Expansion of magnetic fields in toroidal harmonics

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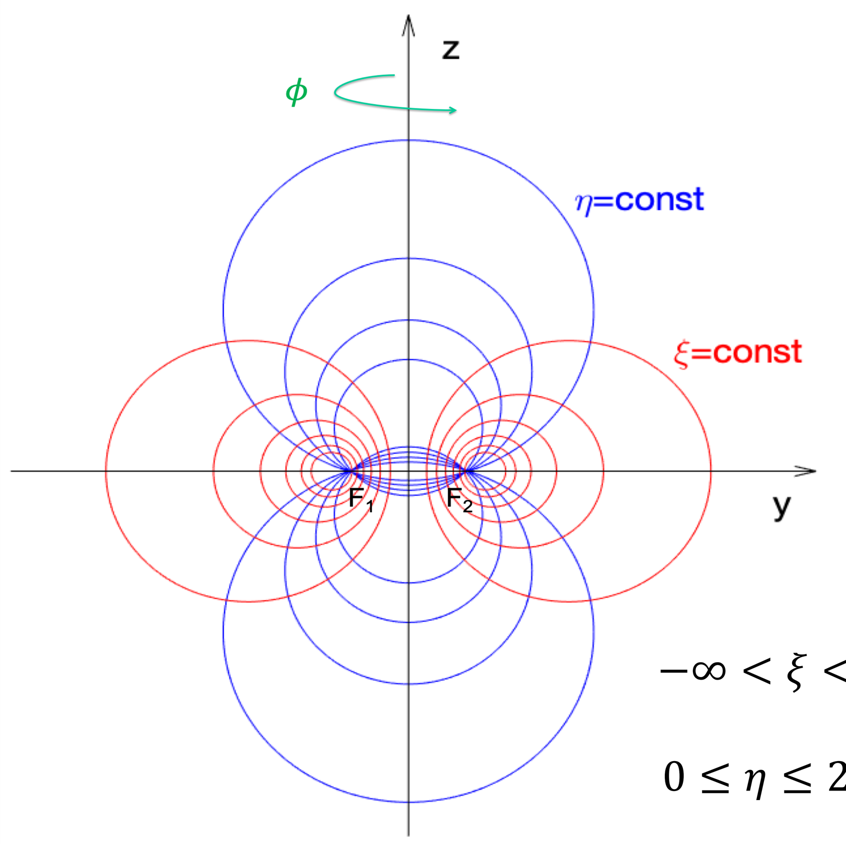
M. Breschi[1], A. Cristofolini[1], L. Gambini[1], L. Bottura[2], E. Felcini[2]
[1]University of Bologna – Bologna, Italy, [2]CERN – Geneva, Switzerland

Summary – Toroidal magnetic configurations are widely exploited in industry and scientific research, involving a vast spectrum of applications, such as thermonuclear fusion, particle detectors, SMES systems and medical devices. To properly design and analyse these systems, it is crucial to determine the magnetic field generated by different configurations. The multipole expansion theory can be applied to the analysis of toroidal configurations, by solving the Laplace equation for the magnetic scalar potential in toroidal coordinates. Contrarily to the case of accelerator magnets with straight axis, in this case the correlation between the current distribution and the field harmonics cannot easily be identified. This paper proposes a methodology for the computation of field harmonics in toroidal coordinates, which is validated by comparison with the results obtained through the Biot-Savart law. This work was carried out in the frame of the GaToroid project [1] undergoing at CERN.

Multipole Expansion in Toroidal Coordinates

The most suitable coordinate system for the multipole expansion in toroidal harmonics is the Toroidal Coordinate System (ξ, η, ϕ), obtained by rotating the two dimensional Bipolar Coordinate System (ξ, η) around the axis which separates the foci F_1 and F_2.



$-\infty < \xi < \infty$ → «radius»

$0 \le \eta \le 2\pi$ → «poloidal angle»

$0 \le \phi \le 2\pi$ → «toroidal angle»

The procedure followed for the evaluation of the field harmonics is based on the Laplace equation_solution for the magnetic scalar potential ψ, in toroidal coordinates.

$$\mathbf{H} = -\nabla\psi \qquad \nabla^2\psi = 0$$

Boundary conditions: void toroidal chamber, finite value for any ξ inside the torus, periodicity along ϕ, periodicity along η

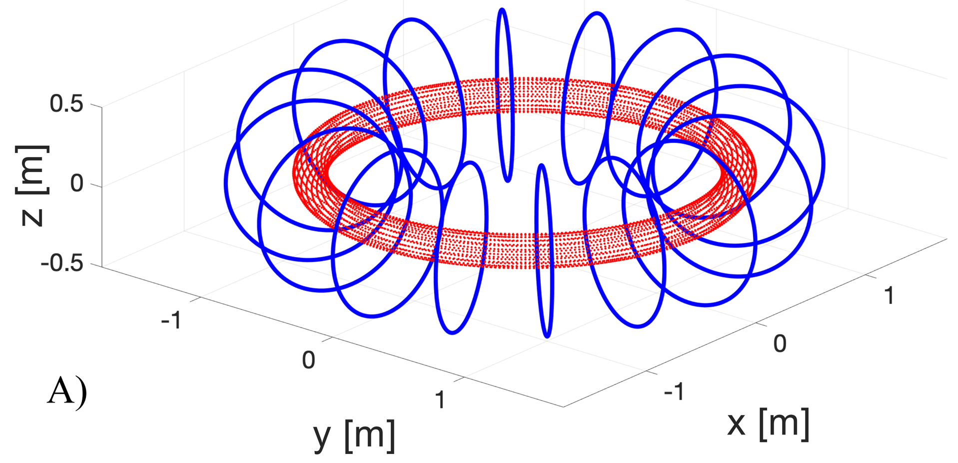
$$\psi(\xi,\eta,\phi) = M_{00}^{\phi}\phi + \sqrt{\cosh(\xi)-\cos(\eta)}\sum_{m=0}^{M}\sum_{n=0}^{N} Q_{m-\frac{1}{2}}^{n}(\cosh(\xi))\left[M_{m,n}^{cc}\cos(n\phi)\cos(m\eta) + M_{m,n}^{cs}\cos(n\phi)\sin(m\eta) + M_{m,n}^{sc}\sin(n\phi)\cos(m\eta) + M_{m,n}^{ss}\sin(n\phi)\sin(m\eta)\right]$$

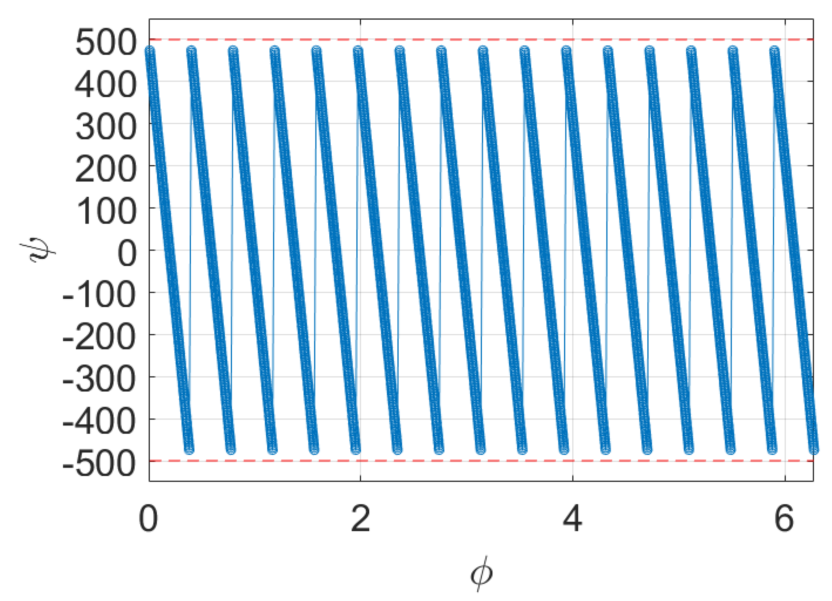
- Ideal contribution: $M_{00}^{\phi} = \frac{(Number\ of\ coils)(Current\ in\ each\ coil)}{2\pi}$
  known a priori, removed to improve the computation accuracy: $\psi^* = \psi - M_{00}^{\phi}\phi$
- Multipolar coefficients: $M_{m,n}^{cc}$, $M_{m,n}^{cs}$, $M_{m,n}^{sc}$, $M_{m,n}^{ss}$

Computation of the magnetic scalar potential

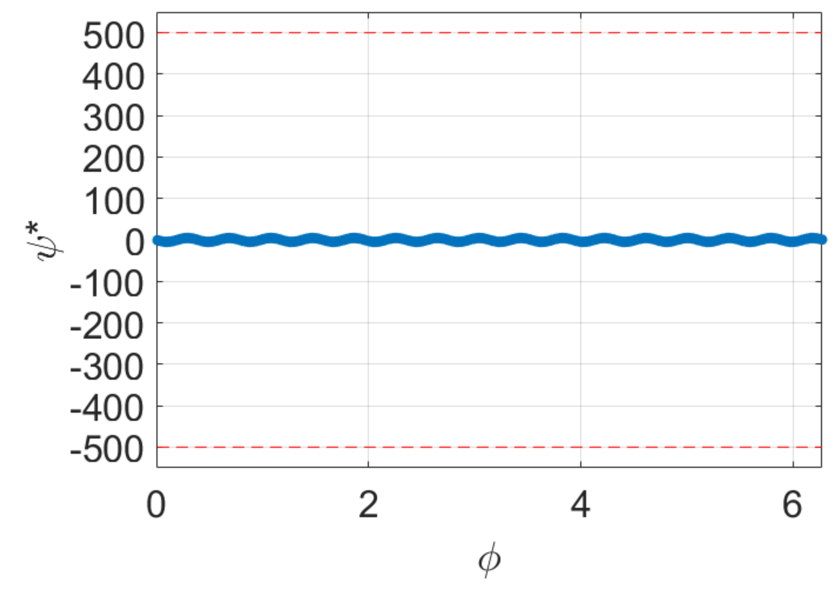
The scalar potential ψ at each point of the reference grid is computed as the sum of the contributions of all coils. The computation is based on the solid angles methodology [2], through an analytical approach which is applied to a discretization of each coil.

The red points represent the reference grid.





With ideal contribution



Without ideal contribution

Procedure for the Multipolar Coefficients Evaluation

The multipolar coefficients are computed from the magnetic scalar potential at a fixed radial coordinate $\xi = \xi_0$, for L values of the poloidal angle η and S values of the toroidal angle ϕ.

Fitting Phase 1 $\xi = \xi_0$, $\eta = \eta_l$,ϕ_s=2π s/S,
where s = 0, 1, ..., S – 1

$$\psi^*(\xi_0,\eta_l,\phi) = \sum_{n=0}^{N}\left(C_n(\xi_0,\eta_l)\cos(n\phi) + D_n(\xi_0,\eta_l)\sin(n\phi)\right)$$

Fitting Phase 2 $\xi = \xi_0$, η_l = 2π l/L,
where l = 0, 1, ..., L – 1

$$D_n(\xi_0,\eta_l) = \sqrt{\cosh(\xi_0)-\cos(\eta_l)} \times \sum_{m=0}^{M} Q_{m-\frac{1}{2}}^{n}(\cosh(\xi_0))\left[M_{m,n}^{sc}\cos(m\eta_l) + M_{m,n}^{ss}\sin(m\eta_l)\right]$$

The initial fitting phase 1 provides as an output the values of the coefficients Cn (ξ_0, η_l) and Dn (ξ_0, η_l), whereas phase 2 provides the values of the coefficients $M_{m,n}^{sc}$ and $M_{m,n}^{ss}$.

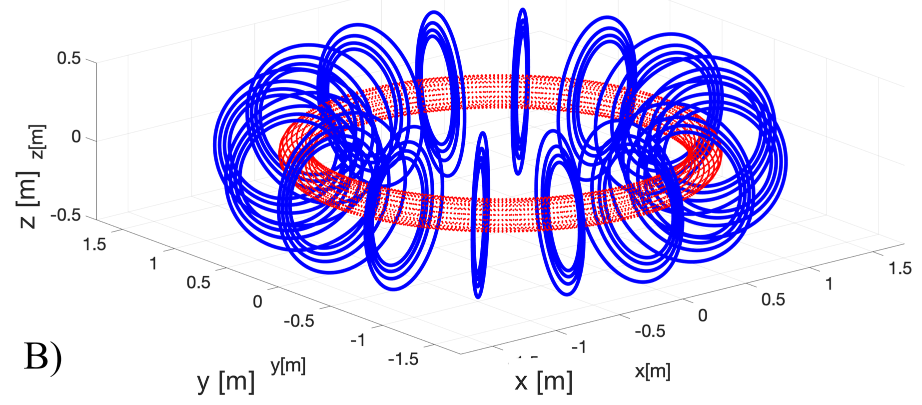
Validation and results

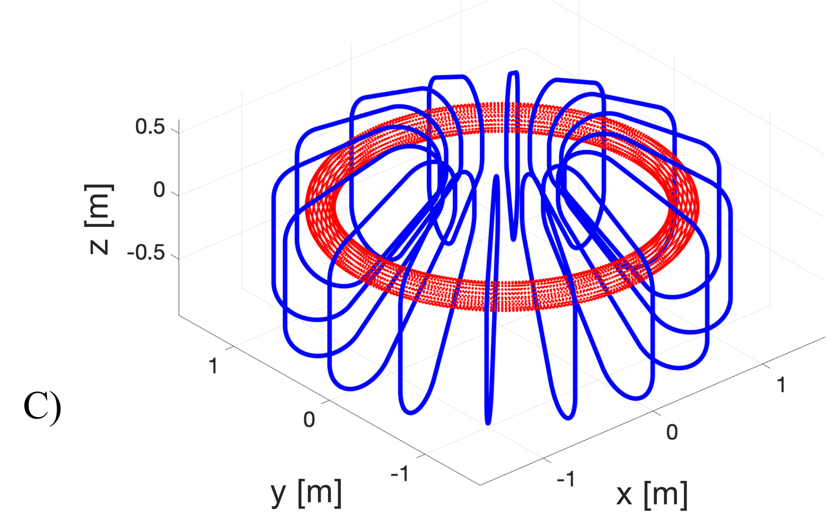
Analyzed configurations:

A) 16 circular coils. B) 16 circular coils with grading (4 coils in each section, 64 in total). C) 16 GaToroid coils.

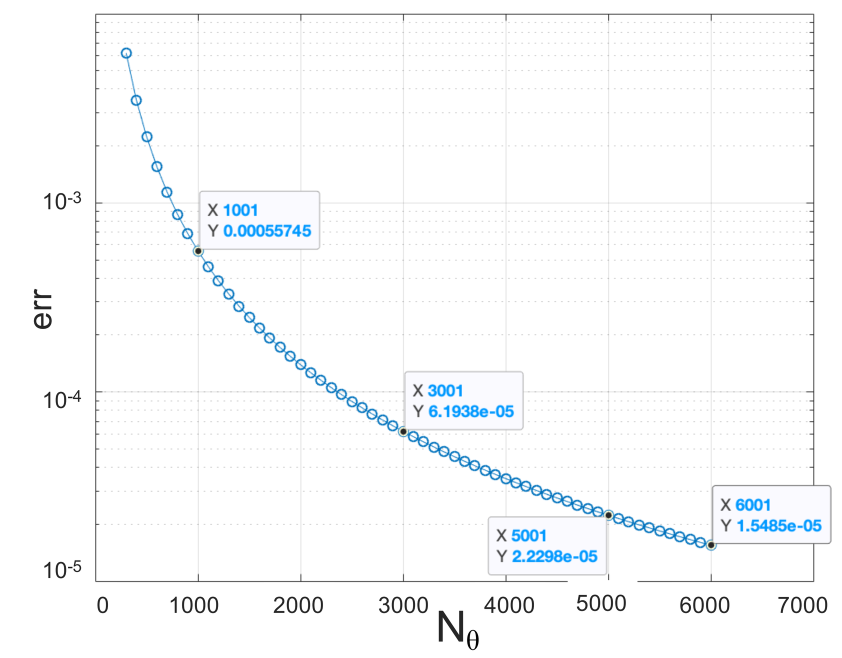
TABLE I
CONFIGURATIONS PARAMETERS

Parameter	Case A	Case B	Case C
Number of coils	16	16	16
Number of coils per section	1	4	1
Current in each coil [A]	1000	1000	1000
Toroid major radius [m]	1.25	0	-
Toroid minor radius [m]	0.5	0	-
Reference grid major radius [m]	1.25	1.25	1.25
Reference grid minor radius [m]	0.1	0.05	0.05
Index m	0:10	0:20	0:30
Index n	0:80	0:96	0:112



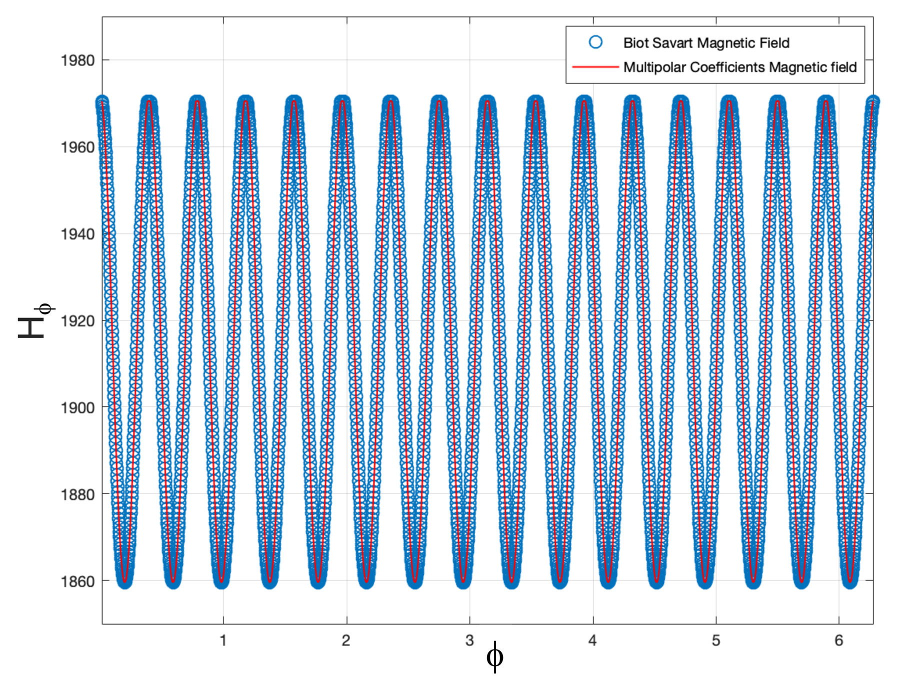


A fine discretization of each coil is required to reach a high computation accuracy.

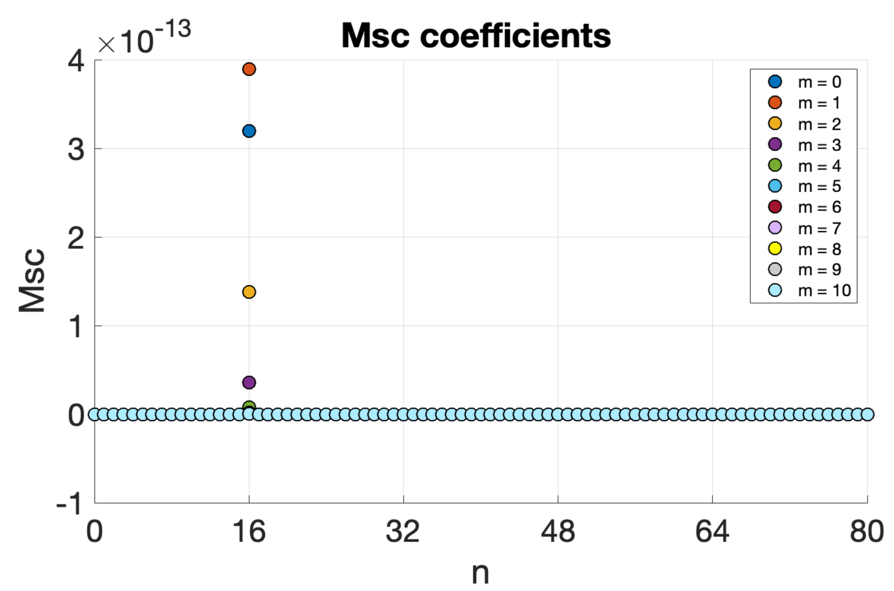


The computed multipolar coefficients were used to determine the magnetic field components in toroidal coordinates.
The procedure was validated by comparison with magnetic field computations based on the Biot-Savart law.

The system periodicity with 16 coils is clearly identified by the computed multipolar components (case A).



Magnetic field component H_ϕ in Case A, at fixed η = 0.6 rad; maximum relative error 1.4×10^{-5} %.



Coefficients of the multipolar expansion in toroidal coordinates for Case A

References

[1] L. Bottura, "A Gantry and apparatus for focussing beams of charged particles ," Patent, EP 18173426.0., May 2018

[2] S. Russenschuck, Field Computation for Accelerator Magnets: Analytical and Numerical Methods for Electromagnetic Design and Optimization, Wiley-VCH Verlag GmbH & Co, 2011