Breaking down a 6D GHGUT

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Outline

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Numerical exploration

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Hosotani Mechanism

Given a theory with a gauge field $A_\mu(x)$, we can construct the **Wilson line** as a function of A_μ around a path $\mathscr{P}$ running from $y \to z$

$$W(z, y) = \exp\left\{-ie \int_{\mathscr{P}} dx^\mu A_\mu(x)\right\}.$$

The Wilson line is for non-smooth manifolds **the holonomy equivalent**.

Suppose we have a theory defined on the **orbifold** $\mathscr{M}_4 \times S^1/\mathbb{Z}_2$, then depending on,

- Boundary conditions
- Matter content

the Wilson lines along S^1 are non-trivial and develop phases $\theta^{\hat{a}}$ corresponding to scalar degrees of freedom.

Suppose we now have a group symmetry G that is broken down to G' via boundary conditions P_0, P_1, U corresponding to,

$$U : y \to y + 2\pi R, \qquad P_0 : y \to -y,$$
$$P_1 : y - \pi R \to y + \pi R,$$

The physical d.o.f corresponding to Wilson line phases $\theta^{\hat{a}}$ are,

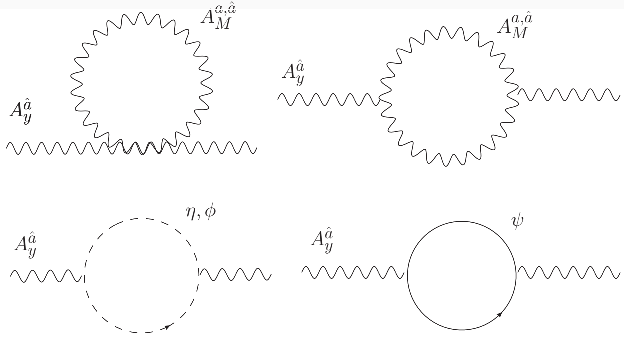
$$W(y, y + 2\pi R) = \mathcal{P} \exp \left\{ ig \int_0^{2\pi R} dy A_y(x, y) \right\} = \exp \left(i\theta^{\hat{a}} \lambda_{\hat{a}} \right),$$
$$\left\{ \theta^{\hat{a}} = g\pi R A_y^{\hat{a}}, \quad \hat{a} \in \mathcal{H}_W \right\},$$

defined by the anti-commuting set $\mathcal{H}_W$,

$$\mathcal{H}_W = \left\{ \frac{\lambda^{\hat{a}}}{2} \quad ; \quad \{\lambda^{\hat{a}}, P_0\} = \{\lambda^{\hat{a}}, P_1\} = 0 \right\}$$

via $\{\lambda^a\}$ the set of generators of G.

Wilson line phases $\theta^{\hat{a}}$ appear as *degenerate vacua at the classical level.*



The degeneracy is lifted by quantum corrections via $V_{\text{eff}}(\theta_H)$.

If the effective potential is minimised at a non-trivial configuration of Wilson line phases,

$$\min V_{\text{eff}}(\theta_H) \equiv \langle \theta_H \rangle \neq 0,$$

the *gauge symmetry is either spontaneously broken or restored.*

The **physical symmetry** of the theory is determined by the *combination of the boundary conditions and the expectation values of Wilson line phases.*

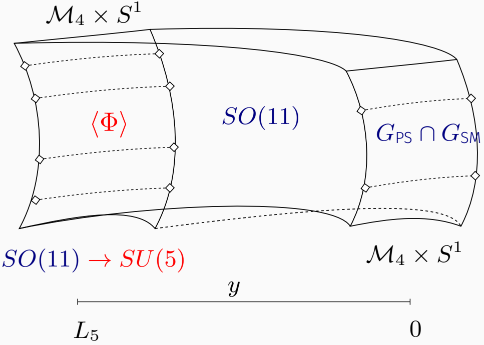
$$\{\langle \theta_H \rangle; P_0, P_1, U\}$$

Conclusion
The Hosotani mechanism achieves symmetry breaking and provides a Higgs candidate which is the extra dimensional component of A_M.

6D model

Space-time and gauge symmetry

The model was proposed by *Hosotani et al.* in *hep-ph/1710.04811*.



Formulated in a *hybrid 6D* compactified space with a $SO(11)$ gauge symmetry.

$$ds^2 = e^{-2\sigma(y)}(\eta_{\mu\nu}dx^{\mu}dx^{\nu} + d\nu^2) + dy^2,$$

- The Electroweak (EW) coordinate $y \in [0, L_5]$
- The GUT coordinate $\nu \in [0, 2\pi R_6]$

Matter Fields

The matter content consists of 6D and 5D fields transforming under $SO(11)$.

6D fields

Gauge Bosons: $A_M(x, y, \nu)$

Spinors: $\Psi^{\alpha}_{\mathbf{32}}(x, y, \nu)$

Dirac Vectors: $\Psi^{\beta}_{\mathbf{11}}(x, y, \nu)$ $\Psi'^{\beta}_{\mathbf{11}}(x, y, \nu)$

5D fields

Brane Spinor Scalar: $\Phi_{\mathbf{32}}(x, \nu)$

Brane Symplectic Majorana: $\chi^{\beta}_{\mathbf{1}}(x, \nu)$

5D / 6D Actions

Pure Bulk: $S^{\text{bulk}}_{\Psi\Psi A_M}$ $S^{\text{bulk}}_{\text{Yang-Mills}}$

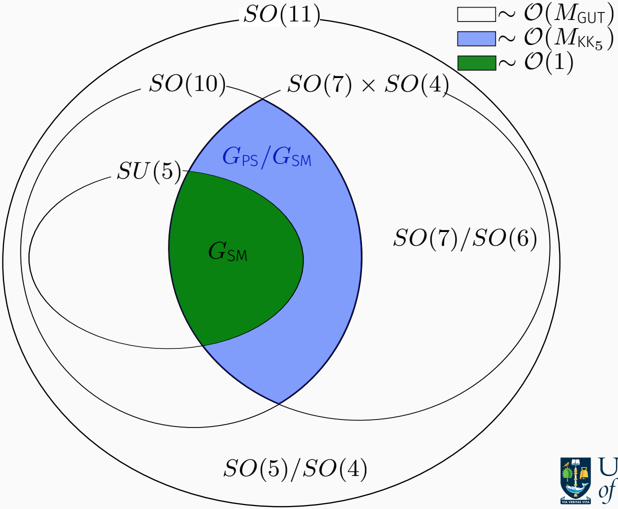
Brane-Bulk: $S^{\text{brane}}_{\chi\chi A_M}$ $S^{\text{brane}}_{\Phi\Phi A_M}$ $S^{\text{brane}}_{\Psi\Phi\chi}$

Symmetry Breaking Schematic

Parity Assignments and the **UV brane scalar** break $SO(11)$ on the *IR brane*:

$$SO(11) \to G_{\text{PS}} \cap SU(5) = G_{\text{SM}}$$

Which is broken to $SU(3)_C \times U(1)_{\text{EM}}$ by the Hosotani mechanism.



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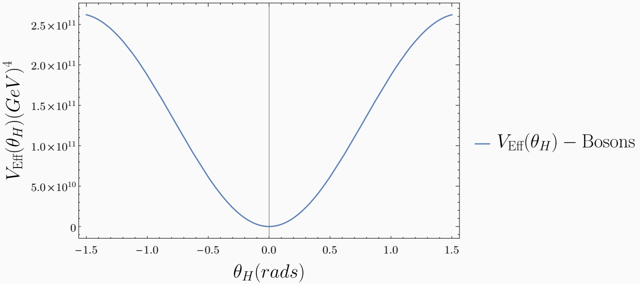
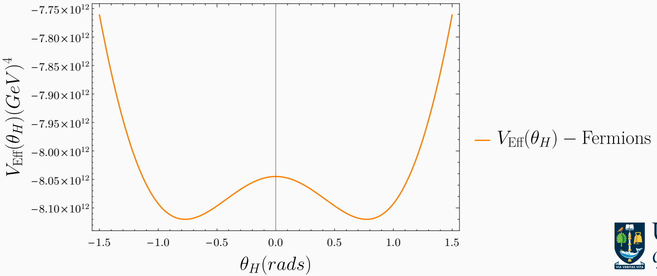
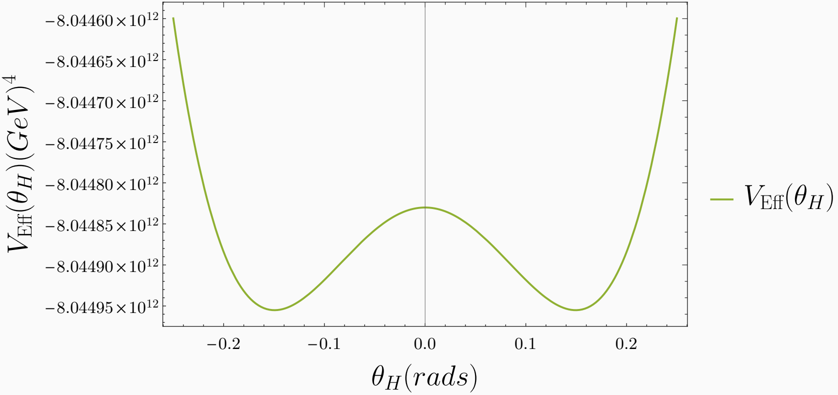
The set of parameters $\mathcal{P}$ determine the Effective potential contributions from the *Fermions* and *Bosons*





Effective Potential $V_{\text{Eff}}(\theta_H)$

The full effective potential develops a non-trivial minimum $\langle\theta_H\rangle$, which determines the mass spectrum on the IR brane



Numerical exploration

Numerical Scans

We aim to find as many Standard Model like solutions as possible by sampling the UV model **stochastically** via the *controlling parameters*,

$$\mathcal{P} = \left\{ \underbrace{k, z_L,}_{\text{AdS5}} \quad \underbrace{c_0, c_0', c_1, c_2,}_{\text{5D masses}} \quad \underbrace{\mu_1, \tilde{\mu}_2, \mu_{11}, \mu_{11}',}_{\text{Bulk-Brane Couplings}} \quad \underbrace{M, m_B}_{\text{5D Majorana Masses}} \right\}.$$

The scanning algorithm consists of two stages in the following order:

1. A uniform random sampling of the phase space **with** / **without** vetting specified by model constraints.
2. A differential evolution algorithm based on a global χ^2_G measure based on some SM values experimental values.

Differential evolution algorithm introduced by *Storn & Price* in *doi.org/10.1023/A:1008202821328*, consists of 3 stages for each generation G for a target parameter vector $\mathbf{x}_i^G$:

Mutation
Create a *mutation vector* out of 3 other $r_1, r_2, r_3 \neq i$ members of G,

$$\mathbf{m}_i^G = \mathbf{x}_{r_1} + F(\mathbf{x}_{r_2} - \mathbf{x}_{r_3})$$

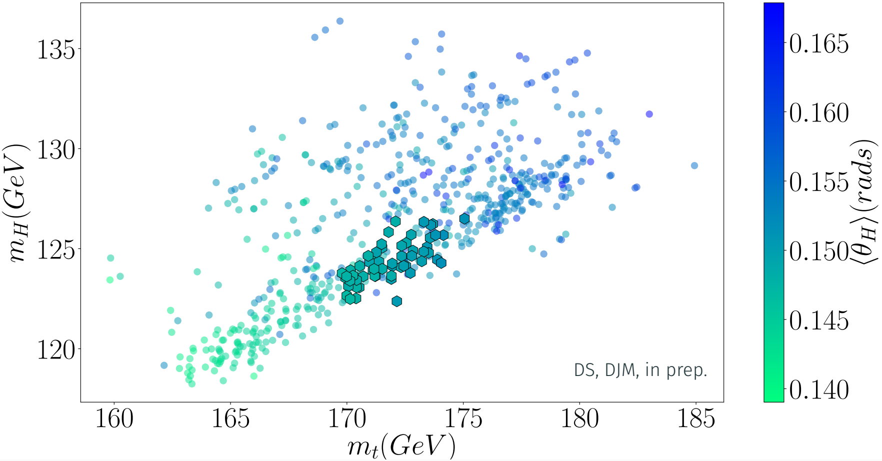
Recombination
Randomly splice the target vector $\mathbf{x}_i^G$ and the mutation vector $\mathbf{m}_i^G$ to form a trial vector,

$$\mathbf{t}_i^{G+1}$$

Selection
Compare the target and trial vector. The lowest χ_G^2 moves to the next generation,

$$\mathbf{x}_i^{G+1} = \begin{cases} \mathbf{t}_i^{G+1} & \text{,if} \quad \chi_G^2(\mathbf{t}_i^{G+1}) < \chi_G^2(\mathbf{x}_i) \\ \mathbf{x}_i^G & \text{,otherwise .} \end{cases}$$

We can recreate the original solution from Hosotani et al. !

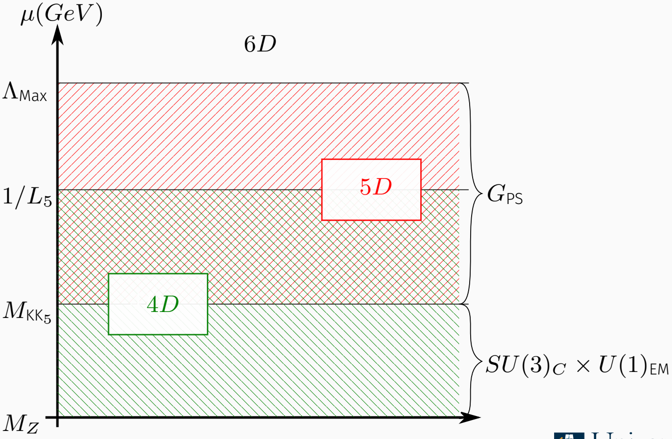


RGE analysis and Higgs phenomenology

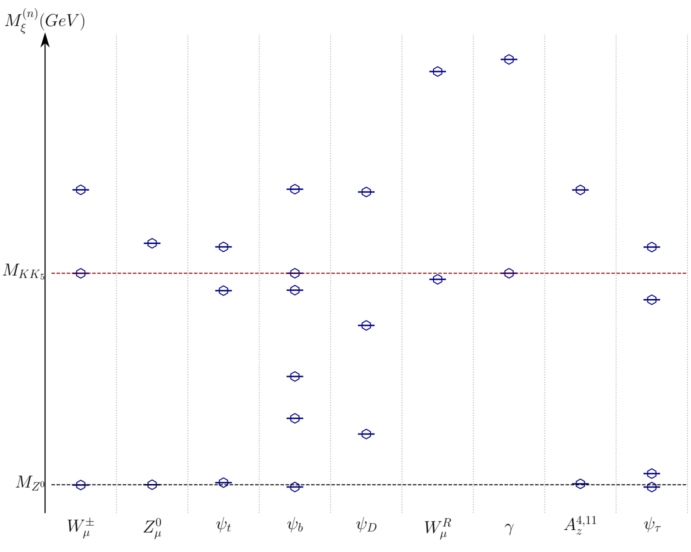
Overall Picture

- Look at gauge coupling unification and Weinberg Angle evolution.
- Examine the phenomenology of new states.
- Examine Higgs production cross sections.

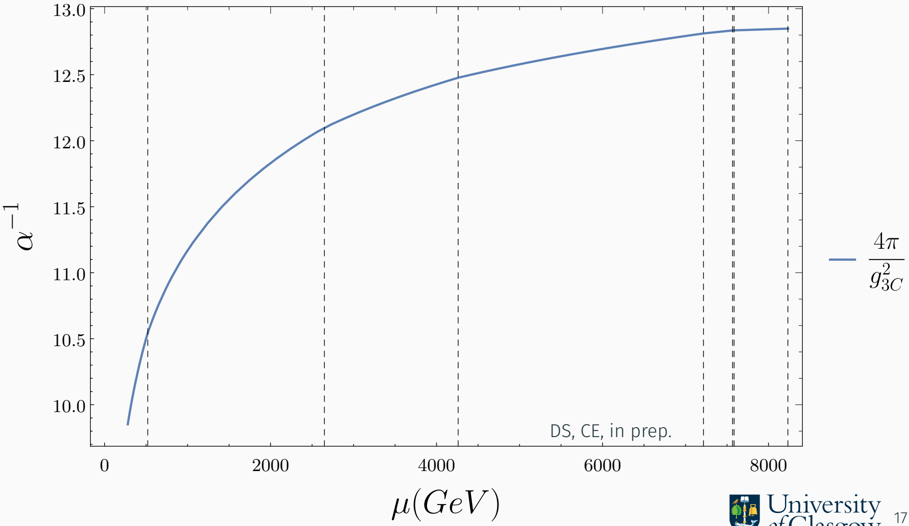
We want to build a **tower of EFTs**.



4D RGE runnings have to take into account the additional KK states.

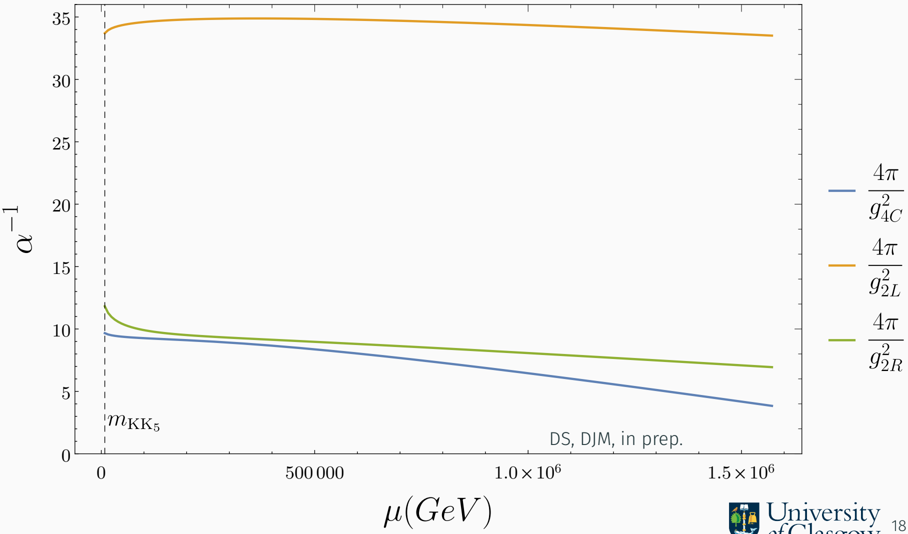


We want to run the 4D RGEs for $SU(3)_C \times U(1)_{EM}$ turning on an additional contribution whenever we hit a Kaluza-Klein excitation, until we hit M_{KK_5}.



5D Running

When we hit M_{KK_5}, switch to a 5D G_{PS} running with formalism from Choi et al. [hep-th/0208071]



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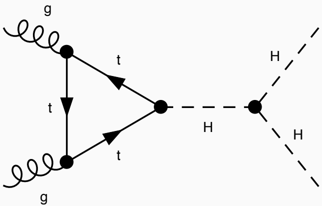
At the cutoff of the 5D theory we want to look at:

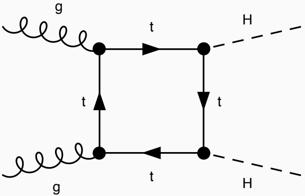
- What happens with the gauge couplings in the full 6D theory?
- How do we relate the 6D bulk gauge couplings $SO(11)$ to the 5D $SU(5)$ one at Λ_{Max}.
- Is the evolution of the Weinberg angle consistent with its $SU(5)$ prediction?

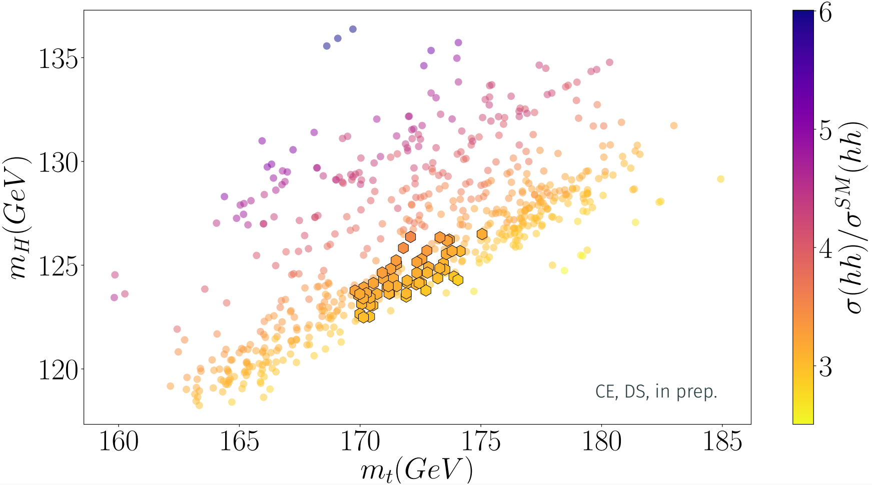
Higgs sector has an enhanced **trilinear coupling** and *similar top Yukawa* w.r.t. the SM,

$$\tau_H \sim \frac{\partial^3 V_{\text{eff}}(\theta_H)}{\partial \theta_H^3} \sim 6 \cdot \tau_H^{SM} \qquad y_T \sim y_T^{SM} \cdot \cos(\theta_H),$$

which is manifest in $gg \to HH$ via topologies,






$m_H(GeV)$
135
130
125
120
160
165
170
175
180
185
$m_t(GeV)$
CE, DS, in prep.
6
5
4
3
$\sigma(hh)/\sigma^{SM}(hh)$

Conclusions

- We've explored a dynamic way of determining the possible solutions of the 6D GHGUT .
- Employed differential evolution to zero in on to SM-like regions.
- Set out a method to explore gauge coupling unification and Weinberg angle evolution.
- Set out the basis for the Higgs sector phenomenology.

Thank you for your attention!

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Backup Slides.

Holonomy and Wilson Loops

Suppose we have a $SU(2)$ theory defined on the 5D manifold $\mathscr{M}_4 \times S^1$, with a constant gauge field,

$$A(y) = A^3(y)\frac{1}{2}\tau^3 = \langle A^3 \rangle \frac{1}{2}\tau^3$$

The *holonomy* T measures the extent to which parallel transport across a smooth manifold changes a geometrical quantity,

$$T = \exp\left(i \oint_{S^1} dy \langle A^3 \rangle \frac{1}{2}\tau^3\right) = \exp\left(i\langle A^3 \rangle \pi R \tau^3\right)$$

The non-trivial transport function affects the $5D$ wave equation for a field ϕ and it's Kaluza- Klein decomposition, resulting in a **mass shift**:

$$m_n = \left| \frac{n}{R} - \frac{1}{2}\langle A^3 \rangle \tau^3 \right|$$

The non-trivial holonomy represents physical degrees of freedom that *cannot be gauged away.*

$V_{\text{eff}}(\theta_H)$ contribution example

Looking at the top quark contribution, it has an equation of motion,

$$S_L(1, \lambda, c_0) S_R(1, \lambda, c_0) + \sin^2 \frac{\theta_H}{2} = 0 \tag{1}$$

The top quark has an effective potential contribution:

$$V_{\text{eff}}^{\text{Top}}(\theta_H) = -\frac{(k z_L^{-1})^4}{(4\pi)^2} \int_0^\infty dq q^3 \ln\left(1 + Q_0(q) \sin^2 \frac{\theta_H}{2}\right),$$

where :

$$Q_0(q) = \frac{1}{S_L(1, iqz_L^{-1}, c_0) S_R(1, iqz_L^{-1}, c_0)}$$

We add up all the contributions to form $V_{\text{eff}}(\theta_H)$,

$$V_{\text{eff}}(\theta_H) = V_{\text{eff}}^{W^\pm} + V_{\text{eff}}^{Z^0} + V_{\text{eff}}^{A} + \sum_{\text{Fermions}} V_{\text{eff}}.$$

Find $\langle\theta_H\rangle$ numerically, along with the 1st solution λ_1 for Equation 1,

$$m_{\text{Top}} = k \cdot \lambda_1 \big|_{\theta_H = \langle\theta_H\rangle} \qquad m_{\text{Higgs}}^2 = \frac{1}{f_H} \frac{\partial^2 V_{\text{eff}}}{\partial \theta_H^2}\bigg|_{\theta_H = \langle\theta_H\rangle}$$

Uniform Sampling

The uniform sampling, along with its quick vetting is done the following stages:

1. We select a subset of parameters $\mathcal{P}_1 \subset \mathcal{P}$,
$$\mathcal{P}_1 = \{k, z_L\}$$
2. We then sample uniformly within their respective bounds, after which we check the consistency condition/s in $\mathcal{C}_1$ defined for the subset $\mathcal{P}_1$. E.g.
$$\mathcal{C}_1 = \{C_1^1\}$$
given k, z_L the Kaluza Klein mass scale should be above the current LHC upper mass limit for RS1 models of 4.1 TeV:
$$C_1^1 : \qquad \frac{\pi k}{z_L - 1} \geq 4.1\,\text{TeV}$$
    - If the point passes the consistency condition/s then we move on to the next stage.
    - If the point fails we go back and sample again for $\mathcal{P}_1$.
3. We do this procedure until we've sampled all of $\mathcal{P}$ and satisfied all constraints $\mathcal{C}$

Global χ^2 measure

Consists of SM central mass values and uncertainty measurements, along with an introduced 1% theory uncertainty:

$$\chi_G^2 = \sum_{\xi} \frac{(\mu^{\xi}_{\text{SM}} - m^{\xi}_{\text{Gen}})^2}{(\sigma^{\xi}_{\text{SM}})^2 + (\sigma^{\xi}_{\text{Theory}})^2}$$

where ξ stands for the Higgs, $W^{\pm}$ bosons, top quark, bottom quark, tau lepton and Neutrino :

$$m_H = 125.09 \pm 0.16 (GeV), \quad m_{W^{\pm}} = 80.379 \pm 0.012 (GeV),$$
$$m_t = 172.44 \pm 0.9 (GeV), \quad m_b = 4.18 \pm 0.04 (GeV)$$
$$m_{\tau} = 1.776 \pm 0.00012 (GeV)$$

Differential Evolution - 1

Once we have an initial *random* population we proceed with the differential evolution algorithm introduced by *Storn & Price* in *doi.org/10.1023/A:1008202821328*, based on:

- Parallelisable algorithm based on generational selection.
- Selects the best points via the 'greedy criterion'.
- Designed to find global minima for non-continuous functions, many minima functions.
- Consists of 3 stages *Mutation, Recombination, Selection*.

Start off with a population of $N \geq 4$ points in the phase space for a generation G, defined by the D_x dimensional parameter vectors

$$\{\mathbf{x}_i^G\} \qquad i = 1, 2, \ldots, N$$

Mutation

Cycle through the **target** vectors $\mathbf{x}_i^G$ out of the population of generation G, and pick 3 other distinct random vectors $\mathbf{x}_{r_1,r_2,r_3}^G$ from G, with $r_1 \neq r_2 \neq r_3 \neq i$ for each i.

Form a **mutation** vector $\mathbf{m}_i^G$ out of the 3,

$$\mathbf{m}_i^G = \mathbf{x}_{r_1}^G + F \cdot (\mathbf{x}_{r_2}^G - \mathbf{x}_{r_3}^G)$$

where $F \in [0, 2]$ is the constant amplification factor.

Recombination

Recombination aims to keep successful solutions from the previous generation and improve on them by combining the target vector $\mathbf{x}_i^G$ and the mutation vector $\mathbf{m}_i^G$.

First pick a random index $I_{\text{rndChoice}} \in \{1, 2, \dots D_x\}$. We then form a **trial** vector $\mathbf{t}_i^{G+1}$ which has components :

$$(\mathbf{t}_i^{G+1})_j = \begin{cases} (\mathbf{m}_i^G)_j & \text{,if rand}(U[0,1])_j \leq C_R \text{ or } j = I_{\text{rndChoice}} \\ (\mathbf{x}_i^G)_j & \text{,otherwise} \end{cases}$$

where rand$(U[0,1])_j$ is a random sampling for each index j, and C_R is the constant decision factor.

Selection

We now compare the two parameter vectors, i.e. the target $\mathbf{x}_i^G$ and the trial $\mathbf{t}_i^{G+1}$, by evaluating the model for the trial vector, against the target via χ^2_G.

We admit to the new generation the trial vector if it's χ^2_G is smaller than the target's, otherwise the target is kept.

$$\mathbf{x}_i^{G+1} = \begin{cases} \mathbf{t}_i^{G+1} & \text{,if} \quad \chi^2_G(\mathbf{t}_i^{G+1}) < \chi^2_G(\mathbf{x}_i) \\ \mathbf{x}_i^G & \text{,otherwise .} \end{cases}$$

This is the admission via the greedy criterion. Mutation, recombination and selection is done until we hit a lower threshold of

$$\chi^2_G = 9.236$$