# Inflation and the Higgs

Antonio Riotto
University of Geneva

Planck 2019

# Inflation

A period of accelerated expansion: approximate de Sitter

$$ds^2 = dt^2 - e^{2Ht} d\mathbf{x}^2 = \frac{1}{H^2\tau^2}\left(d\tau^2 - d\mathbf{x}^2\right)$$

dS has symmetries of a 3D euclidean CFT on the boundary:
end of inflation $\tau = 0$

## 4D bulk isometries

Translations

Spatial rotations

Dilations

$$\tau \to \lambda\tau, \;\; \mathbf{x} \to \lambda\mathbf{x}$$

Special conformal transformations

$$\begin{aligned} \tau &\to \tau - 2\,\tau\,(\mathbf{b}\cdot\mathbf{x}), \\ \mathbf{x} &\to \mathbf{x} + \mathbf{b}\,(-\tau^2 + \mathbf{x}^2) - 2\,\mathbf{x}\,(\mathbf{b}\cdot\mathbf{x}) \end{aligned}$$

## 3D boundary

$\tau = 0$

CFT

Translations

Spatial rotations

Spatial dilations

$$\mathbf{x} \to \lambda\,\mathbf{x}$$

Special conformal transformations

$$\mathbf{x} \to \mathbf{x} + \mathbf{b}\,\mathbf{x}^2 - 2\,\mathbf{x}\,(\mathbf{b}\cdot\mathbf{x})$$

# dS has symmetries of a 3D euclidean CFT on the boundary

$$\phi(\mathbf{x},\tau) \sim (-\tau)^{\Delta}\,\phi(\mathbf{x})$$

$$\Delta = \frac{3}{2} - \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$$

$$D\phi(\mathbf{x},\tau) = -i(\tau\partial_\tau + x^i\partial_i)\phi(\mathbf{x},\tau)$$

$$D\phi(\mathbf{x}) = -i(\Delta + x^i\partial_i)\phi(\mathbf{x})$$

# dS-CFT correspondence

A. Strominger, 2001

Bulk dS fields correspond to primary fields on the boundary


bulk

$$f(\mathbf{k}_i) = \int \frac{\mathrm{d}\tau}{\tau^4} \prod_i G(\mathbf{k}_i, \tau)$$

boundary

$$f(\mathbf{k}_i) = \left\langle \prod_i \mathcal{O}(\mathbf{k}_i) \right\rangle_{\mathrm{CFT}}$$

Massless fields correspond to conformal dimension 3 boundary fields

Graviton corresponds to boundary stress tensor operator

# Massless scalar fields

Massless scalar fields correspond to conformal dimension 3 boundary fields

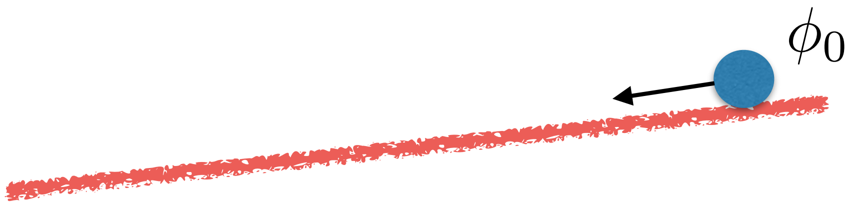
$$\langle \delta\phi_{\mathbf{k}} \delta\phi_{-\mathbf{k}} \rangle' = -\frac{1}{2\,\mathrm{Re}\langle \mathcal{O}_{\mathbf{k}} \mathcal{O}_{-\mathbf{k}} \rangle'} \sim \frac{1}{k^3}$$

$$\mathcal{P}_{\delta\phi}(k) = \frac{k^3}{2\pi^2} \left|\delta\phi_{\mathbf{k}}\right|^2 = \left(\frac{H}{2\pi}\right)^2$$

Scale-invariance due to the symmetries of dS

# Inflation needs to end

$$S \supset \int \mathrm{d}^4x \sqrt{-g} \left[ \frac{1}{2} (\partial\phi)^2 - V(\phi) \right]$$




De Sitter slightly broken:

*expect a small breaking of the 3D CFT symmetries*

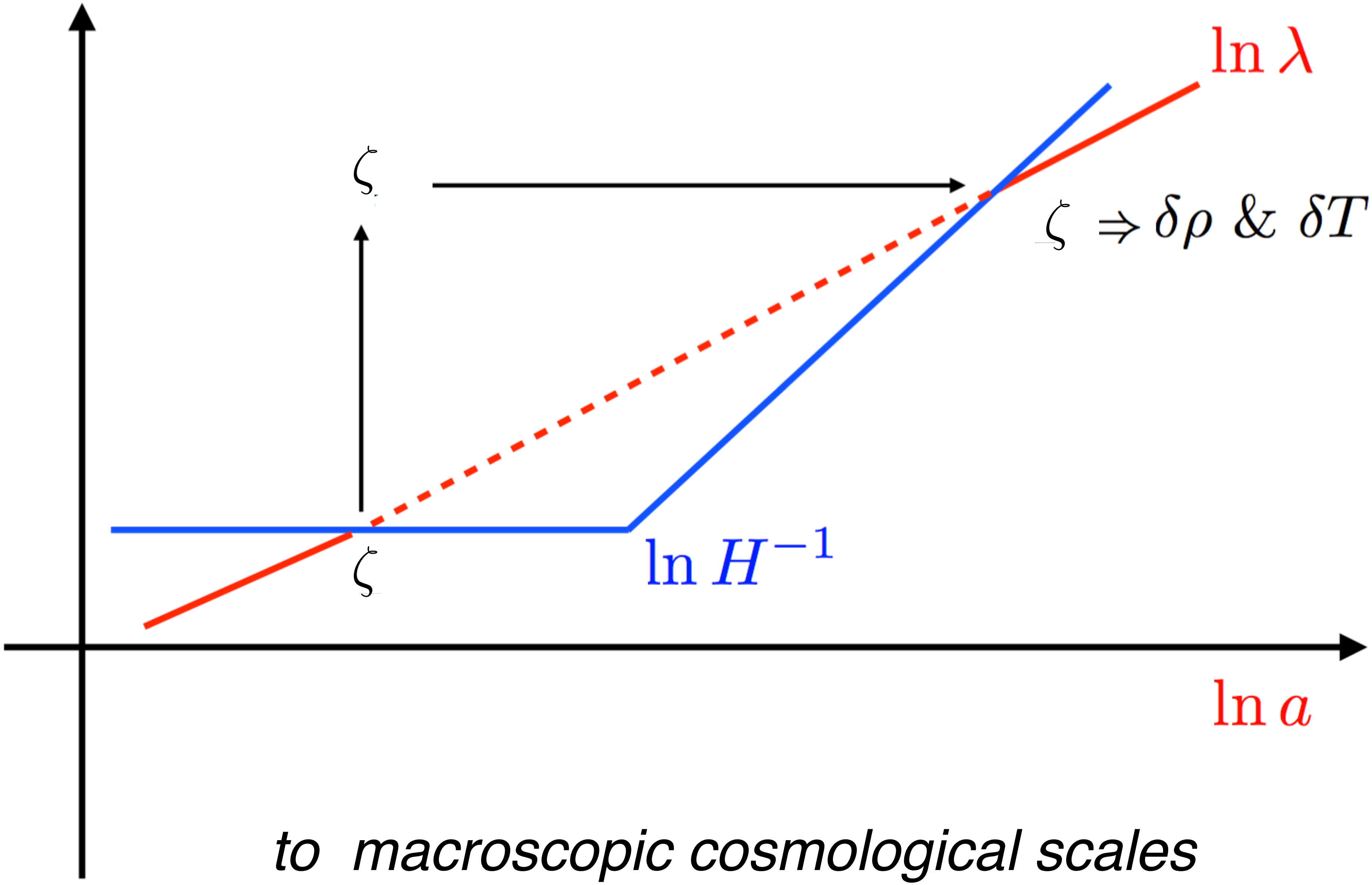
$$-\dot{H} \ll H^2$$

- In the high energy physics language, the slightly broken scale invariance is associated to a *pseudo Nambu-Goldstone boson* representing fluctuations in the clock

- Different regions have

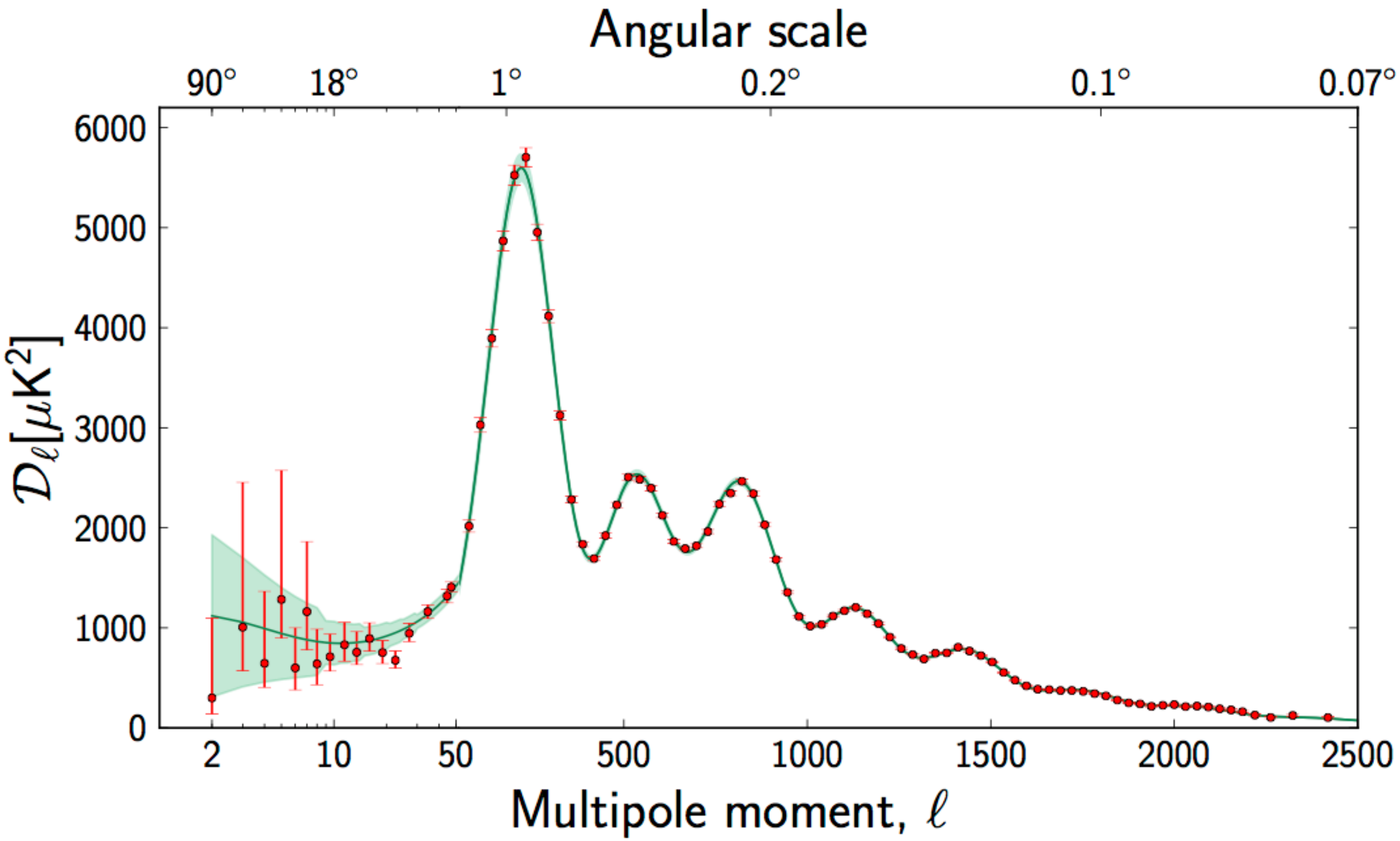
$$\zeta \sim \frac{\delta a}{a} \sim H\delta t \sim H\frac{\delta\phi}{\dot{\phi}_0}$$

*From microscopic quantum scales*




*to macroscopic cosmological scales*

# CMB & Planck




$$\mathcal{P}_\zeta \sim k^{n_\zeta - 1} : \quad n_\zeta = 0.9655 \pm 0.0062$$

# Gravity waves from inflation

$$\mathrm{d}s^2 = \frac{1}{H^2\tau^2}\left[\mathrm{d}\tau^2 - (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\mathrm{d}x^j\right]$$

Tensor modes correspond to the energy-momentum tensor of the boundary 3D CFT, which has conformal weight 3, hence tensor mode spectrum is scale-invariant

$$\langle h^{s_1}_{\mathbf{k}} h^{s_2}_{-\mathbf{k}}\rangle' = -\frac{1}{2\,\mathrm{Re}\langle T^{s_1}_{\mathbf{k}} T^{s_2}_{-\mathbf{k}}\rangle'} \sim \frac{1}{k^3}$$

# Gravity waves from inflation

$$\mathrm{d}s^2 = \frac{1}{H^2\tau^2}\left[\mathrm{d}\tau^2 - (\delta_{ij} + h_{ij})\,\mathrm{d}x^i\mathrm{d}x^j\right]$$

$$\mathcal{P}_{\mathrm{GW}} = \frac{k^3}{2\pi^2}|h_{\mathbf{k}}|^2 \sim \frac{H^2}{M_{\mathrm{Pl}}^2} \sim \frac{E_{\mathrm{inf}}^4}{M_{\mathrm{Pl}}^2}$$

Symmetries fix also the graviton interactions up to a free parameter

$$S_{\mathrm{g}} = \frac{M_{\mathrm{Pl}}^2}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,\left(R-6H^2\right) + \frac{M_{\mathrm{Pl}}^2 L^4}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,W^{\mu\nu}{}_{\rho\sigma}\,W^{\rho\sigma}{}_{\delta\tau}\,W^{\delta\tau}{}_{\mu\nu}$$

which correspond to the freedom of the stress energy-momentum tensor in 3D CFT to be determined by free scalars relative to free fermions

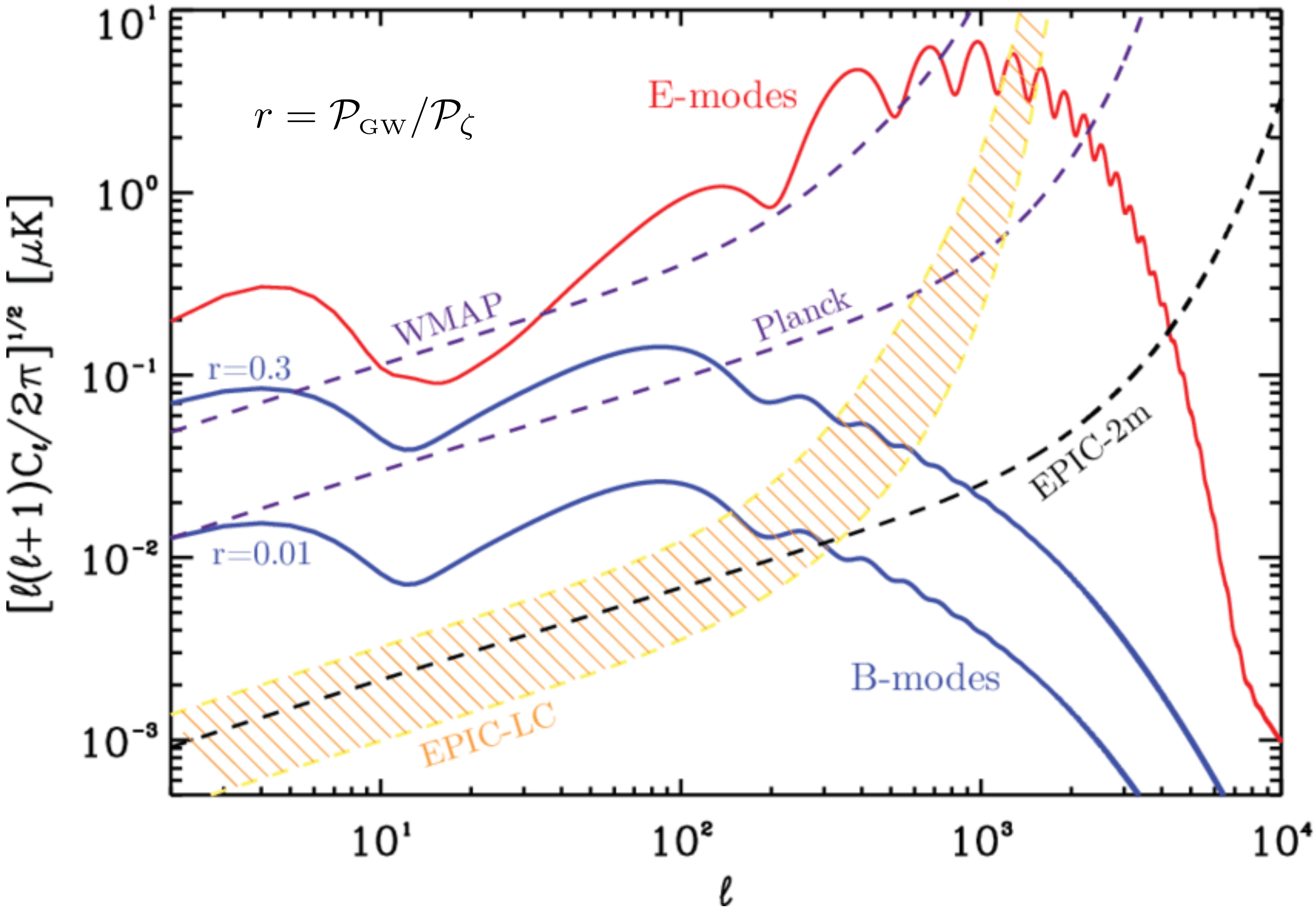
Example: massless interacting gauge fields with all spins

non-linear classical equations found by Vasiliev

4D de Sitter solution plus highly constraining mathematical structure

$$HL = \left(\frac{8}{270}\right)^{1/4} \simeq 0.4$$

D. Anninos et al., 2019




$$[\ell(\ell+1)C_{B\ell}/2\pi]^{1/2} \simeq 0.024(E_{\rm inf}/10^{16}\,{\rm GeV})\,\mu{\rm K}$$

# Detecting gravity waves from inflation implies measuring the energy scale of inflation

Current bounds on gravity waves from Planck 2018+

$$r = \mathcal{P}_{\rm GW}/\mathcal{P}_\zeta < 0.064 \Rightarrow H < 6 \cdot 10^{13}\,\mathrm{GeV}$$

Future bounds on gravity waves (LiteBIRD)

$$r = \mathcal{P}_{\rm GW}/\mathcal{P}_\zeta > 5 \cdot 10^{-4} \Rightarrow H > 6 \cdot 10^{12}\,\mathrm{GeV}$$

Inflation

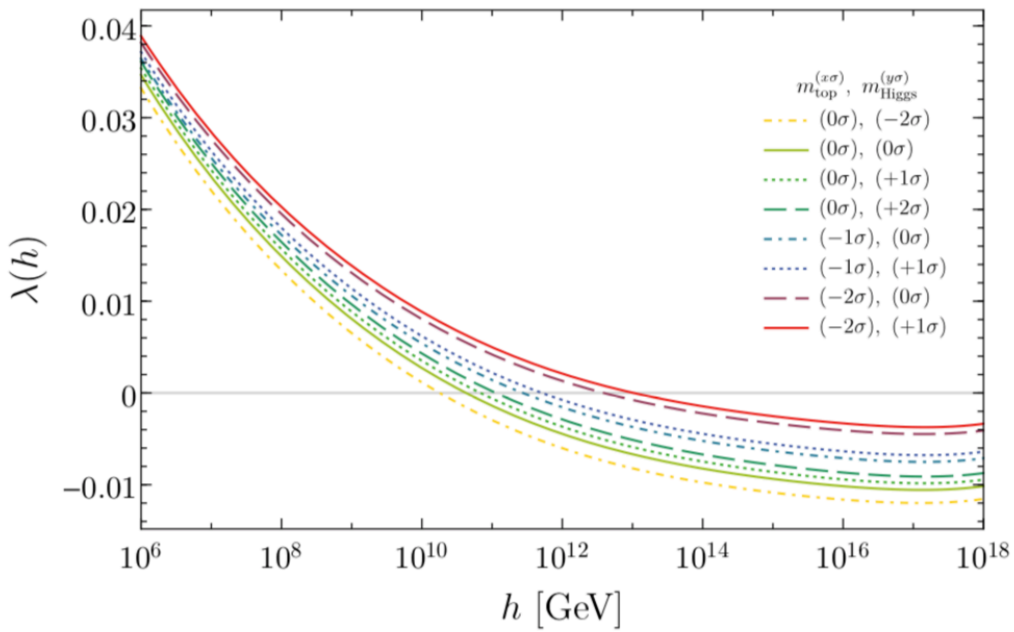
Higgs

Light scalar degree of freedom

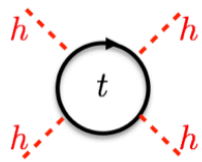
# SM Higgs potential and its instability

$$V(h) = -\frac{1}{2}\mu^2 h^2 + \frac{\lambda}{4} h^4$$

$$h \gg M_W : V(h) = \frac{\lambda(h)}{4} h^4$$

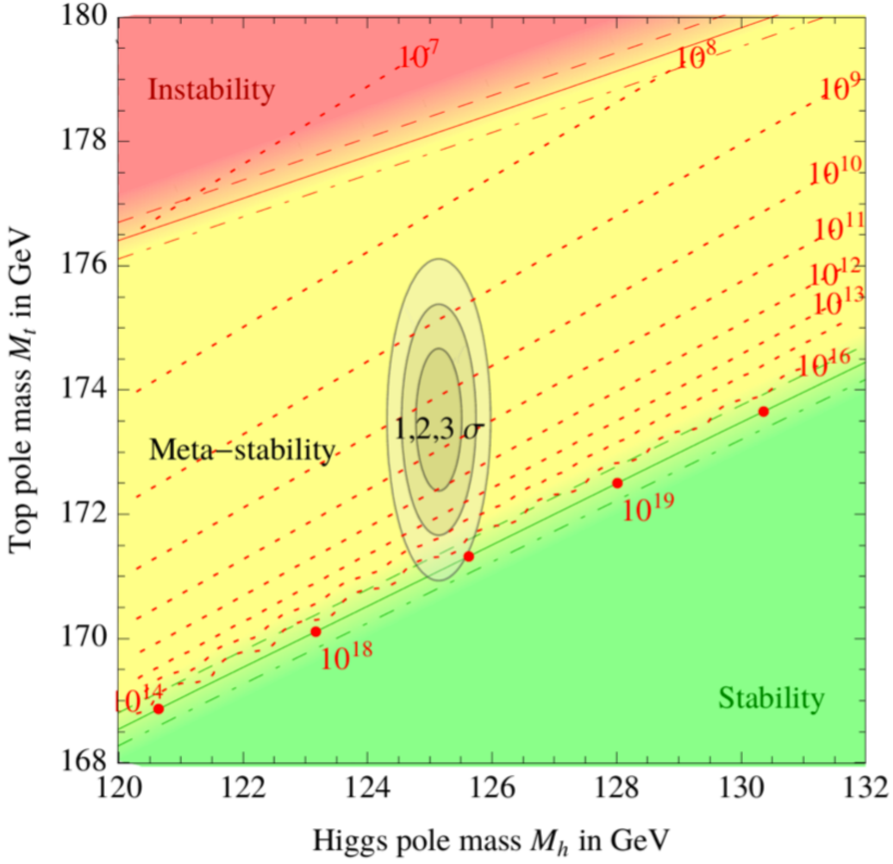


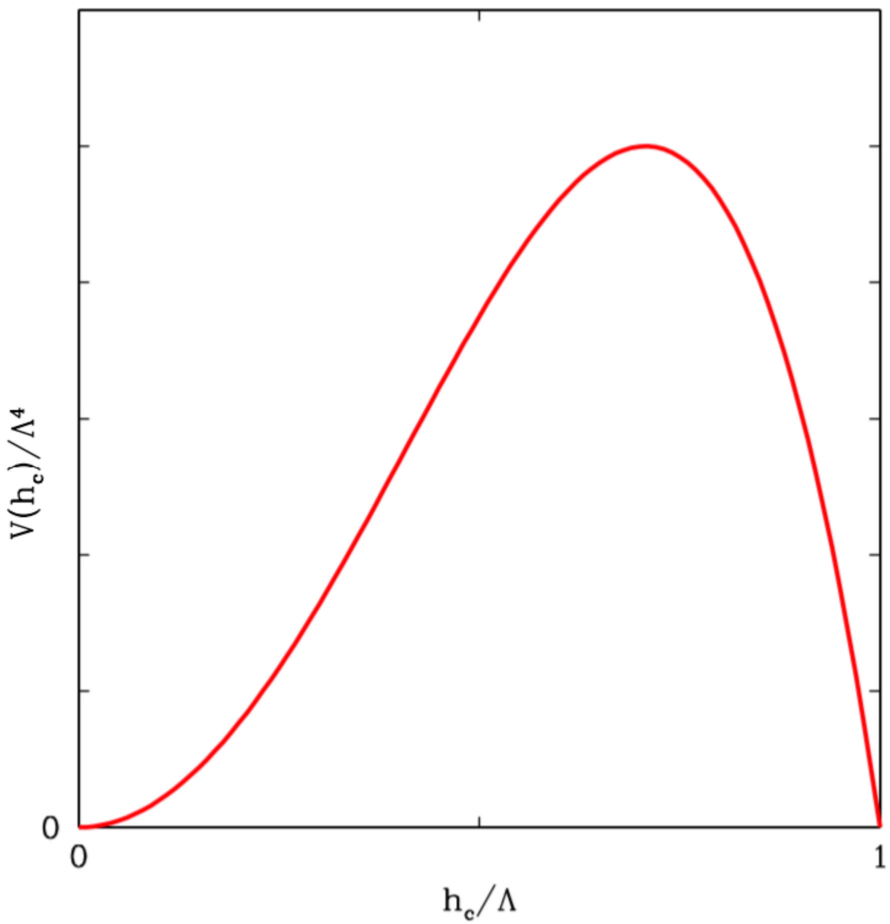

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\ln\mu} = -\frac{6}{16\pi^2} h_t^4 + \cdots =$$




$+ \cdots$

# We live in a metastable vacuum




Buttazzo et al., 2013

# The Higgs during inflation

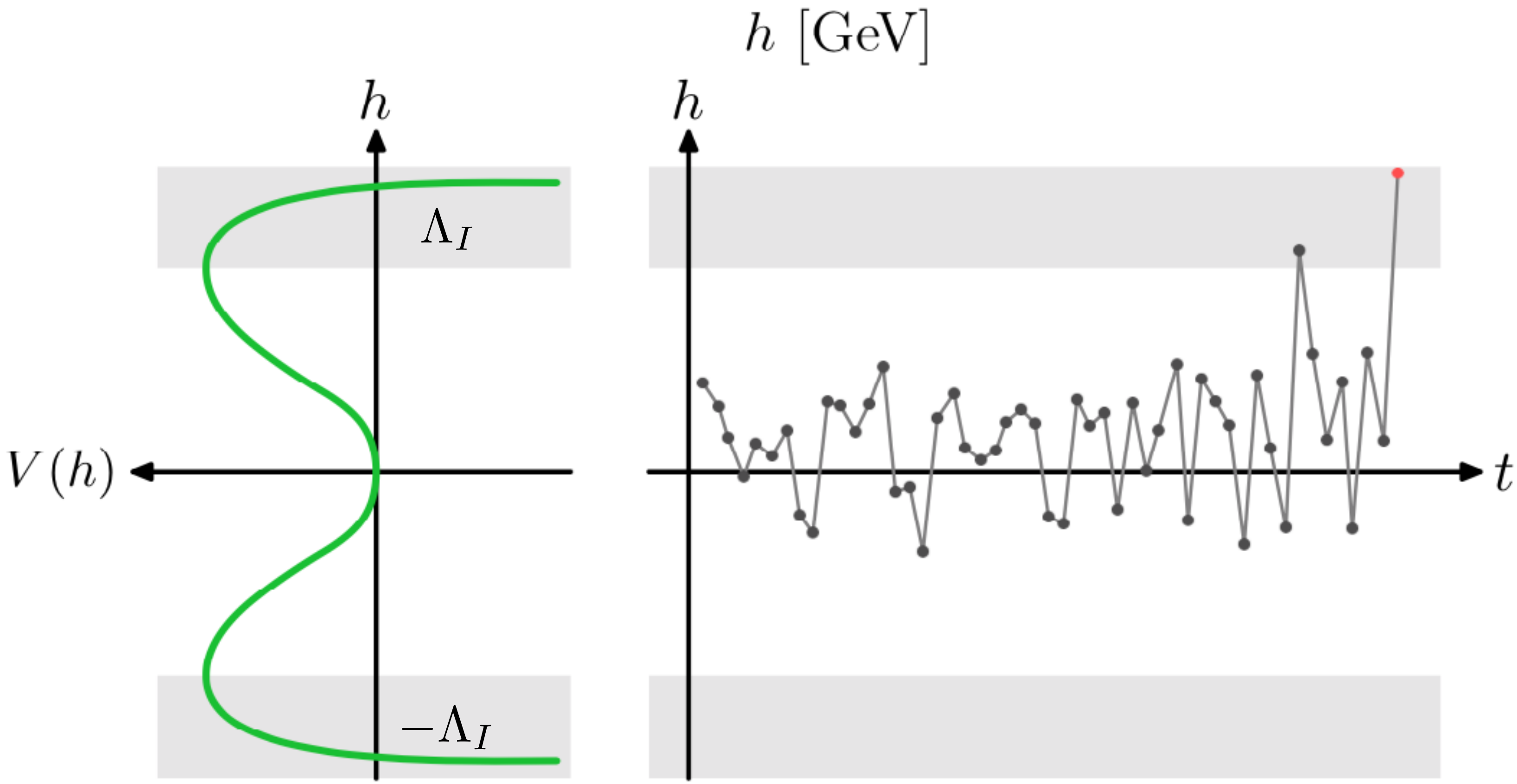
All light scalar fields are quantum mechanically excited,
*the Higgs being not an exception*

The classical value of the Higgs receives quantum kicks and diffuses away from our electroweak vacuum

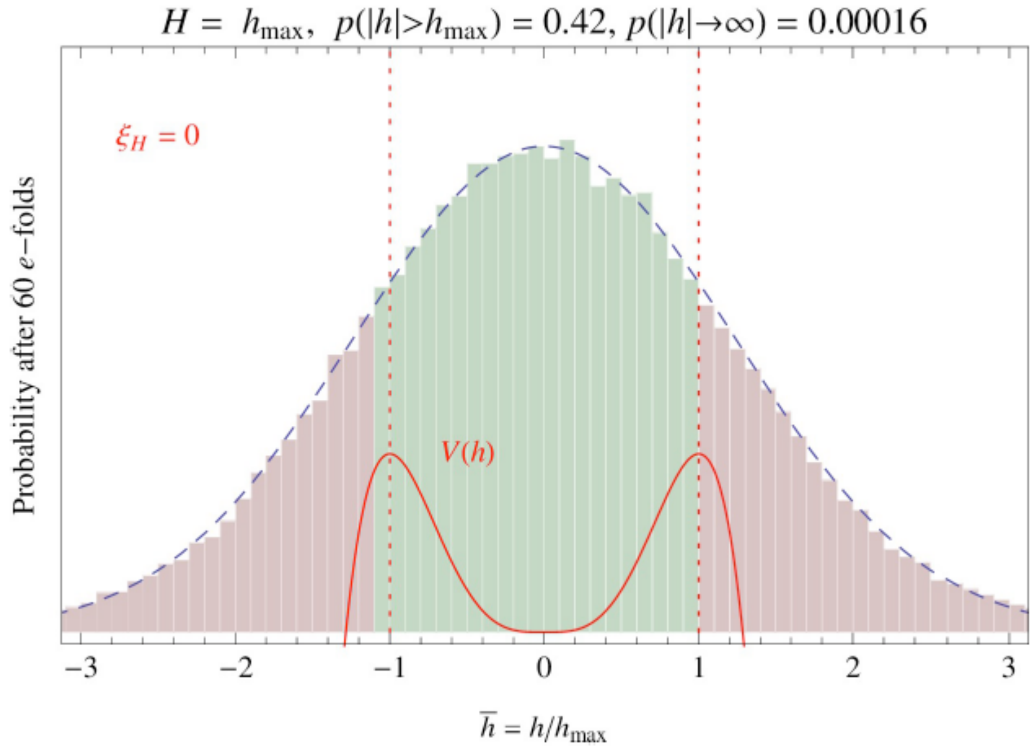
$$\ddot{h}_c + 3H\dot{h}_c + V'(h_c) = \xi$$

$$\langle \xi(t)\xi(t') \rangle = \frac{H^3}{4\pi^2}\delta(t - t')$$

J.R. Espinosa, G.F. Giudice, and A.R., 2007




J.R. Espinosa, G. Giudice and A.R., 2007




J.R. Espinosa et al., 2015

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial h}\left[\frac{\partial}{\partial h}\left(\frac{H^3}{8\pi^2}P\right) + \frac{1}{3H}V'P\right]$$

The Higgs randomly wanders beyond the barrier
in the unbounded from below AdS region

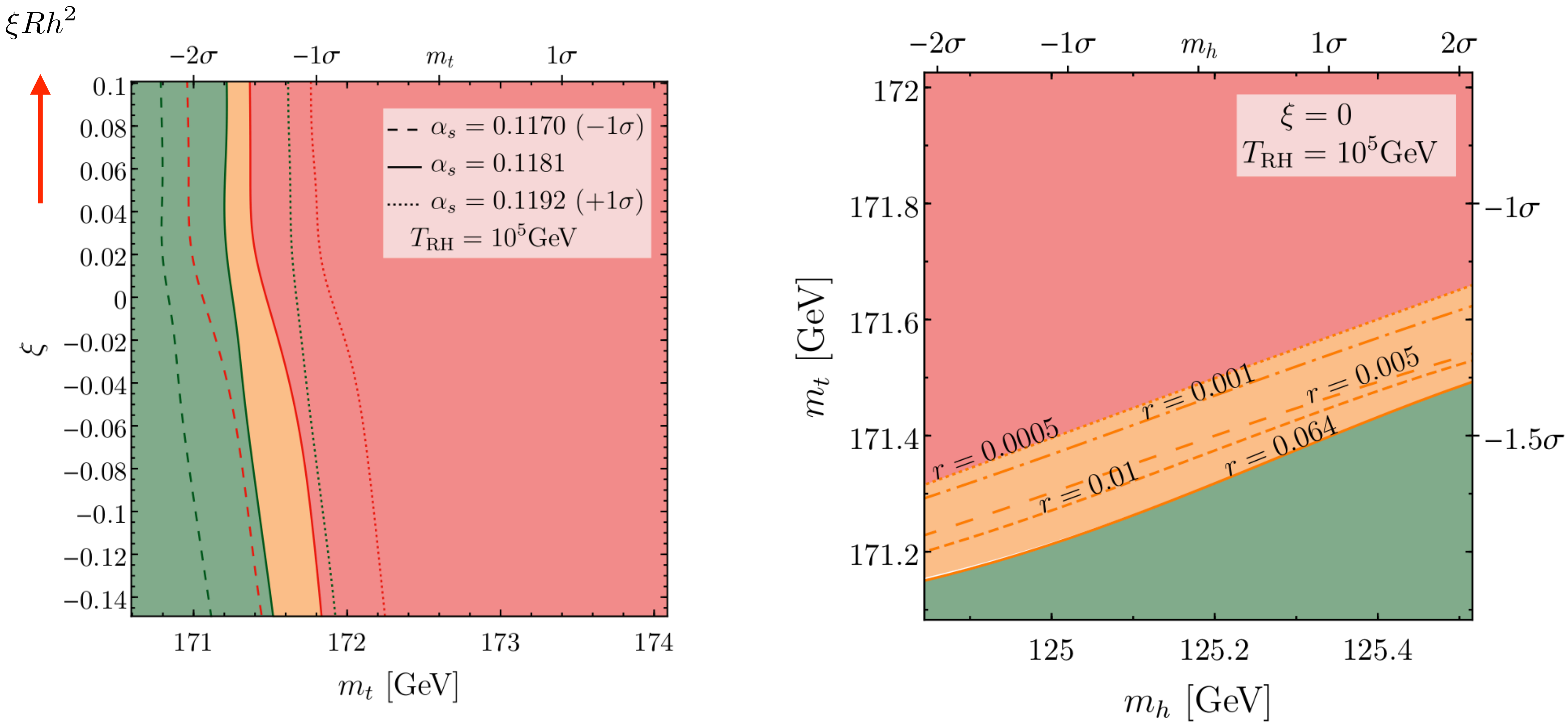
- If Higgs excited, islands of Higgs AdS may be created
- Non trivial GR effects make these islands expand and engulf the all universe, they have huge negative energy density
- Need to avoid this: hard for large Hubble rates

Current bounds on gravity waves from Planck 2018+

$$r = \mathcal{P}_{\rm GW}/\mathcal{P}_\zeta < 0.064 \Rightarrow H < 6 \cdot 10^{13}\,\mathrm{GeV}$$

Future bounds on gravity waves (LiteBIRD)

$$r = \mathcal{P}_{\rm GW}/\mathcal{P}_\zeta > 5 \cdot 10^{-4} \Rightarrow H > 6 \cdot 10^{12}\,\mathrm{GeV}$$



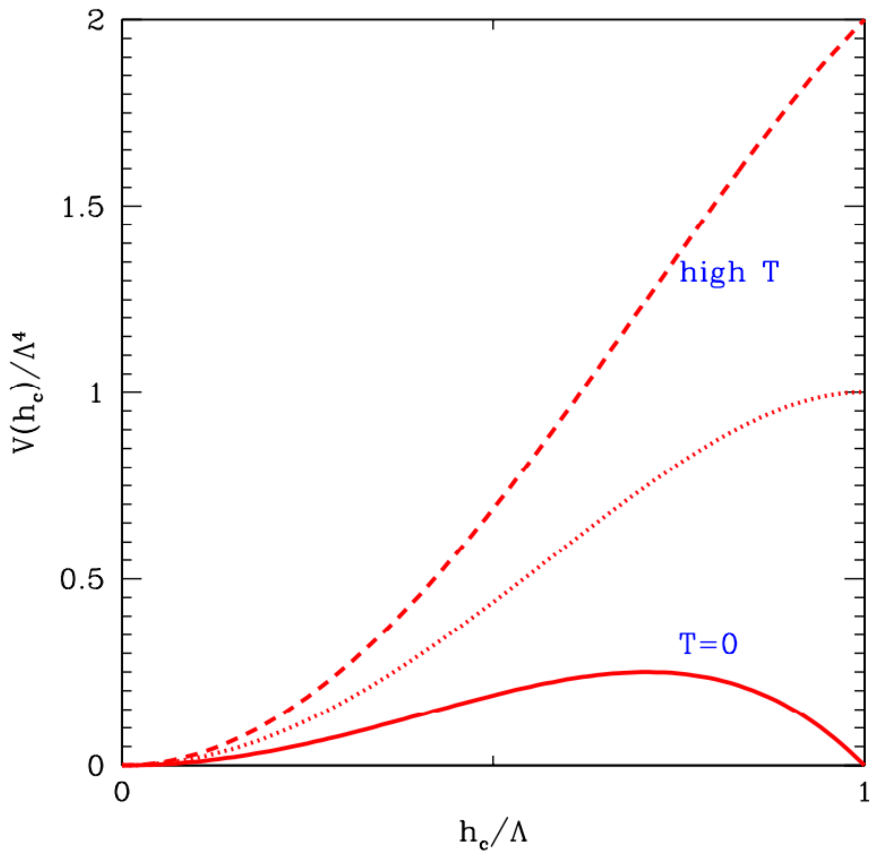

Green: regions allowed independently of any future detection

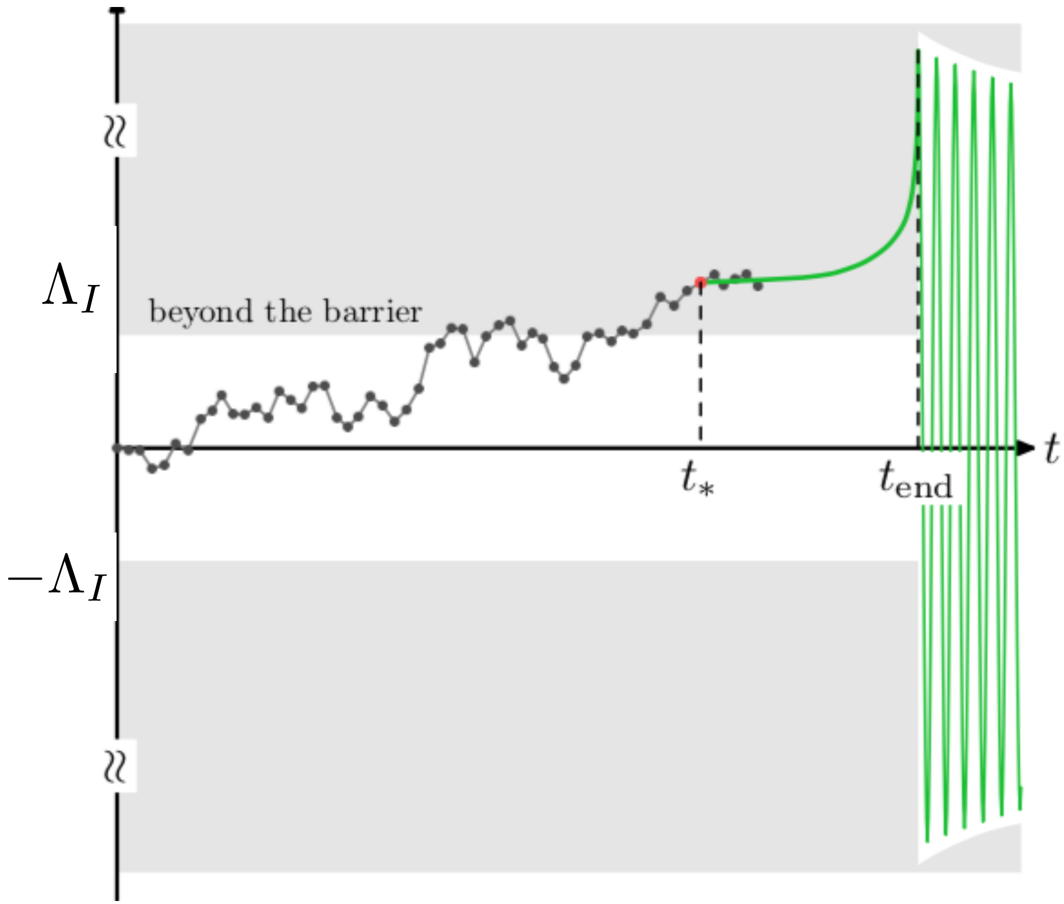
Red: regions ruled out if there is a future detection of GWs

Orange : transition region

G. Franciolini et al., 2018

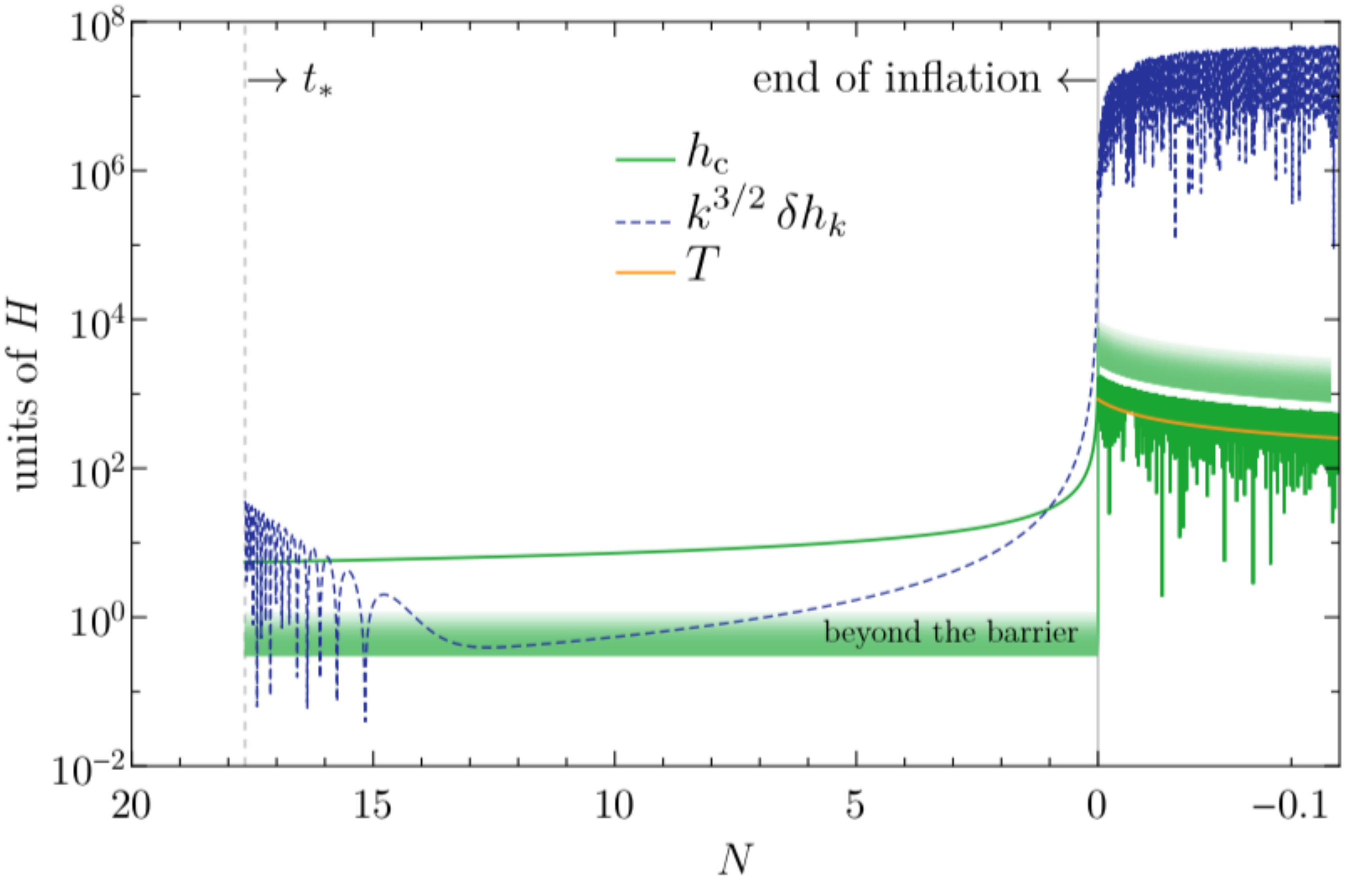
# The Higgs as a bonus: dark matter from the SM



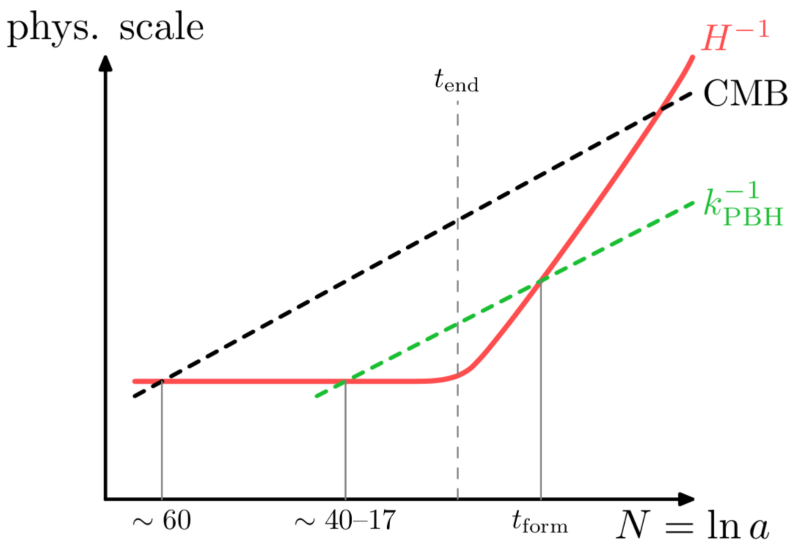



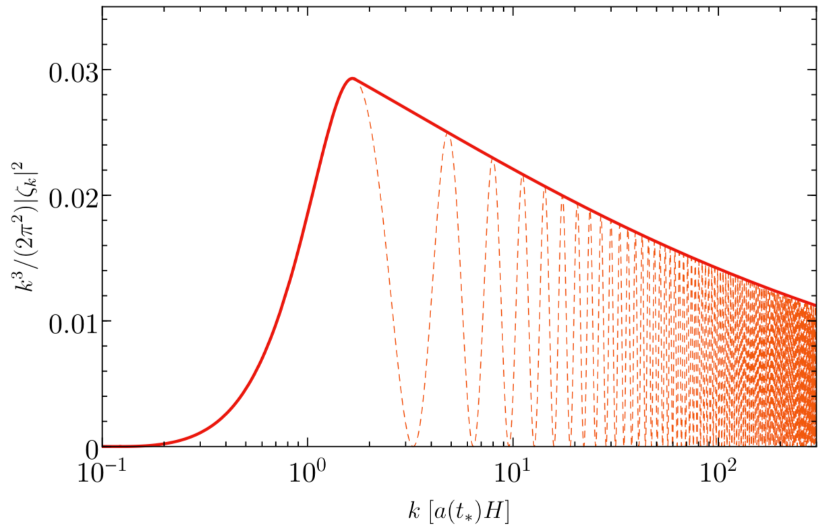

J.R. Espinosa, D. Racco and A.R., hep-ph/1710.1196, PRL

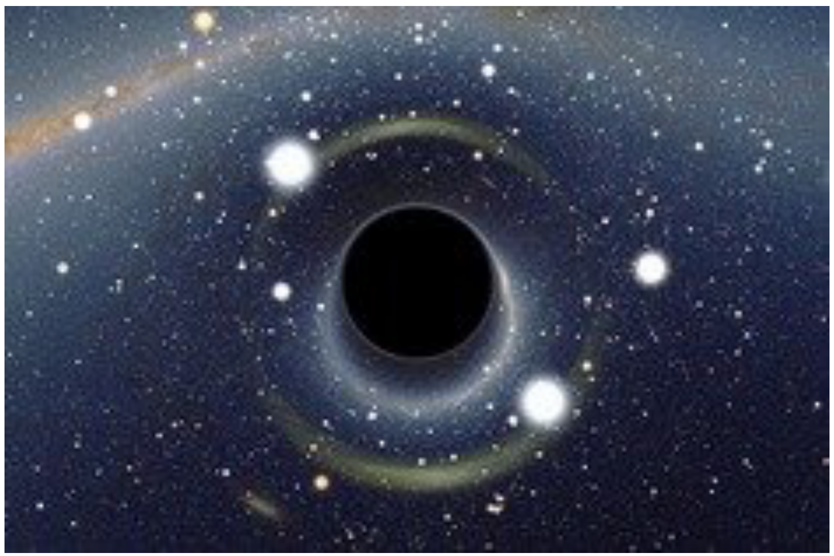
# Higgs perturbations grow during inflation



# Upon horizon re-entry the perturbations form primordial black holes



# Dark matter as PBHs from the SM

Upon horizon re-entry, the small-scale curvature perturbations induced by the Higgs can form PBHs if they are sizeable enough

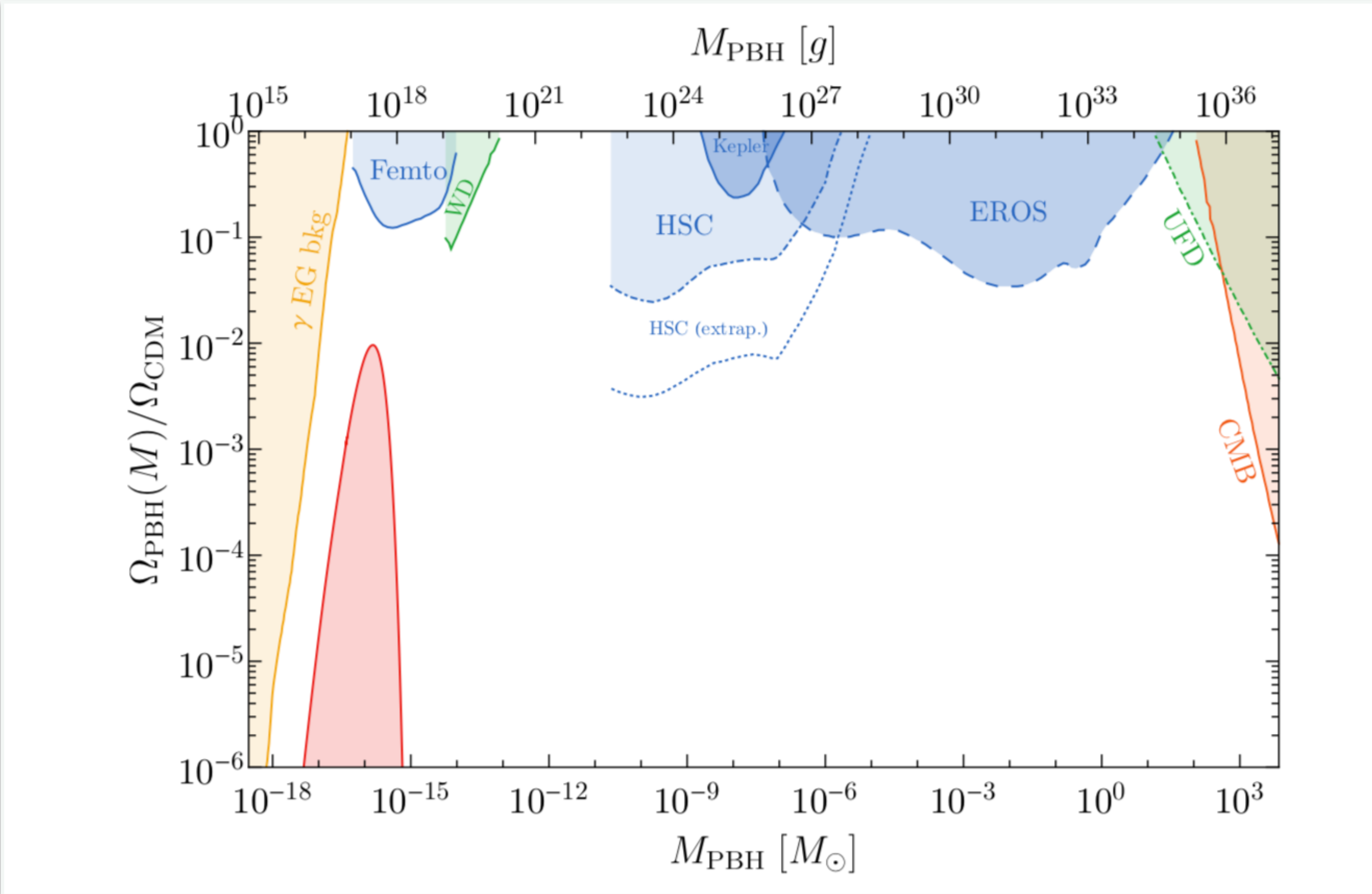
This happens if the density contrast is above a critical value

$$\Delta(\vec{x}) = \frac{4}{9a^2H^2}\nabla^2\zeta(\vec{x}) \gtrsim \Delta_{\rm c} \simeq 0.45$$

The mass of the PBH is roughly the mass contained inside the Hubble volume and the mass fraction at formation time is

$$\beta(M) = \int_{\Delta_{\rm c}}^{\infty} \frac{{\rm d}\Delta}{\sqrt{2\pi}\,\sigma_\Delta} e^{-\Delta^2/2\sigma_\Delta^2}$$

$$\sigma_\Delta^2(M) = \int_0^\infty {\rm d}\ln k\, W^2(k,R)\mathcal{P}_\Delta(k)$$



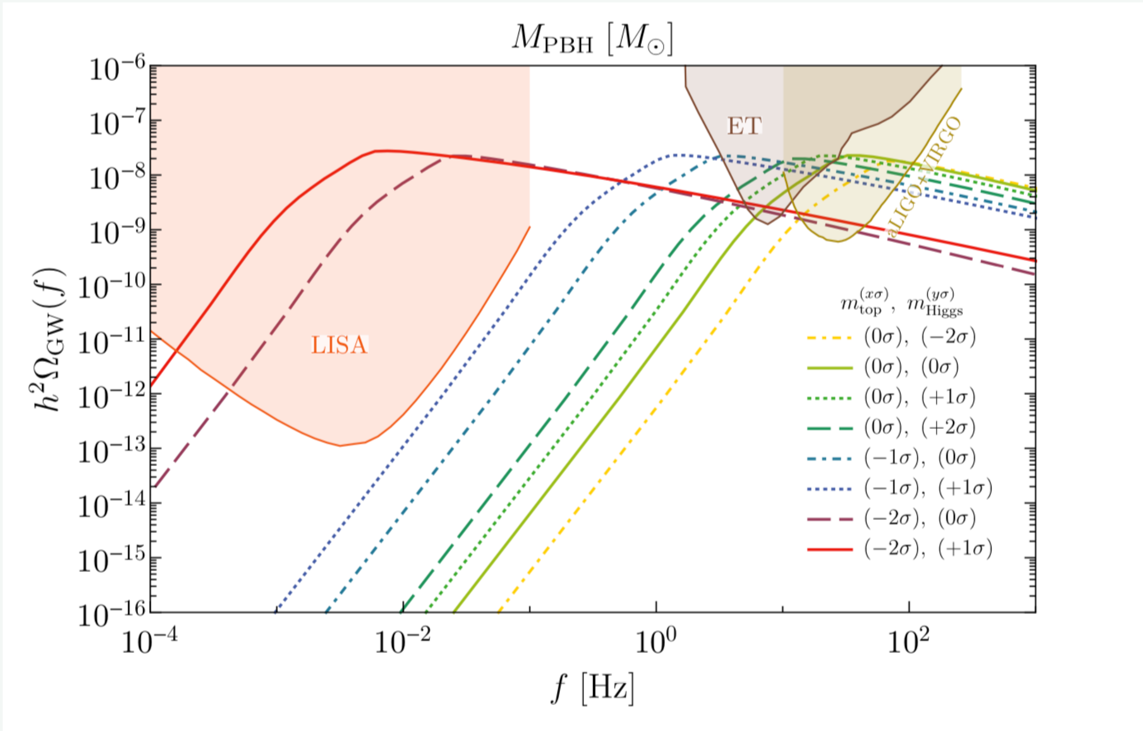

$$\frac{\Omega_{\rm PBH}(M)}{\Omega_{\rm CDM}} = \frac{\beta(M)}{1.6\cdot 10^{-16}}\left(\frac{\gamma}{0.2}\right)^{3/2}\left(\frac{g_*(T_f)}{106.75}\right)^{-1/4}\left(\frac{M}{10^{-15}M_\odot}\right)^{-1/2}$$

J.R. Espinosa, D. Racco and A.R., 2017, PRL

# Gravitational waves from the SM instability

Meanwhile, the same small-scale curvature perturbation induced by the Higgs can source, at second-order, gravitational waves

$$\Box h^s_{ij} = S_{ij} \qquad\qquad S_{ij} = \mathcal{O}(\partial_i\zeta\partial_j\zeta)$$




$$\Omega_{\rm GW}(f) \simeq 3\cdot 10^{-8}\left(\frac{f}{f_*}\right)^{n_T}$$

$$n_T = \begin{cases} 3 & \text{for } f < f_*, \\ -0.6 & \text{for } f > f_*. \end{cases}$$

$$\Lambda_I = 3\cdot 10^{11}\left(\frac{f_*}{\rm Hz}\right)^{-0.65}\ {\rm GeV}$$

J.R. Espinosa, D. Racco and A.R., 2018

# Conclusions

- Inflation is in agreement with all current data
- One next step to take: detection of gravity waves produced during inflation and measurement of the scale of inflation
- The presence of the SM Higgs during inflation might imply a low energy scale inflation and no gravity waves
- Turning this the other way around, no need for physics beyond the SM to explain dark matter, which is in the form of PBHs thanks to the Higgs instability
- Primordial gravitational waves from the Higgs instability