Clockwork mechanism in heterotic M-theory

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based on: S.H. Im, H.P. Nilles and M.O. JHEP 1901 (2019) 151 and work in progress

Outline

- Motivation
- Clockwork mechanism and General Linear Dilaton
- Realization in minimal heterotic M-theory (Hořava-Witten model)
- Heterotic M-theory with vector multiplets
- Axions
- Summary

Motivation

- **Understanding the origin of large hierarchies of scales (and small couplings) is a major challenge in theoretical physics**

- **Some of proposed mechanisms based on extra dimensions**
  - **large extra dimensions (LED)**
  - **warped extra dimensions (RS)**
  - **linear dilaton model (LD)**
- **They may be considered as various General Linear Dilaton (GLD) models**
  - **generalizations of continuous version of clockwork mechanism**

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- **They may be considered as various General Linear Dilaton (GLD) models**
  - **generalizations of continuous version of clockwork mechanism**
- **Which of such models have UV-completions? (containing gravity and simple and/or interesting)**
- **Which may be derived from fundamental higher-dimensional theories like string- or M-theory?**

Clockwork mechanism:

- device to obtain light degrees of freedom
  with (strongly) suppressed couplings
  within theory without small fundamental parameters

- generalization of aligned axion mechanism — Kim, Nilles, Peloso, 2004
- name suggested in — Kaplan, Rattazzi, 2015
- related to deconstruction
- generalization of discrete clockwork to continuous one proposed in — Giudice, McCullough, 2016
- problems of such generalization — Craig, Garcia, Sutherland, 2017
- description of General Continuous Clockwork (GCCW) — Choi, Im, Shin, 2017

Discrete clockwork

Discrete scalar clockwork action ($q > 1$)

$$\int \mathrm{d}^4x \left[\sum_{i=0}^{N} \frac{1}{2} (\partial_\mu \phi_i)^2 + \sum_{i=0}^{N-1} \frac{1}{2} m^2 (\phi_{i+1} - q\phi_i)^2 \right]$$

Mass matrix

$$m^2 \begin{pmatrix} 1 & -q & 0 & \dots & 0 \\ -q & 1+q^2 & -q & \dots & 0 \\ 0 & -q & 1+q^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 1+q^2 & -q \\ 0 & 0 & 0 & -q & q^2 \end{pmatrix}$$

has one massless eigenstate $\chi_0 = \mathcal{N} \sum_{i=0}^{N} \frac{\phi_n}{q^i}$

component at each successive site is q times smaller than at the previous site

for large N: coupling of χ_0 at 0-th and N-th sites are very different

Continuous clockwork

Sites $i = 0 \ldots N$ may be interpreted as points in 5-th dimension

Continuum limit

$$\int \mathrm{d}^5x\, e^{2ky} \left[\frac{1}{2}(\partial_\mu \Phi)^2 + \frac{1}{2} e^{2py} (\partial_5 \Phi)^2\right]$$

may be obtained from simple Lagrangian

$$\int \mathrm{d}^5x \sqrt{-g}\, \frac{1}{2} \partial_\alpha \Phi \partial^\alpha \Phi$$

in the warped background

$$\mathrm{d}s^2 = e^{\frac{4}{3}ky} \left(e^{\frac{2}{3}py} \eta_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{-\frac{4}{3}py} \mathrm{d}y^2\right)$$

$\mu, \nu, \ldots = 0, 1, 2, 3 \qquad \alpha, \beta, \ldots = 0, 1, 2, 3, 5 \qquad x^5 \equiv y$

General Linear Dilaton

gravity + dilaton + cosmological constants on 5D orbifold $M^4 \times S^1/\mathbb{Z}_2$

$$\mathcal{S}_5 = M_5^3 \int \mathrm{d}^5x \sqrt{-g} \left(\tfrac{1}{2}\mathcal{R}_5 - \tfrac{1}{2}\partial_\alpha S \partial^\alpha S - \Lambda_b \, e^{-2(\hat{c}/\sqrt{3})S} - e^{-(\hat{c}/\sqrt{3})S} \left[\Lambda_0 \frac{\delta(y)}{\sqrt{g_{55}}} + \Lambda_\pi \frac{\delta(y-\pi R)}{\sqrt{g_{55}}} \right] \right)$$

4D flat background solution if: $-\Lambda_0 = \Lambda_\pi = \pm 6 \sqrt{\frac{2}{3}\left(\frac{\Lambda_b}{\hat{c}^2-4}\right)}$

general linear dilaton background: $(\hat{c}/\sqrt{3})S = \frac{2}{3}(k-p)y$

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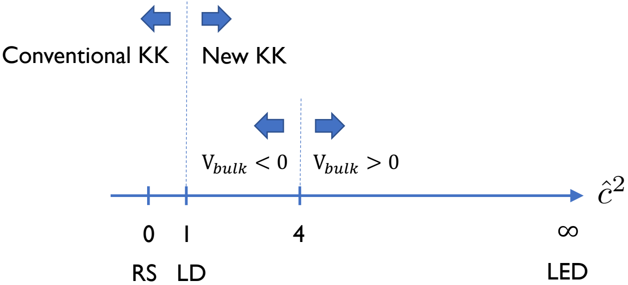
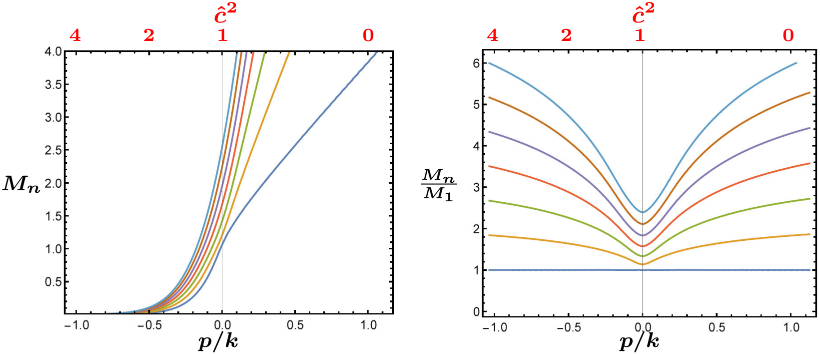


General Linear Dilaton

KK spectrum in GLD models **Choi, Im, Shin, 2017**



- **extra suppression of KK masses for** $\hat{c}^2 > 1$

$$\frac{p}{k} = 2\,\frac{1-\hat{c}^2}{2+\hat{c}^2} \qquad\qquad \hat{c}^2 > 1 \;\Rightarrow\; p/k < 0$$

- **RS:** $\hat{c}^2 = 0 \qquad p/k = 1$
- **LD:** $\hat{c}^2 = 1 \qquad p/k = 0$

Hořava-Witten model

Strongly coupled $E_8 \times E_8$ heterotic M-theory $\rightarrow$ 11D SUGRA

$$\mathcal{S}_{11} = \frac{1}{2\kappa^2} \int_{\mathcal{M}^{11}} \mathrm{d}^{11}x \sqrt{-g} \left(\star\mathcal{R} - G \wedge \star G - 2\sqrt{2}\, C \wedge G \wedge G \right)$$
$$- \frac{1}{8\pi\kappa^2} \left(\frac{\kappa}{4\pi} \right)^{2/3} \sum_{i=1}^{2} \int_{\mathcal{M}^{10}_{(i)}} \mathrm{d}^{10}x \sqrt{-g} \left(\mathrm{tr} F_{(i)} \wedge F_{(i)} - \frac{1}{2} \mathrm{tr} \mathcal{R} \wedge \mathcal{R} \right)$$

compactification on warped orbifold $M^4 \times X^6 \times S^1/\mathbb{Z}_2$

supersymmetry $\rightarrow$ non-zero flux: $\quad G_{ABCD} = -\frac{\mu}{48} \epsilon_{ABCD}{}^{EF} \omega_{EF}$

$$\mu \equiv \frac{\sqrt{2}}{\pi V_0} \left(\frac{\kappa}{4\pi} \right)^{2/3} \int_{X^6} \omega \wedge \left(\mathrm{tr} F_{(1)} \wedge F_{(1)} - \frac{1}{2} \mathrm{tr} \mathcal{R} \wedge \mathcal{R} \right)$$

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Standard embedding (Hořava-Witten) $\mu < 0$:

- volume V of Calabi-Yau X^6 decreases with $|x^{11}|$
- $\Rightarrow$ upper bound on length of 11-th dimension πR_{11}
- $M_W \lll (\pi R_{11})^{-1} < M_{11} < M_{\mathrm{Pl}}$

Minimal heterotic M-theory

Non-standard embedding with $\mu > 0$:

- volume V of Calabi-Yau X^6 increases with $|x^{11}|$
- length of 11-th dimension πR_{11} may be quite large
- hierarchy problem of the weak vs Planck scale may be addressed

$$M_{\rm Pl}^2 \approx \mathcal{O}(10)\, M_{11}^2 (M_{11} \pi R_{11})^2$$

Relation typical for $N = 2$ flat extra dimensions

$M_{11} \sim \mathcal{O}(1)$ TeV, $\pi R_{11} \lesssim 100\,\mu$m
enough to obtain the correct value of $M_{\rm Pl}$

Minimal heterotic M-theory

Compactification of 11D on Calabi-Yau X^6 with $h_{(1,1)} = 1$ (only universal hypermultiplet and gravity multiplet)

$\Rightarrow$ gravity-modulus system described by the GLD action

$$\mathcal{S}_5 = \frac{1}{\kappa_5^2}\int_{\mathcal{M}^5} \mathrm{d}^5x\sqrt{-g}\left[\frac{1}{2}\mathcal{R}_5 - \frac{1}{2}\partial_\alpha S\partial^\alpha S - \Lambda_b\, e^{-(2\hat{c}/\sqrt{3})S}\right] - \frac{1}{\kappa_5^2}\sum_{i=1,2}\int_{\mathcal{M}^4_{(i)}} \mathrm{d}^4x\sqrt{-g}\,\Lambda_{(i)}e^{-(\hat{c}/\sqrt{3})S}$$

with: $\hat{c}^2 = 6\,,\quad \Lambda_b = \frac{\mu^2}{384}\,,\quad \Lambda_{(1)} = -\Lambda_{(2)} = \frac{\mu}{4\sqrt{2}}$

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$\hat{c}^2 > 1 \Rightarrow$ non-conventional spectrum of KK states

$$M_n^2 \approx \mathcal{O}(10^3) \, n^2 \, M_{11}^2 \left(\frac{M_{11}}{M_{\mathrm{Pl}}} \right)^{5/2}$$

masses as for $N = 8/5 = 1.6$ flat extra dimensions
(but non-degenerate)

Heterotic M-theory with vector multiplets (in 5D)

- compactified on CY space with the Hodge number $h_{(1,1)} > 1$
- $h_{(1,1)}$ Kähler moduli t^i defined by $\omega = t^i \omega_i$
- intersection numbers $d_{ijk} \equiv \frac{1}{V_0} \int_{X^6} \omega_i \wedge \omega_j \wedge \omega_k$
- $h_{(1,1)}$ flux parameters

$$\mu_i \equiv \frac{\sqrt{2}}{\pi V_0} \left(\frac{\kappa}{4\pi} \right)^{2/3} \int_X \omega_i \wedge \left(\mathrm{tr} F_{(1)} \wedge F_{(1)} - \frac{1}{2} \mathrm{tr} \mathcal{R} \wedge \mathcal{R} \right)$$

Heterotic M-theory with vector multiplets

Simple example: $h_{(1,1)} = 2$, **only** $d_{112} \neq 0$

- $\mu_1 \neq 0 \qquad \mu_2 \neq 0$ (same as for $h_{(1,1)} = 1$):
  - $\hat{c}^2 = 6$
  - Planck mass as for $N = 2$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu$m $\Rightarrow$ $M_{11} = \mathcal{O}(1)$ TeV
  - KK spectrum similar to that of $N = 1.6$ flat extra dimensions

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- $\mu_1 \neq 0 \qquad \mu_2 = 0$:
  - $\hat{c}^2 = 7$
  - Planck mass as for $N = 1.8$ flat extra dimensions
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- $\mu_1 = 0 \qquad \mu_2 \neq 0$:
  - $\hat{c}^2 = 10$
  - Planck mass as for $N = 1.5$ flat extra dimensions
  - $\pi R_{11} \sim 100\mu\text{m} \quad \Rightarrow \quad M_{11} = \mathcal{O}(100)$ TeV
  - KK spectrum similar to that of $N = 4/3$ flat extra dimensions

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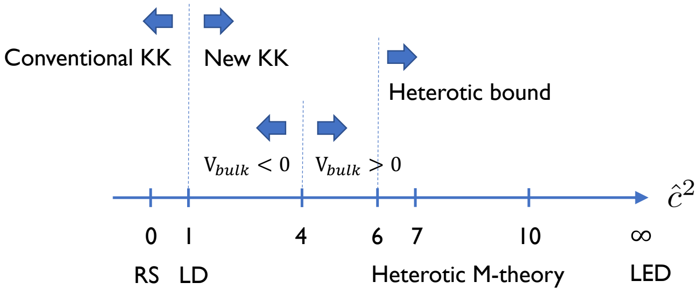
Warped product of one large (flat) and six curved extra dimensions

Heterotic M-theory with vector multiplets

It is not difficult to see that in general heterotic M-theory $\hat{c}^2 \geq 6$



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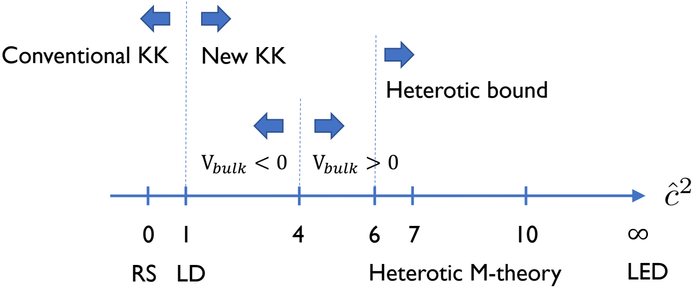

Smaller values of $\hat{c}^2 = 1,\, 4$ were obtained in 5D SUGRA
with decoupled universal hypermultiplet

Kehagias, Riotto, 2017; Antoniadis et al., 2017

Problematic from the point of view of higher dimensional string- or M-theory (LD $\hat{c}^2 = 1$ may be related to 6D "Little String Theory")

Axions

Axion-like fields in heterotic string- and M-theory

- **origin**
  - one (MI) a dual in 4D to tensor field strength in 10D ($H_{\mu\nu\rho}$) or 11D ($G_{\mu\nu\rho\,11}$) SUGRA
  - $h_{(1,1)}$ (MD) b^i coming from harmonic (1,1) forms on CY in tensor fields (B_{mn} in 10D, $C_{mn\,11}$ in 11D)

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  - b^i:
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- decay constants
  - in heterotic string (10D): all of order $\sim 10^{16}$ GeV
  - in heterotic M-theory (11D):
    - may be much smaller in the case of non-standard embedding (due to clockwork mechanism)
    - exactly one of the axions suitable to solve the strong CP problem

Summary

- General Linear Dilaton models (5D)
  - 2-parameter class of potential solutions to the hierarchy problem (using continuous clockwork mechanism)
  - there are consistent UV-completions of GLD models but probably only for a very limited discrete set of parameters
- Heterotic M-theory may be such 11D UV-completion
  - non-standard embedding (of spin connection in the gauge group) is necessary
  - minimal version (modification of Hořava-Witten model)
    - Planck-scale hierarchy as for 2 flat extra dimensions
    - KK spectrum as for 1.6 flat extra dimensions
    - M_{11} scale not very much higher than the weak scale is possible
  - heterotic bound: $\hat{c}^2 \geq 6$
  - models constructed only for $\hat{c}^2 = 6,\ 7,\ 10$
- Previously found 5D SUGRA models with $\hat{c}^2 = 1,\ 4$: uplift to higher dimensional string- or M-theory seems to be problematic
- One axion in minimal version of heterotic M-theory may have properties necessary to solve the strong CP problem