange):

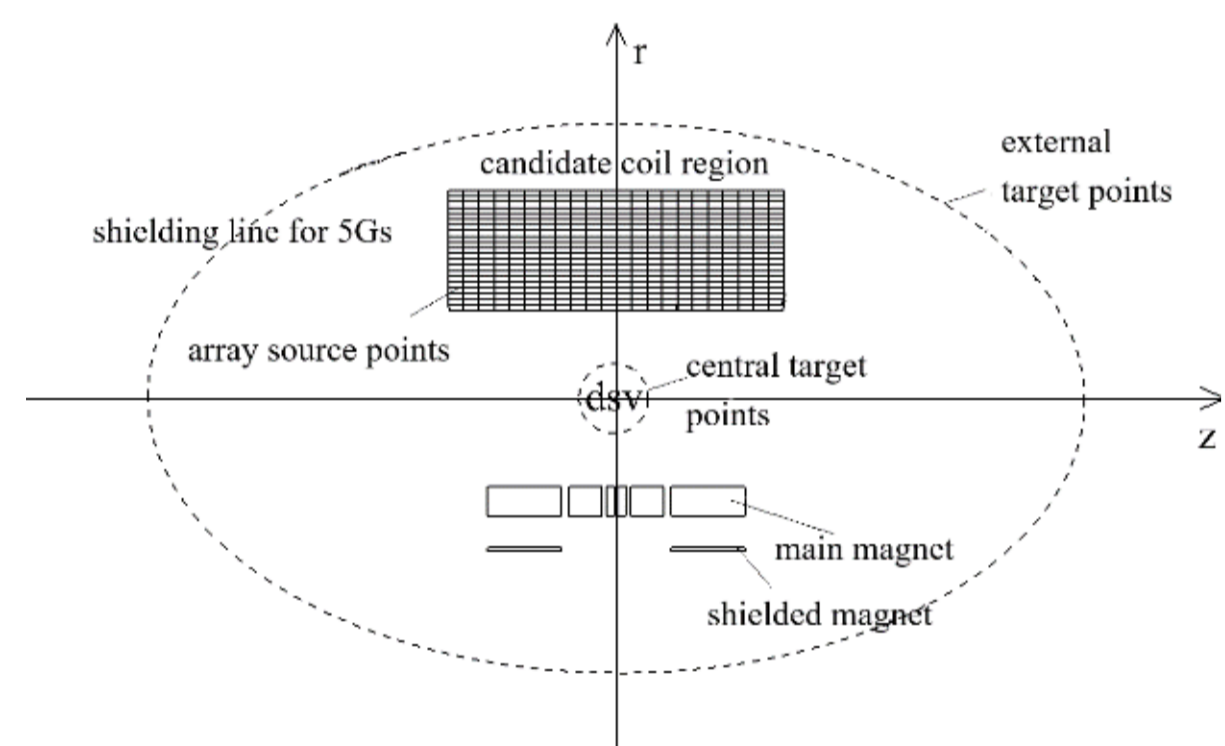


Fig. 1. Space arrangement of magnets, DSV and 5Gs stray field.

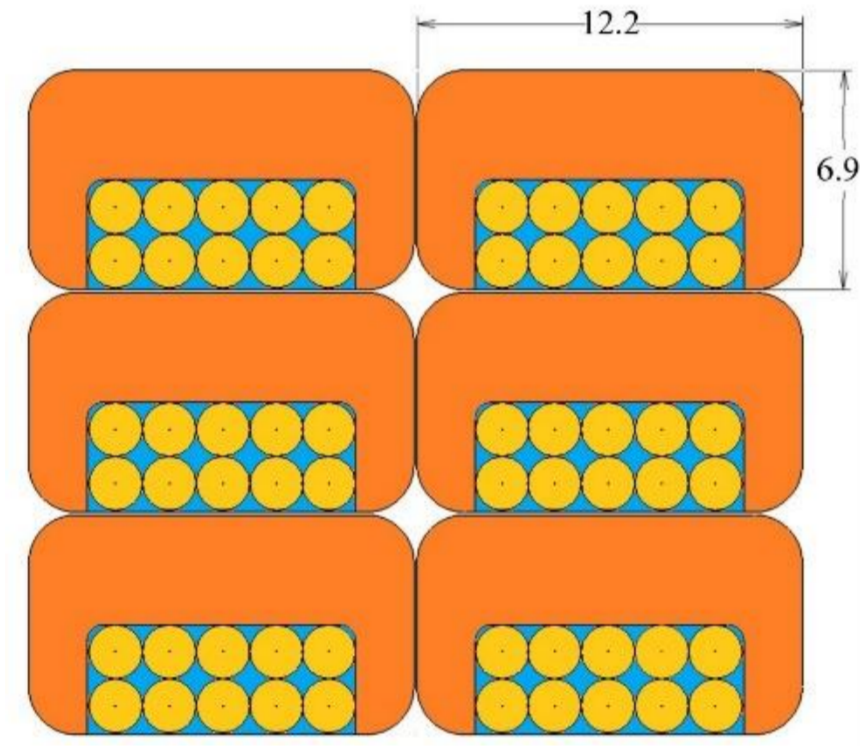


Fig. 2. Structure of Nb<sub>3</sub>Sn Rutherford cable-in-channel (RIC) conductor.

The magnetic field contribution of the  $j$ -th conductor current to the  $i$ -th central or external target point is:  $B_{ij} = b_{ij}I_j$ . In this way, the magnetic field equation for the target points on the DSV edge and shielding line can be established as:

$$\begin{aligned} B_{zdsv} &= A_{mk} \cdot I \\ B_{zstray} &= B_{nk} \cdot I \\ B_{rstray} &= C_{nk} \cdot I \end{aligned} \quad (1)$$

The formulation of linear programming can be obtained as follows: Where  $F$  is the objective function to describe by minimizing the volume of the superconductors.  $r_j$  and  $I_j$  denote radius and current of the  $j$ -th coil.  $J$  denotes the setting of current density in a superconductor.  $B_0$  is the desired magnetic field strength,  $\varepsilon$  refers to the homogeneity factor, and  $I_{max}$  is the maximum current for one conductor.

$$\text{Min } F = \frac{2\pi}{J} \sum_{j=1}^k r_j |I_j|$$

Subject to:

$$\begin{aligned} A_{mk}I &\leq B_0(1 + \varepsilon) \\ -A_{mk}I &\leq -B_0(1 - \varepsilon) \\ |B_{nk}I| &\leq B_{zstray} \\ |C_{nk}I| &\leq B_{rstray} \\ |I| &\leq I_{max} \end{aligned} \quad (2)$$

The initial configuration of the current region after LP is shown in Fig. 3. (a). The red region represents the main magnet with a positive current, blue represents the shielded magnet with a negative current, while most of the areas with no current are green. Since the region with current is irregular, reshaping and repositioning process is necessary. And after every adjustment, the settings of LP should be redefined to get the final rectangular section of the magnet shown in Fig. 3. (b). It is obviously seen that the axial edge of the shield magnet shall not exceed the end of the main magnet for the convenience of engineering installation and fixation.

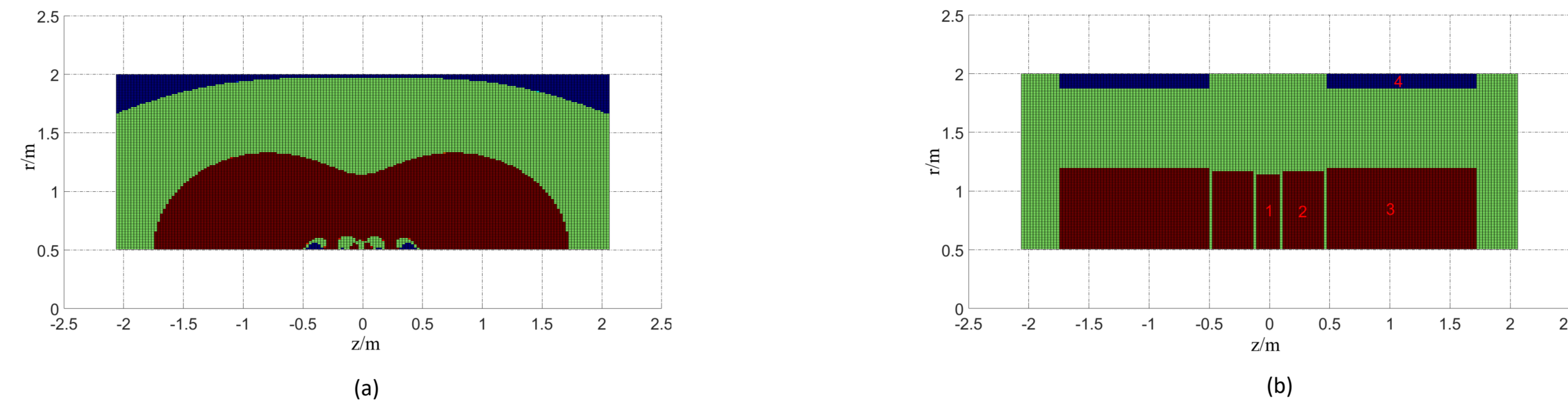


Fig. 3. Current distribution of Linear Programming (a) initial configuration (b) after adjustment.

The main purpose of LP is to provide initial magnet configuration as shown in Fig. 3. (b) for the subsequent non-linear optimization. The better the initial value is, the faster the convergence rate of the non-linear optimization will be.

### 2.2 Non-Linear Programming

The axial component of the central magnetic field at a point  $(r, \theta)$  derived from one current-carrying circular coil coaxial with the z-axis is given by:

$$B_z = \frac{\mu_0 I}{2r} \sum_{n=0}^{\infty} \sin\theta' P_{n+1}^1(\cos\theta') \cdot \left(\frac{r}{a}\right)^{n+1} P_n(\cos\theta) \quad r < a \quad (3)$$

Where  $a, \theta'$  denote the radius and azimuth angle of the circular current. If the coaxial coils are configured symmetrically along the mid-plane with equal current, all odd terms of  $B_z$  will be eliminated but all even terms will duplicate because:

$$P_{n+1}^1(\cos\theta) = (-1)^{n+2} P_{n+1}^1(\cos(\pi - \theta)) \quad (4)$$

Then the remaining terms are as follows: The term  $n=0$  represents the constant term of the magnetic field  $B_0$  which should be reserved, and the other terms need to be eliminated as many as possible.

$$B_z = \mu_0 I \sin\theta' P_{n+1}^1(\cos\theta') a^{-(n+1)} \cdot r^n P_n(\cos\theta) \quad r < a, n = 0, 2, 4 \dots \quad (5)$$

In this way, the sphere harmonics elimination strategy can be established, where  $W_n$  refers to the remaining source terms.

$$W_n = \sum_{j=1}^K \mu_0 I \sin\theta_j P_{n+1}^1(\cos\theta_j) \left(\frac{1}{a_j}\right)^{n+1} \quad r < a, n = 0, 2, 4 \dots \quad (6)$$

$K$  refers to the number of the current centers which is determined by the total number of actual superconducting strands of all the Rutherford cable in channel (RIC) conductors shown in Fig. 2. This method considers the detailed current-carrying locations which makes the MRI magnet design more feasible and accurate. At last the formulation of non-linear programming can be obtained as follows:

$$\begin{aligned} \text{Min } Q &= [(W_0 - B_0) + W_2 + W_4 + W_6 + W_8] \times 10^6 \\ \text{Subject to:} \\ \sqrt{B_{zstray}^2 + B_{rstray}^2} - B_{shield} &< 0 \\ r_{min}(i) &\leq r_{inner}(i) \leq r_{min}(i) + dr(i) & i = 1, 2, 3, 4 \\ r_{outer}(i) &= r_{inner}(i) + Nr(i) \cdot tw & i = 1, 2, 3, 4 \\ z_{right}(i-1) &\leq z_{left}(i) & i = 2, 3 \\ z_{right}(i) &= z_{left}(i) + Nz(i) \cdot nw & i = 1, 2, 3, 4 \\ z_{right}(4) &\leq z_{right}(3) \\ 0 &\leq z_{left}(1) \end{aligned} \quad (7)$$

Where  $Q$  is the objective function to describe by minimizing the central field inhomogeneity.  $i$  refers to the mark of magnet shown in Fig. 3. (b). Since the coil is symmetrical, only the right half part will be considered, and the coil region marked 1 denote the section of  $z \geq 0$ .  $B_{shield}$  is the limited value of the stray field,  $r_{inner}$  and  $r_{outer}$  denote the inner and outer radius, while  $z_{left}$  and  $z_{right}$  are the left and right axial

positions of each coil. And  $r_{min}$  is the minimum inner radius for each coil, and  $dr$  is the allowable radial position adjustment.  $Nr$  and  $tw$  denote the number of conductor layers and the width of the conductor in the radial direction.  $Nz$  and  $nw$  represent the number of conductor turns and the length of the conductor in the axial direction. The shield coils share the same power supply with the main coils but carry current in the opposite direction.

## 4. Optimization Results

The homogeneity over the 45cm DSV region and the distribution of the stray field calculated from the final magnet structure are shown in Fig. 4. (a) and (b) respectively. As shown in these figures, after optimization of non-linear programming the inhomogeneity over the 45cm DSV has been reduced to around 1ppm and the 5 gauss line of the stray field are restricted to a cylinder with 10.5m radius and 11.5m half-length. All of these results can meet our design requirements.

