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# UV COMPLETE CHIRAL FROGGATT-NIELSEN MODELS

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## Outline

- Mass hierarchies and Froggatt-Nielsen models: simplest  $U(1)$  examples
- Global versus gauge symmetry
- Field theory models with chiral heavy fermions
- Flavorful axion
- Conclusions

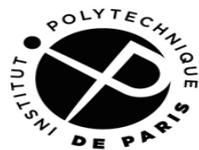


# Fermion mass hierarchies and Froggatt-Nielsen models

- Standard Model gives **no hint** on the **hierarchies** of fermion masses and mixings.

$$\frac{m_u}{m_t} \sim 10^{-5} \qquad \frac{m_\nu}{m_t} \sim 10^{-13}$$

$$V_{CKM} \sim \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$



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An old profilic idea (Froggatt-Nielsen, 79) :

flavor U(1), spontaneously broken **symmetry**.

Fermions of different generations have different charges

Fields:  $Q_i$  ,  $U_i^c$  ,  $D_i^c$

Charges  $q_i$  ,  $u_i^c$  ,  $d_i^c$   $\longrightarrow$  **horizontal/family symmetry**

One needs charged « flavons »  $\Phi$  breaking the symmetry.

**Renormalizable level**: only top Yukawa (and the bottom for two-Higgs doublet models/SUSY) exists.



The other Yukawas are generated through **non-renormalizable operators**

$$\mathcal{L}_{Yuk} = y_{ij}^u \left(\frac{\Phi}{M}\right)^{-\frac{q_i+u_j+h_2}{X_\Phi}} Q_i U_i^c h_2 + y_{ij}^d \left(\frac{\Phi}{M}\right)^{-\frac{q_i+d_j+h_1}{X_\Phi}} Q_i D_i^c h_1 + \dots$$

after symmetry breaking

$$m_{ij}^u = y_{ij}^u \epsilon^{-\frac{q_i+u_j+h_2}{X_\Phi}} v_2, \quad m_{ij}^d = y_{ij}^d \epsilon^{-\frac{q_i+d_j+h_1}{X_\Phi}} v_1$$

where  $\epsilon = \frac{\langle \Phi \rangle}{M} \sim \lambda = 0.22$

Quarks masses and mixings are given by ( $q_{13} = q_1 - q_3$   $X_\Phi =$

$$\frac{m_u}{m_t} \sim \epsilon^{q_{13}+u_{13}}, \quad \frac{m_c}{m_t} \sim \epsilon^{q_{23}+u_{23}}, \quad \frac{m_d}{m_b} \sim \epsilon^{q_{13}+d_{13}}, \quad \frac{m_s}{m_b} \sim \epsilon^{q_{23}+d_{23}}$$

$$\sin \theta_{12} \sim \epsilon^{q_{12}}, \quad \sin \theta_{13} \sim \epsilon^{q_{13}}, \quad \sin \theta_{23} \sim \epsilon^{q_{23}}.$$

Good fit to data  $\Rightarrow$  **larger charges** for the **lighter generations**

$$q_1 > q_2 > q_3, \quad u_1 > u_2 > u_3, \quad d_1 > d_2 > d_3$$

$$m_t \sim 1$$

$$m_c \sim \epsilon^4$$

$$m_u \sim \epsilon^8$$

$$m_b \sim \epsilon^3$$

$$m_s \sim \epsilon^{5 \div 6}$$

$$m_d \sim \epsilon^{7 \div 8}$$

$$m_\tau \sim \epsilon^3$$

$$m_\mu \sim \epsilon^5$$

$$m_e \sim \epsilon^9$$

$$V_{us} \sim \epsilon$$

$$V_{ub} \sim \epsilon^3$$

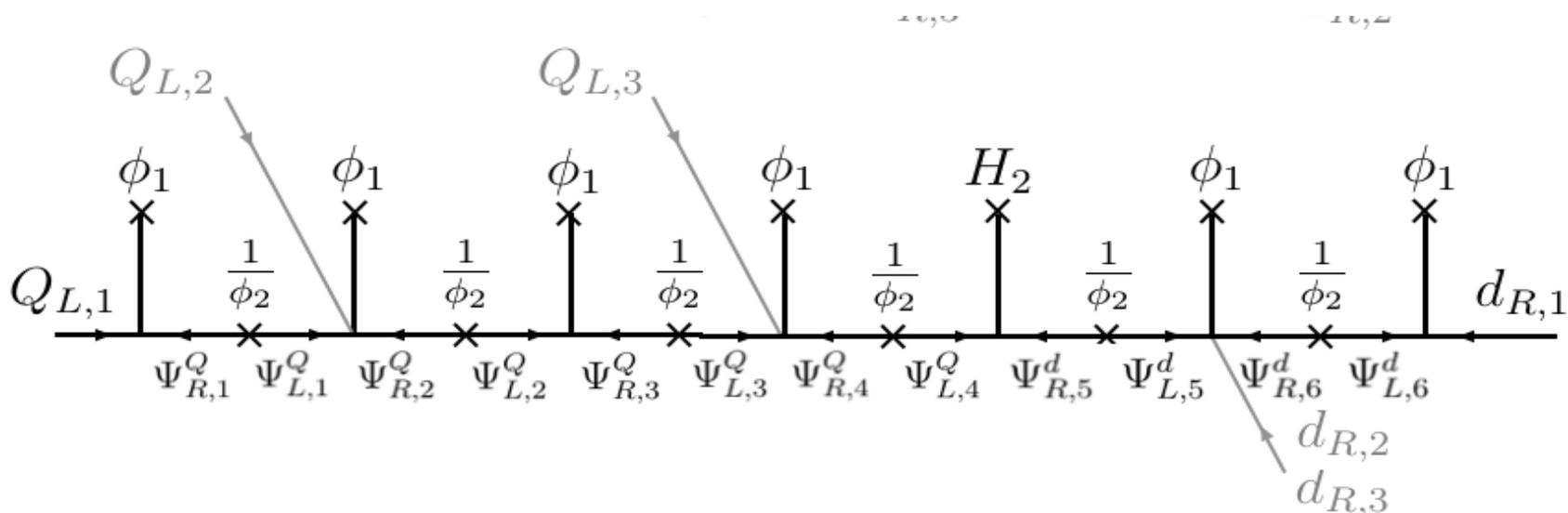
$$V_{cb} \sim \epsilon^2$$

- **Origin** of non-renormalizable operators ?

- String theory/supergravity

In this case  $M = M_P$

- **Mixing** of **light fermions** with **heavy fermions** of mass  $M \sim \langle \Phi_2 \rangle$



All the papers we are aware of assume heavy fermions to be **vector-like** wrt  $U(1)_X \longrightarrow X_{\Phi_2} \equiv X_2 = 0$

- $U(1)_X$  can be **global** or **local**

In both cases we can consider **mixed anomalies**

$$U(1)_X - G_a - G_b \longrightarrow A_a \sim \text{Tr}(X T_a^2)$$

where  $G_{a,b} = SU(3)_c, SU(2)_L, U(1)_Y$

If heavy fermions are vector-like, mixed anomalies are completely determined by the **SM fermions**.

**Minimal models:** one spurion and holomorphic couplings: relations mass matrices-mixed anomalies (Ibanez-Ross; Binetruy-Ramond...)

$$\det (Y_U Y_D^{-2} Y_L^3) = \varepsilon^{3/2(A_2 + A_1 - 2A_3)}$$

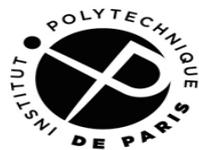
$$\det (Y_U Y_D) = \varepsilon^{A_3 + 3(h_1 + h_2)},$$

which can also be combined into

$$\det (Y_D^{-2} Y_L^2) = \varepsilon^{A_2 + A_1 - \frac{8}{3}A_3 - 2(h_1 + h_2)}$$

- **Minimal number** of heavy fermions equal to the exponent of the determinant of the mass matrix (Leurer-Nir, Seiberg) 

$$2N_Q^H + N_U^H + N_D^H = A_3 + 3(h_1 + h_2) \quad \text{etc}$$



## Global versus gauge symmetry

Fermion masses/mixings for one spurion and holomorphic couplings strongly imply **non-vanishing mixed anomalies**.

OK if  $U(1)_X$  is **global**. In this case, the model has an axion with flavor-dependent couplings to fermions : **flavorful axion**

(Wilczek; Calibbi, Goertz, Redigolo, Ziegler, Zupan; Ema, Hamaguchi, Moroi, Nakayama)

**Stronger couplings** to light quarks:  $\frac{q_i}{V} \partial_m a \bar{q}_i \gamma^m q_i$

Couplings to gauge fields similar to DFKZ models

$$\frac{E}{N} = \frac{A_1 + A_2}{A_3} \sim \frac{8}{3}$$



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However, quantum Gravity/String Theory does not like global symmetries: typically explicitly **broken by gravity** and nonperturbative effects.

If  $U(1)_X$  is gauged, models with one spurion and heavy vector-like fermions generally incompatible with anomaly cancelation 

- Stringy origin, **Green-Schwarz mechanism**
- Field theory, but heavy **chiral fermions**



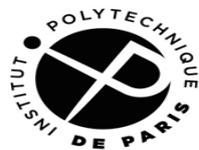
# Field theory models with chiral heavy fermions

$$-\mathcal{L}_{\text{mass}} = y_i \Phi_2 \Psi_i \Psi_i^c$$

$X_2 \neq 0$   $\longrightarrow$  Heavy fermions vector-like wrt SM gauge group, but **chiral** wrt  $U(1)_X$

**Anomaly cancels** between heavy and light fermions

$$A_a^{SM} + A_a^{\text{heavy}} = 0$$



Use one flavon  $\Phi_1$  and a second for fixing the heavy fermion masses  $M = \langle \Phi_2 \rangle$

$$\mathcal{L}_{Yuk} = y_{ij}^u \left( \frac{\Phi_1}{\Phi_2} \right)^{\frac{q_i + u_j + h_2}{X_2 - X_1}} Q_i U_i^c h_2 + \dots$$

with  $\frac{V_1}{V_2} \sim \sin \theta_c \sim 0.22$

It can be shown that the contribution of heavy fermions to the SM gauge couplings running is completely determined by the mixed anomaly of **light SM fermions**



Contribution of  
 heavy fermions

$$\frac{1}{g_a^2}(\mu) = \frac{1}{g_a^2}(\Lambda) - \frac{b_a^{SM}}{8\pi^2} \ln \frac{\Lambda}{\mu} + \frac{A_a^{SM}}{X_2} \ln \frac{\Lambda}{V_2}$$

In MSSM, the heavy fermions contribution preserves unification if

$$A_2^{SM} = A_3^{SM} = \frac{3}{5} A_1^{SM}$$

which is also the preferred relation from the mass matrices viewpoint.

- There is an analogy with the stringy Green-Schwarz case: below heavy fermion masses the field theory model is similar to the stringy one



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The simplest way to satisfy

$$A_2^{\text{heavy}} = A_3^{\text{heavy}} = \frac{3}{5} A_1^{\text{heavy}}$$

with heavy fermions in « complete SU(5) representations »  
( different  $U(1)_X$  charges) .

We have **explicit examples** with anomaly-cancellation. Strong **perturbativity limits** on the heavy-fermion masses, similar to the vector-like FN models (Calibbi, Lalak, Pokorski, Ziegler)

$$M \geq 10^{14} \text{ GeV}$$

## A flavorful axion

The model has a **physical axion**

$$\Phi_i = (V_i + r_i) e^{\frac{ia_i}{V_i}} \longrightarrow$$

$$\theta_X \sim X_1 V_1 a_1 + X_2 V_2 a_2 \quad \text{Goldstone eaten by the gauge boson}$$

$$a_{PQ} \sim X_2 V_2 a_1 - X_1 V_1 a_2 \quad \text{Peccei-Quinn axion}$$

PQ symmetry is protected by the gauge symmetry, **accidental**.

Any **difference in low-energy couplings** compared to  
 « standard » flavorful axion ?

Yes, since below the scale of heavy fermions the effective action of the form

anomaly heavy fermions

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{Yuk}} - A_a^{SM} \frac{a_{PQ}}{f_a} F_a \tilde{F}_A$$

The low-energy couplings to gauge fields are then determined **both** by heavy and light fermions . We find

Couplings  
to gauge fields

$$\frac{C_a}{C_{q_i}} = \frac{A_a^{SM}}{q_i} \frac{X_1^2 V_1^2 + X_2^2 V_2^2}{X_1 V_1^2 + X_2 V_2^2}$$

Couplings  
to quarks

contribution heavy fermions

Flavorful axion couplings correspond to  $X_2 = 0$



## Conclusions

- Field-theory gauged FN models with chiral heavy fermions **more natural**: dynamical mass, low-energy anomalies, axion
- Structure of mass matrices for minimal Yukawas naturally **preserves gauge coupling unification** in MSSM
- Low-energy axion couplings **slightly modified** compared to the flavorful axion.
- We have **explicit examples**, perturbativity in minimal setup favors **large** axion decay constants  $f \geq 10^{14} \text{ GeV}$

**THANK YOU**



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# Backup slides