Fine-tuning in models with extended Higgs sectors

Ruud Peeters
University of Groningen

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With: Daniël Boer, Sybrand Zeinstra

Outline

Motivation: claims of fine-tuning at tree-level in left-right symmetric model[1]

[1] Barenboim, Ruis 1998
Deshpande et al. 1991
Dekens, Boer 2014

Outline

Motivation: claims of fine-tuning at tree-level in left-right symmetric model[1]

Fine-tuning & measures

Left-right symmetric model (LRSM)

Seesaw relation

Conclusion

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Fine-tuning

Large cancellations necessary between independent quantities

Undesirable property of beyond the Standard Model theory

Hypothetical example: $m_Z^2 = \mu_1^2 - \mu_2^2$

μ parameters at very high scale -> Fine-tuning needed

Quantify amount of fine-tuning with a measure

Only interested in tree-level fine-tuning, will not discuss hierarchy problem!

Fine-tuning measures

Most common measure: Barbieri-Giudice measure

Compare masses to parameters

$$\Delta_{BG} = \max_i \left| \frac{\partial \log m^2}{\partial \log p_i} \right| = \max_i \left| \frac{p_i}{m^2} \frac{\partial m^2}{\partial p_i} \right|$$

For previous example: $\Delta_{BG} = \max \left\{ \left| \frac{\mu_1^2}{m_Z^2} \right|, \left| \frac{\mu_2^2}{m_Z^2} \right| \right\}$

Log(Δ) ~ # digits that have to be tuned

Fine-tuning measures

Relation of interest: no masses (minimization of potential)

Dekens measure: use relation between parameters

$$\Delta_D = \max_{i,j} \left| \frac{\partial \log q_j}{\partial \log p_i} \right| = \max_{i,j} \left| \frac{p_i}{q_j} \frac{\partial q_j}{\partial p_i} \right|$$

q_i are parameters eliminated using minimum equations

Left-right symmetric model

$$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

Neutrino masses via type-I, type-II seesaw

Additional sources of CP violation

Explain C/P violation as low-energy phenomena

Stepping stone to GUT

Higgs sector LRSM

Bidoublet: ϕ

Left-handed triplet: Δ_L

Right-handed triplet: Δ_R

$$\langle \Delta_R \rangle \sim v_R \sim 10 \text{ TeV}$$
$$\langle \phi \rangle \sim \kappa, \kappa' \sim 100 \text{ GeV}$$
$$\langle \Delta_L \rangle \sim v_L \sim \text{eV}$$

Large hierarchy!

$$
\begin{aligned}
V_H^P = & -\mu_1^2 \mathrm{Tr}(\phi\phi^\dagger) - \mu_2^2 \left[\mathrm{Tr}(\phi\tilde{\phi}^\dagger) + \mathrm{Tr}(\tilde{\phi}\phi^\dagger)\right] - \mu_3^2 \left[\mathrm{Tr}(\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R^\dagger)\right] \\
& + \lambda_1 \left[\mathrm{Tr}(\phi\phi^\dagger)\right]^2 + \lambda_2 \left(\left[\mathrm{Tr}(\phi\tilde{\phi}^\dagger)\right]^2 + \left[\mathrm{Tr}(\tilde{\phi}\phi^\dagger)\right]^2\right) + \lambda_3 \mathrm{Tr}(\phi\tilde{\phi}^\dagger)\mathrm{Tr}(\tilde{\phi}\phi^\dagger) \\
& + \lambda_4 \mathrm{Tr}(\phi\phi^\dagger)\left[\mathrm{Tr}(\phi\tilde{\phi}^\dagger) + \mathrm{Tr}(\tilde{\phi}\phi^\dagger)\right] + \rho_1 \left(\left[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\right]^2 + \left[\mathrm{Tr}(\Delta_R\Delta_R^\dagger)\right]^2\right) \\
& + \rho_2 \left[\mathrm{Tr}(\Delta_L\Delta_L)\mathrm{Tr}(\Delta_L^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R)\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger)\right] + \rho_3 \mathrm{Tr}(\Delta_L\Delta_L^\dagger)\mathrm{Tr}(\Delta_R\Delta_R^\dagger) \\
& + \rho_4 \left[\mathrm{Tr}(\Delta_L\Delta_L)\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R)\mathrm{Tr}(\Delta_L^\dagger\Delta_L^\dagger)\right] \\
& + \alpha_1 \mathrm{Tr}(\phi\phi^\dagger)\left[\mathrm{Tr}(\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R^\dagger)\right] \\
& + \alpha_2 \left(e^{i\delta_2}\left[\mathrm{Tr}(\phi\tilde{\phi}^\dagger)\mathrm{Tr}(\Delta_R\Delta_R^\dagger) + \mathrm{Tr}(\tilde{\phi}\phi^\dagger)\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\right] + h.c.\right) \\
& + \alpha_3 \left[\mathrm{Tr}(\phi^\dagger\Delta_L\Delta_L^\dagger\phi) + \mathrm{Tr}(\phi\Delta_R\Delta_R^\dagger\phi^\dagger)\right] + \beta_1 \left[\mathrm{Tr}(\phi^\dagger\Delta_L\phi\Delta_R^\dagger) + \mathrm{Tr}(\phi^\dagger\Delta_L^\dagger\phi\Delta_R)\right] \\
& + \beta_2 \left[\mathrm{Tr}(\tilde{\phi}^\dagger\Delta_L\phi\Delta_R^\dagger) + \mathrm{Tr}(\phi^\dagger\Delta_L^\dagger\tilde{\phi}\Delta_R)\right] + \beta_3 \left[\mathrm{Tr}(\phi^\dagger\Delta_L\tilde{\phi}\Delta_R^\dagger) + \mathrm{Tr}(\tilde{\phi}^\dagger\Delta_L^\dagger\phi\Delta_R)\right]
\end{aligned}
$$

Seesaw relation

From minimization of potential:

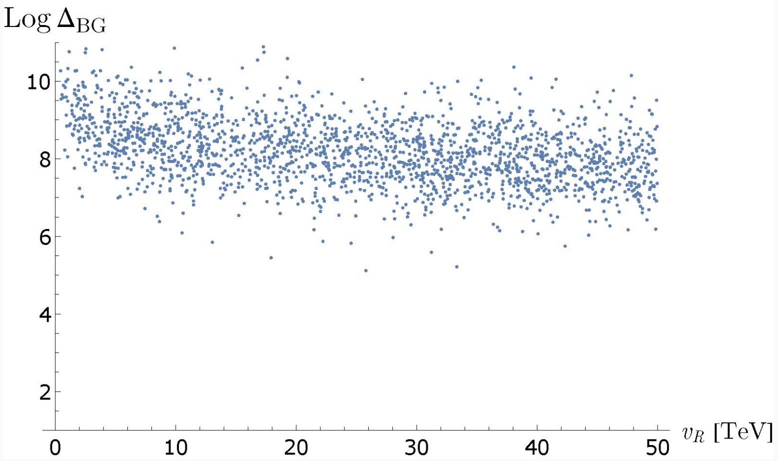
$$2\rho_1 - \rho_3 = \frac{1}{v_L v_R}(\kappa\kappa'\beta_1 + \kappa^2\beta_2 + \kappa'^2\beta_3)$$

Filling in values for vevs:

$$\rho_1 \approx 10^9(\beta_1 + \beta_2 + \beta_3) + \frac{\rho_3}{2}$$

Natural parameters ⟶ Fine-tuning

$$\beta_1 + \beta_2 + \beta_3 = \mathcal{O}(10^{-9})$$



Seesaw relation

But is this actual fine-tuning?

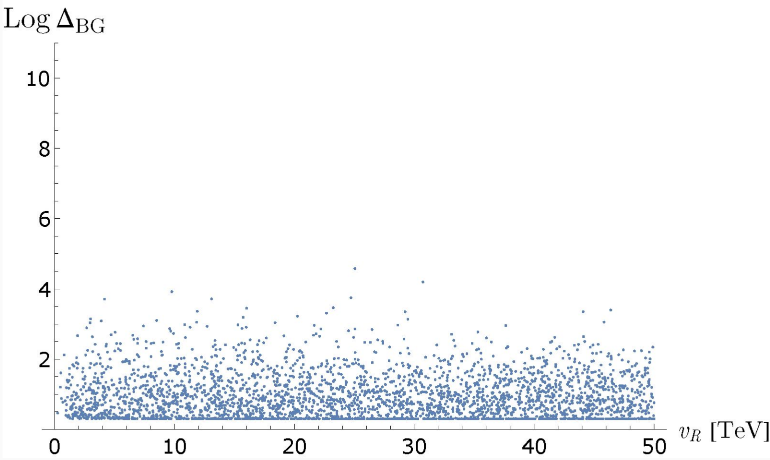
No ‘preferred’ parameter

Solve for different parameter:

$$\beta_1 = -\beta_2 - \beta_3 + 10^{-9}(2\rho_1 - \rho_3)$$

ρ term just a small correction

No fine-tuning necessary!



Seesaw relation

What is happening here?

Minimization of potential: constraint that has to be satisfied!

No preference for one parameter

Small correction vs. large fine-tuning

Small corrections occur everywhere, but not fine-tuned!

Conclusion

Dangerous to claim fine-tuning in minimization conditions!

Minimization conditions are fundamentally different from e.g. mass

If minimization conditions can be solved with coupling constants O(1), without introducing fine-tuning: minimization conditions not a source of fine-tuning!

Possible to have a large hierarchy, without introducing fine-tuning (at tree-level)

Thank you for your attention!

Choice of dependent parameters

Example: $A = x B + C$, $x << 1$, $B, C \sim 1$

$$\Delta_D = \max_{i,j} \left| \frac{\partial \log q_j}{\partial \log p_i} \right| = \max_{i,j} \left| \frac{p_i}{q_j} \frac{\partial q_j}{\partial p_i} \right|$$

xB is a small correction

set q = A, p = B, C, $\Delta_D = \max \left\{ \left| \frac{xB}{A} \right|, \left| \frac{C}{A} \right| \right\} \approx \max\{x, 1\} = 1$

But when using q = B, p = A,C:

$$B = \frac{A - C}{x}$$

$$\Delta_D = \max \left\{ \left| \frac{A}{xB} \right|, \left| \frac{C}{xB} \right| \right\} \approx \max\{1/x, 1/x\} = 1/x \gg 1$$

Problematic case

Setting β_2, β_3 to zero:

$$2\rho_1 - \rho_3 = 10^9 \beta_1$$

No way around unnaturally small value for β_1

Not fine-tuning, still a problem