Self-consistent gradient corrections to false vacuum decay for a U(1) gauge theory

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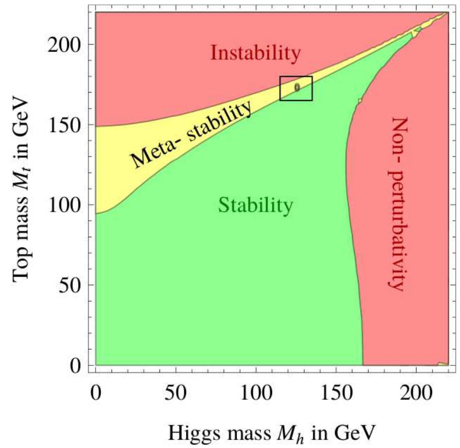
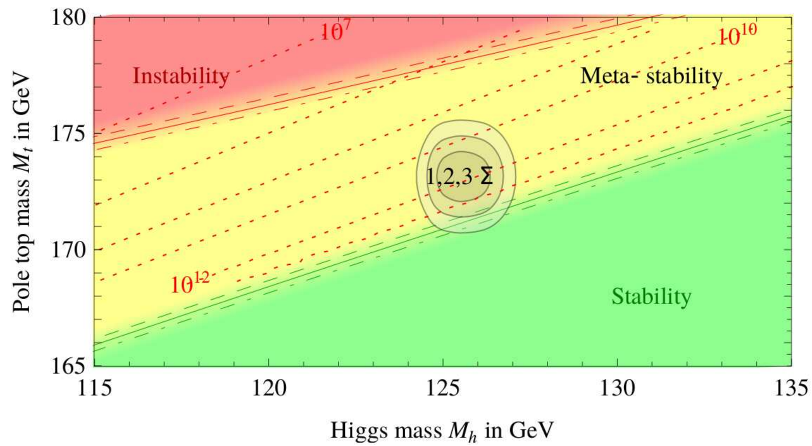
1. Motivation

The Standard Model of particle physics with its current measurements, specifically the masses of the Higgs and the top quark, place our universe in a metastable region of the Higgs field potential [Degrassi et al., 2012, Buttazzo et al., 2013].





The metastability region is characterized by more than one local minimum allowing for *tunneling phenomena* associated with regions of different phases[Coleman, 1977, Callan and Coleman, 1977].

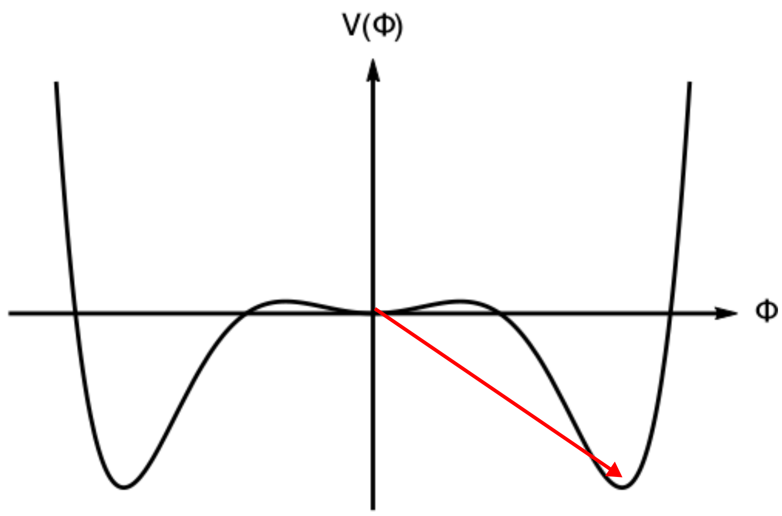
1. Background

Tunneling Phenomena

Energetically forbidden regions can be accessed in QM through tunneling. Probabilities of excitations are exponentially suppressed.

Vacuum state decay

The expectation value of a field at tree-level corresponds to a local minimum.
A theory with more than one local minimum presents different *vacuum sectors*.
In certain scenarios they can be connected by specific euclidean solutions of the classical e.o.m..



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2. Previous Studies

Semi-classical field theory

Approximate the euclidean vacuum to vacuum transition amplitude for a theory,

$$Z[0] = \langle \varphi_+ | e^{-HT/\hbar} | \varphi_+ \rangle = \mathcal{N} \int \mathcal{D}\phi \, e^{-S_E[\phi]}, \quad (1)$$

using the extremal point of the action, which satisfies the boundary conditions at hand.

Effective action in the standard model

[Hung, 1979], [Cabibbo et al., 1979] investigate necessary conditions such that the vacuum is stable. Modern studies assume a metastable situation and compute the decay rates to use as bounds[Isidori et al., 2001].

A relation between the decay rate and the effective action exists[Garbrecht and Millington, 2015]

$$\Gamma/V = 2\,|\mathrm{Im}\, e^{-\Gamma^{(n)}[\varphi^{(n)}]/\hbar}|/(TV), \quad (2)$$

Are these results gauge independent?

The effective potential is known to be gauge-dependent[Jackiw, 1974], however quantities that depend solely on extremal points are not (see [Nielsen, 1975, Aitchison and Fraser, 1984], [Metaxas and Weinberg, 1996, Lalak et al., 2016], [Endo et al., 2017, Plascencia and Tamarit, 2016].)

2. Previous Studies

Corrections due to gradient effects

This study uses a spatially non-homogeneous background, the bounce, in an abelian Higgs-like gauge field with a metastable vacuum at tree-level.

> Similar studies have been carried out focusing on different cases or aspects such as Dirac fields, Electroweak phase transition and scale invariant potentials, etc. (see [Baacke, 1990], [Baacke and Junker, 1994], [Sürig, 1998] [Garbrecht and Millington, 2015], [Garbrecht and Millington, 2017], [Ai et al., 2018]).

We will focus on the numerical implementation for future gauge dependence studies.

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3. Our toy model

Consider the following Lagrangian in Euclidean space-time

$$\mathscr{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + (D_\mu \phi)^* (D_\mu \phi) + U(\phi^* \phi) + \mathscr{L}_{\mathrm{G.F.}} + \mathscr{L}_{\mathrm{ghost}}, \tag{3}$$

where

$$U(\phi^* \phi) = \alpha \phi^2 + \lambda \phi^4 + \lambda_6 \phi^6 \tag{4}$$

$$\mathscr{L}_{\mathrm{G.F.}} = \frac{1}{2\xi} (\partial_\mu A_\mu - \zeta g \phi G)^2, \tag{5}$$

and leads to the partition function

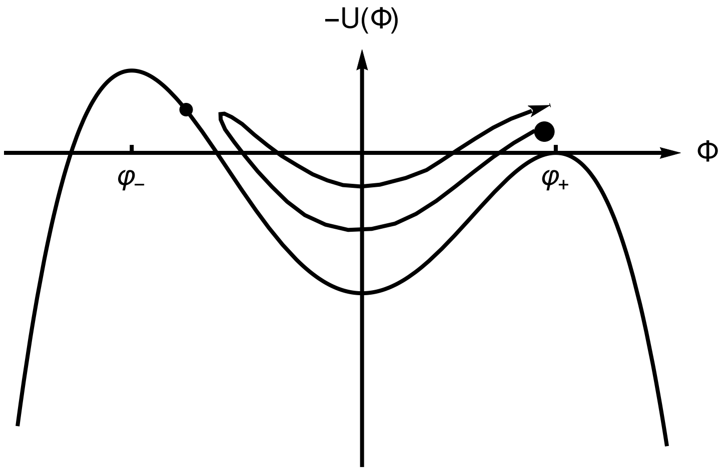
$$Z[J, K_\mu, \bar{\psi}, \psi] = \int \mathscr{D}\phi \mathscr{D}A_\mu \mathscr{D}\eta \mathscr{D}\bar{\eta} \exp\Bigg(-\frac{1}{\hbar} \int \mathrm{d}^4 x [\mathscr{L} - J(x)\phi(x) - K_\mu(x) A_\mu(x) - \bar{\psi}(x)\eta(x) - \bar{\eta}(x)\psi(x)] \Bigg). \tag{6}$$

3. The bounce solution

The thin wall approximation

The classical bounce $\varphi_b(r)$ is obtained from the classical equation of motion by looking at $O(4)$ invariant solutions.
We employ a thin wall approximation where the dissipative term is ignored, with $r^2 = \mathbf{x}^2 + x_4^2$ one has:

$$-\frac{\mathrm{d}^2\varphi}{\mathrm{d}r^2} - \frac{3}{r}\frac{\mathrm{d}\varphi}{\mathrm{d}r} + U'(\varphi) = 0 \tag{7}$$



Expansion around the bounce

One finds 1-loop propagators for the theory expanded around φ_b, that is we make the substitution:

$$\phi = \frac{1}{\sqrt{2}}(\varphi_b + \hat{\Phi} + iG). \tag{8}$$

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4. 1-PI Effective action

The computation of the 1-PI effective action formally gives the expression,

$$\Gamma^{(1)}[\varphi^{(1)}] = S[\varphi^{(1)}] + \frac{\hbar}{2}\log\frac{\det\mathscr{M}^{-1}_{\hat{\Phi}}(\varphi^{(1)})}{\det\mathscr{M}^{-1}_{\hat{\Phi}}(0)} + \frac{\hbar}{2}\log\frac{\det\mathscr{M}^{-1}_{(A_\mu,G)}(\varphi^{(1)})}{\det\mathscr{M}^{-1}_{(A_\mu,G)}(0)} - \hbar\log\frac{\det\mathscr{M}^{-1}_{(\bar{\eta},\eta)}(\varphi^{(1)})}{\det\mathscr{M}^{-1}_{(\bar{\eta},\eta)}(0)}, \tag{9}$$

where $\varphi^{(1)}$ will later be expanded in $\varphi_b + \hbar\delta\varphi$, and it is as well the Coleman-Weinberg potential (CW) times a volume factor, when evaluated at homogeneous field values. The corresponding operators:

$$\mathscr{M}^{-1}_{\hat{\Phi}}(\varphi^{(1)}) = -\Delta + \alpha + 3\lambda\,(\varphi^{(1)})^2 + \frac{15\lambda_6}{4}(\varphi^{(1)})^4\,, \tag{10}$$

$$\mathscr{M}^{-1}_{(A_\mu,G)}(\varphi^{(1)}) = \begin{pmatrix} (-\Delta + g^2(\varphi^{(1)})^2)\,\delta_{\mu\nu} + \frac{\xi-1}{\xi}\partial_\mu\partial_\nu & \left(\frac{\zeta+\xi}{\xi}\right) g\,(\partial_\mu\varphi^{(1)}) + \left(\frac{\zeta-\xi}{\xi}\right) g\,\varphi^{(1)}\,\partial_\mu \\ 2\,g\,(\partial_\nu\varphi^{(1)}) + \left(\frac{\xi-\zeta}{\xi}\right) g\,\varphi^{(1)}\,\partial_\nu & -\Delta + \alpha + \lambda\,(\varphi^{(1)})^2 + \frac{3\lambda_6}{4}(\varphi^{(1)})^4 + \frac{\zeta^2 g^2 (\varphi^{(1)})^2}{\xi} \end{pmatrix}, \tag{11}$$

$$\mathscr{M}^{-1}_{(\bar{\eta},\eta)}(\varphi^{(1)}) = -\Delta + \zeta\,g^2\,(\varphi^{(1)})^2. \tag{12}$$

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4. Self-consistent Green's function method

1. Obtain φ_b the bounce as described above.
2. Solve for the tree-level Green's functions in position space

$$\left.\frac{\delta^2 S}{\delta\Phi(x)\delta\Phi(z)}\right|_{\phi=\varphi_b} G(\varphi_b; z, y) = \delta^{(4)}(x-y). \tag{13}$$

3. Compute the "$\log\det$" terms from the 1-PI effective action $\Gamma^{(1)}[\varphi_b]$.
4. The tadpole function for a field ϕ on the background φ_b,

$$\Pi(\varphi_b; x)\varphi_b(x) = \frac{\delta}{\delta\varphi(x)} \log \frac{\det M^{-1}_{\varphi_b}[\varphi(x)]}{\det M^{-1}_{\varphi_b}[0]} \tag{14}$$

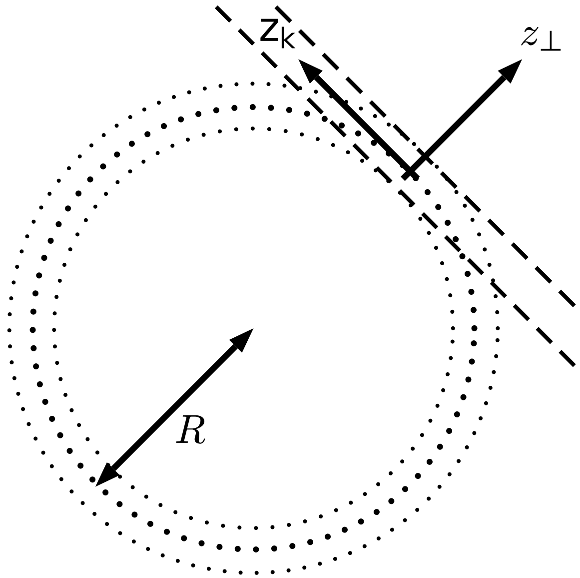
   gives the corrections to the bounce, $\delta\varphi$, according to

$$-\partial_z^2 \delta\varphi(x) + U'(\varphi_b(x) + \delta\varphi(x)) + \hbar\Pi(\varphi_b; x)\varphi_b(x) = 0 \tag{15}$$

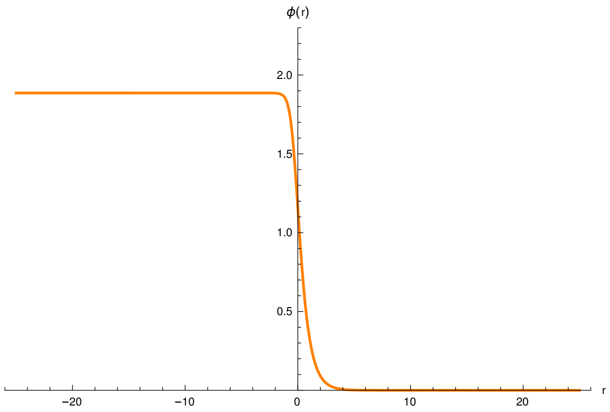
5. Substituting $\delta\varphi$ back into the action yields quadratic corrections to the decay rate.

4. Simplifications and Assumptions

Planar Wall (3 + 1 Decomposition)



Fourier transform the directions tangential to the bubble wall to simplify the system of equations. This allows us to express Green's functions as sums over 3-momentum where the integrands can be readily computed numerically.



4. Simplifications and Assumptions II

Step 0: Handling the easy Gauge

$\xi = \zeta = 1$ decouples most of the components of A_μ and lets us test the method in a setting where only A_4 and the Goldstone boson G are coupled.

Iterative treatment to obtain the Green's functions

Split the operator into a diagonal and off-diagonal part,

$$\mathscr{M}^{-1}_{(A_4,G);\mathbf{k}}(\varphi_b;z) = \mathscr{M}_0^{-1}(z) + \delta\mathscr{M}^{-1}(z) = \begin{pmatrix} M_{\mathbf{k}}^{-1}(\varphi_b(z)) & 0 \\ 0 & N_{\mathbf{k}}^{-1}(\varphi_b(z)) \end{pmatrix} + \begin{pmatrix} 0 & 2g(\partial_z\varphi_b) \\ 2g(\partial_z\varphi_b) & 0 \end{pmatrix},$$

iterate over the solution to the diagonal part to include gradient corrections.

This simultaneously expands on the coupling g and the gradients of the background.

5. Implementation I: Functional determinants

The s-parameter

We deform the operator through an auxiliary parameter s, as done by [Baacke and Junker, 1994].

Given a differential operator $\mathscr{M}^{-1}$ then its deformation is $\mathscr{M}_s^{-1} = \mathscr{M}^{-1} + s$.

If G is a Green's function for $\mathscr{M}^{-1}$ with spectral decomposition $\sum_n \frac{f_n(x) f_n^*(y)}{\lambda_n}$ then the spectral decomposition for the deformed operator is $\sum_n \frac{f_n^*(y) f_n(x)}{\lambda_n + s}$ and one can write

$$\log \frac{\det M^{-1}(\varphi)}{\det M^{-1}(\chi)} = -\int_0^\infty \mathrm{d}s \int \mathrm{d}^4 x \, G_s(\varphi; x, x) - G_s(\chi; x, x) \tag{16}$$

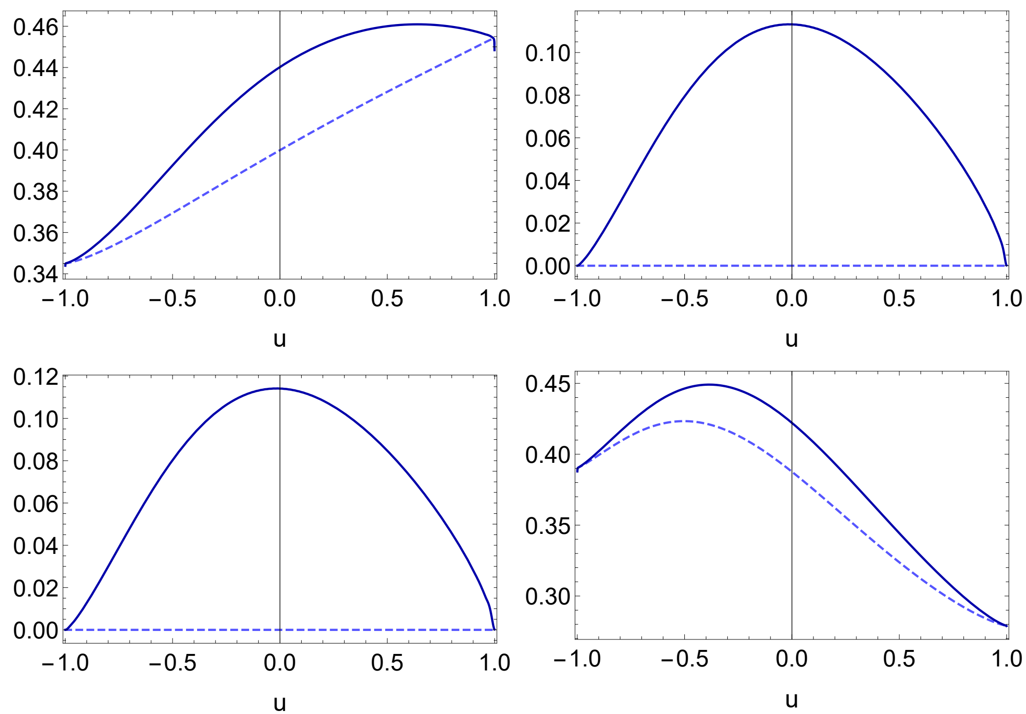
We take the 3+1 decomposition and numerically integrate up to some cutoff in tangential momentum and the s-parameter in our case:

$$\log \frac{\det M^{-1}(\varphi_b)}{\det M^{-1}(0)} = -\int_0^{\Lambda_s^2} \mathrm{d}s \int_{-\infty}^{\infty} \mathrm{d}z \int \mathrm{d}^3\mathbf{x} \int_0^{\Lambda_k} \mathrm{d}k \, \frac{k^2}{2\pi^2} \mathrm{tr}\left(G_{s,\mathbf{k}}(\varphi_b; z, z) - G_{s;\mathbf{k}}(0; z, z)\right) \tag{17}$$

5. Implementation II: Numerical methods

Numerical computation

- Find a bounce solution for tuned CW-potential, center the wall and extend values to infinity.
- Fix a value of $|\mathbf{k}|$, solve diagonals numerically for $G^{(0)}(x, x')$ and iterate for corrections.
- Compute quantities of interest such as determinants and tadpole contributions.



Observations

- Corrections need sampling in two dimensions of a numerical integral, one iteration needs around 15k sec with 6 cores.
- A large region in $|\mathbf{k}|$ must be scanned.

5. Problems

Tuning the potential

- One must ensure the validity of the thin wall approximation by picking potentials that present degenerate vacua.
- A finite functional determinant when integrated along the bubble wall requires degeneracy at the CW-level.

Renormalizing

- The model is not renormalizable without introducing higher-dimensional operators.
- The CW-potential at a fixed field value allows to extract the divergences and write down coupling counterterms.
- The wavefunction renormalization is obtained through Feynman diagrams with momentum-dependent vertexes involving the gauge field.

6. Conclusions and Outlook

So far...

- The decay rate in the model involving a gauge-field and a complex scalar is computed together with corrections by taking into account gradients of the background using a self-consistent prescription.
- A numerical treatment has been developed to compute the quantities involved.
- We renormalize quantities through the use of the CW potential and WKB expansions.

What's next...

- Consider different gauge-parameters. (Might need new strategies)
- Study cases including the abandoning the planar and/or thin wall approximations.
- Consider more realistic models, where metastable vacua appear through radiative corrections.

... perhaps "our universe is simply one of those things that happen from time to time." Edward Tryon

Thank you!

Backup slides

Decay rate expanded

$$\frac{\Gamma}{V} \propto \exp\left(-\frac{1}{\hbar}\left(B + \hbar B^{(1)} + \hbar^2 B^{(2)}\right)\right) \tag{18}$$

Boundary conditions for the bounce

$$t = \pm\infty, \phi(t, \mathbf{x}) = \phi_{fv} \qquad \text{and} \qquad |\mathbf{x}| \to \infty, \phi \to \phi_{fv} \tag{19}$$

Conclusions from the previous studies:

- Scalar loops increase *B* and cause faster decay.
- Fermion loops decrease *B* and prolong the decay.

Parameters used for the current study:

$$\alpha = 2 \quad \lambda = -2.02546 \quad \beta = \frac{1}{2} \quad g = \frac{1}{2} \tag{20}$$