Model-independent upper limits on lepton number violating states from neutrino mass



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Problem: How are neutrino masses generated?

- Neutrino oscillations imply that neutrinos are massive.
- @ At least one neutrino has a mass larger or equal to 0.05 eV.
- However in the SM neutrinos are massless: need BSM physics.
- Hint: Lowest dimension effective operator $O_W = LLHH$ (D=5, Weinberg) violates lepton number (L) in 2 units.
- After EWSB, naturally light Majorana neutrino masses.
- Which is the UV completion of Ow chosen by Nature?

Concents

I- Mechanisms for neutrino masses II- Upper limits on scale of new particles III- Lower limits on scale of new particles IV- Summary and conclusions

I- Mechanisms for neutrino masses

Mechanisms

- Tree level. Only a few: seesaws I/II/III. Simple, GUT connection, leptogenesis, but huge scales imply very hard to test and hierarchy problem.
- Radiative. In principle more testable, but hundreds of them. Classified by:
- 1. Topologies at a loop order (up to 3 loops)
- 2. L=2 EFT operators beyond Weinberg operator.

Tree Level: seesaws

Minkowski, Yanagida, Gell-Mann, Mohapatra, Glashow...

yLHN, mNN

yLHΣ, mΣΣ yLΔL, μ HΔ[†]H



SSI SSII SSIII

LOOP LEVEL MODELS

Zee, Cheng-Li, Babu, Ma, Bonnet, Cepedello, Aristizabal-Sierra, Krauss, Aoki...



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Examples of Loop models zee, Cheng-Li, Babu

Singly-charged scalar: fLLh+

Zee model + $y\bar{e}\Phi^{\dagger}L + \mu h^{-}H\Phi$ Zee-Babu model + $geek^{++} + \mu k^{++}h^{-}h^{-}$





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LER CPETALOTS

Babu-Leung, De Gouvea-Jenkins

-> Zee model _ Zee-Babu model $O_2 = L^i L^j L^k e^c H^l \epsilon_{ii} \epsilon_{kl}, \quad O_{3a} = L^i L^j Q^k d^c H^l \epsilon_{ij} \epsilon_{kl}, \quad O_{3b} = L^i L^j Q^k d^c H^l \epsilon_{ik} \epsilon_{jl},$ $O_{4a} = L^i L^j \bar{Q}_i \bar{u}^c H^k \epsilon_{jk}, \quad O_{4b} = L^i L^j \bar{Q}_k \bar{u}^c H^k \epsilon_{ij}, \qquad O_8 = L^i \bar{e}^c \bar{u}^c d^c H^j \epsilon_{ij}.$ $O_9 = L^i L^j L^k e^c \hat{L}^l e^c \epsilon_{ij} \epsilon_{kl},$ $O_{10} = L^i L^j L^k e^c Q^l d^c \epsilon_{ij} \epsilon_{kl},$ $O_{11a} = L^i L^j Q^k d^c Q^l d^c \epsilon_{ij} \epsilon_{kl},$ $O_{11b} = L^i L^j Q^k d^c Q^l d^c \epsilon_{ik} \epsilon_{il},$ $O_{12a} = L^i L^j \bar{Q}_i \bar{u}^c \bar{Q}_j \bar{u}^c,$ $O_{12b} = L^i L^j \bar{Q}_k \bar{u}^c \bar{Q}_l \epsilon_{ij} \epsilon^{kl},$ $O_{14a} = L^i L^j \bar{Q}_k \bar{u}^c Q^k d^c \epsilon_{ij},$ $O_{13} = L^i L^j \bar{Q}_i \bar{u}^c L^k e^c \epsilon_{ik},$ $O_{15} = L^i L^j L^k d^c \bar{L}_i \bar{u}^c \epsilon_{ik},$ $O_{16} = L^i L^j \bar{e}^c d^c \bar{e}^c u^c \epsilon_{ij},$ $O_{17} = L^i L^j d^c d^c \bar{d}^c \bar{u}^c \epsilon_{ii},$ $O_{18} = L^i L^j d^c u^c \bar{u}^c \bar{u}^c \epsilon_{ij},$ $O_{19} = L^i Q^j d^c d^c \bar{e}^c \bar{u}^c \epsilon_{ii},$ $O_{20} = L^i d^c \bar{Q}_i \bar{u}^c \bar{e}^c \bar{u}^c.$

EFT EStimate

De Gouvea-Jenkins







Operator $\mathcal{O}_2 = LLL\bar{e}H$

Estimate chirality flip $m_{\nu} \simeq \frac{c_2 V V^2}{16\pi^2 \Lambda}$ Loop factor UV model: Zee $m_{\nu} \simeq \frac{f m_{\tau}^2 \mu}{16 \pi^2 m_{h^+}^2}$



Questions

A. How can we classify the plethora of models?

B. What are the most testable ones, with the lightest states?

C. Is any class of models already ruled-out?

D. Can we study the phenomenology without going to a particular model?

II- Upper limits on scale of new particles

Main idea

- 1. m_{ν} requires at least one new particle X (mass M) coupled to SM lepton/s, carrying L (and maybe B).
- 2. QFT: L is violated (by two units) via new operators at Λ , which encode the (model-dependent) UV physics.
- 3. Majorana neutrino masses, $m_{\nu} \propto 1/\Lambda$, are generated.

4. $m_{\nu} > 0.05 \, \text{eV} \& M \le \Lambda \Longrightarrow$ conservative upper bound on M.

Bounds apply to all models where X is the lightest state.

Example at tree level

@ SM bilinear LH (seesaw type I):

1. New particle: fermion singlet N with Y=0 and L=-1.

- 2. L is violated (by two units) via MNN (+ yLHN).
- 3. Neutrino masses, $m_{\nu} = y^2 v^2 / M$, are generated.
- 4. $m_{\nu} > 0.05 \, \text{eV} \& y \le 1 \implies$ conservative upper bound:

 $M \leq 10^{15} \,\mathrm{GeV}$

Possible new particles

 $LH \longrightarrow N(SSI), \Sigma(SSIII)$ $LL \longrightarrow \Delta (SSII), h (Zee)$ *ee* $\longrightarrow k$ (Zee – Babu) $LH^{\dagger} \longrightarrow \dots$ $\bar{e}H^{\dagger} \longrightarrow \dots$ $\bar{e}\sigma_{\mu}L^{\dagger}\longrightarrow\ldots$

Particles generating tree Level neutrino masses

$X \sim (SU(3)_c, SU(2)_L, U(1)_Y)_{S/F/V}^{L, 3B}$

Seesaw type L=2 operators

Particle	$\Delta \mathcal{L} = 0$	$ \Delta \mathcal{L} = 2$	*	$\operatorname{BL}^{\bullet}$	l	$m_{m u}$	Upper bound
$\bar{N} \sim (1, 1, 0)_F^{-1, 0}$	$yar{N}HL$	$Mar{N}ar{N}$	I	${\mathcal O}_1$	0	$\frac{y^2 v^2}{M}$	$M \lesssim 10^{15} { m GeV}$
$\Delta \sim (1, 3, 1)_S^{-2, 0}$	$y L \Delta L$	$\mu H \Delta^{\dagger} H$	II	\mathcal{O}_1	0	$\frac{y \mu v^2}{M^2}$	$M \lesssim 10^{15} { m GeV}$
$\bar{\Sigma}_0 \sim (1, 3, 0)_F^{-1, 0}$	$y \bar{\Sigma}_0 L H$	$Mar{\Sigma}_0ar{\Sigma}_0$	III	\mathcal{O}_1	0	$\frac{y^2 v^2}{M}$	$M \lesssim 10^{15} { m GeV}$
$L_1 \sim (1 \ 2 \ -1/2)^{1,0}$	$m ar{L}_1 L$	$\frac{c}{\Lambda}L_1HLH$		${\mathcal O}_1$	0	$\frac{c m}{M} \frac{v^2}{\Lambda}$	$M \lesssim 10^{15} { m GeV}$

Particles generating loop level neutrino masses

SU(2) U(1) L, 3B

Loop order

		,	Zee	ŀ		,	Lee-Badu
	Particle	$\Delta \mathcal{L} = 0$	$ \Delta \mathcal{L} = 2$	BL	l	$m_{ u}$	Upper bound
n 2	$\bar{N} \sim (1, 1, 0)_F^{-1, 0}$	$yar{N}HL$	$Mar{N}ar{N}$	\mathcal{O}_1	0	$\frac{y^2 v^2}{N}$	$M \lesssim 10^{15} { m GeV}$
5	$\Delta \sim (1, 3, 1)_S^{-2, 0}$	$y L \Delta L$	$\muH\Delta^\dagger H$	${\mathcal O}_1$	0	$\frac{y}{M^2}$	$M \lesssim 10^{15}~{\rm GeV}$
N V	$\bar{\Sigma}_0 \sim (1, 3, 0)_F^{-1, 0}$	$y \bar{\Sigma}_0 L H$	$M\bar{\Sigma}_0\bar{\Sigma}_0$	${\mathcal O}_1$	0	$\frac{y^2 v^2}{M}$	$M \lesssim 10^{15} { m GeV}$
y A	$L_1 \sim (1, 2, -1/2)_{E}^{1,0}$	$mar{L}_1L$	$\frac{c}{\Lambda}L_1HLH$	\mathcal{O}_1	0	$\frac{c m}{M} \frac{v^2}{\Lambda}$	$M \lesssim 10^{15} \text{ GeV}$
	-1 $(-,-,-)F$	$y H^{\dagger} \overline{e} L_1$	$rac{c}{\Lambda^2}L_1ar{u}d^\dagger L^\dagger$	\mathcal{O}_8^\dagger	2	$\frac{c yy_u y_d y_l}{(4\pi)^4} \frac{v^2}{\Lambda}$	$M \lesssim 10^7 { m GeV}$
1	$h \sim (1, 1, 1)_S^{-2, 0}$	yLLh	$rac{c}{\Lambda}h^{\dagger}\overline{e}LH$	\mathcal{O}_2	1	$\frac{c y y_l}{(4\pi)^2} \frac{v^2}{\Lambda}$	$M \lesssim 10^{10}~{\rm GeV}$
	$k \sim (1, 1, 2)_S^{-2, 0}$	$yar{e}^\daggerar{e}^\dagger k$	$rac{c}{\Lambda^3}k^{\dagger}L^{\dagger}L^{\dagger}L^{\dagger}L^{\dagger}L^{\dagger}$	\mathcal{O}_9^\dagger	2	${c y y_l^2 \over (4\pi)^4} {v^2 \over \Lambda}$	$M \lesssim 10^6 { m GeV}$
	$\bar{E} \sim (1, 1, 1)^{-1,0}$	$yar{E}LH^\dagger$	$\frac{c}{\Lambda^4} LEHQ^{\dagger} \bar{u}^{\dagger} H$	${\cal O}_6$	2	$rac{c y y_u}{(4\pi)^4} rac{v^2}{\Lambda}$	$M \lesssim 10^{10} \ {\rm GeV}$
2	Σ $(1,1,1)_F$	$mar{E}e$	$\frac{c}{\Lambda^3}EL^{\dagger}L^{\dagger}L^{\dagger}H^{\dagger}$	\mathcal{O}_2^\dagger	1	$\frac{c m}{M} \frac{y_l}{(4\pi)^2} \frac{v^2}{\Lambda}$	$M \lesssim 10^{10} { m GeV}$
5	$\bar{\Sigma}_1 \sim (1,3,1)_F^{-1,0}$	$y H^{\dagger} \bar{\Sigma}_1 L$	$\frac{c}{\Lambda^2}LHH\Sigma_1H$	${\cal O}_1'^1$	2	$rac{c y}{(4\pi)^4} rac{v^2}{\Lambda}$	$M \lesssim 10^{10} \ {\rm GeV}$
2	$L_2 \sim (1, 2, -3/2)_F^{1,0}$	$y H\overline{e}L_2$	$\frac{c}{\Lambda^2} \bar{L}_2 L L L$	${\mathcal O}_2$	1	$\frac{c y y_l}{(4\pi)^2} \frac{v^2}{\Lambda}$	$M \lesssim 10^{11} { m GeV}$
	$X_2 \sim (1, 2, 3/2)_V^{-2, 0}$	$y \bar{e}^{\dagger} \bar{\sigma}^{\mu} L X_{2\mu}$	$\frac{c}{\Lambda} \bar{u}^{\dagger} \bar{\sigma}^{\mu} \bar{d} X^{\dagger}_{2\mu} H$	\mathcal{O}_8	2	$\frac{cyy_uy_dy_e}{(4\pi)^4}\frac{v^2}{\Lambda}$	$M \lesssim 10^7 ~{\rm GeV}$

Particles with B (leptoquarks)

zadiative

 $X \sim (SU(3)_c, SU(2)_L, U(1)_Y)_{S/F/V}^{L, 3B}$ L=2 operators , Loop order

Particle	$\Delta \mathcal{L} = 0$	$ \Delta \mathcal{L} = 2$	BL	l	$m_{ u}$	Upper bound
$\tilde{R}_2 \sim (3, 2, 1/6)_S^{-1, 1}$	$y\overline{d}L ilde{R}_2$	$\frac{c}{\Lambda} \tilde{R}_2^{\dagger} Q L H$	\mathcal{O}_{3_b}	1	$\frac{c y y_d}{(4\pi)^2} \frac{v^2}{\Lambda}$	$M \lesssim 10^{11}~{\rm GeV}$
$R_2 \sim (3, 2, 7/6)_S^{-1, 1}$	$y \bar{e}^{\dagger} Q^{\dagger} R_2$	$rac{c}{\Lambda^3}R_2^{\dagger}L^{\dagger}LL\bar{d}^{\dagger}$	${\cal O}_{10}^\dagger$	2	$\frac{c y y_d y_l}{(4\pi)^4} \frac{v^2}{\Lambda}$	$M \lesssim 10^7~{\rm GeV}$
	$y \bar{u} L R_2$	$rac{c}{\Lambda^3}R_2^{\dagger}L^{\dagger}LL\bar{d}^{\dagger}$	\mathcal{O}_{15}^\dagger	3	$\frac{c y y_d y_u g^2}{2(4\pi)^6} \frac{v^2}{\Lambda}$	$M \lesssim 10^6~{\rm GeV}$
$S_1 \sim (\overline{3}, 1, 1/3)_S^{-1, -1}$	$y LQS_1$	$rac{c}{\Lambda}S_1^\dagger LH\overline{d}$	\mathcal{O}_{3_b}	1	$\frac{c y y_d}{(4\pi)^2} \frac{v^2}{\Lambda}$	$M \lesssim 10^{11}~{\rm GeV}$
	$y \bar{u}^{\dagger} \bar{e}^{\dagger} S_1$	$rac{c}{\Lambda}S_1^{\dagger}LHar{d}$	\mathcal{O}_8	2	$\frac{c y y_l y_u y_d}{(4\pi)^4} \frac{v^2}{\Lambda}$	$M \lesssim 10^7~{\rm GeV}$
$S_3 \sim (\overline{3}, 3, 1/3)_S^{-1, -1}$	$y LS_3Q$	$rac{c}{\Lambda}\overline{d}LS_3^{\dagger}H$	\mathcal{O}_{3_b}	1	$\frac{c y y_d}{(4\pi)^2} \frac{v^2}{\Lambda}$	$M \lesssim 10^{11}~{\rm GeV}$
$\tilde{S}_1 \sim (\bar{3}, 1, 4/3)_S^{-1, -1}$	$yar{e}^\daggerar{d}^\dagger ilde{S}_1$	$\frac{c}{\Lambda^3} \tilde{S}_1^\dagger L^\dagger L^\dagger L^\dagger Q^\dagger$	${\cal O}_{10}^\dagger$	2	$\frac{c y y_d y_l}{(4\pi)^4} \frac{v^2}{\Lambda}$	$M \lesssim 10^7 ~{\rm GeV}$
$V_{2} \sim (\bar{2} \ 2 \ 5 \ 6)^{-1,-1}$	$y\bar{d}^{\dagger}\bar{\sigma}^{\mu}V_{2\mu}L$	$\frac{c}{\Lambda^5}Q^{\dagger}\bar{\sigma}^{\mu}LV^{\dagger}_{2\mu}H\bar{e}LH$	\mathcal{O}_{23}	3	$\frac{c y y_d y_l}{(4\pi)^6} \frac{v^2}{\Lambda}$	$M \lesssim 10^4 { m GeV}$
$V_2 \sim (3, 2, 3/6)_V$	$y Q \sigma^{\mu} V_{2\mu} \bar{e}^{\dagger}$	$\frac{c}{\Lambda^5}Q^{\dagger}\bar{\sigma}^{\mu}LV^{\dagger}_{2\mu}H\bar{e}LH$	$\mathcal{O}_{44_{a,b,d}}$	3	$\frac{c y g^2}{2(4\pi)^6} \frac{v^2}{\Lambda}$	$M \lesssim 10^7~{\rm GeV}$
$\tilde{V}_2 \sim (\bar{3}, 2, -1/6)_V^{-1, -1}$	$y \bar{u}^{\dagger} \bar{\sigma}^{\mu} \tilde{V}_{2\mu} L$	$\frac{c}{\Lambda}Q^{\dagger}\bar{\sigma}^{\mu}LH\tilde{V}^{\dagger}_{2\mu}$	\mathcal{O}_{4_a}	1	$\frac{c y y_u}{(4\pi)^2} \frac{v^2}{\Lambda}$	$M \lesssim 10^{12}~{\rm GeV}$
$U_1 \sim (3 \ 1 \ 2/3)^{-1,1}$	$y Q^{\dagger} \bar{\sigma}^{\mu} U_{1\mu} L$	$\frac{c}{\Lambda} \bar{u}^{\dagger} \bar{\sigma}^{\mu} L H U_{1\mu}^{\dagger}$	\mathcal{O}_{4_a}	1	$\frac{c y y_u}{(4\pi)^2} \frac{v^2}{\Lambda}$	$M \lesssim 10^{12}~{\rm GeV}$
(0, 1, 2/0)V	$y \bar{d} \sigma^{\mu} U_{1\mu} \bar{e}^{\dagger}$	$\frac{c}{\Lambda} \bar{u}^{\dagger} \bar{\sigma}^{\mu} L H U_{1\mu}^{\dagger}$	\mathcal{O}_8	2	$\frac{c y y_u y_d y_l}{(4\pi)^4} \frac{v^2}{\Lambda}$	$M \lesssim 10^7~{\rm GeV}$
$U_3 \sim (3, 3, 2/3)_V^{-1, 1}$	$y Q^{\dagger} \bar{\sigma}^{\mu} U_{3\mu} L$	$\frac{c}{\Lambda} \bar{u}^{\dagger} \bar{\sigma}^{\mu} L U^{\dagger}_{3\mu} H$	\mathcal{O}_{4_a}	1	$\frac{c y y_u}{(4\pi)^2} \frac{v^2}{\Lambda}$	$M \lesssim 10^{12}~{\rm GeV}$
$\tilde{U}_1 \sim (3, 1, 5/3)_V^{-1, 1}$	$y \bar{u} \sigma^{\mu} \bar{e}^{\dagger} \tilde{U}_{1\mu}$	$\frac{c}{\Lambda^5} \bar{u}^\dagger \bar{\sigma}^\mu L H \tilde{U}^\dagger_{1\mu} \bar{e} L H$	\mathcal{O}_{46}	3	$\frac{c y g^2}{2(4\pi)^6} \frac{v^2}{\Lambda}$	$M \lesssim 10^7 ~{\rm GeV}$



Naturalness limits much stronger, but less robust.

III- Lower limits on scale of new particles

Phenomenology

- o Driven by renormalizable interaction:
- 1. Violation of lepton flavor, universality, PMNS unitarity.
- 2. Direct searches at colliders.
- o Driven by non-renormalizable part:
- A. L=2 processes, like neutrinoless double beta decay.
- B. B violation, like nucleon decays.
- C. Washout of BAU

Neutrinoless double beta decay

Ibarra, De Gouvea, Blennow, Rodejohann, Bonnet...

- a New contributions may be significant for:
- 1. SSI/III, if new fermion singlets $M_R \sim \mathcal{O}(\text{GeV})$

2. New D=7 operators, if $\Lambda \leq \mathcal{O}(100)$ TeV

Like $\mathcal{O}_8 = ueLdH$, generated by L_1, X_2, S_1, U_1

B violation (LG)

Weinberg, Weldon, Nath, Barr, Babu, Arnold, Dorsner...

Di-quark couplings generate tree-level nucleon decays:

$$S_{1} = (3,1,1/3): \quad y_{1}S_{1} ue + y_{2}S_{1}^{\dagger} ud$$

$$\Gamma(p \to \pi^{0}e^{+}) \simeq \frac{|y_{1}|^{2}|y_{2}|^{2}}{8\pi} \frac{m_{p}^{5}}{M_{S_{1}}^{4}} < \frac{1}{10^{33} \text{ y}}$$

$$\Longrightarrow M_{S} \gtrsim 10^{16} \text{ GeV}$$

Therefore, S1 cannot generate neutrino masses.

IV-Summary and conclusions



Summary plot

Tree level

Loop level



Conclusions

- Simple way of organising the plethora of neutrino models in "just" 20 categories (14 after nucleon decay limits).
- o Robust limits on all possible new states involved in m_{ν} .
- a Useful framework to study phenomenology.
- Nucleon decays rule-out some scenarios.
- Most promising states: doubly-charged scalars (<1000 TeV).



Backenup

LOOP LEVEL ESCIMALE De Gouvea

Weinberg operator induced via L=2 operators.
Matching at loop level. Estimate of m_ν:
1. Each loop: 1/(16π²)
2. SM chirality-flips: y_τ, y_t
3. W-bosons: g²/2



Weinberg, Weldon, Nath, Barr, Babu, Arnold, Dorsner...

 $S_1 \bar{d}\bar{u}, \ S_{1,3} Q^{\dagger} Q^{\dagger}, \ \bar{u}\bar{\sigma}^{\mu} V_{2\mu} Q^{\dagger}, \ \bar{d}\sigma^{\mu} \tilde{V}_{2\mu} Q^{\dagger} \Rightarrow M \gtrsim 10^{16} \text{ GeV}$ $\tilde{S}_1 \bar{u}\bar{u} \Rightarrow p \rightarrow e^+ e^- \bar{\nu}_e \pi^+ M \gtrsim 10^{11} \text{ GeV}$ $\tilde{R}_2 Q H^{\dagger} Q / \Lambda', \quad H^{\dagger} R_2 \bar{d}^{\dagger} \bar{d}^{\dagger} / \Lambda', \quad \bar{d}^{\dagger} \sigma_{\mu} H^{\dagger} Q U^{\mu}_{1,3} / \Lambda' \qquad B+L$ $p \rightarrow K^+ \nu$ $\Lambda' = M_p \Rightarrow M \gtrsim 10^7 \text{ GeV}$

Mashoul of BAU

Harvey, Turner

@ L=2 operators + sphalerons may erase the BAU, unless: $\Gamma(T_{\mathscr{B}-\mathscr{L}}) \leq H(T_{\mathscr{B}-\mathscr{L}})$ $\implies \Lambda \gtrsim [M_p T_{\mathscr{B}-\mathscr{L}}^{2d-9}/(20 \,\mathrm{PS}_n)]^{1/(2d-8)}$ $T_{\mathscr{B}-\mathscr{L}} = 10^{6}, 10^{10}, 10^{13} \text{ GeV} \Longrightarrow \overset{\Lambda_{d=5} \gtrsim 10^{11}, 10^{13}, 10^{14} \text{ GeV}}{\Lambda_{d>5} \gtrsim 10^{7}, 10^{10}, 10^{13} \text{ GeV}}$

Strong limits on scale Λ , dependent on B-L scale.

Higgs naturalness: scalars



 $\int_{H} \int_{H} \int_{H} \delta m_{H}^{2} \simeq -\left(\frac{\lambda}{16\pi^{2}}\right) N_{w} N_{c} M^{2} \ln\left(\frac{M^{2}}{\Lambda^{2}}\right)$



 $\int_{H} \int_{W,Z} \int_{W,Z} \int_{S} \int_{S} \delta\lambda \simeq \left(\frac{3}{32\pi^2}\right) (Y^2 g'^4 + C_2 g^4) \ln\left(\frac{M^2}{\Lambda^2}\right)$

Higgs naturalness: fermions



$$\delta m_H^2 \simeq \left(\frac{1}{4\pi^2}\right) N_c |y|^2 M^2 \ln\left(\frac{1}{4\pi^2}\right) N_c |y|^2$$

SSI. Vissani, Casas



$$\delta m_{H}^{2} \simeq \left(\frac{M^{2}}{32\pi^{4}}\right) N_{c} (3Dg^{4} + N_{w}Y^{2}g^{\prime 4}) \ln\left(\frac{M^{2}}{\Lambda^{2}}\right)$$
SSIII, Farina