

An Improved Self-Consistent Model and its Application to Estimate the Critical Current of REBCO Magnet



Tianyong Gong, Guangtong Ma, Ruichen Wang, Songlin Li, Chunxing Yao, Zhen Luo, Xingchao Nie
 Applied Superconductivity Lab., State Key Lab. of Traction Power, Southwest Jiaotong University, Chengdu, China.
 School of Electrical Engineering, Southwest Jiaotong University, Chengdu, China.

Presenter: G. T. Ma

E-mail: gtma@swjtu.edu.cn, gongtianyong178@163.com



INTRODUCTION

Prior to the fabrication of a superconducting magnet, it is crucial to estimate the critical current of the magnet. Several methods have been proposed to estimate the critical current of REBCO magnet, such as the load line method, the current-voltage curve method and the self-consistent model. In the self-consistent model, the non-linear E-J relationship of coated superconductor can be taken into account and only a static electromagnetic problem needs to be solved. Thus, the self-consistent model is very efficient to estimate the critical current with high accuracy.

However, there are several issues about the self-consistent model. For instance, (i) this model is not suitable to a three-dimensional case; (ii) a large number of turns of the REBCO magnet make the modelling extremely complicated and the calculation time-consuming. To address these two issues, this paper will improve the self-consistent model by resorting to a fractionation method and the homogenized technique. With this improved model, the critical current of a generalized racetrack REBCO magnet can be efficiently estimated.

IMPROVED SELF-CONSISTENT MODEL

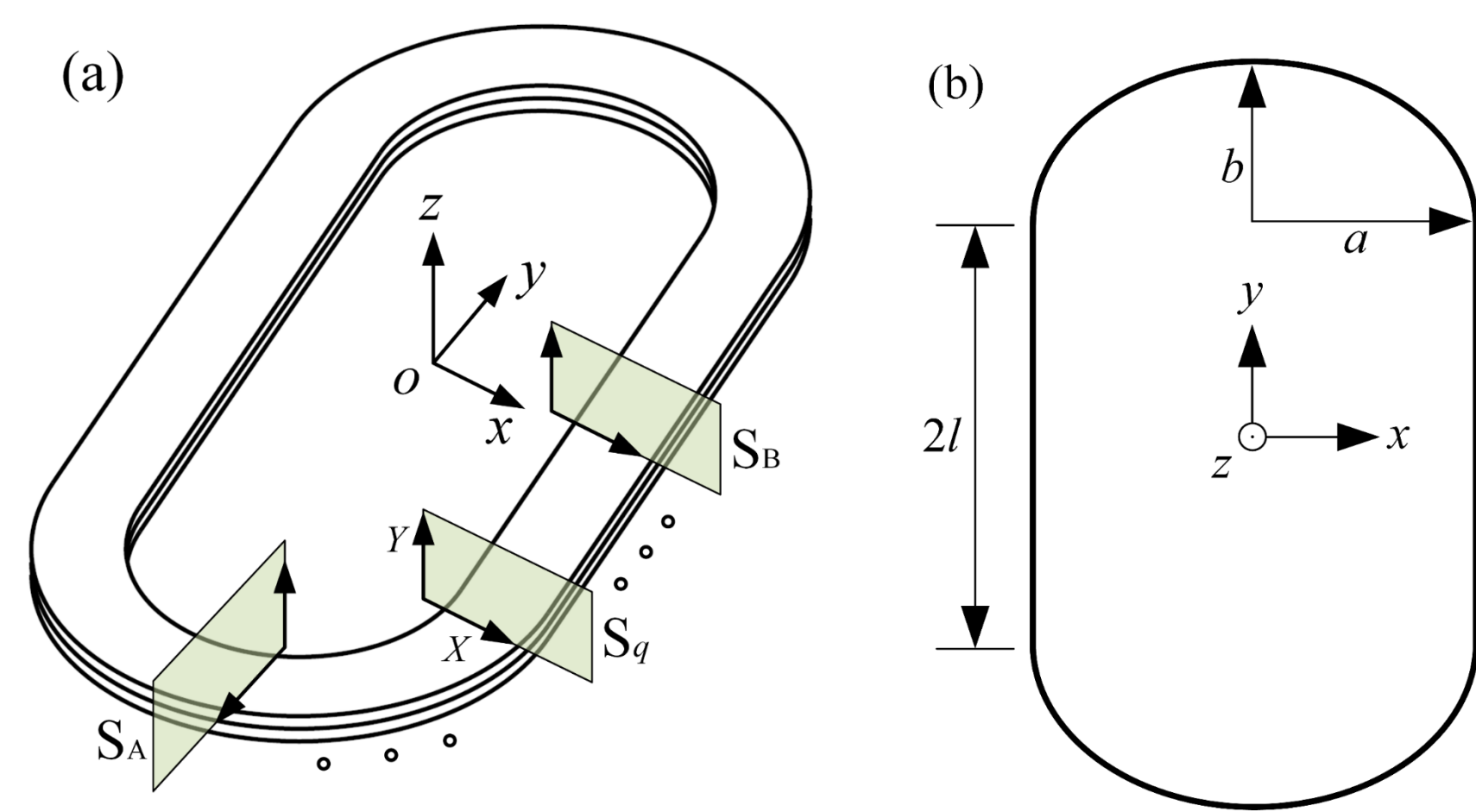


Fig. 1. (a) The layout of a racetrack magnet composed of two single-pancake magnets, and (b) the center line of the racetrack magnet which has a generalized racetrack structure with the straight section and the elliptical section. The Cartesian coordinate system is adopted and the origin coincides with the center of the magnet. These cross-section marked as S_q ($q = 1, 2, \dots, m$) will be adopted to estimate the critical current of the magnet by resorting to the self-consistent model.

The thin-sheet approximation is adopted to treat the coated conductor as a surface. So, the analytical expressions of the self-field can be obtained from these current-carrying surfaces,

$$B_{e,i} = \frac{\mu_0 K}{4\pi} \int_{-\pi}^{\pi} [E_i(\theta, w) - E_i(\theta, -w)] d\theta,$$

where μ_0 is the vacuum permeability; K denotes the surface current density, here $K = Ia/(2w)$ and Ia represents the transport current of the magnet; where the subscript i represents the x , y or z direction. The functions E_i are defined to be,

$$E_x(\theta, w) = \sum_{k=1}^N \frac{b_k \cos \theta}{\sqrt{(x - a_k \cos \theta)^2 + (b_k \sin \theta \pm l - y)^2 + (z - w)^2}},$$

$$E_y(\theta, w) = \sum_{k=1}^N \frac{a_k \sin \theta}{\sqrt{(x - a_k \cos \theta)^2 + (b_k \sin \theta \pm l - y)^2 + (z - w)^2}},$$

$$E_z(\theta, w) = \sum_{k=1}^N \frac{((y \mp l)a_k \sin \theta + xb_k \cos \theta - a_k b_k)(z - w)}{R^3 - R(z - w)^2},$$

where R is the distance between the source point and the field point. The functions a_k and b_k are defined to be,

$$a_k = a + 0.5D(2k - N - 1), b_k = b + 0.5D(2k - N - 1)$$

where $+l$ is selected when $0 < \theta \leq \pi$ corresponding to the $+y$ part of the elliptical section and $-l$ is selected when $-\pi \leq \theta \leq 0$ corresponding to the $-y$ part of the elliptical section.

Making the magnetic vector potential A_z as the governing variable, the governing equation of the self-consistent model is,

$$\frac{\partial^2 A_z}{\partial X^2} + \frac{\partial^2 A_z}{\partial Y^2} = -\mu_0 J_c \left(\frac{\partial A_z}{\partial Y} - \frac{\partial A_z}{\partial X} \right) P.$$

Both criteria, the MAX criterion and AVG criterion are adopted to estimate the critical current,

$$P_{MAX} = \frac{E_{c,0}}{E_c} \left| \frac{E_{c,0}}{E_c} \right|^{n-1} \quad \text{and} \quad \frac{\sum_{i=1}^N P_i |P_i|^{m-1} I_i}{\sum_{i=1}^N I_i} = \frac{E_{c,0}}{E_c}.$$

The critical currents of the magnet based on the MAX criterion and AVG criterion can be estimated by

$$I_{c,MAX} = \min \{ I_{c,1}, I_{c,2}, \dots, I_{c,m} \}, \quad I_{c,AVG} = \left(\prod_{q=1}^m I_{c,q} \right)^{-m},$$

where $I_{c,q}$ is the critical current of q -th cross-section.

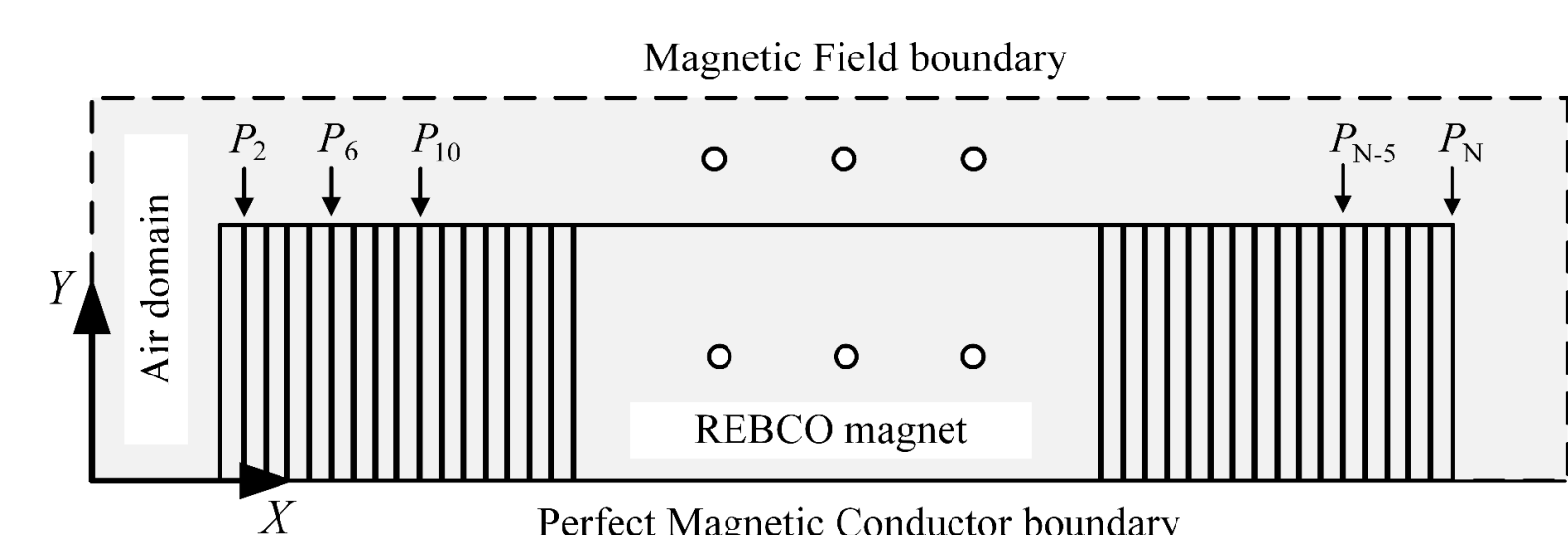


Fig. 2. The self-consistent model of a racetrack REBCO magnet, where the magnetic fields on the Magnetic Field boundary are computed by the analytical expressions of the self-field.

RESULTS AND DISCUSSION

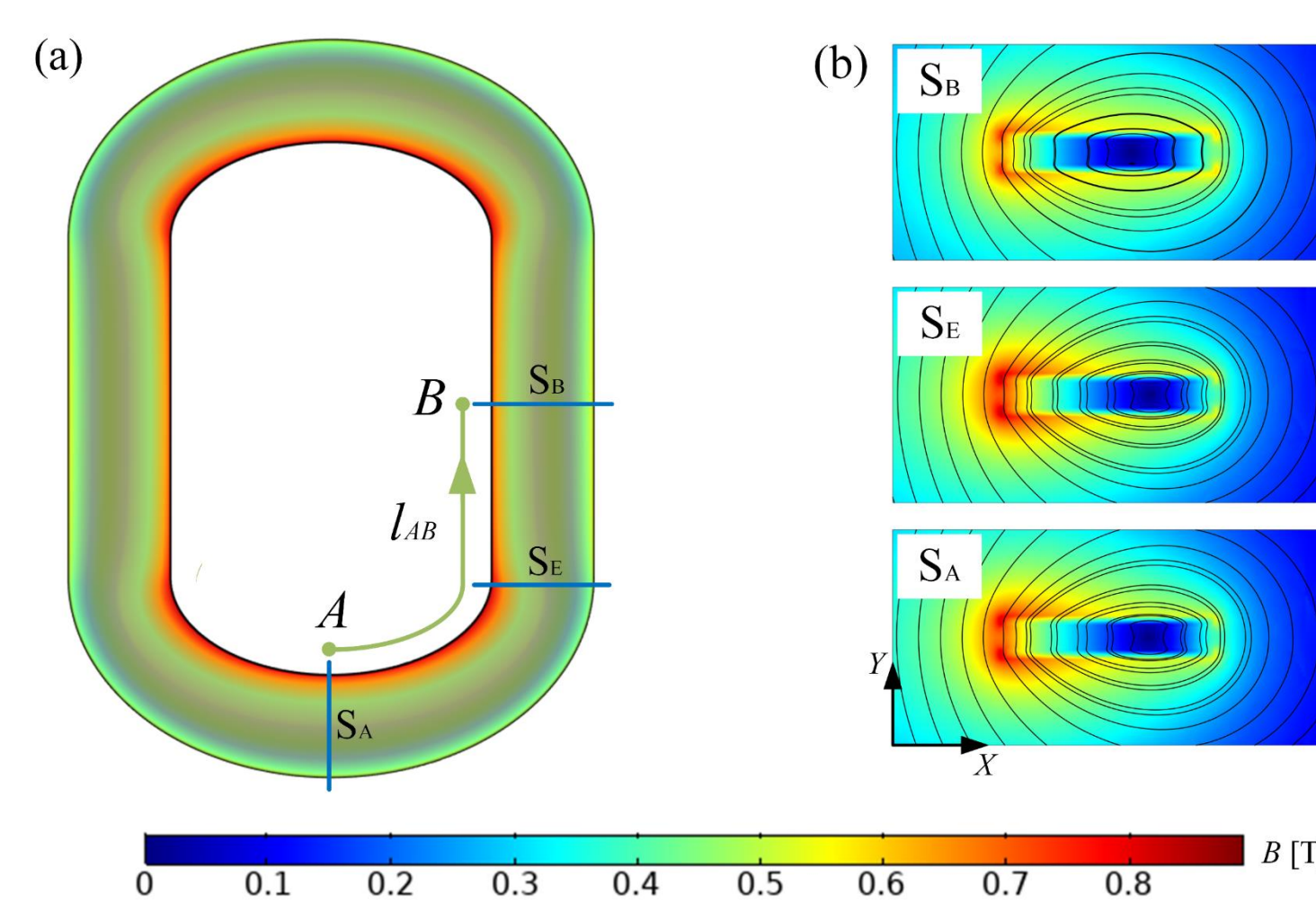


Fig. 3. The magnetic field distributions of (a) a generalized racetrack magnet, where the ratio of minor axis b to major axis a is 0.7 and (b) three specific cross-sections, marked as S_A , S_B and S_E .

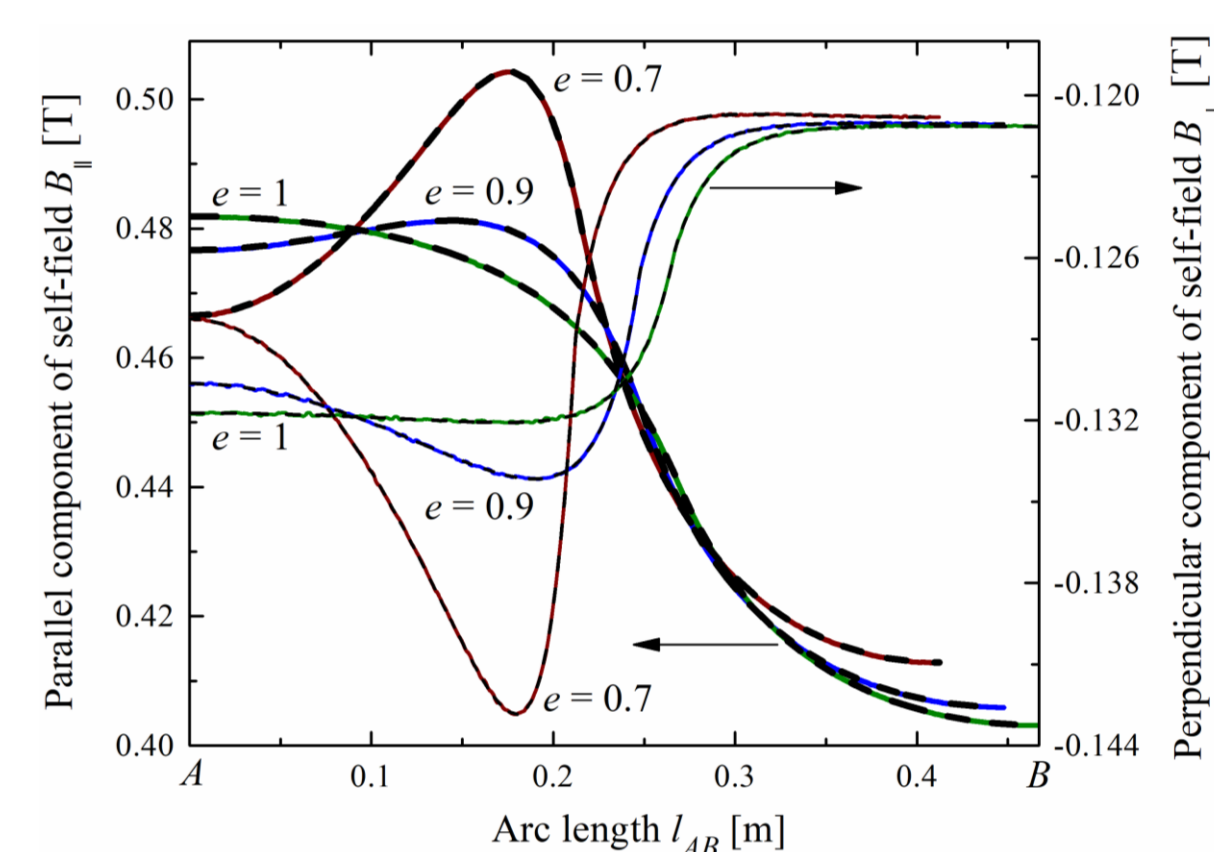


Fig. 4. The parallel and perpendicular components of the magnetic fields, where the dotted lines represent the analytical results and the solid lines represent those of the 3-D FEM model.

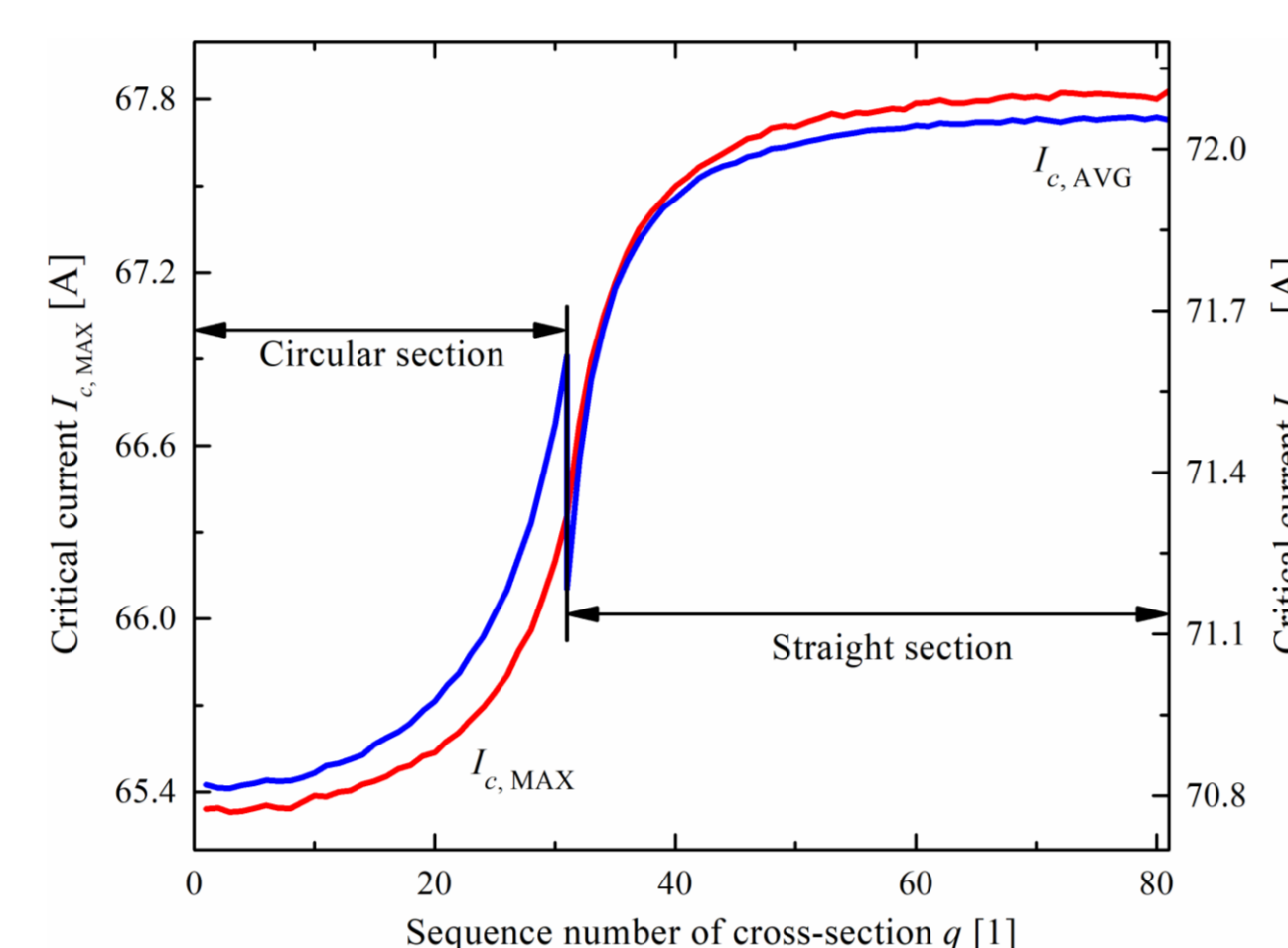


Fig. 5. The critical currents of the racetrack REBCO magnet evaluated by the different cross-sections, where the circular section is divided by 31 cross-sections and the straight section is divided by 51 cross-sections.

TABLE I. CRITICAL CURRENTS OF A RACETRACK MAGNET

Objects	P-model ^a	T-A model ^b	Relative error [%]
Circular section I_c [A]	71.0	70.2	1.14
Straight section I_c [A]	71.97	72.0	0.04
Racetrack magnet	71.44	71.1	0.48

^a The critical current is based on the AVG criterion.

^b The critical current is estimated based on the V-I curve method.

TABLE II. CRITICAL CURRENTS OF A RACETRACK MAGNET

Items	Original P-model	Homogenized P-model	Relative error [%]
$I_{c,MAX}$ [A]	45.12 (529 s)	45.30 (20 s)	0.4
$I_{c,AVG}$ [A]	45.85 (341 s)	45.85 (20 s)	0

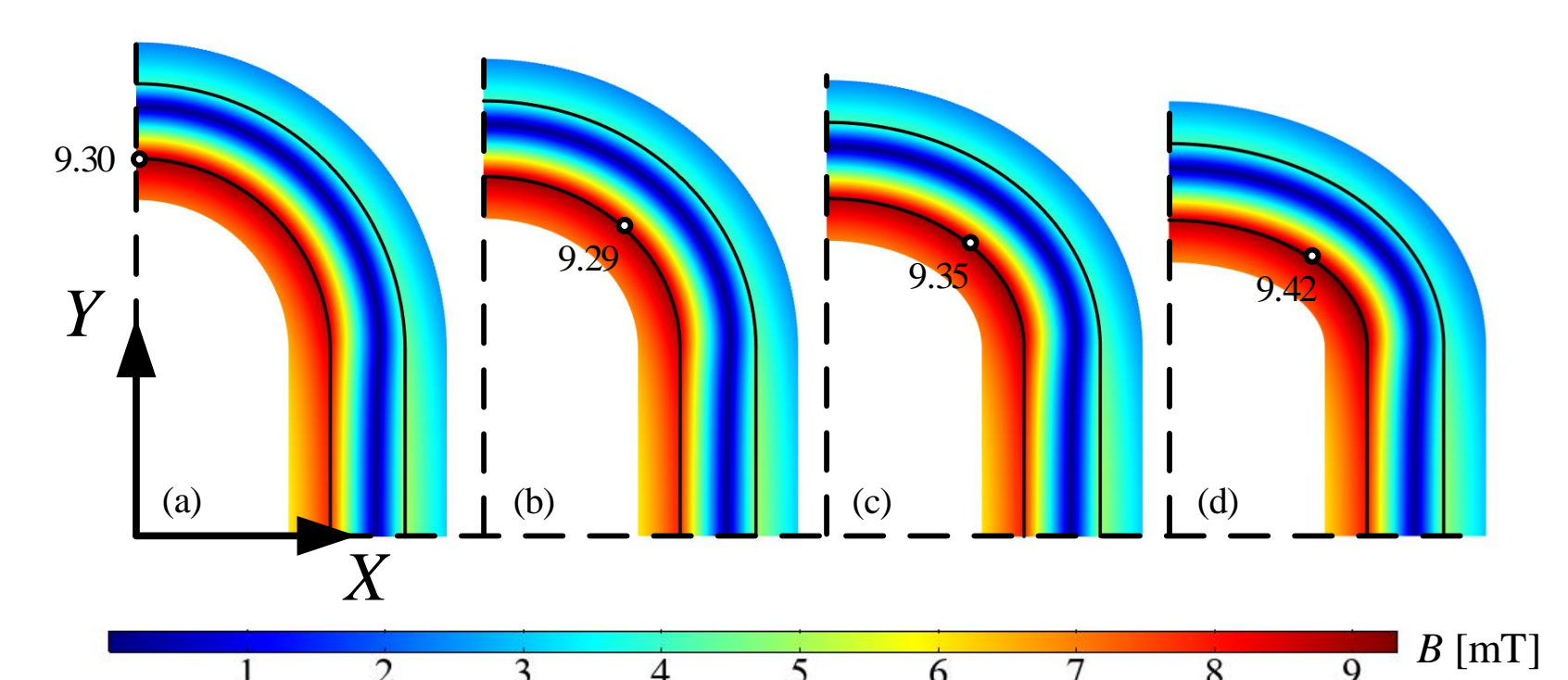


Fig. 6. The magnetic field distributions of the referred REBCO magnet, where the black lines represent the outline of the racetrack magnet, (a) $e = 1$, (b) $e = 0.9$, (c) $e = 0.8$, (d) $e = 0.7$. The maximum magnetic fields of the magnet carrying 1 A transport current are portrayed in the corresponding positions.

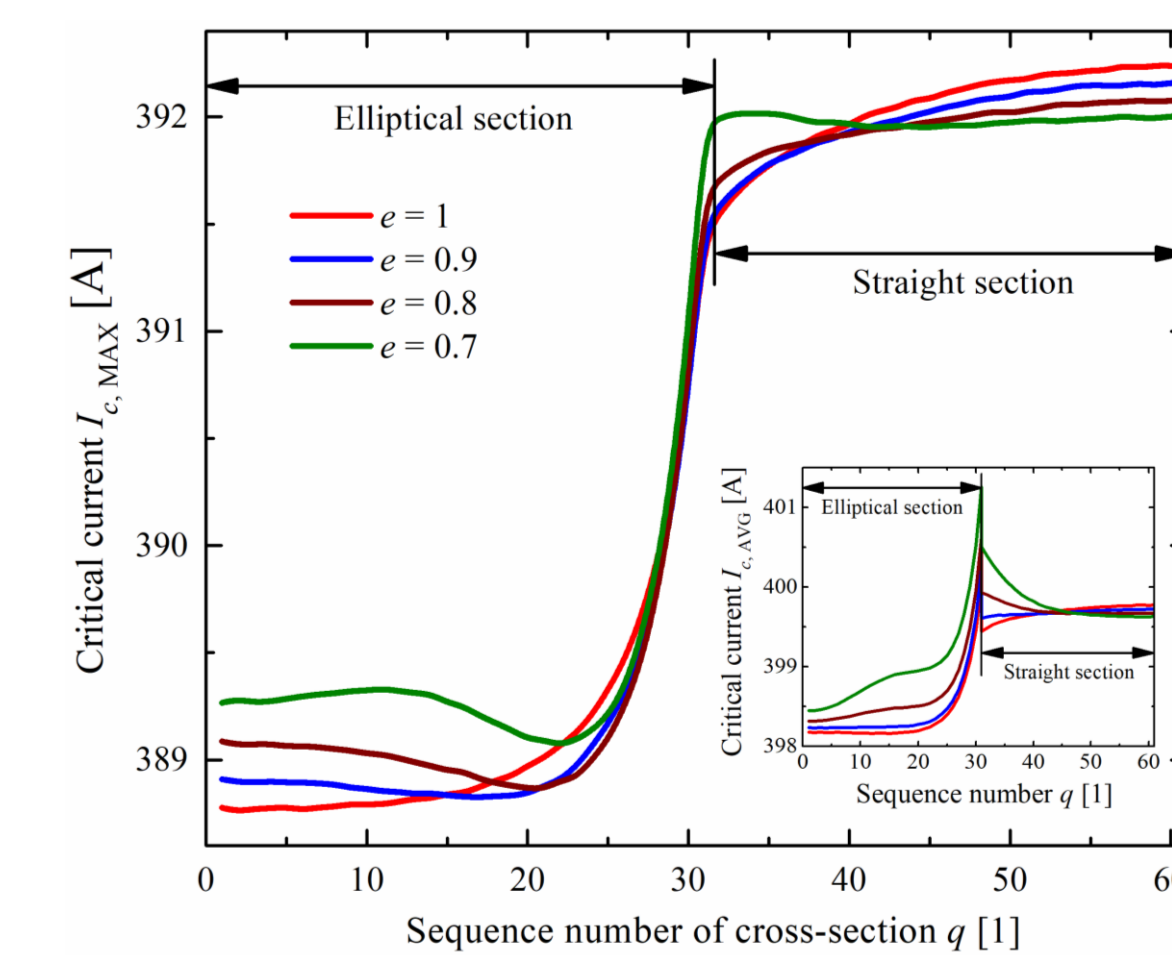


Fig. 7. The critical currents of the referred magnet with the varied ratios. This figure implies that the critical current of the generalized racetrack REBCO magnet is not reduced as the decrease of the ratio, namely, the elliptical section has no significant effect on the critical current of a generalized racetrack REBCO magnet.

CONCLUSIONS

First, on the basis of the Biot-Savart's law, the self-field of a generalized racetrack magnet is analyzed. The results show that the position of the maximum magnetic field varies as the ratio of the minor axis to major axis of the elliptical section of a generalized racetrack magnet.

Secondly, there are obvious distinctions on the critical currents of the different cross-sections of a racetrack REBCO magnet, which depends on the magnetic field distributions.

Thirdly, the improved self-consistent model shows a fast computational speed with the accuracy preserved, so it is very promising for this model to carry out the design optimization of a large-scale superconducting magnet.

Lastly, the critical current of a generalized racetrack REBCO magnet is estimated and it is found that the elliptical section hardly brings about an attenuation of the critical current.