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A Numerical Method to Calculate Spatial Harmonic Coefficients of Magnetic Fields generated by Screening Currents in an HTS Magnet

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Abstract - This paper presents a numerical method, named ``segmentation method'', to calculate spatial harmonic coefficients of magnetic fields generated by both transport and screening currents in a high temperature superconductor (HTS) magnet. Once screening current distributions in the individual turn of an HTS magnet are obtained using finite element analysis based on the edge-element H-formulation and the domain homogenization technique, the current flowing domain is divided into multiple ``segments'' and equivalent loop currents in each segment is calculated using an equivalent current transformation technique. Then, harmonic coefficients of magnetic fields by the individual loop currents are calculated analytically and the final harmonic coefficients are obtained by superposition. To verify the proposed segmentation method, an HTS magnet comprising a stack of 12 single pancake coils was assumed and its field gradients *without* consideration of screening current were calculated up to the 10th order by the so-called Garrett's method (proven) and the proposed segmentation method. Accordance between the two results validates the reliability of the proposed method. Then, field gradients of magnetic fields of the same magnet generated by both transport and screening currents were calculated using the proposed method and the conventional inverse calculation method. While the inverse method shows significant errors in high order field gradients, the segmentation method delivers reasonable values of field gradients. The results suggest that the proposed segmentation method may be used to calculate field gradients of an HTS magnet with the non-uniform screening current distribution considered.

◆ Introduction

- Prerequisite condition of ***uniform current densities*** in a solenoid magnet for a well-known method, Garrett's method, to calculate spatial harmonic coefficients
  - → Current densities in an HTS magnet may be ***non-uniform***
- Development of a numerical method for spatial harmonic coefficient calculation
  - → A method which is able to consider non-uniform current densities

◆ Calculation Approach: "segmentation method" based on equivalent loop current transformation technique and harmonic analysis

- **Equivalent "loop" current transformation** of calculated current densities of a "segment", Figure 1

$$I_\phi^{equiv} = J_a S_a + J_b S_b + J_c S_c$$

- Harmonic analysis of an ideal "loop" current: an analytic solution of magnetic field (B_z^{loop}) expressed with associated Legendre polynomials

$$B_z^{loop}(\boldsymbol{r}) = \sum_{n=0}^{\infty} \frac{\mu_0}{2} I_\phi \frac{\sin\alpha}{r_0^{n+1}} P_{n+1}^1(\cos\alpha)\ r^n\ P_n^0(\cos\theta) \rightarrow B_z^{loop}(z) = \sum_{n=0}^{\infty} \frac{Zn}{n!} z^n$$

- Spatial harmonic coefficients of magnetic field generated by equivalent "loop" currents

$$B_z^{loops}(z) = \sum_{seg=1}^{2\times p\times q} \sum_{n=0}^{\infty} \frac{Zn_{seg}}{n!} z^n = \sum_{n=0}^{\infty} \frac{Zn}{n!} z^n$$

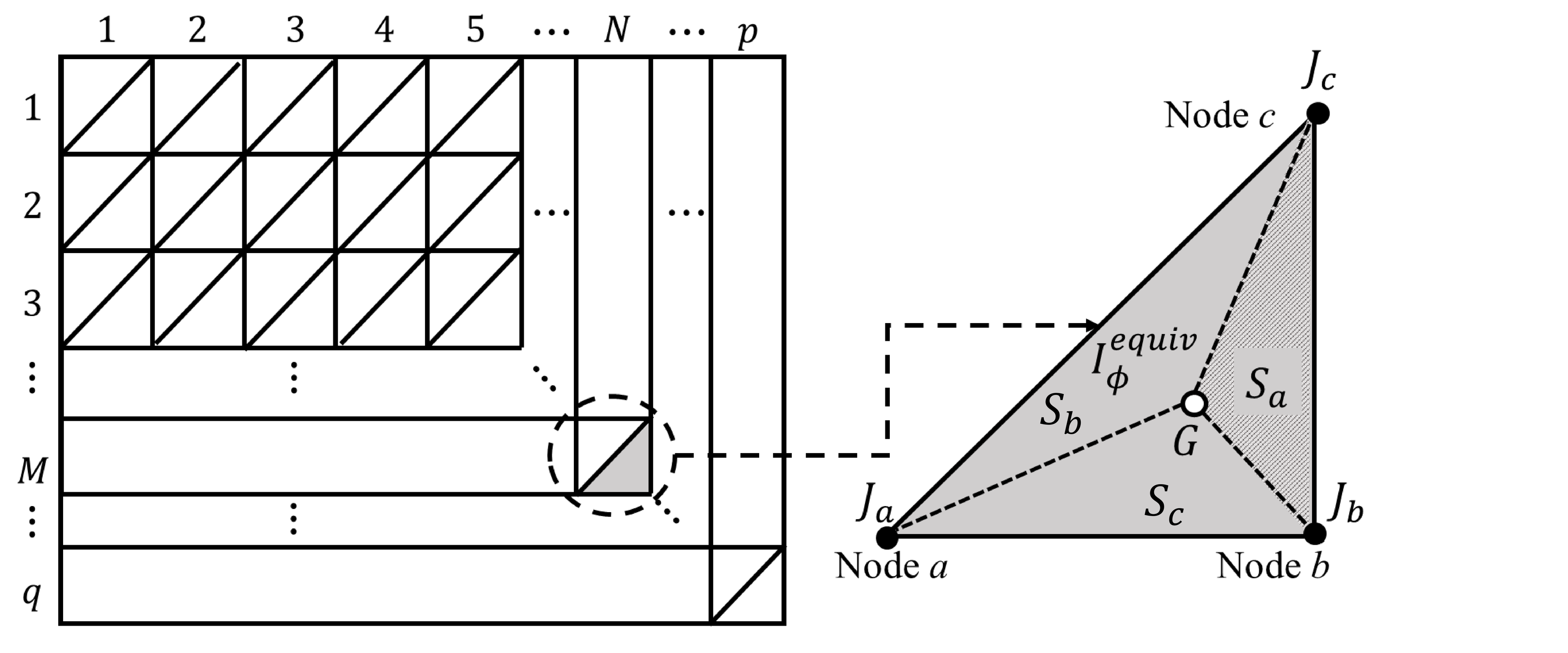

Figure 1. Schematic diagram of a segmented domain. An "analysis domain" consists of $2 \times p \times q$ triangle segments. Three current densities, J_a, J_b, and J_c, at three nodes of a triangle segment can be transformed into an equivalent "loop current, I_{equiv}^{ϕ}, located at the center of mass, G, of a triangle segment.

◆ Verification of the Segmentation Method: a solenoid case without screening current

- Harmonic coefficients calculation of a solenoid magnet consisting of 12 single pancakes

TABLE I
DESIGN PARAMETERS OF AN IDEAL SOLENOID MAGNET

Magnet parameters	Unit	
Conductor width, w	[mm]	4.10
Conductor thickness, th	[mm]	0.12
Inner radius, a_1	[mm]	50
Outer radius, a_2	[mm]	59.36
The number of single pancake (SP)		12
SP - SP spacer	[mm]	0.20
Total height	[mm]	51.40
Operating current, I_{op}	[A]	100
Uniforn current density, J_{uni}	[A/mm^2]	203.25
Simulation parameters	Unit	
Critical current, I_c	[A]	300
The index number, n		21
The number of segmentation, p and q	[#]	100 - 1000

Table1. It presents key parameters of a solenoid magnet arbitrarily chosen for verification of our segmentation method. The magnet consists of a stack of 12 single pancake coils wound with 4.1 mm wide and 0.12 mm thick REBCO tapes. The inner radius, outer radius, and height are, respectively, 50, 59,3 and 51.4 mm. 0.2 mm pancake-to-pancake spacers are assumed. Both p and q in Fig. 1 are ranged 100 - 1000 in this case study

TABLE II
SPATIAL HARMONIC COEFFICIENTS OF MAGNETIC FIELDS GENERATED BY THE SOLENOID MAGNET WITH UNIFORM CURRENT DENSITIES CALCULATED WITH GARRETT'S METHOD AND THE PROPOSED METHOD

Coeff.	Unit	Garrett's method	Segmentation method
$Z0$	[T]	0.974	0.974
$Z2$	[T/cm^2]	$-0.657 \cdot 10^{-1}$	$-0.657 \cdot 10^{-1}$
$Z4$	[T/cm^4]	$0.155 \cdot 10^{-1}$	$0.155 \cdot 10^{-1}$
$Z6$	[T/cm^6]	$-0.295 \cdot 10^{-2}$	$-0.295 \cdot 10^{-2}$
$Z8$	[T/cm^8]	$-0.894 \cdot 10^{-2}$	$-0.894 \cdot 10^{-2}$
$Z10$	[T/cm^{10}]	$0.334 \cdot 10^{-1}$	$0.334 \cdot 10^{-1}$

Table2. Comparisons between the proven model, Garrett's method, and the "segmentation method" are shown. Accordance between the two results may verify the reliability of our segmentation method for calculations of spatial harmonic coefficients of magnetic fields for a given current distribution.

- Zonal harmonic coefficients error analysis: the number of segmentation (p, q) dependence

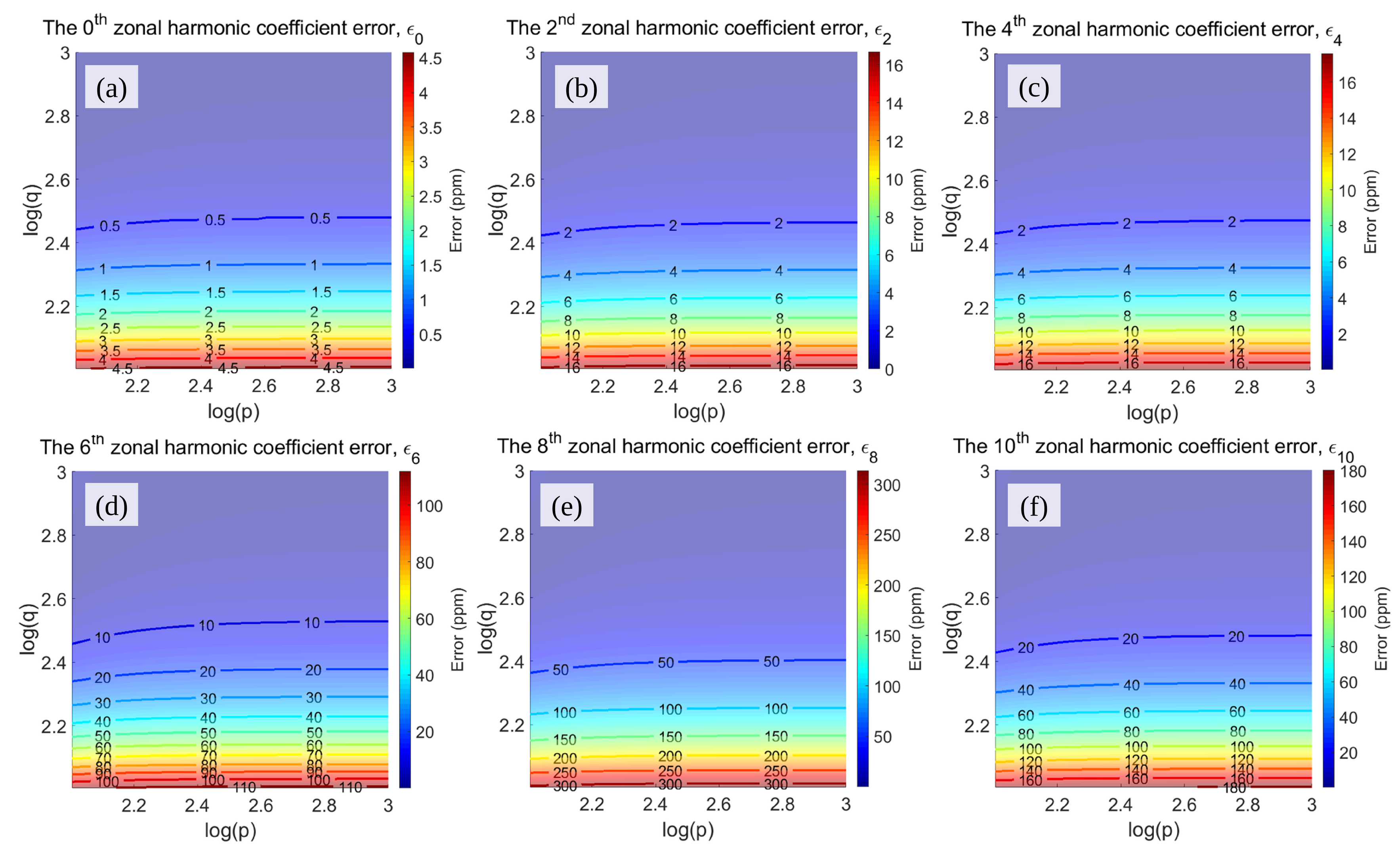

Figure 2. Calculation results of p and q dependent errors. (a) – (f) stand for, respectively, the zeroth, the second , the fourth, the sixth, the eighth, and the tenth order zonal harmonic coefficient error.

◆ Case Study : HTS magnet with screening current (SC) considered

- Calculation results: axial magnetic field with SC considered, and total current densities

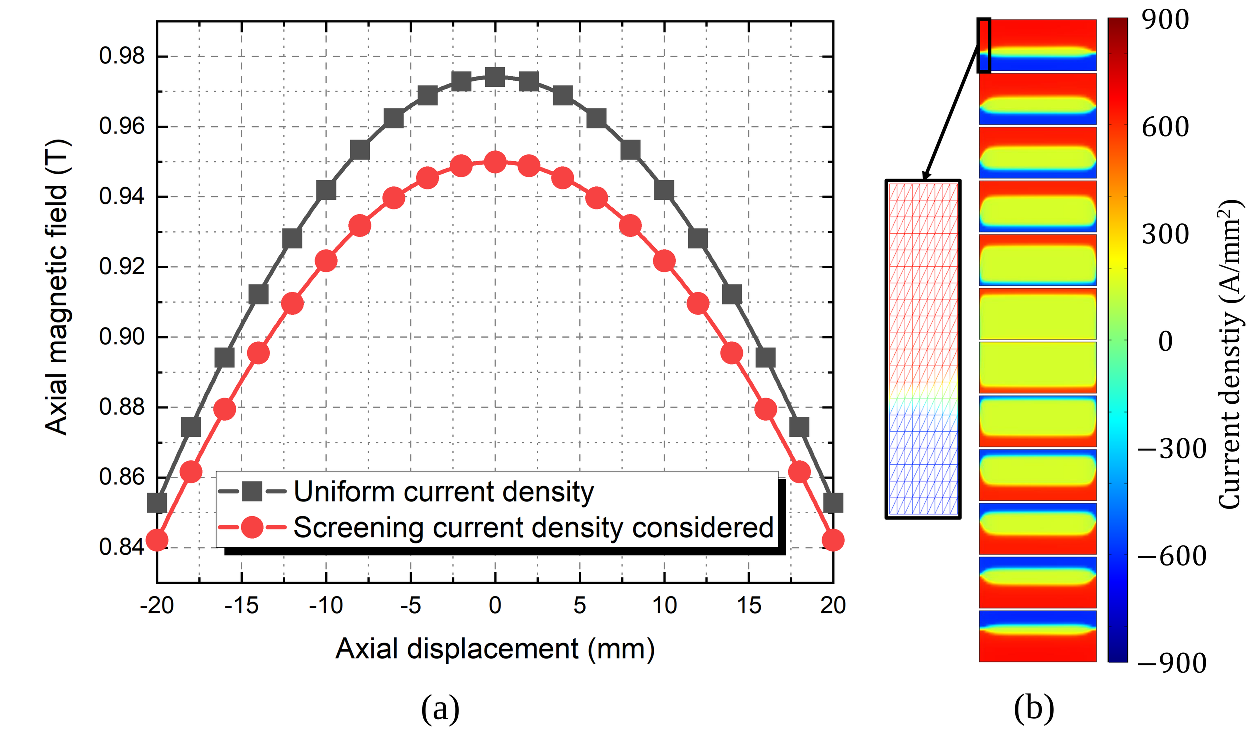

Figure 3. Calculation results of magnetic fields and total current densities: (a) black square symbol line indicates calculated magnetic field with an assumption of uniform current density, while red circle symbol line with an assumption of non-uniform current densities with SCs considered; and (b) shows calculated total current densities (transport + screening current), J_{total}, of individual turns in an HTS magnet.

TABLE III
SPATIAL HARMONIC COEFFICIENTS OF MAGNETIC FIELDS GENERATED BY THE HTS MAGNET WITH SCF CONSIDERED

Field	Coeff.	Unit	Segmentation method	Inverse method
B_z^{scf}	$Z0$	[T]	0.95	0.95
	$Z2$	[T/cm^2]	$-0.57 \cdot 10^{-1}$	$-0.57 \cdot 10^{-1}$
	$Z4$	[T/cm^4]	$0.96 \cdot 10^{-2}$	$1.0 \cdot 10^{-2}$
	$Z6$	[T/cm^6]	$-0.29 \cdot 10^{-2}$	$-1.1 \cdot 10^{-2}$
	$Z8$	[T/cm^8]	$-0.15 \cdot 10^{-1}$	$2.1 \cdot 10^{-1}$
	$Z10$	[T/cm^{10}]	$0.33 \cdot 10^{-1}$	$20 \cdot 10^{-1}$

Table3. presents spatial harmonic coefficients calculated by two different methods: (1) the segmentation method proposed in this paper; and (2) the inverse calculation method, commonly used in previous reports, where field gradients are inversely calculated using field mapping results.

◆ Conclusion

- Accordance shown in Table 2 between the two results validates the reliability of the proposed method to calculate harmonic field coefficients for a given current distribution.
- The results suggest the potential of our segmentation method to accurately calculate spatial harmonic coefficients of magnetic fields of an HTS magnet with the non-uniform screening current distribution taken into consideration.

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