The effective potential in 5D warped models

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Based on a work in collaboration with

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Introduction: 5D Warped models and the radion

- 5D warped models have been widely used for model building in particle physics.

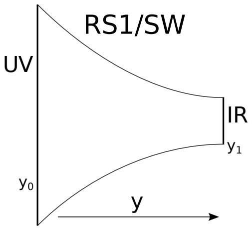
$$M_P^2 \sim \sqrt{\frac{M_5^9}{\Lambda_5}}$$

$$m_{4d} \sim e^{-(A_{IR}-A_{UV})} m$$

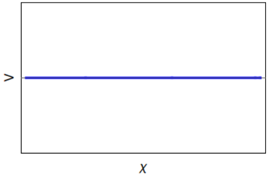
- Goldberger-Wise-like mechanism to stabilize the extra-dimension size:

$$A_{IR} - A_{UV} \sim 30 - 35.$$

- Massless radion $\rightarrow$ Light radion (dilaton).
- Original GW requires some small fine-tuning in the IR brane tension (small backreaction). [Goldberger, Wise; 99]
- CPR mechanism: no fine-tuning and allows for large backreaction in the IR: soft-wall models.
  [Bellazzini, Csaki, Hubisz, Serra, Terning; 13]
  [Coradeschi, Lodone, Pappadopulo, Rattazzi, Vitale; 13]
- Easiest realization: include a GW field with small negative mass squared $m_\phi^2 \sim -10^{-1,-2}\Lambda_5/M_5^3$.



$$ds^2 = e^{-2A}dx^2 + dy^2$$



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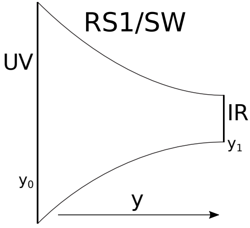

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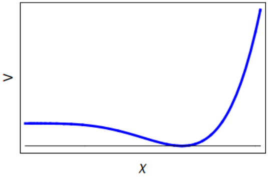
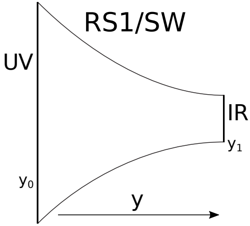


- At finite temperature, these models undergo a phase transition to an AdS-Schwarzschild solution. [Creminelli, Nicolis, Rattazzi; 01]
- Holographically dual to a CFT at finite temperature.
- Phase transition at the early universe: source of gravitational waves. [Randall, Servant; 06] [Megías, Quirós; 18]
- If the low energy theory is describe by the SM+radion (large gap between the radion and first KK modes):

$\downarrow$

- Radion potential correctly describes the phase transition.

Techniques to accurately calculate the radion potential.



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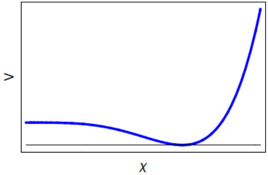

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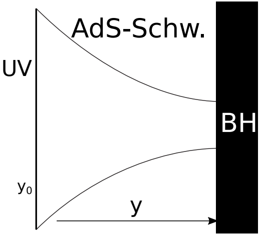

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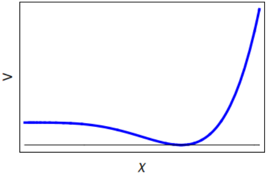

The effective action

- Action:

$$S = \int dx^4\, dy\, \sqrt{|g|} \left[-\frac{1}{\kappa^2} R + \mathcal{L}_{\rm bulk}(\phi) + \dots \right] - \int d^4x \sqrt{|g_0|}\, U_0 \Big|_{y_0} - \int d^4x \sqrt{|g_1|}\, U_1 \Big|_{y_1}$$

- Effective action for the radion and the 4D metric:

$$e^{iS_{\rm eff}[h_{\mu\nu},\chi]} = \int \mathcal{D}\omega\, \delta[h_{\mu\nu} - H_{\mu\nu}(\omega)]\delta[\chi - X(\omega)] e^{iS[\omega]}$$

$$\xrightarrow{\text{tree level}} S_{eff}[h_{\mu\nu},\chi] = \min_{\substack{X(\omega)=\chi \\ H(\omega)=h}} S[\omega].$$

- $\mathcal{L}_{\rm eff}(h_{\mu\nu},\chi) = -V_{\rm eff}(\chi) - \frac{1}{2} C_{\rm eff}(\chi) h^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{1}{2M_P^2} K_{\rm eff}(\chi) R[h] + \dots$

- Different choices of the interpolating fields $\to$ equivalent theories.

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Interpolating radion choice

- Two examples of interpolating radions:

- Using the warp factor:

$$X = \frac{1}{2}\sqrt{g_{\mu\nu}|_1 \; g^{\mu\nu}|_0} = e^{A|_0 - A|_1}$$

- Breaking of the UV and IR BC for A.

$$V_{\text{eff}} = \underbrace{e^{-4A}\left(U_0 - \frac{6}{\kappa^2}A'\right)\Big|_{y_0}}_{V_{UV}} + \underbrace{e^{-4A}\left(U_1 + \frac{6}{\kappa^2}A'\right)\Big|_{y_1}}_{V_{IR}}$$

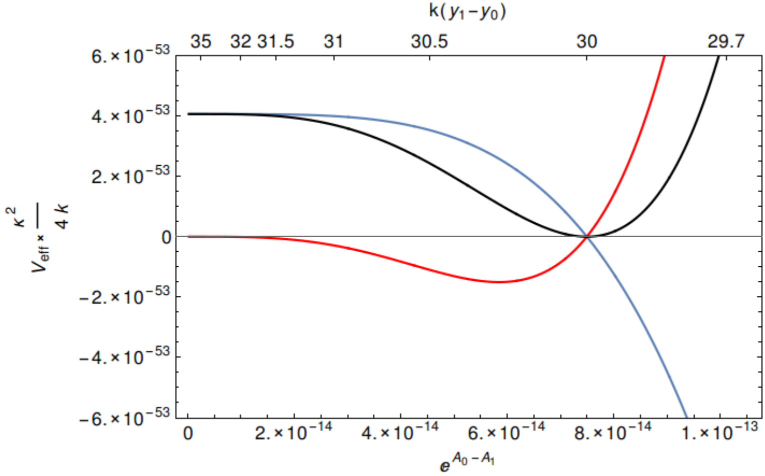
- Using the physical distance:

$$X = \int dy \sqrt{g_{55}}$$

- Breaking of the UV BC for A and one of the bulk EoMs.

$$V_{\text{eff}} = \underbrace{e^{-4A}\left(U_0 - \frac{6}{\kappa^2}A'\right)\Big|_{y_0}}_{V_{UV}}$$

Numerical calculation ($m_\phi^2 = -0.4k^2$)



- High numerical precision to compute V_{UV} (> 50 digits).
- Several approximations used in the literature.

[Bellazzini, Csaki, Hubisz, Serra, Terning; 13], [Megías, Quirós; 18]

The limit $M_P \to \infty$

- If $y_0 \to -\infty$, then $M_P \to \infty$, and gravity is decoupled.
- Given the large hierarchy of the phenomenologically relevant models, such approximation seems to be legitimate.
- Asymptotic expansion of the fields:

$$\begin{aligned}\phi(y) &= \phi_{(1,0)} e^{\Delta_- ky} + \phi_{(2,0)} e^{2\Delta_- ky} + \ldots + \phi_{(0,1)} e^{\Delta_+ ky} + \phi_{(0,2)} e^{2\Delta_+ ky} + \ldots \\ &\quad + \phi_{(1,1)} e^{(\Delta_- + \Delta_+)ky} + \ldots + \phi_{(n,m)} e^{(n\Delta_- + m\Delta_+)ky} + \ldots, \\ A(y) &= ky + A_0 + \ldots + A_{(n,m)} e^{(n\Delta_- + m\Delta_+)ky} + \ldots \qquad \Delta(\Delta - 4) = m_\phi^2\end{aligned}$$

- However, when $y_0 \to -\infty$, the on shell action diverges $\to$ holographic renormalization is required. [de Haro, Solodukhin, Skenderis; 00]
- Result for the effective potential:

$$V_{\text{eff}} = \underbrace{e^{-4A} \left[\frac{6}{\kappa^2} A' + U_2(\phi) \right] \Big|_{y=y_2}}_{V_{IR}} + \underbrace{k\Delta_-(\Delta_- - 2)\phi_{(1,0)}\phi_{(0,1)}}_{V_{UV}}$$

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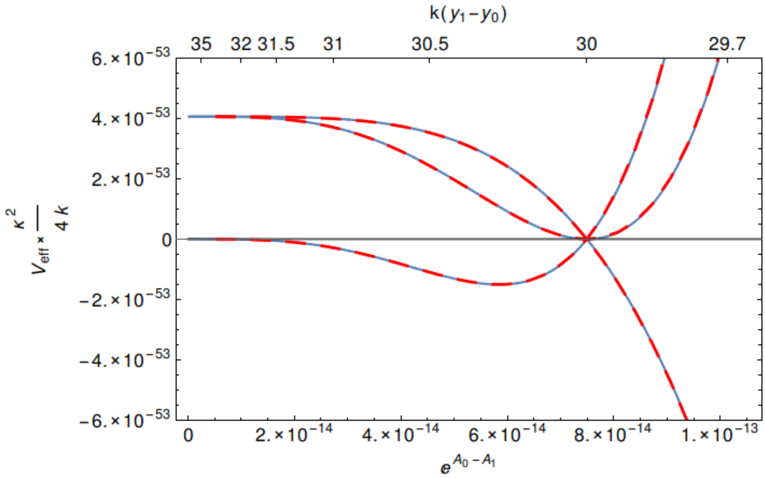
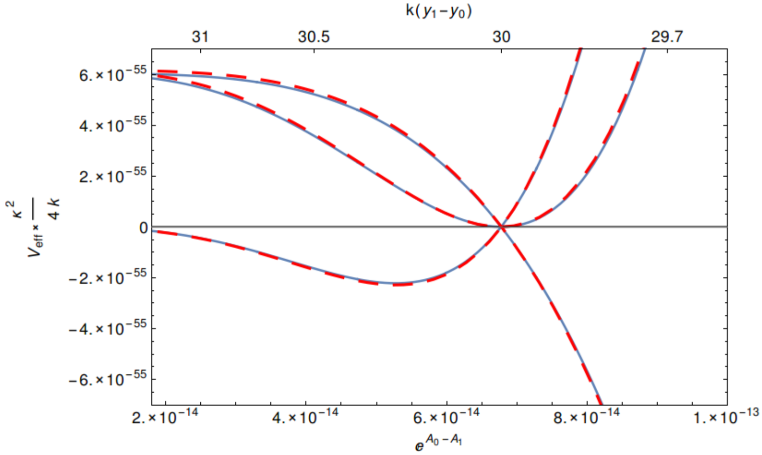


- Problematic if $m_\phi^2 \to 0$.
- Solution: expand EoMs around $m_\phi^2 = 0$ and solve perturbatively.

Numerical calculation ($m_\phi^2 = -0.04k^2$)



Kinetic term

- $C_{\text{eff}}(\chi)$ can be computed up to $O\left(\frac{\chi^2, m_{\text{rad}}^2}{m_{KK}^2}\right)$ once the radion wave function is known.
- The mass of the radion is calculated expanding linearly the perturbation of the EoM and solving the eigenvalue problem.
- An approximation to the mass can be obtained with the effective potential once the field is canonically normalized.
- The comparison of the mass calculated with different methods can be used as a check.
- Using exact radion wave functions we find agreements of $1\% - 3\%$.

Conclusions

- The potential for the radion in warped 5D models is relevant, not only to enlighten the stabilization mechanism, but also to study possible phase transitions that these models predict at the early universe.
- One purpose of this project is to analyze different techniques to compute the effective potential for the radion.
- Exact numerical calculations are tedious because they require to keep track of many significant digits.
- Here I have presented one possibility to simplify this exact numeric calculation: take the limit $M_P \to \infty$.
- The V_{UV} does not vanish in this limit, but gives a relevant contribution to the full potential.