## Kaons in the Standard Model

## (Perturbative Corrections)

Based on work in collaboration with:
Andrzej Buras, Sebastian Jäger \& Matthias Jamin [1507.06345] Maria Cerda-Sevilla, Sebastian Jäger \& Ahmet Kokulu [1611.08276]
[And based on older calculations with Joachim Brod, Emanuel Stamou and Ulrich Haisch]

New Physics in Kaon and Beam Dump IoP Meeting Birmingham, 3 December 2018

Martin Gorbahn


## Content

## Semi-leptonic Decays

$\mathrm{K} \rightarrow \pi \bar{v} v$
$\varepsilon^{\prime}{ }_{\text {K }} / \varepsilon_{\text {K }}$

## CKM Factors I

## Semi-leptonic decays $\left(\mathrm{V}_{\mathrm{us}}\right): \lambda=\mathcal{O}(0.2)$



$$
V_{i j}=\mathcal{O}\left(\begin{array}{ccc}
1 & \lambda & \lambda^{3} \\
\lambda & 1 & \lambda^{2} \\
\lambda^{3} & \lambda^{2} & 1
\end{array}\right)
$$

Remaining Wolfenstein Parameters: A, $\eta$ \& $\varrho$
$\operatorname{Im} V_{t s}^{*} V_{t d}=-\operatorname{Im} V_{c s}^{*} V_{c d}=\mathcal{O}\left(\lambda^{5}\right)$
$\operatorname{Re} V_{u s}^{*} V_{u d}=-\operatorname{Re} V_{c s}^{*} V_{c d}=\mathcal{O}\left(\lambda^{1}\right)$

$$
\operatorname{Re} V_{t s}^{*} V_{t d}=\mathcal{O}\left(\lambda^{5}\right)
$$

## Leptonic and Semileptonic

Observables: $\mathrm{K}(\pi) \rightarrow \mathrm{l} \bar{\nu}_{l} \quad \& \quad \mathrm{~K} \rightarrow \pi \mathrm{l} \bar{v}_{l}$

$$
\begin{aligned}
& \text { Lattice } \\
& \Gamma\left(K_{\ell 3(\gamma)}\right)=\frac{G_{F}^{2} m_{K}^{5}}{192 \pi^{3}} C_{K} \underbrace{}_{\text {eww }}{\left|V_{u s}\right|^{2} f_{+}(0)^{2}}^{\ell} I_{K}^{\ell}\left(\lambda_{+, 0}\right)\left(1+\delta_{S U(2)}^{K}+\delta_{\mathrm{em}}^{K \ell}\right)^{2} \\
& \text { Perturbative } \\
& \text { [Cirigliano, Giannotti, } \\
& \text { Neufeld `08] } \downarrow \\
& \text { Isospin breaking effects: } \\
& \text { Flavianet `10 }
\end{aligned}
$$

## QED x QCD corrections



## $S_{\text {ew }}=1.023$ from running of semileptonic operator

Does not contain any QCD corrections

QCD corrections [Brod, Gorbahn] can be significant if run to a very low scale.

Scale dependence will cancel when matched to QED \& QCD calculation of form factors

## CKM Unitarity

$$
\begin{array}{ll}
\Gamma\left(\mathrm{K}_{\mathrm{l} 3}\right) & \left|\mathrm{V}_{\mathbf{u s}}\right| \mathrm{f}_{+}(0)=0.2163(5) \\
\frac{\Gamma\left(\mathrm{K}_{\mathrm{l} 2}\right)}{\Gamma\left(\pi_{\mathrm{l} 2}\right)} & \frac{\left|\mathrm{V}_{\mathrm{us}}\right| \mathrm{f}_{\mathrm{K}}}{\left|\mathrm{~V}_{\mathbf{u d}}\right| \mathrm{f}_{\pi}}=0.2758(5) \\
& \left|\mathrm{V}_{\mathbf{u d}}\right|^{2}+\left|\mathrm{V}_{\mathbf{u s}}\right|^{2}+\left|\mathrm{V}_{\mathbf{u b}}\right|^{2}=1
\end{array}
$$

3 Equations, 4 Unknowns ( $\mathrm{V}_{\mathrm{us}}, \mathrm{V}_{\mathrm{us}}, \mathrm{f}_{+}(0), \mathrm{f}_{\mathrm{K}} / \mathrm{f}_{\pi}$ )
$\mathrm{f}_{+}(0)$ from Lattice
gives $f_{K} / f_{\pi} V_{u s}, V_{u d}$
$\mathrm{f}_{\mathrm{K}} / \mathrm{f}_{\pi}$ from Lattice gives $f_{+}(0) V_{u s}, V_{u d}$

## CKM Unitarity (Model Independent)

[Cirigliano et. al. `09]

$$
\Lambda_{N P} \gg M_{W} \quad \text { Neglect } \quad \mathcal{O}\left(\frac{M_{W}}{\Lambda_{N P}}\right) \quad \text { corrections }
$$

Use $\mathrm{SU}(2) \otimes \mathrm{U}(1)$ invariant operators [Buchmüller-Wyler ${ }^{`} 06$ ] (plusU(3) ${ }^{5}$ flavour symmetry)

$$
\mathrm{O}_{\mathrm{lq}}^{(3)}=\left(\bar{l} \gamma^{\mu} \sigma^{\mathrm{a}} \mathrm{l}\right)\left(\overline{\mathrm{q}} \gamma_{\mu} \sigma^{\mathrm{a}} \mathbf{q}\right) \quad \mathrm{O}_{\mathrm{ll}}^{(3)}=\frac{1}{2}\left(\overline{\mathrm{l}} \boldsymbol{\gamma}^{\mu} \sigma^{\mathrm{a}} \mathrm{l}\right)\left(\overline{\mathrm{l}} \gamma_{\mu} \sigma^{\mathrm{a}} \mathrm{l}\right)
$$

Constrained from EW precision data [Han, Skiba `05]

Redefine

$$
\mathrm{G}_{\mathrm{F}}(\mu \rightarrow \mathrm{ev} \bar{v}) \rightarrow \mathrm{G}_{\mathrm{F}}\left(1-2 \bar{\alpha}_{\mathrm{ll}}^{(3)}\right) \longrightarrow \mathrm{G}_{\mathrm{F}}^{\mu}
$$

$$
\mathrm{G}_{\mathrm{F}}(\mathrm{~d} \rightarrow \mathrm{u} e \bar{v}) \rightarrow \mathrm{G}_{\mathrm{F}}\left(1-2 \bar{\alpha}_{\mathrm{lq}}^{(3)}\right) \longrightarrow \mathrm{G}_{\mathrm{F}}^{S L}
$$

## CKM Unitarity (Model Independent)

「Cirigliann et. al. `n91


CKM
Unitarity

## CKM Factors II


$\operatorname{Im} V_{t s}^{*} V_{t d}=-\operatorname{Im} V_{c s}^{*} V_{c d}=\mathcal{O}\left(\lambda^{5}\right) \quad \operatorname{Im} V_{u s}^{*} V_{u d}=0$

$$
\operatorname{Re} V_{u s}^{*} V_{u d}=-\operatorname{Re} V_{c s}^{*} V_{c d}=\mathcal{O}\left(\lambda^{1}\right) \quad \operatorname{Re} V_{t s}^{*} V_{t d}=\mathcal{O}\left(\lambda^{5}\right)
$$

Kaon observables $\propto \mathrm{V}_{\mathrm{ts}}{ }^{*} \mathrm{~V}_{\mathrm{td}} \rightarrow$ suppressed in SM sensitive to flavour violating NP
Kaon observables $\propto \mathrm{V}_{\mathrm{us}}{ }^{*} \mathrm{~V}_{\mathrm{ud}}$ or $\mathrm{V}_{\mathrm{cs}}{ }^{*} \mathrm{~V}_{\mathrm{cd}} \rightarrow$ dominated by QCD, useful for extracting low energy constants

## CKM Factors in Kaon physics



> Using the GIM mechanism, we can eliminate either $\mathrm{V}_{\mathrm{cs}}{ }^{*} \mathrm{~V}_{\mathrm{cd}}$ or $\mathrm{V}_{\mathrm{us}}{ }^{*} \mathrm{~V}_{\mathrm{ud}} \rightarrow-\mathrm{V}_{\mathrm{cs}}{ }^{*} \mathrm{~V}_{\mathrm{cd}}-\mathrm{V}_{\mathrm{ts}}{ }^{*} \mathrm{~V}_{\mathrm{td}}$

Z-Penguin and Boxes (high virtuality): power expansion in: $\mathrm{A}_{\mathrm{c}}-\mathrm{A}_{\mathrm{u}} \propto 0+\mathrm{O}\left(\mathrm{m}_{\mathrm{c}}{ }^{2} / \mathrm{M}_{\mathrm{w}}{ }^{2}\right)$
$\gamma / \mathrm{g}$-Penguin (momentum expansion + e.o.m.): power expansion in: $\mathrm{A}_{\mathrm{c}}-\mathrm{A}_{\mathrm{u}} \propto \mathrm{O}\left(\log \left(\mathrm{m}_{\mathrm{c}}{ }^{2} / \mathrm{m}_{\mathrm{u}}{ }^{2}\right)\right)$

## $\mathrm{K} \rightarrow \pi \bar{v} v$



$$
x_{i}=\frac{m_{i}^{2}}{M_{W}^{2}}
$$

$$
\sum_{i} V_{i s}^{*} V_{i d} F\left(x_{i}\right)=V_{t s}^{*} V_{t d}(F(\underbrace{}_{t})-F\left(x_{\mathfrak{u}}\right))+V_{c s}^{*} V_{c d}\left(F\left(x_{c}\right)-F\left(x_{u}\right)\right)
$$

Top (SD),
Charm (Renormalisation $\lambda^{5} \overline{M_{W}^{2}}$ Group Improved) \& Light Quarks
(Non-Perturbative)


$$
\mathrm{Q}_{\nu}=\left(\bar{s}_{\mathrm{L}} \gamma_{\mu} \mathrm{d}_{\mathrm{L}}\right)\left(\bar{v}_{\mathrm{L}} \gamma^{\mu} \nu_{\mathrm{L}}\right)
$$

Matrix element from $K_{13}$ decays (Isospin symmetry: $\mathrm{K}^{+} \rightarrow \pi^{0} \mathrm{e}^{+} v$ )
[Mescia, Smith]

## Top quark contribution



$$
x_{i}=\frac{m_{i}^{2}}{M_{W}^{2}}
$$

$$
\sum_{i} V_{i s}^{*} V_{i d} F\left(x_{i}\right)=V_{t s}^{*} V_{t d}\left(F\left(x_{t}\right)-F\left(x_{u}\right)\right)+V_{c s}^{*} V_{c d}\left(F\left(x_{c}\right)-F\left(x_{u}\right)\right)
$$

$$
\text { Quadratic GIM } \lambda^{\lambda^{5} \frac{m_{t}^{2}}{M_{W}^{2}}}
$$

Matching (NLO +EW):
[Misiak, Urban; Buras, Buchalla;
Brod, MG, Stamou ${ }^{111]}$
$\mathrm{Q}_{\boldsymbol{v}}=\left(\bar{s}_{\mathrm{L}} \gamma_{\mu} \mathrm{d}_{\mathrm{L}}\right)\left(\bar{v}_{\mathrm{L}} \gamma^{\mu} \nu_{\mathrm{L}}\right)$

After 2011 uncertainty at 1\%

## $\mathrm{K}^{+} \rightarrow \pi^{+} \bar{v} v$ at $\mathrm{M}_{\mathrm{W}}$



$$
x_{i}=\frac{m_{i}^{2}}{M_{W}^{2}}
$$

$$
\sum_{i} V_{i s}^{*} V_{i d} F\left(x_{i}\right)=V_{t s}^{*} V_{t d}\left(F\left(x_{t}\right)-F\left(x_{i l}\right)\right)+V_{c s}^{*} V_{c d}\left(F\left(x_{c}\right)-F\left(x_{u}\right)\right)
$$

$$
\text { Quadratic GIM } \lambda^{5} \frac{m_{t}^{2}}{M_{W}^{2}} \lambda \frac{m_{c}^{2}}{M_{W}^{2}} \ln \frac{M_{W}}{m_{c}} \lambda \frac{\Lambda_{\mathrm{CCD}}^{2}}{M_{W}^{2}}
$$

Matching (NLO +EW):
[Misiak, Urban; Buras, Buchalla;
Brod, MG, Stamou ${ }^{111]}$
Operator
$\mathrm{Q}_{\nu}=\left(\bar{s}_{\mathrm{L}} \gamma_{\mu} \mathrm{d}_{\mathrm{L}}\right)\left(\bar{v}_{\mathrm{L}} \gamma^{\mu} v_{\mathrm{L}}\right)$
Mixing (RGE)

## $\mathrm{K}^{+} \rightarrow \pi^{+} \bar{v} v$ charm contribution


$\mathrm{K}^{+} \rightarrow \pi^{+} \bar{v} v$ from $\mathrm{M}_{\mathrm{W}}$ to $\mathrm{m}_{\mathrm{C}}$
$\mathrm{P}_{\mathrm{c}}$ : charm quark contribution to $\mathrm{K}^{+} \rightarrow \pi^{+} \bar{v} v(30 \%$ to BR) Series converges very well (NNLO: $10 \% \rightarrow 2.5 \%$ uncertainty)

NNLO + EW $\begin{gathered}{[\text { Buras, MG, Haisch, }} \\ \text { Nierste; Brod MG] }\end{gathered}$



No GIM below the charm quark mass scale higher dimensional operators UV scale dependent One loop ChiPT calculation approximately cancels this scale dependence $\quad \delta P_{c, u}=0.04 \pm 0.02$
[Isidori, Mescia, Smith `05]
Explorative (unphysical) Lattice calculation: $\delta \mathrm{P}_{\mathrm{c}, \mathrm{u}}=0.0040( \pm 13)( \pm 32)(-45)$ [Bai et.al. ${ }^{\text {17 }}$ ]

## Expressions for $\mathrm{K} \rightarrow \pi \bar{v} v$

$$
\begin{aligned}
& \mathcal{B}\left(K^{+} \rightarrow \pi^{+} \nu \bar{\nu}\right)=\kappa_{+}\left(1+\Delta_{\mathrm{EM}}\right) \cdot {\left[\left(\frac{\operatorname{Im} \lambda_{t}}{\lambda^{5}} X\left(x_{t}\right)\right)^{2}\right.} \\
&\left.+\left(\frac{\operatorname{Re} \lambda_{c}}{\lambda} P_{c}(X)+\frac{\operatorname{Re} \lambda_{t}}{\lambda^{5}} X\left(x_{t}\right)\right)^{2}\right] \\
& \mathcal{B}\left(K_{L} \rightarrow \pi^{0} \nu \bar{\nu}\right)=\kappa_{L} \cdot\left(\frac{\operatorname{Im} \lambda_{t}}{\lambda^{5}} X\left(x_{t}\right)\right)^{2}
\end{aligned}
$$

New Physics without extra light degrees of freedom can be absorbed into

$$
X\left(x_{t}\right)->X\left(x_{t}\right)+X_{N P}
$$

## $\mathrm{K} \rightarrow \pi \bar{v} v:$ Error Budget

Updating old analysis [Brod et.al. `11] using input from PDG `17

| $\mathrm{B}_{\mathrm{L}} \cdot 10^{11}$ | Central: | 2.778 | $\mathrm{~B}_{+} \cdot 10^{11}$ | Central: | 8.401 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Error: | -0.386 | 0.418 | Error: | -0.831 | 0.882 |
| A | -0.34 | 0.374 | A | -0.723 | 0.781 |
| $\mathrm{X}_{\mathrm{t}}$ | -0.068 | 0.069 | $\delta \mathrm{P}_{\mathrm{cu}}$ | -0.245 | 0.249 |
| $\eta$ | -0.169 | 0.174 | $\rho$ | -0.225 | 0.216 |
| $\kappa_{1}$ | -0.016 | 0.002 | $\rho$ | -0.184 | 0.186 |
| $\lambda$ | -0.001 | 0.001 | $\mathrm{P}_{\mathrm{c}}$ | -0.143 | 0.144 |
|  |  |  | $\mathrm{X}_{\mathrm{t}}$ | $K_{+}$ | -0.041 |
|  |  |  | $\eta$ | -0.039 | 0.041 |
|  |  |  |  |  |  |
|  |  |  |  | -0.002 | 0.002 |

## $\mathrm{K} \rightarrow \pi \bar{v} v:$ Error Budget

Updating old analysis [Brod et.al. ${ }^{` 11]}$ using input from CKMfitter `16

| $\mathrm{B}_{\mathrm{L}} \cdot 10^{11}$ | Central: | 2.874 | $\mathrm{~B}_{+} \cdot 10^{11}$ | Central: | 8.345 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Error: | -0.195 | 0.161 | Error: | -0.468 | 0.409 |
| A | -0.152 | 0.1 | A | -0.308 | 0.202 |
| $\mathrm{X}_{\mathrm{t}}$ | -0.07 | 0.071 | $\delta \mathrm{P}_{\mathrm{cu}}$ | -0.243 | 0.247 |
| $\eta$ | -0.099 | 0.104 | $\rho$ | -0.093 | 0.089 |
| $\kappa_{1}$ | -0.017 | 0.002 | $\mathrm{P}_{\mathrm{c}}$ | -0.183 | 0.185 |
| $\lambda$ | 0. | 0. | $\mathrm{X}_{\mathrm{t}}$ | -0.142 | 0.143 |
|  |  |  | $\kappa_{+}$ | -0.04 | 0.04 |
|  |  |  | $\eta$ | -0.023 | 0.024 |
|  |  |  | $\lambda$ | -0.001 | 0.001 |

## Simple New Physics

 $B R\left(K_{L}\right) 10^{11}$Assume $X_{N P}=\left|X_{N P}\right| e^{i \delta}$
$B R\left(K_{L}\right) 10^{11}$


## CP violation in Kaons

CP violation in mixing, interference \& decay $\rightarrow$ non-zero

$$
\eta_{+-}=\frac{\left\langle\pi^{+} \pi^{-} \mid K_{L}^{0}\right\rangle}{\left\langle\pi^{+} \pi^{-} \mid K_{S}^{0}\right\rangle} \quad \eta_{00}=\frac{\left\langle\pi^{0} \pi^{0} \mid K_{L}^{0}\right\rangle}{\left\langle\pi^{0} \pi^{0} \mid K_{S}^{0}\right\rangle}
$$

Only CP violation in mixing ( $\operatorname{Re} \varepsilon$ ), interference of mixing and decay $\left(\operatorname{Im} \varepsilon, \operatorname{Im} \varepsilon^{\prime}\right)$ and direct CP violation $\left(\operatorname{Re} \varepsilon^{\prime}\right)$

$$
\epsilon_{K}=\left(\eta_{00}+2 \eta_{+-}\right) / 3 \quad \epsilon^{\prime}=\left(\eta_{+-}-\eta_{00}\right) / 3
$$

Using: $\quad \lambda_{i j}=\frac{q}{p} \frac{\left\langle\pi^{i} \pi^{j} \mid \bar{K}^{0}\right\rangle}{\left\langle\pi^{i} \pi^{j} \mid K^{0}\right\rangle} \quad$ and $\quad\left|1-\lambda_{i j}\right| \ll 1$

$$
\epsilon^{\prime} \approx \frac{1}{6}\left(\lambda_{00}-\lambda_{+-}\right)+\frac{1}{12}\left(\lambda_{00}-\lambda_{+-}\right)\left(2-\lambda_{00}-\lambda_{+-}\right)+\ldots
$$

## Formula for $\varepsilon^{\prime} / \varepsilon$

$\mathrm{a}_{0}, \mathrm{a}_{2} \& \mathrm{a}_{2}{ }^{+}$from experiment $\left\langle\pi^{0} \pi^{0} \mid K^{0}\right\rangle=a_{0} e^{i \chi_{0}}+a_{2} e^{i \chi_{2}} / \sqrt{2}$ [Cirigliano, et.al. `11] $\mathrm{a}_{0} \& \mathrm{a}_{2}$ : isospin amplitudes for isospin conservation

$$
\left\langle\pi^{+} \pi^{-} \mid K^{0}\right\rangle=a_{0} e^{i \chi_{0}}-a_{2} e^{i \chi_{2}} \sqrt{2}
$$

$$
\left\langle\pi^{+} \pi^{0} \mid K^{+}\right\rangle=3 a_{2}^{+} e^{i \chi_{2}^{+}} / 2
$$

Current theory gives us only: $A_{I}=\left\langle(\pi \pi)_{I}\right| \mathcal{H}_{\text {eff }}|K\rangle$
Normalise to $\mathrm{K}^{+}$decay $\left(\omega_{+}, \mathrm{a}\right)$ and $\varepsilon_{\mathrm{K}}$, expand in $\mathrm{A}_{2} / \mathrm{A}_{0}$ and CP violation:

$$
\operatorname{Re}\left(\frac{\epsilon^{\prime}}{\epsilon}\right) \simeq \frac{\epsilon^{\prime}}{\epsilon}=-\frac{\omega_{+}}{\sqrt{2}\left|\epsilon_{K}\right|}\left[\frac{\operatorname{Im} A_{0}}{\operatorname{Re} A_{0}}\left(1-\hat{\Omega}_{\mathrm{eff}}\right)-\frac{1}{a} \frac{\operatorname{Im} A_{2}}{\operatorname{Re} A_{2}}\right]
$$

[Buras, MG, Jäger, Jamin `15]
Adjusted to keep electroweak penguins in $\operatorname{Im} \mathrm{A}_{0}$ [Cirigliano, et.al. ${ }^{71]}$

## Current-Current \& CKM

Study Unitarity \& CKM Elements to get Im $\mathrm{A}_{\mathrm{I}} \& \operatorname{Re} \mathrm{~A}_{\mathrm{I}}$

We use unitarity to eliminate

$$
V_{c s}^{*} V_{c d}=-V_{u s}^{*} V_{u d}-V_{t s}^{*} V_{t d} Q_{2}^{c}
$$

Current-current interactions:
Two contributions if $\mu>\mathrm{m}_{\mathrm{c}}$.

$\left(\propto \mathrm{V}_{\mathrm{ts}}{ }^{*} \mathrm{~V}_{\mathrm{td}}\right.$ and $\left.\propto \mathrm{V}_{\mathrm{us}}{ }^{*} \mathrm{~V}_{\mathrm{ud}}\right) \quad V_{u s}^{*} V_{u d} Q_{1 / 2}^{u}+V_{c s}^{*} V_{c d} Q_{1 / 2}^{c} \rightarrow$

$$
V_{u s}^{*} V_{u d}\left(Q_{1 / 2}^{u}-Q_{1 / 2}^{c}\right)-V_{t s}^{*} V_{t d} Q_{1 / 2}^{c}
$$

For $\mu<\mathrm{m}_{\mathrm{c}}: \mathrm{V}_{\mathrm{ts}}{ }^{*} \mathrm{~V}_{\mathrm{td}}$ is absent: $\quad V_{u s}^{*} V_{u d} Q_{1 / 2}^{u}$

## Penguin \& CKM

Penguins: $f\left(m_{u}\right)-f\left(m_{c}\right)=0$ :
Only $\mathrm{V}_{\text {ts }}{ }^{*} \mathrm{~V}_{\text {td }}$ contribution

$\left\{V_{u s}^{*} V_{u d} f\left(m_{u}\right)+V_{c s}^{*} V_{c d} f\left(m_{c}\right)+V_{t s}^{*} V_{t d} f\left(m_{t}\right)\right\} Q_{\text {Penguin }} \rightarrow$ $\left\{V_{u s}^{*} V_{u d}\left[f\left(m_{u}\right)-f\left(m_{c}\right)\right]+V_{t s}^{*} V_{t d}\left[f\left(m_{t}\right)-f\left(m_{c}\right)\right]\right\} Q_{\text {Penguin }}$ $\mu>\mathrm{m}_{\mathrm{c}}: \mathrm{V}_{\mathrm{ts}}{ }^{*} \mathrm{~V}_{\mathrm{td}} \mathrm{Q}^{{ }^{1} / 2}$ mixes into $\mathrm{V}_{\mathrm{ts}}{ }^{*} \mathrm{~V}_{\mathrm{td}} \mathrm{Q}_{\text {Penguin }}($ like usual).
$\mu>\mathrm{m}_{\mathrm{c}}: \mathrm{V}_{\mathrm{us}}{ }^{*} \mathrm{~V}_{\mathrm{ud}}\left(\mathrm{Q}^{\mathrm{u}_{1 / 2}}-\mathrm{Q}^{\mathrm{c}} 1 / 2\right)$ does not mix into $Q_{\text {Penguin }}$.
$\mu<\mathrm{m}_{\mathrm{c}}$ : Match $\mathrm{V}_{\mathrm{ts}}{ }^{*} \mathrm{~V}_{\mathrm{td}} \mathrm{Q}^{\mathrm{c}_{1 / 2}}$ onto $\mathrm{V}_{\mathrm{ts}}{ }^{*} \mathrm{~V}_{\mathrm{td}} \mathrm{Q}_{\text {Penguin }}$
$\rightarrow \mathrm{CP}$ violation from $Q_{\text {Penguin }}$
$\rightarrow C P$ conserving from $Q^{u_{1 / 2}}$ (plus small $Q_{\text {Penguin }}$ )

## Effective Hamiltonian

Currently we use the effective Hamiltonian below the charm:

$$
\mathcal{H}_{\mathrm{eff}}=\frac{G_{F}}{\sqrt{2}} V_{u d} V_{u s}^{*} \sum_{i=1}^{10}\left(z_{i}(\mu)+\tau y_{i}(\mu)\right) Q_{i}(\mu), \quad \tau \equiv-\frac{V_{t d} V_{t s}^{*}}{V_{u d} V_{u s}^{*}}
$$

current-current
QCD \&

$$
\begin{aligned}
Q_{1,2 / \pm} & =\left(\bar{s}_{i} u_{j}\right)_{V-A}\left(\bar{u}_{k} d_{l}\right)_{V-A} \\
Q_{3, \ldots, 6} & =\left(\bar{s}_{i} d_{j}\right)_{V-A} \sum_{q=u, d, s}\left(\bar{q}_{k} q_{l}\right)_{V \pm A}
\end{aligned}
$$

electroweak
penguins

$$
Q_{7, \ldots, 10}=\left(\bar{s}_{i} d_{j}\right)_{V-A} \sum_{q=u, d, s} e_{q}\left(\bar{q}_{k} q_{l}\right)_{V \pm A}
$$

We have $z_{i} \& y_{i}$ at NLO [Buras et.al., Ciuchini et. al. '92 ${ }^{`} 93$ ]
And now also a Lattice QCD calculation of: $\left\langle(\pi \pi)_{\mathrm{I}}\right| \mathrm{Q}_{\mathrm{i}}|\mathrm{K}\rangle=\left\langle\mathrm{Q}_{\mathrm{i}}\right\rangle_{\mathrm{I}}$ by RBC-UKQCD [Blum et. al., Bai et. al. `15]

# $\operatorname{Im} A_{2} / \operatorname{Re} A_{2}-(V-A) x(V-A)$ 

$\mathrm{A}_{2}$ only contributes in the ratio $\operatorname{Im} \mathrm{A}_{2} / \operatorname{Re} \mathrm{A}_{2}$
Let us first consider only (V-A)x(V-A) operators:

$$
\begin{aligned}
Q_{1}=\left(\bar{s}_{\alpha} u_{\beta}\right)_{V-A}\left(\bar{u}_{\beta} d_{\alpha}\right)_{V-A} & Q_{2}=(\bar{s} u)_{V-A}(\bar{u} d)_{V-A} \\
Q_{9}=\frac{3}{2}(\bar{s} d)_{V-A} \sum_{q=u, d, s, c, b} e_{q}(\bar{q} q)_{V-A} & Q_{10}=\frac{3}{2}\left(\bar{s}_{\alpha} d_{\beta}\right)_{V-A} \sum_{q=u, d, s, c, b} e_{q}\left(\bar{q}_{\beta} q_{\alpha}\right)_{V-A}
\end{aligned}
$$

Isospin limit: $2<\mathrm{Q}_{9}>_{2}=2<\mathrm{Q}_{10}>_{2}=3<\mathrm{Q}_{1}>_{2}=3<\mathrm{Q}_{2}>_{2}$
$\operatorname{Re} \mathrm{A}_{2}:\left(\mathrm{z}_{1}+\mathrm{z}_{2}\right)<\mathrm{Q}_{1}+\mathrm{Q}_{2}>_{2}=\mathrm{z}_{+}<\mathrm{Q}_{+}>_{2} \quad \operatorname{Im} \mathrm{~A}_{2}: \mathrm{y}_{9}<\mathrm{Q}_{9}>_{2}+\mathrm{y}_{10}<\mathrm{Q}_{10}>_{2}$

$$
\left(\frac{\operatorname{Im} A_{2}}{\operatorname{Re} A_{2}}\right)_{V-A}=\operatorname{Im} \tau \frac{3\left(y_{9}+y_{10}\right)}{2 z_{+}}, \quad \tau=\frac{V_{t s}^{*} V_{t d}}{V_{u s}^{*} V_{u d}}
$$

# $\operatorname{Imm}_{0} / \operatorname{Re~}_{0}-(\mathrm{V}-\mathrm{A}) \times(\mathrm{V}-\mathrm{A})$ 

More operators contribute to $\operatorname{Im} \mathrm{A}_{0} / \operatorname{Re} \mathrm{A}_{0}$

$$
\operatorname{Re} A_{0}=\frac{G_{F}}{\sqrt{2}} V_{u d} V_{u s}^{*}\left(z_{+}\left\langle Q_{+}\right\rangle_{0}+z_{-}\left\langle Q_{-}\right\rangle_{0}\right)
$$

Fierz relations for (V-A)x(V-A) give, e.g.: $\left\langle\mathrm{Q}_{4}\right\rangle_{0}=\left\langle\mathrm{Q}_{3}\right\rangle_{0}+2\langle\mathrm{Q}-\rangle_{0}$

$$
\left(\frac{\operatorname{Im} A_{0}}{\operatorname{Re} A_{0}}\right)_{V-A}=\operatorname{Im} \tau \frac{2 y_{4}}{(1+q) z_{-}}+\mathcal{O}\left(p_{3}\right)
$$

Is only a function of Wilson coefficients and of the ratio

$$
q=\left(z_{+}(\mu)\left\langle Q_{+}(\mu)\right\rangle_{0}\right) /\left(z_{-}(\mu)\left\langle Q_{-}(\mu)\right\rangle_{0}\right)
$$

Expression with $\mathrm{p}_{3}=\left\langle\mathrm{Q}_{3}\right\rangle_{0} /\left\langle\mathrm{Q}_{4}\right\rangle_{0}$ and EW penguins given in [Buras, MG, Jäger \& Jamin `15]

# (V-A)x(V+A) Contributions 

$\mathrm{Q}_{6} \& \mathrm{Q}_{8}$ give the leading contribution to $\operatorname{Im} A_{0} \& \operatorname{ImA} A_{2}$ respectively

$$
\begin{aligned}
& \left(\frac{\operatorname{Im} A_{0}}{\operatorname{Re} A_{0}}\right)_{6}=-\frac{G_{F}}{\sqrt{2}} \operatorname{Im} \lambda_{t} y_{6} \frac{\left\langle Q_{6}\right\rangle_{0}}{\operatorname{Re} A_{0}} \\
& \left(\frac{\operatorname{Im} A_{2}}{\operatorname{Re} A_{2}}\right)_{8}=-\frac{G_{F}}{\sqrt{2}} \operatorname{Im} \lambda_{t} y_{8}^{\text {eff }} \frac{\left\langle Q_{8}\right\rangle_{2}}{\operatorname{Re} A_{2}}
\end{aligned}
$$

Here: Take Re $\mathrm{A}_{0}$ from data
One can re-express $<\mathrm{Q}_{6}>_{0} \&<\mathrm{Q}_{8}>_{2}$ in terms of $\mathrm{B}_{6} \& \mathrm{~B}_{8}$

## Prediction for $\varepsilon^{\prime} / \varepsilon$

$\mathrm{I}=2$ Similarly for $(\mathrm{V}-\mathrm{A}) \mathrm{x}(\mathrm{V}-\mathrm{A})$ :
$\frac{\varepsilon^{\prime}}{\varepsilon}=10^{-4}\left[\frac{\operatorname{Im} \lambda_{\mathrm{t}}}{1.4 \cdot 10^{-4}}\right]\left[a\left(1-\hat{\Omega}_{\mathrm{eff}}\right)\left(-4.1(8)+24.7 B_{6}^{(1 / 2)}\right)+1.2(1)-10.4 B_{8}^{(3 / 2)}\right]$
$(\mathrm{V}-\mathrm{A}) \mathrm{x}(\mathrm{V}+\mathrm{A})$ Matrix elements $\mathrm{B}_{6}=0.57(19)$ and $\mathrm{B}_{8}=0.76(5)$
from Lattice QCD [Blum et. al., Bai et. al. ${ }^{15}$ ]

$$
\begin{aligned}
& \left(\frac{\epsilon^{\prime}}{\epsilon}\right)_{\mathrm{SM}}=1.9(4.5) \times 10^{-4} \\
& \left(\frac{\epsilon^{\prime}}{\epsilon}\right)_{\exp }=16.9(2.3) \times 10^{-4}
\end{aligned}
$$

Similar findings by Kitahara et.al. 16

| quantity | error on $\varepsilon^{\prime} / \varepsilon$ |
| :---: | :---: |
| $B_{6}^{(1 / 2)}$ | 4.1 |
| NNLO | 1.6 |
| $\hat{\Omega}_{\text {eff }}$ | 0.7 |
| $p_{3}$ | 0.6 |
| $B_{8}^{(3 / 2)}$ | 0.5 |
| $p_{5}$ | 0.4 |
| $m_{s}\left(m_{c}\right)$ | 0.3 |
| $m_{t}\left(m_{t}\right)$ | 0.3 |

## NLO vs NNLO

Theory prediction only at NLO at the moment
Convergence at $\mathrm{m}_{\mathrm{c}}$ is not clear - should calculate next order

Long term use Lattice QCD
Also the error estimate does not include $\mathrm{O}\left(\mathrm{p}^{2} / \mathrm{m}_{\mathrm{c}}{ }^{2}\right)$ corrections which for $\mathrm{K} \rightarrow \pi \pi$ are expected to be small

## Status of $\varepsilon^{\prime} / \varepsilon$ NNLO

| Energy | Fields | Order |
| :---: | :---: | :--- |
| $\mu_{\mathrm{W}}$ | $\mathrm{g}, \gamma, \mathrm{W}, \mathrm{Z}, \mathrm{h}$, <br> $\mathrm{u}, \mathrm{d}, \mathrm{s}, \mathrm{c}, \mathrm{b}, \mathrm{t}$ | NNLO $\left.\mathrm{Q}_{1}-\mathrm{Q}_{6} \& \mathrm{Q}_{8 \mathrm{~g}} \mathrm{i}\right)$ <br> NNLO EW Penguins (traditional Basis) |
| $R$ | $\gamma, \mathrm{~g}, \mathrm{u}, \mathrm{d}, \mathrm{s}, \mathrm{c}, \mathrm{b}$ | NNLO $\mathrm{Q}_{1}-\mathrm{Q}_{6} \& \mathrm{Q}_{8 \mathrm{~g}}$ iii) |

## RG-invariant factorisation

Traditional the contribution of running $\left(U\left(\mu, \mu_{0}\right)\right)$ and matching $(M(\mu))$ are combined as:

$$
\begin{aligned}
\langle\vec{Q}\rangle^{(3)}\left(\mu_{L}\right) \vec{C}^{(3)}\left(\mu_{L}\right)= & \langle\vec{Q}\rangle\left(\mu_{L}\right) U^{(3)}\left(\mu_{L}, \mu_{c}\right) M^{(34)}\left(\mu_{c}\right) U^{(4)}\left(\mu_{c}, \mu_{b}\right) \\
& M^{(45)}\left(\mu_{b}\right) U^{(5)}\left(\mu_{b}, \mu_{W}\right) \vec{C}^{(5)}\left(\mu_{W}\right)
\end{aligned}
$$

Alternatively we can also factorise as

$$
\begin{aligned}
\langle\vec{Q}\rangle^{(3)}\left(\mu_{L}\right) \vec{C}^{(3)}(\mu)= & \langle\vec{Q}\rangle\left(\mu_{L}\right)^{(3)} u^{(3)}\left(\mu_{L}\right) \\
& u^{(3)^{-1}}\left(\mu_{c}\right) M^{(34)}\left(\mu_{c}\right) u^{(4)}\left(\mu_{c}\right) \\
& u^{(4)^{-1}}\left(\mu_{b}\right) M^{(45)}\left(\mu_{b}\right) u^{(5)}\left(\mu_{b}\right) \\
& u^{(5)^{-1}}\left(\mu_{W}\right) \vec{C}^{(5)}\left(\mu_{W}\right)
\end{aligned}
$$

or write in terms of scheme and scale independent quantities:

# RG-invariant factorisation 

All hatted quantities $\langle\hat{\vec{Q}}\rangle^{(3)}, \hat{M}^{(34)}, \hat{M}^{(45)}$ and $\hat{\vec{C}}^{(5)}$ and also their products

$$
\hat{\vec{C}}^{(3)}=\hat{M}^{(34)} \hat{M}^{(45)} \hat{\vec{C}}^{(5)}
$$

are formally scheme and scale independent.
The matrix elements $\langle\hat{\vec{Q}}\rangle$ satisfy $d=4$ Fierz identities.
$\hat{\vec{C}}^{(3)}$ is $\mu$ independent, but shows residual $\mu$ dependence.
Plot this for the $\hat{y}\left(\mu_{c}\right)$ (the ones $\propto \operatorname{Im}\left(V_{t s}^{*} V_{t d}\right)$ ): and for $\hat{z}\left(\mu_{c}\right)$ (relevant for $\operatorname{Re} \mathrm{A}_{0}$ and $\operatorname{Re} \mathrm{A}_{2}$ )
Use different RGE running (numerical or via $\Lambda_{\mathrm{MS}}$ ) from $\alpha_{s}\left(\mathrm{M}_{z}\right)$ at LO, NLO \& NNLO

The Real Part of $\mathrm{A}_{0} \& \mathrm{~A}_{2}$ is dominated by $\mathrm{z}_{+} \& \mathrm{z}_{-}$
$\operatorname{Re} A_{2}=\hat{z}_{+}\left\langle\hat{Q}_{+}\right\rangle_{2}$
$\operatorname{Re} A_{0}=\hat{z}_{+}\left\langle\hat{Q}_{+}\right\rangle_{0}+\hat{z}_{-}\left\langle\hat{Q}_{-}\right\rangle_{0}$

The residual $\mu_{\mathrm{c}}$ dependence reduces order by order

At NLO there is still a dependence on the implementation of $\alpha_{\mathrm{s}}$ Running.

Shift probably due to running down from $\mathrm{M}_{\mathrm{Z}}$


## Transform Lattice RISMOM

 matrix elements to $\hat{q}$ scheme$\operatorname{Re} A_{0}=33.2 \times 10^{-8} \mathrm{GeV}$
$\operatorname{Re} \mathrm{A}_{2}=1.48 \times 10^{-8} \mathrm{GeV}$

## Lattice input to $\operatorname{Re} \mathrm{A}_{0}$ has still

 $20 \%$ / $25 \%$ stat / sys. uncertainty



## QCD Penguin scale uncertainty is reduced from NLO to NNLO







Plot residual $\mu_{c}$ dependence of the QCD contribution to $\varepsilon^{\prime} / \varepsilon$ Uncertainty is significantly reduced by going to NNLO There are still improvements:
e.g. better $\alpha_{\mathrm{s}}$ implementation \& better incorporation of subleading corrections - will not change the overall picture

## Conclusion

Perturbative calculations for $\mathrm{K} \rightarrow \pi \bar{v} v$ under very good control, with only sub-leading non-perturbative effects.

Ongoing Lattice efforts improve the estimate of nonperturbative effects for $K \rightarrow \pi \bar{v} v$.

What is more interesting:
$\mathrm{K}^{+} \rightarrow \pi^{+} \bar{v} v$ at $3 \%$ or $\mathrm{K}_{\mathrm{L}} \rightarrow \pi^{0} \bar{v} v$
Perturbative NNLO calculation removes large part of the perturbative uncertainty in $\varepsilon^{\prime}$ к.

Interesting tension with experiment.

## CKM Unitarity (Model Independent)



