A Novel and Fast Method for Inductance and Force Calculation of Multi-Coaxial Coils for NMR Magnet Design

MT 26 International Conference on Magnet Technology
Vancouver, Canada | 2019

Mon-Af-Po1.13-06 [26]

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Introduction

Motivation

- A comparative study is important to validate result.
- More lower computational cost for optimization purposes based on accurate result is also meaningful.

Approach

(1) The inductance and force formulas were derived by the magnetic field based on Biot-Savart law, and each of values was calculated by self-developed MATLAB functions enabling double integrations employing Legendre-Gauss quadrature with no concern of singularity; (2) The calculation result are compared with the traditional method; (3) As a simple implementation of our approach, the self-inductance and axial force of the MIT 1-GHz NMR magnet design were calculated, and the running time were compared with those obtained by conventional methods.

Basic Formula

A. Magnetic Field Calculation for Any Interested Area

As shown in Fig.1 the expression of the components of magnetic flux density **B** at point $P(\rho, \varphi, z)$ can be given by double integration then summed by each coil, one can find them as follows:

$$B_\rho = \sum_{i=1}^{N} \frac{\mu_0 J_i}{2\pi} \int_{a_{1i}}^{a_{2i}} \int_{b_{1i}}^{b_{2i}} \frac{1}{\rho\sqrt{(a+\rho)^2+(z-b)^2}} \left[-K + \frac{a^2+\rho^2+(z-b)^2}{(a-\rho)^2+(z-b)^2} E\right] da\, db$$

$$B_\varphi = 0$$

$$B_z = \sum_{i=1}^{N} \frac{\mu_0 J_i}{2\pi} \int_{a_{1i}}^{a_{2i}} \int_{b_{1i}}^{b_{2i}} \frac{1}{\rho\sqrt{(a+\rho)^2+(z-b)^2}} \left[K + \frac{a^2-\rho^2-(z-b)^2}{(a-\rho)^2+(z-b)^2} E\right] da\, db$$

The Legendre-Gauss Quadrature are used for the double integral by programmed calculation function *rzBI()* on MATLAB.

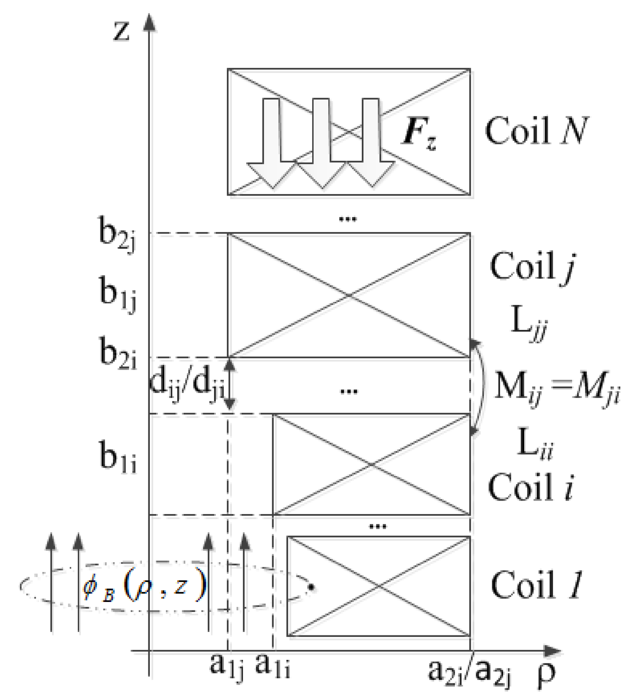

Fig. 1. A Quater of Multi-Coils Solenoid Schematic Diagram

B. Inductance Calculation

The magnetic flux ϕ_B is the surface integral of the normal component of the magnetic field B passing through that surface, which can be found as:

$$d\phi_B = B \cdot dS$$

As shown in Fig.1, for an axisymmetric model, the flux linking a loop with radius ρ is

$$\phi_B(\rho, z) = \int_{\theta=0}^{\theta=2\pi} \int_{r=0}^{r=\rho} B_z(r) r dr\, d\theta = 2\pi \int_{r=0}^{r=\rho} B_z(r) r dr$$

Where $\phi_B(\rho, z)$ is a flux function of any round loop located on solenoid cross section and $B_z(r)$ is axial components of $\boldsymbol{B}$, it can be measured by rzBI() function.

Thus, the total flux linkage λ linked to any coil is

$$\lambda = N \frac{1}{(a_{2i}-a_{1i})(b_{2i}-b_{1i})} \int_{\rho=a_{1i}}^{\rho=a_{2i}} \int_{z=b_{1i}}^{z=b_{2i}} \phi_B(\rho, z) d\rho\, dz$$

Last, the self-inductance and mutual inductance between coil *i* and coil *j* are:

$$L_{ii} = \frac{\lambda_{ii}}{I_i} = \frac{N_i \int_{\rho=a_{1i}}^{\rho=a_{2i}} \int_{z=b_{1i}}^{z=b_{2i}} \phi_B(\rho, z) d\rho\, dz}{(a_{2i}-a_{1i})(b_{2i}-b_{1i}) I_i}$$

$$M_{ii} = \frac{\lambda_{ij}}{I_i} = \frac{N_j \int_{\rho=a_{1j}}^{\rho=a_{2j}} \int_{z=b_{1j}}^{z=b_{2j}} \phi_B(\rho, z) d\rho\, dz}{(a_{2j}-a_{1j})(b_{2j}-b_{1j}) I_i}$$

C. Axial Force Calculation

To determine the axial force, the Lorentz force can be expressed as:

$$d\boldsymbol{F} = \boldsymbol{J} \times \boldsymbol{B} dv$$

where

$$\boldsymbol{J} \times \boldsymbol{B} = \begin{vmatrix} a_\rho & a_\varphi & a_z \\ 0 & |J| & 0 \\ B_\rho & B_\varphi & B_z \end{vmatrix} \quad \text{and} \quad |J_i| = \frac{N I_i}{(a_{2i}-a_{1i})(b_{2i}-b_{1i})}$$

Then, Integrate over the solenoid volume, one can get the axial force as follows:

$$F = \int_{vol} J \times B dv = \int_{vol} \left(|J| B_z(\rho, z) a_\rho - |J| B_\rho(\rho, z) a_z\right) dv$$
$$= \int_{a_1}^{a_2} \int_{b_1}^{b_2} 2\pi \frac{NI}{(b_2-b_1)(a_2-a_1)} \rho B_z(\rho, z) a_\rho d\rho\, dz$$
$$- \int_{a_1}^{a_2} \int_{b_1}^{b_2} 2\pi \frac{NI}{(b_2-b_1)(a_2-a_1)} \rho B_\rho(\rho, z) a_z d\rho\, dz = F_\rho - F_z$$

Because of axisymmetric of F_ρ on the overall solenoid, so $F_\rho = 0$, and the axial force F_z is:

$$F_z = \int_{a_1}^{a_2} \int_{b_1}^{b_2} 2\pi \frac{NI}{(b_2-b_1)(a_2-a_1)} \rho B_\rho(\rho, z) a_z d\rho\, dz$$

Numerical Validation

To verify the accuracy of the proposed method, the following set of example are applied.

A. Self-inductance

TABLE I
SELF-INDUCTANCE CALCULATION COMPARISON

r (m)	Analytical (mH)	MagNet™ (mH)	Proposed Method (mH)
0.1	254.91	254.13	254.95
0.2	509.82	508.27	509.91
0.3	764.73	762.41	764.86
0.4	1019.64	1016.54	1019.82
0.5	1274.55	1270.68	1274.77

As for $r = 0.1m$, the proposed method difference ratio compared to analytical result is 0.016%, however the FEM method is 0.306%.

B. Mutual-Inductance

TABLE II
MUTUAL-INDUCTANCE CALCULATION COMPARISON

d_{12} (m)	Kajikawa (mH)	R.Ravard (mH)	Proposed Method (mH)
0	0.776	0.7753904	0.7748074
0.005	0.571	0.5712872	0.5713019
0.01	0.435	0.4348317	0.4348257
0.02	0.267	0.2667799	0.2667747
0.05	0.0823	0.0823239	0.0823220
0.1	0.021	0.0209659	0.0209653
0.2	0.00386	0.0038579	0.0038578
0.3	0.0013	0.00130199	0.00130196
0.5	0.000312	0.0003119	0.0003119

The maximum relative calculation difference ratio 0.15%, and minimum difference ratio is 0 within 7 decimal places.

C. Axial Force

TABLE III
MUTUAL-INDUCTANCE CALCULATION COMPARISON

d_{12} (m)	Filament (mN)	R.Ravard (mN)	Garrett (mN)	Proposed Method (mN)
-0.3	0	0	0	0
-0.15	70.105646	70.401487	70.409863	70.409874
0	76.707864	77.003143	77.012527	77.012535
0.05	66.487895	66.730553	66.738826	66.738831
0.3	22.268595	22.279495	22.283131	22.283130
1	1.6785743	1.6715419	1.6723566	1.6723564
5	0.0068950	0.0068087	0.0068580	0.0068580

The axial distance $d_{12} = -0.3$, the axial force is zero, and when the axial distance $d_{12} = 0$, it has maximum axial force.

Case Study: 1GHz NMR Magnet

An MIT 1-GHz (23.5T) NMR magnet design is analyzed in this study. The magnet parameters are shown in Fig.2. The half magnet has 20 coils that are count from bottom to top coil.

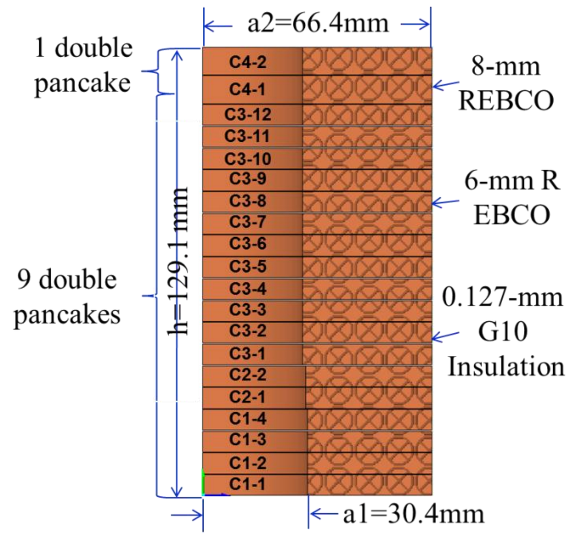

Fig. 2. A quarter cross-section model of the MIT 1-GHz (23.5-T) NMR magnet

A. Inductance

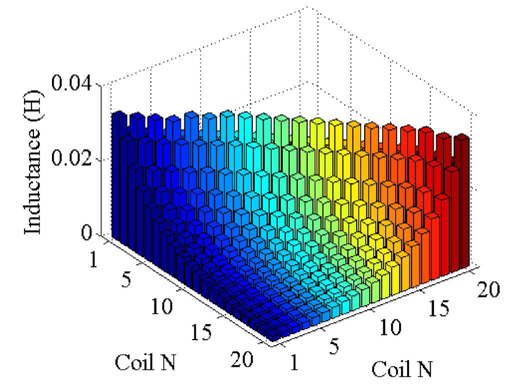

Fig.3. The inductance calculation Result

B. Axial Force

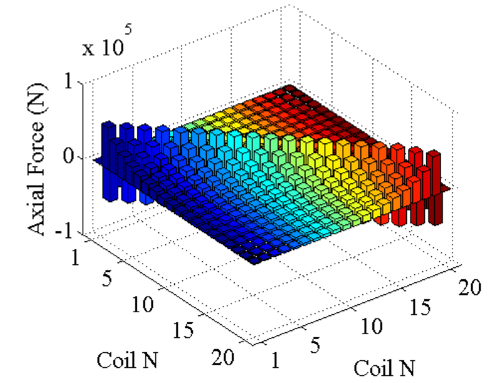

Fig.4. The air force calculation Result

The total inductance of the 23.5T magnet is 10.1954H, compared to Garrett method 10.1928H and FEM method 10.1546H.

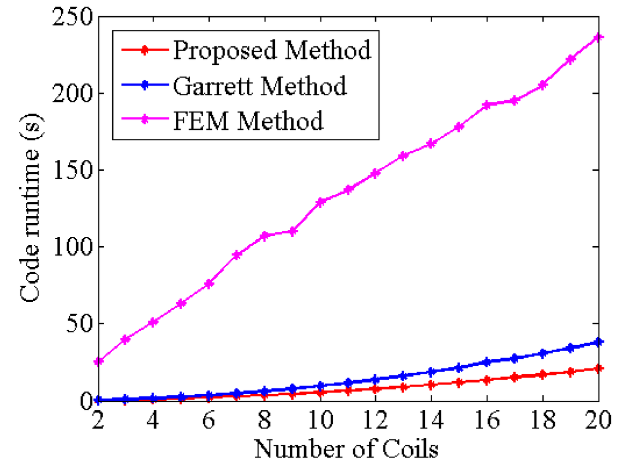

Fig.5. The code runtime with respect to coil number

The proposed method with 5 Gaussian points calculation time is 20.97s, Garrett method is 37.99s, FEM method is 236.2s.

Conclusion

Result

- The results obtained by suggested method are in excellent agreement with other methods.
- This procedure are more general to calculate most solenoids.
- This approach has an obvious advantage for computational time advantage in multi-coils NMR system.
- The method can calculate inductance and force separately.