Effective Field Theories in R_ξ gauges[1]

Michael Paraskevas

Faculty of Physics, University of Warsaw (IFT UW)

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[1] *M. Misiak, MP, J. Rosiek, K. Suxho and B. Zglinicki, JHEP* **1902**, *051 (2019), [arXiv:1812.11513]*

Motivation

- As current experimental evidence indicate, a sizeable energy gap between the new physics scale and the electroweak scale is present.
- In this region, the most convenient calculational framework is an Effective Field Theory with only the SM degrees of freedom, the so-called SMEFT[2,3].

[2] W. Buchmuller and D. Wyler, (1986).

[3] B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek, (2010).

- Practical calculations within the (dim-6) SMEFT require introducing convenient gauge-fixing terms.

- In particular, it has been shown[4,5] that effects of higher-dimensional operators should be taken into account in the definition of R_ξ-gauges. Otherwise one can end up with tree-level mixing in the gauge bosons, goldstones and ghosts propagators.
  e.g.,

$$\frac{C^{\varphi WB}}{\Lambda^2}(\varphi^\dagger \sigma^A \varphi) W^A_{\mu\nu} B^{\mu\nu} \rightarrow \left(\frac{C^{\varphi WB} v^2}{\Lambda^2}\right)(\partial_\mu W^3_\nu)(\partial_\mu B_\nu) + \ldots$$

$\Rightarrow$ Z-A mixing at tree level (ξ-dependent)!

[4] A. Dedes, W. Materkowska, MP, J. Rosiek and K. Suxho, JHEP 1706 (2017) 143
[5] A. Helset, MP and M. Trott, Phys. Rev. Lett. 120 (2018) 251801

- **Result of R_ξ-SMEFT:** All propagators keep their SM-form (ie., no tree-level mixing) and the effect of dim-6 operators appears only in interactions.

- **Purpose of R_ξ-EFT:** *Apply R_ξ* **beyond dim-6** *level and* **beyond SM** *content.*

The EFT framework

Let us consider an EFT that arises after decoupling[6] of heavy particles at scale Λ and assume that the UV-theory at that scale is perturbative.
The dynamics of light fields ($m \ll \Lambda$) at low energy scales ($E \ll \Lambda$) are described by the effective Lagrangian,

$$\mathcal{L} = \mathcal{L}^{(4)} + \sum_{k=1}^{\infty} \frac{1}{\Lambda^k} \sum_i C_i^{(k+4)} Q_i^{(k+4)}.$$

- $\mathcal{L}^{(4)}$ is the dimension-four ("renormalizable") part of $\mathcal{L}$,
- $Q_i^{(k+4)}$ stand for dimension-$(k+4)$ local operators built out of light fields and their derivatives.
- $C_i^{(k+4)}$ are their respective couplings, known as Wilson coefficients.

The EFT expansion is truncated at an arbitrary but fixed order N, i.e., terms of order $1/\Lambda^{N+1}$ or higher are neglected.

[6] T. Appelquist and J. Carazzone,(1975).

Fundamental blocks of an EFT Lagrangian

The fundamental blocks of a general gauge invariant EFT Lagrangian are [7,8]

$$\mathcal{L} = \mathcal{L}[\Phi, F_{\mu\nu}, D_\mu, (\Psi)]$$

- Real scalars in a (possibly reducible) multiplet:

$$\Phi = \varphi + \langle\Phi\rangle \leftrightarrow \Phi_i = \varphi_i + v_i$$
$$D_\mu\Phi = (\partial_\mu + iA^a_\mu T^a)\Phi$$

- Field strength tensor in adjoint of the group - reducible if not simple:

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu A^a_\mu - \partial_\nu A^a_\mu - f^{abc} A^b_\mu A^c_\nu, \\ (D_\rho F_{\mu\nu})^a &= \partial_\rho F^a_{\mu\nu} - f^{abc} A^b_\rho F^c_{\mu\nu} \\ \Big(A^a_{\mu\nu} &\equiv \partial_\mu A^a_\mu - \partial_\nu A^a_\mu \Big) \end{aligned}$$

[7] *footnote in B. Grzadkowski et al., (2010).*

[8] *proof in M. Iskrzyński, MSc thesis.*

Main steps in R_ξ-EFT

- Distinguish which operators are relevant to gauge fixing and which of them are dangerous.
- Eliminate the dangerous ones with Equations of Motion, ie., “send” them beyond truncation order N.
- Introduce a gauge fixing term and a corresponding ghost sector which gives perturbation friendly Feynman Rules.

Distinguishing relevant and dangerous operators

An operator potentially relevant for gauge-fixing has the form,

$$Q^{(n+2m+k)} = \Phi^n F^m D^k$$

It is irrelevant if it has 3 or more objects with vanishing VEVs,

$$\text{e.g.,} \quad (F^T_{\mu\nu} F^{\mu\nu})^2 \rightarrow \text{pure interactions}$$

It is relevant if it contributes to gauge and scalar boson bilinears,

$$\text{e.g.,} \quad (\Phi^T \Phi)^2 (F^T_{\mu\nu} F^{\mu\nu}) \;\rightarrow\; v^4 (A^T_{\mu\nu} A^{\mu\nu})$$

but it is dangerous if it contains higher derivative bilinears

$$\text{e.g.,} \quad (D^\mu D_\mu \Phi)^T (D^\nu D_\nu \Phi) \;\rightarrow\; (\partial^\mu \partial_\mu \Phi)^T (\partial^\nu \partial_\nu \Phi)$$

- Higher derivative bilinears are formally part of the propagators.
- If treated perturbatively they will appear as two point vertices in the feynman rules (e.g., $\sim i(k^2)^2$).
- They will be resummed at tree-level and this will affect the form of the propagators.

They have to be removed!

Eliminating dangerous operators

One can remove the dangerous operators applying (perturbative) field redefinitions making use of the equivalence theorem of S-matrix[9,10]

Equivalently for our purpose using the Equations of Motion (EOM).

[9] H. D. Politzer (1980), C. Arzt (1995), H. Simma (1994).
[10] J. C. Criado and M. Pérez-Victoria, JHEP **1903**, 038 (2019).

Understand the logic through a toy-example:

$$\mathcal{L}_{\rm toy} = (\partial\phi)^2 + m^2\phi^2 + \frac{C^{(6)}}{\Lambda^2}(\partial^2\phi)^2$$

giving the EOM,

$$\partial^2\phi = m^2\phi + \frac{C^{(6)}}{\Lambda^2}\partial^2(\partial^2\phi)$$

Applying EOM one can trade,

$$\frac{C^{(6)}}{\Lambda^2}(\partial^2\phi)^2 = \frac{C^{(6)}}{\Lambda^2}m^2\phi(\partial^2\phi) + \frac{(C^{(6)})^2}{\Lambda^4}(\partial^4\phi)(\partial^2\phi)$$

Both **higher** and **lower** derivative operators can be obtained.

But **higher** derivatives are **always suppressed** by extra powers of $1/\Lambda$.

Returning to the general case, one can apply successive EOM for

$$D_\mu D^\mu \Phi = [\text{Lower-D}] + \mathcal{O}(\Lambda^{-1}) \ , \quad D_\mu F^{\mu\nu} = [\text{Lower-D}] + \mathcal{O}(\Lambda^{-1})$$

together with integration by parts, Bianchi identities, etc., and practically eliminate the "dangerous" operators.

For an EFT considered to order N, the result is:

- The dangerous higher derivative bilinears of gauge bosons and scalars are suppressed as $\frac{1}{\Lambda^{N+1}}$ and therefore can be neglected.
- The only relevant operators are of the form

$$\Phi^n F^m D^k \to \Phi^n D^2, \Phi^n F^2, \Phi^n$$

which can be expressed in the form[11],

$$\mathcal{L}_C = \frac{1}{2}(D_\mu \Phi)_i \; K_{ij}[\Phi] \; (D^\mu \Phi)_j - \frac{1}{4} F^a_{\mu\nu} \; J^{ab}[\Phi] \; F^{b\,\mu\nu} - V[\Phi],$$

Bilinear terms arise when $J[\Phi]$ and $K[\Phi]$ are set to their expectation values,

$$\begin{aligned} K_{ij}[\Phi] \quad &\rightarrow \quad K_{ij} = \mathbf{1}_{ij} + \mathcal{O}_{ij}(Cv/\Lambda), \\ J^{ab}[\Phi] \quad &\rightarrow \quad J^{ab} = \mathbf{1}^{ab} + \mathcal{O}^{ab}(Cv/\Lambda). \end{aligned}$$

with J, K being symmetric and positive definite - possess **inverse** and **square-root**. Then $\mathcal{L}_C$ becomes (recall $A^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$),

$$\mathcal{L}_C = \frac{1}{2}(D_\mu \Phi)^T \, K \, (D^\mu \Phi) - \frac{1}{4} A^T_{\mu\nu} \, J \, A^{\mu\nu} + \ldots (\text{Interactions or } V[\Phi]) \; .$$

[11] A. Helset, MP and M. Trott, (2018).

Introducing gauge-fixing

We obtain the (usual) "unwanted" gauge-goldstone boson mixing term,

$$\frac{1}{2}(D_\mu\Phi)^T\, K\,(D^\mu\Phi) \to -i\left(\partial^\mu A^a_\mu\right)\left[\varphi^T K\, T^a\langle\Phi\rangle\right],$$

modified by the presence of the matrix K.

To compensate for the presence of J, K in the Lagrangian, we modify accordingly the gauge-fixing (GF) and ghost sector as

$$\mathcal{L}_{GF} + \mathcal{L}_{FP} = -\frac{1}{2\xi}\,\mathcal{G}^a J^{ab}\mathcal{G}^b + \bar{N}^a J^{ab} M_F^{bc} N^c \ ,$$

with $\mathcal{G}^a$ **linear** in the fields,

$$\mathcal{G}^a = \partial^\mu A^a_\mu - i\xi(J^{-1})^{ac}\left[\varphi^T K\, T^c\langle\Phi\rangle\right] \ ,$$

and the Fadeev-Popov matrix M_F obtained as usual ($\mathbf{s}\mathcal{G}^a = M_F^{ab} N^b$)

- The unwanted gauge-goldstone mixing is eliminated.

By redefining the fields as follows:

$$\tilde{\varphi} = K^{\frac{1}{2}}\varphi \ , \quad \tilde{A}_\mu = J^{\frac{1}{2}} A_\mu \ , \quad \eta = J^{\frac{1}{2}} N \ , \quad \bar{\eta} = J^{\frac{1}{2}} \bar{N} \ ,$$

- all kinetic terms become canonical.

$$\begin{aligned} \mathcal{L}_C + \mathcal{L}_{GF} &= -\frac{1}{4}\tilde{A}^T_{\mu\nu}\tilde{A}^{\mu\nu} - \frac{1}{2\xi}(\partial^\mu \tilde{A}_\mu)^T(\partial^\nu \tilde{A}_\nu) + \frac{1}{2}\tilde{A}^T_\mu (M^T M)\tilde{A}^\mu \\ &\quad + \frac{1}{2}(\partial_\mu \tilde{\varphi})^T(\partial^\mu \tilde{\varphi}) - \frac{\xi}{2}\tilde{\varphi}^T (M M^T)\tilde{\varphi} \ , \\ \mathcal{L}_{FP} &= \bar{\eta}^T \partial^\mu \partial_\mu \eta + \xi\, \bar{\eta}^T (M^T M)\eta + \ldots(\text{interactions}) \end{aligned}$$

with the (non-square in general), ${M_j}^b \equiv [K^{\frac{1}{2}}(iT^a)\langle\Phi\rangle]_j \, (J^{-\frac{1}{2}})^{ab}$.

With Singular Value Decomposition one can further show,

- for all gauge bosons and ghosts, $(m_A^2)^a = \xi (m_\eta^2)^a$
- for massive gauge and (would-be) goldstone bosons: $(m_A^2)^j = \xi (m_\phi^2)^j$

This is the convenient R_ξ framework of SM(EFT)!

Conclusions

- The effective field theories considered here are QFTs with local symmetries and therefore (in general) require gauge fixing.
- With a proper EOM reduction and careful modifications in the gauge fixing process one can apply the standard R_ξ-gauge to this general class of EFTs.
- Here we discussed the case of common ξ but it is possible to apply different ξ's, one for each gauge boson - this is useful for practical calculations.

Backup - SVD

To diagonalize the mass matrices, one can apply the Singular Value Decomposition

$$M = U^T \Sigma V$$

with orthogonal $U_{m\times m}$, $V_{n\times n}$ and diagonal $\Sigma_{m\times n}$, (i.e., a non-square matrix with ${\Sigma_j}^b = 0$ for $j \neq b$). Then,

$$\begin{aligned} VM^TMV^T &= \Sigma^T\Sigma = \begin{bmatrix} D_p & \\ & 0 \end{bmatrix}_{n\times n} \\ (\xi\times) \qquad UMM^TU^T &= \Sigma\Sigma^T = \begin{bmatrix} D_p & \\ & 0 \end{bmatrix}_{m\times m} \end{aligned}$$

with $p = min(m, n)$.
This suggests that the non-vanishing eigenvalues of gauge-bosons and goldstones are proportional, with ξ being the proportionality factor.