Non-Minimal Flavour Violation in $A_4 \times SU(5)$ SUSY GUTs

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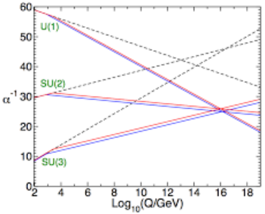
Outline

- Introduction
- SUSY-breaking and Non-Minimal Flavour Violation
- $SU(5)$ Unification and A_4
- This work - NMFV in this scenario
- Results
- Conclusions and Outlook

Introduction

Why SUSY?

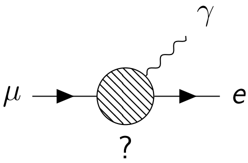
- Still (mostly) cures the hierarchy problem
- Precise gauge coupling unification
- Rich phenomenology, hints for experimentalists



Gauge couplings unify in MSSM[1]

Why flavour physics?

- Many experimental results hint at departure from SM
- Models can predict mixing - how much?



[1] S. Martin, Adv. Ser. Direct. High Energy Phys **18** (1998), hep-ph/9709356

Breaking Supersymmetry

Viable SUSY in nature **must be broken**

General soft-breaking Lagrangian in the MSSM:

$$\mathcal{L}_{\rm soft}^{\rm MSSM} = -\frac{1}{2}\left(M_1 \widetilde{B}\widetilde{B} + M_2 \widetilde{W}\widetilde{W} + M_3 \widetilde{g}\widetilde{g} + \mathrm{h.c.}\right)$$
$$-M_Q^2 \widetilde{Q}^\dagger \widetilde{Q} - M_L^2 \widetilde{L}^\dagger \widetilde{L} - M_U^2 \widetilde{U}^* \widetilde{U} - M_D^2 \widetilde{D}^* \widetilde{D} - M_E^2 \widetilde{E}^* \widetilde{E}$$
$$-\left(A_U \widetilde{U}^* H_u \widetilde{Q} + A_D \widetilde{D}^* H_d \widetilde{Q} + A_E \widetilde{E}^* H_d \widetilde{L} + \mathrm{h.c.}\right)$$
$$- m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - \left(b H_u^* H_d + \mathrm{h.c.}\right)$$

Parameters M_Q, M_L A_U etc. are **3x3 matrices** in ‘flavour space’

Non-Minimal Flavour Violation

Minimal Flavour Violation paradigm $\implies$ diagonal soft parameters.

$$M_Q^2 = \begin{pmatrix} (M_Q)^2_{11} & 0 & 0 \\ \cdot & (M_Q)^2_{22} & 0 \\ \cdot & \cdot & (M_Q)^2_{33} \end{pmatrix} \quad A_U = \begin{pmatrix} (A_U)_{11} & 0 & 0 \\ 0 & (A_U)_{22} & 0 \\ 0 & 0 & (A_U)_{33} \end{pmatrix}$$

Assumption in most analyses, **no theory motivation**

Relax assumption $\implies$ **Non-Minimal Flavour Violation** (NMFV)

$$M_Q^2 = \begin{pmatrix} (M_Q)^2_{11} & (\Delta^Q_{12})^2 & (\Delta^Q_{13})^2 \\ \cdot & (M_Q)^2_{22} & (\Delta^Q_{23})^2 \\ \cdot & \cdot & (M_Q)^2_{33} \end{pmatrix} \quad A_U = \begin{pmatrix} (A_U)_{11} & \Delta^{AU}_{12} & \Delta^{AU}_{13} \\ \Delta^{AU}_{21} & (A_U)_{22} & \Delta^{AU}_{23} \\ \Delta^{AU}_{31} & \Delta^{AU}_{32} & (A_U)_{33} \end{pmatrix}$$

In a unified framework, flavour symmetries can *generate NMFV*

Dimensionless Parametrisation

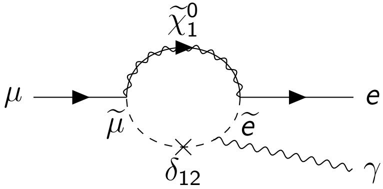
Reformulate NMFV by normalising to diagonal elements of soft matrices:

$$(\delta^Q_{LL})_{ij} = \frac{(\Delta^Q_{ij})^2}{(M_Q)_{ii}(M_Q)_{jj}}, \quad (\delta^U_{RR})_{ij} = \frac{(\Delta^U_{ij})^2}{(M_U)_{ii}(M_U)_{jj}}, \quad (\delta^D_{RR})_{ij} = \frac{(\Delta^D_{ij})^2}{(M_D)_{ii}(M_D)_{jj}},$$

$$(\delta^U_{RL})_{ij} = \frac{v_u}{\sqrt{2}} \frac{\Delta^{AU}_{ij}}{(M_Q)_{ii}(M_U)_{jj}}, \quad (\delta^D_{RL})_{ij} = \frac{v_d}{\sqrt{2}} \frac{\Delta^{AD}_{ij}}{(M_Q)_{ii}(M_D)_{jj}},$$

$$(\delta^L_{LL})_{ij} = \frac{(\Delta^L_{ij})^2}{(M_L)_{ii}(M_L)_{jj}}, \quad (\delta^E_{RR})_{ij} = \frac{(\Delta^E_{ij})^2}{(M_E)_{ii}(M_E)_{jj}}, \quad (\delta^E_{RL})_{ij} = \frac{v_d}{\sqrt{2}} \frac{\Delta^{AE}_{ij}}{(M_L)_{ii}(M_E)_{jj}}$$

Contribution to flavour violating decay $\mu \to e\gamma$ }



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$SU(5)$ Unification

Collect SM fields into irreps. of SU(5):

$$F = \overline{\mathbf{5}} = \begin{pmatrix} d_r^c \\ d_b^c \\ d_g^c \\ e^- \\ -\nu_e \end{pmatrix}_L , \qquad T = \mathbf{10} = \begin{pmatrix} 0 & u_g^c & -u_b^c & u_r & d_r \\ . & 0 & u_r^c & u_b & d_b \\ . & . & 0 & u_g & d_g \\ . & . & . & 0 & e^c \\ . & . & . & . & 0 \end{pmatrix}_L$$

Unification gives equalities between parameters at the GUT scale:

$$\begin{aligned} M_Q^2 &= M_U^2 = M_E^2 \equiv M_T^2 , \\ M_D^2 &= M_L^2 \equiv M_F^2 , \\ A_D &= (A_E)^T \equiv A_{FT} , \\ A_U &\equiv A_{TT} \end{aligned} \qquad \begin{aligned} \delta_{LL}^Q &= \delta_{RR}^U = \delta_{RR}^E \equiv \delta^T , \\ \delta_{RR}^D &= \delta_{LL}^L \equiv \delta^F , \\ \delta_{RL}^D &= (\delta_{RL}^E)^T \equiv \delta^{FT} , \\ \delta_{RL}^U &\equiv \delta^{TT} \end{aligned}$$

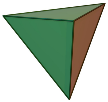
The $A_4 \times SU(5)$ Model

Addition of discrete symmetry unifies three families of the $\overline{\mathbf{5}}$

Representations of A_4:

$$F = \mathbf{3}$$

$$T = \mathbf{1}$$



$\Longrightarrow$

Unified breaking matrices:

$$M_F^2 = \begin{pmatrix} m_F^2 & 0 & 0 \\ 0 & m_F^2 & 0 \\ 0 & 0 & m_F^2 \end{pmatrix}$$

$$M_T^2 = \begin{pmatrix} m_{T_1}^2 & 0 & 0 \\ 0 & m_{T_2}^2 & 0 \\ 0 & 0 & m_{T_3}^2 \end{pmatrix}$$

Break discrete symmetry $\Longrightarrow$ NMFV patterns at the GUT scale - these incite **flavour mixing at low scales**

$A_4 \times SU(5)$ —GUT Scale→ Effective MSSM —SUSY Scale→ Effective SM

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Explorative Study of NMFV

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- MFV not theoretically well motivated
- Flavour violation places additional constraints on models
- Relax minimal assumptions, explore phenomenology

Question

What is the allowed flavour violation in such a scenario?

- Scan over NMFV parameters at the GUT scale simultaneously, run predictions to low scale, and determine degree of mixing permitted

Model Unknowns

MFV Parameters

Masses:

$$m_F \quad m_{T_1} \quad m_{T_2} \quad m_{T_3}$$

$$M_1 \quad M_2 \quad M_3 \quad M_{H_u} \quad M_{H_d}$$

Couplings etc.:

$$(A_{TT})_{33} \quad (A_{FT})_{33} \quad \tan\beta \quad \mu$$

13 fixed inputs, not varied in this analysis

These sets of unknowns specify MSSM SUSY-breaking **entirely**

NMFV Parameters

Off-diagonal matrix elements

$$(\delta^F)_{12} \quad (\delta^F)_{13} \quad (\delta^T)_{23}$$

$$(\delta^T)_{12} \quad (\delta^T)_{13} \quad (\delta^T)_{23}$$

$$(\delta^{TT})_{12} \quad (\delta^{TT})_{13} \quad (\delta^{TT})_{23}$$

$$(\delta^{FT})_{12} \quad (\delta^{FT})_{13} \quad (\delta^{FT})_{21}$$

$$(\delta^{FT})_{23} \quad (\delta^{FT})_{31} \quad (\delta^{FT})_{32}$$

15 numerical inputs, scanned over with flat prior distributions

Fixed MFV reference Points

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	Parameter/Observable	Scenario 1	Scenario 2
MFV Parameters at GUT scale	m_F	5000	5000
	m_{T_1}	5000	5000
	m_{T_2}	200	233.2
	m_{T_3}	2995	2995
	$(A_{TT})_{33}$	-940	-940
	$(A_{FT})_{33}$	-1966	-1966
	M_1	250.0	600.0
	M_2	415.2	415.2
	M_3	2551.6	2551.6
	M_{H_u}	4242.6	4242.6
	M_{H_d}	4242.6	4242.6
	$\tan\beta$	30	30
	μ	-2163.1	-2246.8

Table: GUT scale parameters that define MFV scenarios.

- MFV defined by flavour-conserving params
- 1 inspired by previous work[2], 2 motivated by experimental limits
- Almost-mass-degenerate $\widetilde{\chi}_0^1$ and $\widetilde{\mu}$ to satisfy relic density through coannihilation

[2] A. Belyaev, S.F. King and P. Schaefers, Phys. Rev. D **97** (2018), 1801.00514

NMFV Parameter Scan

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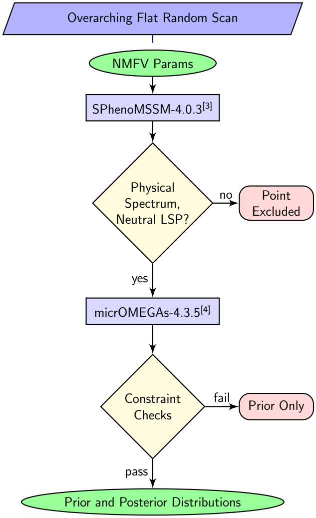

[3] W. Porod, Comput. Phys. Commun. **153** (2003), hep-ph/0301101
[4] G.Belanger et. al., Comput. Phys. Commun. **149** (2002), hp-ph/0112278

Observable	Constraint
m_h	(125.2 ± 2.5) GeV
$\mathrm{BR}(\mu \to e\gamma)$	$< 4.2 \times 10^{-13}$
$\mathrm{BR}(\mu \to 3e)$	$< 1.0 \times 10^{-12}$
$\mathrm{BR}(\tau \to e\gamma)$	$< 3.3 \times 10^{-8}$
$\mathrm{BR}(\tau \to \mu\gamma)$	$< 4.4 \times 10^{-8}$
$\mathrm{BR}(\tau \to 3e)$	$< 2.7 \times 10^{-8}$
$\mathrm{BR}(\tau \to 3\mu)$	$< 2.1 \times 10^{-8}$
$\mathrm{BR}(\tau \to e^-\mu\mu)$	$< 2.7 \times 10^{-8}$
$\mathrm{BR}(\tau \to e^+\mu\mu)$	$< 1.7 \times 10^{-8}$
$\mathrm{BR}(\tau \to \mu^- ee)$	$< 1.8 \times 10^{-8}$
$\mathrm{BR}(\tau \to \mu^+ ee)$	$< 1.5 \times 10^{-8}$
$\mathrm{BR}(B \to X_s\gamma)$	$(3.32 \pm 0.18) \times 10^{-4}$
$\mathrm{BR}(B_s \to \mu\mu)$	$(2.7 \pm 1.2) \times 10^{-9}$
ΔM_{B_s}	(17.757 ± 0.312) ps^{-1}
ΔM_K	$(3.1 \pm 1.2) \times 10^{-15}$ GeV
ϵ_K	2.228 ± 0.29
$\Omega_{\mathrm{DM}}h^2$	0.1198 ± 0.0042

Table: Experimental constraints imposed on the $A_4 \times SU(5)$ parameter space in our study.

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Results: Summary Table

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Parameters	Scenario 1	Scenario 2	Principle Constraints
$(\delta^T)_{12}$	[-0.015, 0.015]	[-0.12, 0.12]†	$\Omega_{\tilde{\chi}_1^0}h^2$, $\mu \to e\gamma$
$(\delta^T)_{13}$	[-0.06, 0.06]†	[-0.3, 0.3]†	$\Omega_{\tilde{\chi}_1^0}h^2$
$(\delta^T)_{23}$	$[0, 0]^*$	[-0.1, 0.1†]	$\Omega_{\tilde{\chi}_1^0}h^2$, $\mu \to 3e$, $\mu \to e\gamma$,
$(\delta^F)_{12}$	[-0.008, 0.008]	[-0.015, 0.015]†	$\mu \to 3e$, $\mu \to e\gamma$
$(\delta^F)_{13}$	[-0.01, 0.01]†	[-0.15, 0.15]†	$\mu \to 3e$, $\mu \to e\gamma$
$(\delta^F)_{23}$	[-0.015, 0.015]†	[-0.15, 0.15]†	$\Omega_{\tilde{\chi}_1^0}h^2$, $\mu \to e\gamma$, $\mu \to 3e$
$(\delta^{TT})_{12}$	[-3, 3.5] $\times 10^{-5}$	[-1, 1.5]† $\times 10^{-3}$	prior, $\Omega_{\tilde{\chi}_1^0}h^2$
$(\delta^{TT})_{13}$	[-6, 7]† $\times 10^{-5}$	[-4, 2.5]† $\times 10^{-3}$	prior, $\Omega_{\tilde{\chi}_1^0}h^2$
$(\delta^{TT})_{23}$	[-0.5, 4]† $\times 10^{-5}$	[-0.25, 0.2]†	prior, $\Omega_{\tilde{\chi}_1^0}h^2$
$(\delta^{FT})_{12}$	[-0.0015, 0.0015]	[-1.2, 1.2]† $\times 10^{-4}$	$\mu \to 3e$, $\Omega_{\tilde{\chi}_1^0}h^2$, $\mu \to e\gamma$
$(\delta^{FT})_{13}$	[-0.002, 0.002]†	[-5, 5] $\times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0}h^2$, $\mu \to 3e$, $\mu \to e\gamma$
$(\delta^{FT})_{21}$	$[0, 0]^*$	[-1.2, 1.2]† $\times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0}h^2$, prior
$(\delta^{FT})_{23}$	[-0.0022, 0.0022]†	[-6, 6]† $\times 10^{-4}$	$\mu \to 3e$, $\Omega_{\tilde{\chi}_1^0}h^2$, $\mu \to e\gamma$
$(\delta^{FT})_{31}$	[-0.0004, 0.0004]†	[-2, 2]† $\times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0}h^2$
$(\delta^{FT})_{32}$	$[0, 0]^*$	[-1.5, 1.5] $\times 10^{-4}$	$\Omega_{\tilde{\chi}_1^0}h^2$

Table: Estimated allowed GUT scale flavour-violation for both reference scenarios and impactful constraints.

Motivation for a Simultaneous Scan

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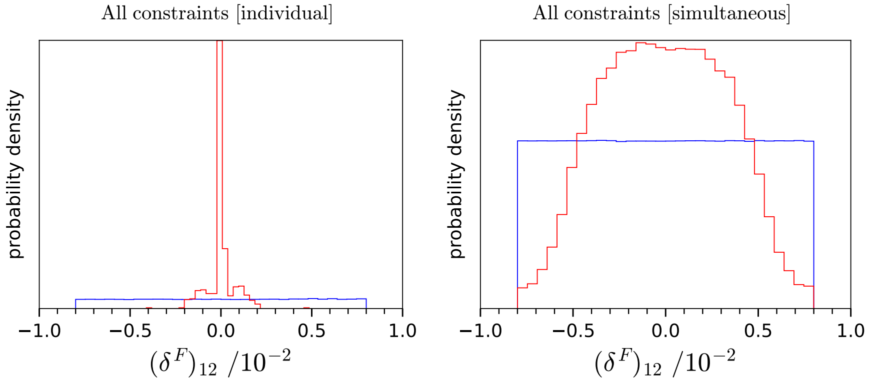

Figure: Comparison of individual VS simultaneous scan in Scenario 1 for $(\delta^F)_{12}$.

Blue shows prior distribution, and red shows posterior after constraints are applied

Results: Constraints

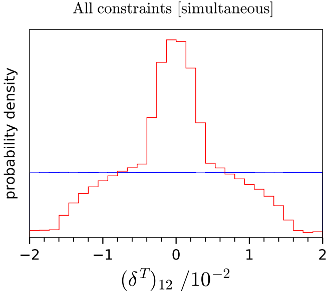

Figure: Prior in blue, posterior in red

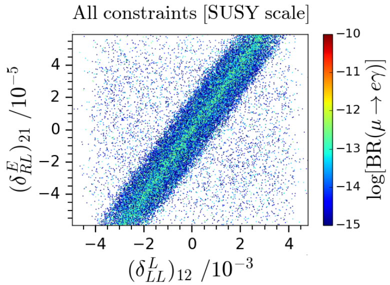

Figure: Correlations between parameters

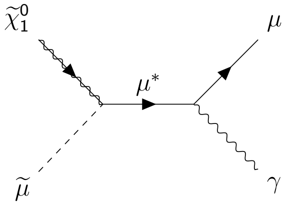

Figure: Dominant co-annihilation channel

Co-annihilation is critial

NMFV can alter $m_{\widetilde{\mu}}$ and significantly alter relic density

Conclusions

- Constrained $SU(5)$ MSSM NMFV parameters in this setup
- Dark matter and lepton flavour experiments place the most stringent limits (*see arXiv:1812.07643 for more info*)

Outlook:

- Study predictions of flavour violation with addition of seesaw mechanism
- Flavoured GUT model descrimination using flavour violation

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- Constrained $SU(5)$ MSSM NMFV parameters in this setup
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- Flavoured GUT model descrimination using flavour violation

Thank you for your attention

Backup: Leptonic Flavour Violation

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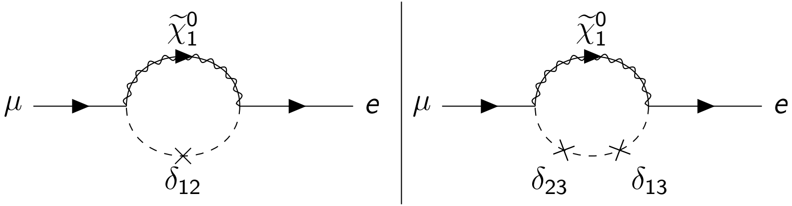
$$\boxed{\begin{array}{c} BR(\mu \to e\gamma) \\ BR(\mu \to 3e) \end{array}} \xrightarrow{???} \boxed{\begin{array}{c} (\delta)_{13} \\ (\delta)_{23} \end{array}}$$

$\mu \to e\gamma$ and $\mu \to 3e$ can have a distinctive constraining effect on $(\delta)_{13}$ and $(\delta)_{23}$ parameters.



Backup: Dark Matter Relic Density

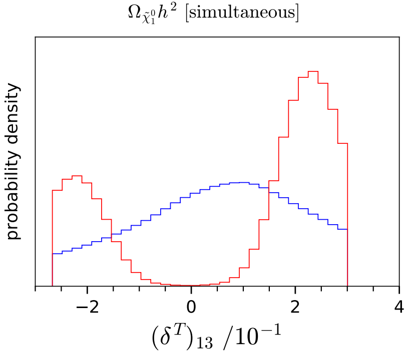


Figure: Dark matter constraint action on $(\delta^T)_{13}$, simultaneous scan over Scenario 2

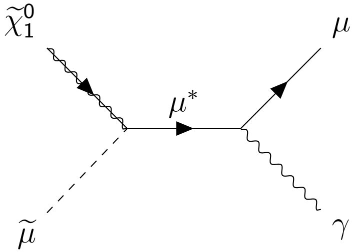


Figure: A co-annihilation channel that contributes to relic abundance

Backup: Correlations, Cancellations

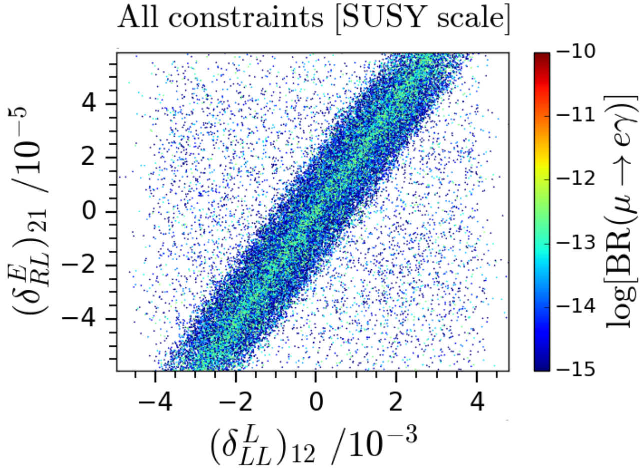


Figure: Correlations plots of $(\delta^F)_{12}$ and $(\delta^{FT})_{12}$ at GUT scale. Results reflect simultaneous scan around Scenario 1.