Flavourful Axion Phenomenology

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The Lamp of the East

"In the golden age of Asia
Korea was one of its lamp bearers,
And that lamp is waiting
To be lighted once again
For the illumination of the East."
- Rabindranath Tagore (1929)

Introduction

- Strong CP problem and PQ symmetry → "Axion"
- It may be identified with a flavour symmetry.

Wilczek, '82; … ; Ema, et.al.; Calibbi, et.al., '16

- Signatures and constraints on flavour violating axion couplings.

F. Bjoerkeroth, EJC, S. King, 1711.05741

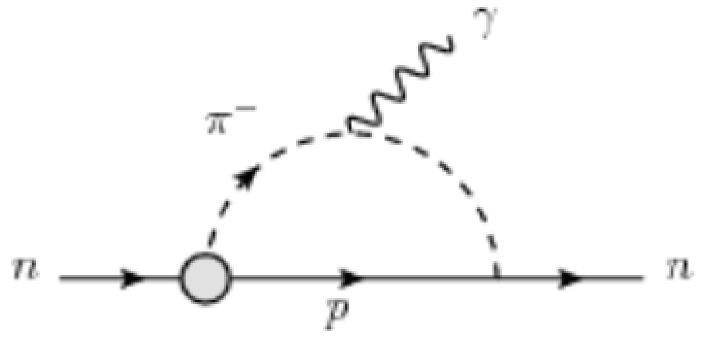
1806.00660

- An (approximate) PQ symmetry as an accidental symmetry in a Pati-Salam unification model with A-to-Z flavour group → correlated flavour observables.

Strong CP problem

- Gauge-invariance allows QCD θ term:

$$\mathcal{L}_\theta = \theta \frac{g_3^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{a\mu\nu}$$



- It is a CP-odd $E \cdot B$ term inducing nucleon EDM.

$$d_n \sim \frac{1}{4\pi^2} \frac{e}{m_N} g^\theta_{\pi NN} g_{\pi NN} \log\left(\frac{m_N}{m_\pi}\right) < 10^{-26} \text{ecm} \Rightarrow \theta < 10^{-10}$$

- Why is θ so small? → Symmetry origin: Peccei-Quinn '77.

PQ mechanism

- Introduce a QCD anomalous global U(1) symmetry which is spontaneously broken at a high scale F_a → axion.
- QCD anomaly induces an axion-gluon-gluon coupling:

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + (\theta + c_G\frac{a}{F_a})\frac{g_3^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^{a\mu\nu} \quad \Rightarrow \bar{\theta} \equiv \theta + c_G\frac{a}{F_a}$$

- QCD condensation generates axion potential driving the theta value to vanish: $V \sim \Lambda^4_{QCD}(1-\cos\bar{\theta}) \Rightarrow \langle\bar{\theta}\rangle = 0$

- Axion mass is predicted to be $m_a \sim \frac{\Lambda^2_{QCD}}{F_a} \sim 10^{-3}\left(\frac{10^{10}{\rm GeV}}{F_a}\right){\rm eV}$

Axion models

- **K**im-**S**hifman-**V**ainshtein-**Z**akharov:

$$\mathcal{L}_{KSVZ} = \lambda_Q S Q Q^c + h.c.$$

PQ charges: $1 \quad 0 \quad -1$

Axion is a Goldstone boson:

$$S = F_a e^{i\frac{a}{F_a}}$$

- **D**ine-**F**ischler-**S**rednicki-**Z**hitinitski:

$$\mathcal{L}_{DFSZ} = y_u q u^c H_u + y_d q d^c H_d + \lambda_H S H_u H_d + h.c.$$

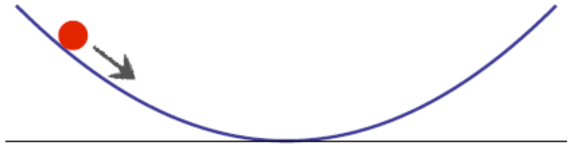
$0 \quad 1 \quad -1 \qquad 1 \quad 0 \quad -1$

- Flavourful PQ symmetry: each generation has different PQ charges

$$\sum_i X(q_i) + X(u_i^c) + X(d_i^c) \neq 0$$

Constraints on F_a

- Lower bound from star cooling: low F_a → efficient axion emission → fast star cooling.
- Upper bound from axion dark matter density: axion potential generated after Λ_{QCD} drives a coherent axion oscillation starting from a initial misaligned field value.



$$10^{10}\mathrm{GeV} \leq F_a \leq 10^{12}\mathrm{GeV}$$

Flavourful axion phenomenology

- PQ charges may be generation-dependent: QCD axion = phase field of flavons

Feng, et.al., '98

$$\phi_i = \frac{v_i}{\sqrt{2}}\, e^{i\frac{x_i\, a}{v_{PQ}}} \qquad a = \sum_i x_i \frac{v_i a_i}{v_{PQ}};\ v_{PQ}^2 = \sum_i x_i^2 v_i^2$$

- Fermion Yukawa couplings take a general form:

$$-\mathcal{L}_{\rm Yuk} = e^{i\frac{a}{v_{PQ}}\left(x_{f_{Li}} - x_{f_{Rj}}\right)} M_{ij}^f\, \overline{f_{Li}} f_{Rj}$$

General flavourful axion couplings

- Take the transformation $f_{L/Ri} \to e^{i\frac{a}{v_{PQ}} x_{f_{L/Ri}}} f_{L/Ri}$ and then go to the mass basis: $f_{L/R} \to U_{fL/R}\, f_{L/R},\; M^f \to U^\dagger_{fL} M^f U_{fR} = m^f$

$$-\mathcal{L} \;\; = \frac{\partial_\mu a}{v_{PQ}} \bar{f}_i \gamma^\mu \left(V^f_{ij} - A^f_{ij} \gamma_5 \right) f_j + \frac{a}{f_a} \left(\frac{\alpha_s}{8\pi} G\tilde{G} + c_{a\gamma} \frac{\alpha}{8\pi} F\tilde{F} \right) + m^f_i \overline{f_{Li}} f_{Rj}$$

$$V^f = \frac{1}{2} \left(U^\dagger_{fL} x_{f_L} U_{fL} + U^\dagger_{fR} x_{f_R} U_{fR} \right)$$

$$A^f = \frac{1}{2} \left(U^\dagger_{fL} x_{f_L} U_{fL} - U^\dagger_{fR} x_{f_R} U_{fR} \right)$$

$f_a = v_{PQ} / N_{DW}$ (N_{DW} = QCD anomaly)

Lepton decays to axion

- LFV decays $l_i \to l_j a$ with the couplings (V^e_{ij}, A^e_{ij}):

$$B(l_i \to l_j a) \equiv \tilde{c}_{l_i \to l_j} \left|C^e_{ij}\right|^2 \left(\frac{10^{12} GeV}{v_{PQ}}\right)^2$$

$$\tilde{c}_{l_i \to l_j} \approx \frac{1}{16\pi\Gamma(l_i)} \frac{m^3_{l_i}}{(10^{12} GeV)^2} \qquad \left|C^e_{ij}\right|^2 = \left|V^e_{ij}\right|^2 + \left|A^e_{ij}\right|^2$$

- Angular distribution:

$$\frac{d\Gamma}{d\cos\theta} = \frac{\left|C^e_{ij}\right|^2}{32\pi} \frac{m^3_{l_i}}{v^2_{PQ}} \left(1 - A\, P_{l_i} \cos\theta\right) \qquad A = \frac{2\Re\left(A^e_{ij} V^{e\,*}_{ij}\right)}{\left|C^e_{ij}\right|^2} = \begin{cases} A = 0 \ \ (\text{isotropy}) \\ A = -1 \ \ (V - A\text{: SM}) \\ A = +1 \ \ (V + A\text{: our model}) \end{cases}$$

Decay	Branching ratio	Experiment	$\tilde{c}_{\ell_1 \to \ell_2}$	v_{PQ}/GeV
$\mu^+ \to e^+ a$	$< 2.6 \times 10^{-6}$	$(A = 0)$ Jodidio *et al* [86]	7.82×10^{-11}	$> 5.5 \times 10^9 \lvert V^e_{21} \rvert$
	$< 2.1 \times 10^{-5}$	$(A = 0)$ TWIST [87]		$> 1.9 \times 10^9 \lvert C^e_{21} \rvert$
	$< 1.0 \times 10^{-5}$	$(A = 1)$ TWIST [87]		$> 2.8 \times 10^9 \lvert C^e_{21} \rvert$
	$< 5.8 \times 10^{-5}$	$(A = -1)$ TWIST [87]		$> 1.2 \times 10^9 \lvert C^e_{21} \rvert$
	$\lesssim 5 \times 10^{-9}*$	Mu3e (future) [88]		$\gtrsim 1 \times 10^{11} \lvert C^e_{21} \rvert$
$\tau^+ \to e^+ a$	$< 1.5 \times 10^{-2}$	ARGUS [89]	4.92×10^{-14}	$> 1.8 \times 10^6 \lvert C^e_{31} \rvert$
$\tau^+ \to \mu^+ a$	$< 2.6 \times 10^{-2}$	ARGUS [89]	4.87×10^{-14}	$> 1.4 \times 10^6 \lvert C^e_{32} \rvert$

Radiative LFV decay: $l_1 \to l_2 a\gamma$

- Cristal Box: $Br(\mu \to e a\gamma) < 1.1 \times 10^{-9} \Rightarrow v_{PQ} > 9.4 \times 10^{8} |C^{e}_{21}|\ GeV$
  $[Br(\mu \to e\gamma) < 4.9 \times 10^{-11}]$

$$\frac{d^2\Gamma}{dx\, dc_\theta} = \frac{\alpha \left|C^{e}_{\ell_1\ell_2}\right|^2 m^3_{\ell_1}}{32\pi^2 v^2_{PQ}} f(x, c_\theta), \quad f(x, c_\theta) = \frac{1 - x(1 - c_\theta) + x^2}{(1-x)(1-c_\theta)}. \qquad x = 2E_{\ell_2}/m_{\ell_1}, \quad c_\theta \equiv \cos\theta_{2\gamma}$$

- Future search?

Decay	Branching ratio	Experiment
$\mu^+ \to e^+\gamma$	$< 4.2 \times 10^{-13}$	MEG [92]
	$\lesssim 6 \times 10^{-14}*$	MEG-II (future) ?? [93]
$\tau^- \to e^-\gamma$	$< 3.3 \times 10^{-8}$	BaBar [95]
$\tau^- \to \mu^-\gamma$	$< 4.4 \times 10^{-8}$	BaBar [95]

Meson decay to axion

- Meson decays $P \to P' a$ from flavourful vector couplings:

$$B\big(P(q_i) \to P'(q_j)\, a\big) = \frac{1}{16\pi\Gamma(P)} \left|V_{ij}^q\right|^2 |f_+(0)|^2 \frac{m_P^3}{v_{PQ}^2} \left(1 - \frac{m_{P'}^2}{m_P^2}\right)^3$$

$$\equiv \tilde{c}_{P\to P'} \left|V_{ij}^q\right|^2 \left(\frac{10^{12} GeV}{v_{PQ}}\right)^2$$

Decay	Branching ratio	Experiment	$\tilde{c}_{P\to P'}$	v_{PQ}/GeV
$K^+ \to \pi^+ a$	$< \mathbf{0.73 \times 10^{-10}}$	E949 + E787 [59]	3.51×10^{-11}	$> 6.9 \times 10^{11} \lvert V_{21}^d \rvert$
	$< 0.01 \times 10^{-10}*$	NA62 (future) [62]		$> 5.9 \times 10^{12} \lvert V_{21}^d \rvert$
	$< 1.2 \times 10^{-10}$	E949 + E787 [58]		
	$< 0.59 \times 10^{-10}$	E787 [73]		
$K_L^0 \to \pi^0 a$	$< \mathbf{5 \times 10^{-8}}$	KOTO [68]	3.67×10^{-11}	$> 2.7 \times 10^{10} \lvert V_{21}^d \rvert$
$(K_L^0 \to \pi^0 \nu\bar{\nu})$	$(< 2.6 \times 10^{-8})$	E391a [66]		
$B^\pm \to \pi^\pm a$	$< \mathbf{4.9 \times 10^{-5}}$	CLEO [71]	5.30×10^{-13}	$> 1.0 \times 10^{8} \lvert V_{31}^d \rvert$
$(B^\pm \to \pi^\pm \nu\bar{\nu})$	$(< 1.0 \times 10^{-4})$	BaBar [74]		
	$(< 1.4 \times 10^{-4})$	Belle [75]		
$B^\pm \to K^\pm a$	$< \mathbf{4.9 \times 10^{-5}}$	CLEO [71]	7.26×10^{-13}	$> 1.2 \times 10^{8} \lvert V_{32}^d \rvert$
$(B^\pm \to K^\pm \nu\bar{\nu})$	$(< 1.3 \times 10^{-5})$	BaBar [76]		
	$(< 1.9 \times 10^{-5})$	Belle [75]		
	$(< 1.5 \times 10^{-6})*$	Belle-II (future) [77]		
$B^0 \to \pi^0 a$			4.92×10^{-13}	
$(B^0 \to \pi^0 \nu\bar{\nu})$	$(< 0.9 \times 10^{-5})$	Belle [75]		$\gtrsim 2.3 \times 10^{8} \lvert V_{31}^d \rvert$
$B^0 \to K^0_{(S)} a$	$< \mathbf{5.3 \times 10^{-5}}$	CLEO [71]	6.74×10^{-13}	$> 1.1 \times 10^{8} \lvert V_{32}^d \rvert$
$(B^0 \to K^0 \nu\bar{\nu})$	$(< 1.3 \times 10^{-5})$	Belle [75]		
$D^\pm \to \pi^\pm a$	< 1		1.11×10^{-13}	$> 3.3 \times 10^{5} \lvert V_{21}^u \rvert$
$D^0 \to \pi^0 a$	< 1		4.33×10^{-14}	$> 2.1 \times 10^{5} \lvert V_{21}^u \rvert$
$D_s^\pm \to K^\pm a$	< 1		4.38×10^{-14}	$> 2.1 \times 10^{5} \lvert V_{21}^u \rvert$
$B_s^0 \to \overline{K}^0 a$	< 1		3.64×10^{-13}	$> 6.0 \times 10^{5} \lvert V_{31}^d \rvert$

Axion-meson mixing

- After QCD condensation,

$$\frac{c_{ij}}{v_{PQ}}\partial_\mu a\,\bar{f}_i\gamma^\mu\gamma_5 f_j \;\Rightarrow\; c_P\frac{f_P}{f_a}\partial_\mu a\partial^\mu P$$

- Kinetic mixing diagonalization & mass re-diagonalizaton:

$$a \to a + c_P\frac{f_P}{f_a}\frac{m_P^2}{m_P^2-m_a^2}P\,, \qquad P \to P - c_P\frac{f_P}{f_a}\frac{m_a^2}{m_P^2-m_a^2}a$$

mass-dependent mixing

Axion-pion mixing

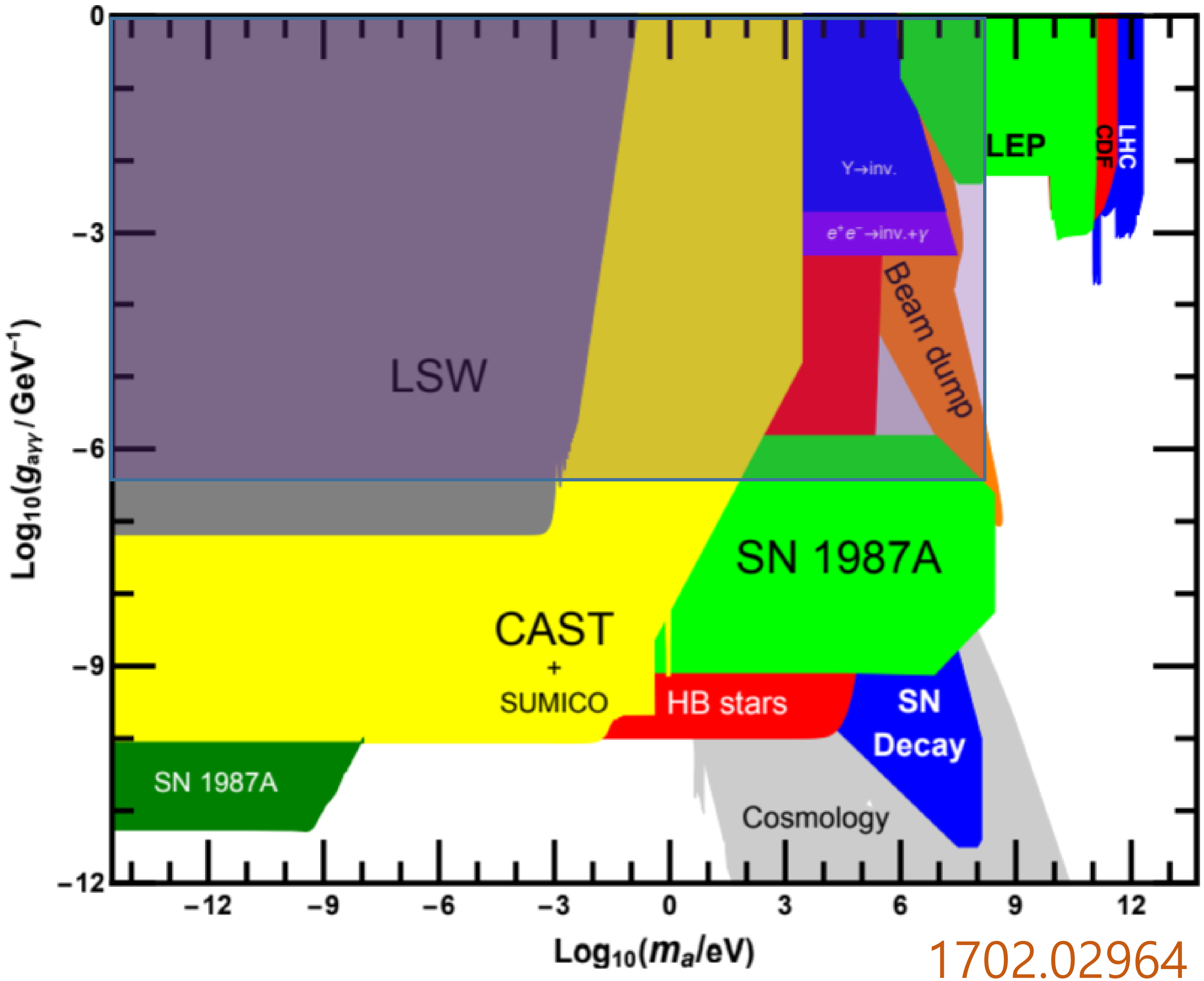
- For QCD axion, the mixing is negligible ($f_P \ll f_a, m_a \ll m_\pi$).
- For ALP, it can lead to sizable contribution to $K^+ \to \pi^+ a$ & $a \to \gamma\gamma$ induced from $K^+ \to \pi^+\pi^0$ & $\pi^0 \to \gamma\gamma$:

$$\Gamma(K^+ \to \pi^+ a) \approx \left(c_\pi \frac{f_\pi}{f_a} \frac{m_a^2}{m_\pi^2 - m_a^2}\right)^2 \Gamma(K^+ \to \pi^+\pi^0)$$

$$\Gamma(a \to \gamma\gamma) \approx \left(c_\pi \frac{f_\pi}{f_a} \frac{m_a^2}{m_\pi^2 - m_a^2}\right)^2 \left(\frac{m_a}{m_\pi}\right)^3 \Gamma(\pi^0 \to \gamma\gamma) \Rightarrow (g_{a\gamma})_{mix} = \frac{\alpha}{\pi} \frac{c_\pi m_a^2}{m_\pi^2 - m_a^2} \frac{1}{f_a}$$

- $B(K^+ \to \pi^+ a) < 10^{-10}$ puts a limit:

$$f_a > 4 \left(\frac{c_\pi m_a^2}{m_\pi^2 - m_a^2}\right) TeV \Rightarrow (g_{a\gamma})_{mix} < 5.8 \times 10^{-7}\ GeV^{-1} \text{ for } m_a < 110 MeV$$


LSW
CAST
+
SUMICO
SN 1987A
HB stars
SN
Decay
Cosmology
SN 1987A
Beam dump
Y→inv.
e⁺e⁻→inv.+γ
LEP
CDF
LHC
Log10(gaγγ/GeV⁻¹)
Log10(ma/eV)
0
−3
−6
−9
−12
−12
−9
−6
−3
0
3
6
9
12

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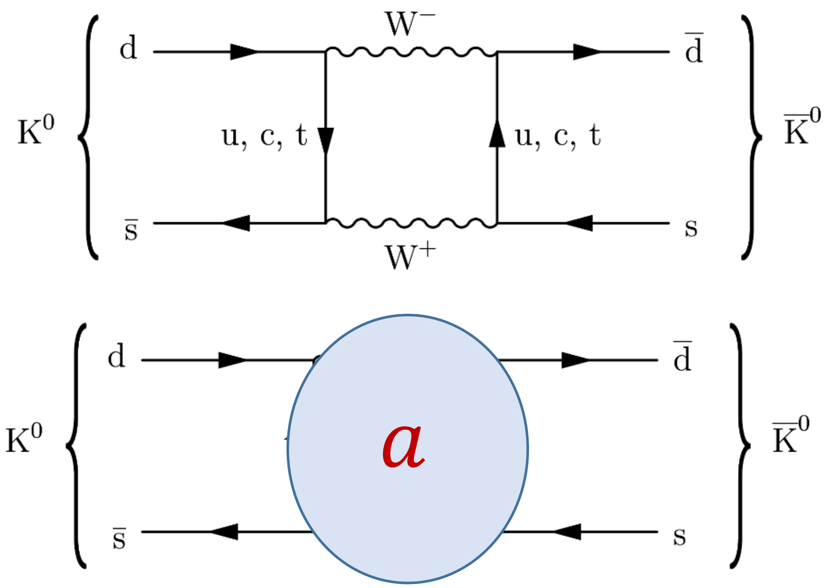
Impact on neutral meson mass splitting

- Flavourful axion couplings ($A^{u,d}_{12,23}$) induces mixing with heavy mesons (K, D, B) contributing to their mass splitting:



$$\Delta m_P \approx |\eta_P|^2 m_P = |c_P|^2 \frac{f_P^2}{v_{PQ}^2} m_P$$

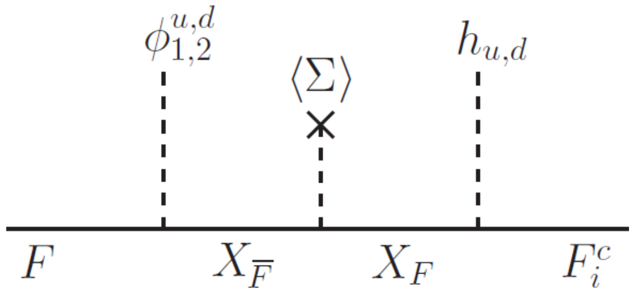
System	$(\Delta m_P)_{\rm exp}/\rm MeV$	$v_{PQ}/\rm GeV$
$K^0 - \overline{K}^0$	$(3.484 \pm 0.006) \times 10^{-12}$	$\gtrsim 2 \times 10^6 \lvert c_{K^0}\rvert$
$D^0 - \overline{D}^0$	$(6.25\,^{+2.70}_{-2.90}) \times 10^{-12}$	$\gtrsim 4 \times 10^6 \lvert c_{D^0}\rvert$
$B^0 - \overline{B}^0$	$(3.333 \pm 0.013) \times 10^{-10}$	$\gtrsim 8 \times 10^5 \lvert c_{B^0}\rvert$
$B_s^0 - \overline{B}_s^0$	$(1.1688 \pm 0.0014) \times 10^{-8}$	$\gtrsim 1 \times 10^5 \lvert c_{B_s^0}\rvert$

A-to-Z flavour Pati-Salam Model

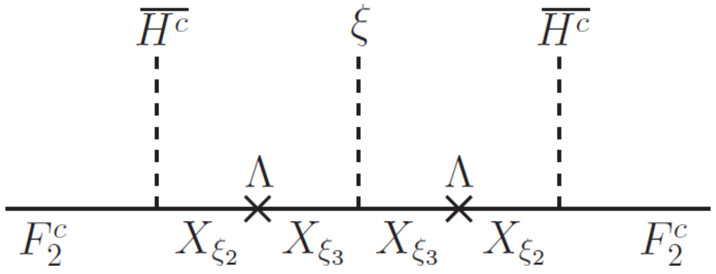
King, '14

- A Pati-Salam unification model where an approximate PQ symmetry, arises accidentally from discrete flavour symmetry $A_4 \times Z_5 \times Z_3 \times Z_5'$:

$$W_Y^{\rm eff} = \lambda_3 (F \cdot h_3) F_3^c + \lambda_{1u} \frac{(F \cdot \phi_1^u) h_u F_1^c}{\langle \Sigma_u \rangle} + \lambda_{2u} \frac{(F \cdot \phi_2^u) h_u F_2^c}{\langle \Sigma_u \rangle} + \lambda_{1d} \frac{(F \cdot \phi_1^d) h_d F_1^c}{\langle \Sigma_{15}^d \rangle} + \lambda_{2d} \frac{(F \cdot \phi_2^d) h_{15}^d F_2^c}{\langle \Sigma_d \rangle} + \lambda_{ud} \frac{(F \cdot \phi_1^u) h_d F_1^c}{\langle \Sigma_d \rangle}$$



$$W_{\rm Maj}^{\rm eff} = \frac{\overline{H^c}\,\overline{H^c}}{\Lambda} \left(\frac{\xi^2}{\Lambda^2} F_1^c F_1^c + \frac{\xi}{\Lambda} F_2^c F_2^c + F_3^c F_3^c + \frac{\xi}{\Lambda} F_1^c F_3^c \right)$$



Protection up to D=10

	Field	G_{PS}	A_4	$\mathbb{Z}_5$	$\mathbb{Z}_3$	$\mathbb{Z}'_5$	R	$U(1)_{PQ}$
Fermions	F	$(4,2,1)$	3	1	1	1	1	0
	$F^c_{1,2,3}$	$(\bar{4},1,2)$	1	$\alpha,\alpha^3,1$	$\beta,\beta^2,1$	$\gamma^3,\gamma^4,1$	1	$-2,-1,0$
	$\overline{H^c}$	$(4,1,2)$	1	1	1	1	0	0
	H^c	$(\bar{4},1,2)$	1	1	1	1	0	0
Flavons	$\phi^u_{1,2}$	$(1,1,1)$	3	α^4,α^2	β^2,β	γ^2,γ	0	$2,1$
	$\phi^d_{1,2}$	$(1,1,1)$	3	α^3,α	β^2,β	γ^2,γ	0	$2,1$
Higgses	h_3	$(1,2,2)$	3	1	1	1	0	0
	h_u	$(1,2,2)$	$1''$	α	1	1	0	0
	h^u_{15}	$(15,2,2)$	1	α	1	1	0	0
	h_d	$(1,2,2)$	$1'$	α^3	1	1	0	0
	h^d_{15}	$(15,2,2)$	$1'$	α^4	1	1	0	0
	Σ_u	$(1,1,1)$	$1''$	α	1	1	0	0
	Σ_d	$(1,1,1)$	$1'$	α^3	1	1	0	0
	Σ^d_{15}	$(15,1,1)$	$1'$	α^2	1	1	0	0
	ξ	$(1,1,1)$	1	α^4	β^2	γ^2	0	2
Messengers	$X_{F''_{1,3}}$	$(4,2,1)$	$1''$	α,α^3	β^2,β	γ^2,γ	1	$2,1$
	$X_{F'_{1,3}}$	$(4,2,1)$	$1'$	α,α^3	β,β^2	γ,γ^2	1	$1,2$
	$X_{\overline{F_i}}$	$(\bar{4},2,1)$	1	α^i	$\beta,\beta,\beta^2,\beta^2$	$\gamma^3,\gamma^3,\gamma^4,\gamma^4$	1	$-2,-2,-1,-1$
	X_{ξ_i}	$(1,1,1)$	1	α^i	$\beta,\beta,\beta^2,\beta^2,1$	$\gamma^3,\gamma,\gamma^4,\gamma^2,1$	1	$-2,1,-1,2,0$
	$\bar{\phi}^u_{1,2}$	$(1,1,1)$	3	α,α^3	β,β^2	γ^3,γ^4	0	$-2,-1$
	$\bar{\phi}^d_{1,2}$	$(1,1,1)$	3	α^2,α^4	β,β^2	γ^3,γ^4	0	$-2,-1$
	$\bar{\xi}$	$(1,1,1)$	1	α	β	γ^3	0	-2

$x_{f_L} = (0,0,0)$
$x_{f_R} = (2,1,0)$
$N_{DW} = 6$

"Family-dependent DFSZ"

Axion-dependent Yukawa

- Flavon fields $\phi_{1,2}^{u,d}$ and right-handed fermions $F_{1,2,3}^{c}$ are charged under the PQ symmetry:

$$-\mathcal{L}_Y = \lambda_3(\bar{f}\cdot\langle h_3\rangle^*)f_{R3} + \frac{\lambda_{1u}v_u}{\sqrt{2}v_{\Sigma_u}}(\bar{u}_L\cdot\langle\varphi_1^u\rangle^*)u_{R1}\exp\left[\frac{-ix_{\varphi_1^u}a}{v_{PQ}}\right]$$

$$+\frac{\lambda_{2u}v_u}{\sqrt{2}v_{\Sigma_u}}(\bar{u}_L\cdot\langle\varphi_2^u\rangle^*)u_{R2}\exp\left[\frac{-ix_{\varphi_2^u}a}{v_{PQ}}\right] + \frac{\lambda_{1d}v_d}{\sqrt{2}v_{\Sigma_d}}(\bar{d}_L\cdot\langle\varphi_1^d\rangle^*)d_{R1}\exp\left[\frac{-ix_{\varphi_1^d}a}{v_{PQ}}\right]$$

$$+\frac{\lambda_{2d}v_d}{\sqrt{2}v_{\Sigma_d}}(\bar{d}_L\cdot\langle\varphi_2^d\rangle^*)d_{R2}\exp\left[\frac{-ix_{\varphi_2^d}a}{v_{PQ}}\right] + \frac{\lambda_{ud}v_d}{\sqrt{2}v_{\Sigma_d}}(\bar{d}_L\cdot\langle\varphi_1^u\rangle^*)d_{R1}\exp\left[\frac{-ix_{\varphi_1^u}a}{v_{PQ}}\right]$$

$$+\left\{d_L\to e_L, d_R\to e_R, \lambda_{1d}\to\tilde{\lambda}_{1d}, \lambda_{2d}\to\tilde{\lambda}_{2d}, \lambda_{ud}\to\tilde{\lambda}_{ud}\right\} + \text{h.c.}.$$

$x_{\varphi_1^{u,d}} = 2$

$x_{\varphi_2^{u,d}} = 1$

Fitting fermion masses and mixing

- Fermion mass matrices from 15 input parameters:

$$M^u = v_u \begin{pmatrix} 0 & b & \epsilon_{13} c \\ a & 4b & \epsilon_{23} c \\ a & 2b & c \end{pmatrix}, \quad M^d = v_d \begin{pmatrix} y_d^0 & 0 & 0 \\ B y_d^0 & y_s^0 & 0 \\ B y_d^0 & 0 & y_b^0 \end{pmatrix}, \quad M^e = v_d \begin{pmatrix} -(y_d^0/3) & 0 & 0 \\ B y_d^0 & x y_s^0 & 0 \\ B y_d^0 & 0 & y_b^0 \end{pmatrix}$$

$$m^\nu = m_a \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} + m_b e^{i\eta} \begin{pmatrix} 1 & 4 & 2 \\ 4 & 16 & 8 \\ 2 & 8 & 4 \end{pmatrix} + m_c e^{i\xi} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

→ Specific flavour prediction for axion

- Best-fit:

Parameter	Value
$a\,/10^{-5}$	$1.246\, e^{4.047i}$
$b\,/10^{-3}$	$3.438\, e^{2.080i}$
c	-0.545
$y_d^0\,/10^{-5}$	$3.053\, e^{4.816i}$
$y_s^0\,/10^{-4}$	$3.560\, e^{2.097i}$
$y_b^0\,/10^{-2}$	3.607

Parameter	Value
$\epsilon_{13}\,/10^{-3}$	$6.215\, e^{2.434i}$
$\epsilon_{23}\,/10^{-2}$	$2.888\, e^{3.867i}$
B	$10.20\, e^{2.777i}$
x	5.880

Parameter	Value
m_a /meV	3.646
m_b /meV	1.935
m_c /meV	1.151
η	2.592
ξ	2.039

Our model Predictions

- Best-fit values for axion-fermion couplings:

$$V^u = -A^u \simeq \begin{pmatrix} 1.0 & 4.3\times10^{-3}e^{-0.05i} & -1.7\times10^{-5}e^{-0.015i} \\ 4.3\times10^{-3}e^{0.05i} & -0.5 & -6.0\times10^{-4} \\ -1.7\times10^{-5}e^{0.015i} & -6.0\times10^{-4} & 7.3\times10^{-7} \end{pmatrix}$$

$$V^d = -A^d \simeq \begin{pmatrix} 0.78 & 0.25 & -0.0065 \\ 0.25 & 0.72 & -0.0057 \\ -0.0065 & -0.0057 & 7.5\times10^{-5} \end{pmatrix}, \qquad V^e = -A^e \simeq \begin{pmatrix} 0.99 & 0.073 & -0.0085 \\ 0.073 & 0.51 & -0.0013 \\ -0.0085 & -0.0013 & 7.5\times10^{-5} \end{pmatrix}$$

- Rare decays to axion:

Process	Branching ratio ($v_{PQ} = 10^{12}$ GeV)	Experimental sensitivity
$K^+ \to \pi^+ a$	2.19×10^{-12}	$\lesssim 1\times10^{-12}$ (NA62 future)
$K_L^0 \to \pi^0 a$	2.29×10^{-12}	$< 5\times10^{-8}$ (KOTO)
$\mu^+ \to e^+ a$	8.3×10^{-13}	$\lesssim 5\times10^{-9}$ (Mu3e future)

- Correlated probes:

$$R_{\mu/K} \equiv \frac{\mathrm{Br}(\mu^+ \to e^+ a)}{\mathrm{Br}(K^+ \to \pi^+ a)} \simeq 4.45\,\frac{|V_{21}^e|^2}{|V_{21}^d|^2} \approx 31\, e^{-1.8\sqrt{x}} \approx 0.38.$$

Conclusion

- Axion solves the strong CP and dark matter problem.
- A specific Pati-Salam unified model is worked out to show an automatic PQ symmetry guaranteed by a flavour symmetry.
- It leads to interesting flavourful axion phenomenology.
- Rare Kaon and muon decays, $K^+ \to \pi^+ a$ & $\mu \to e\, a$, provide sensitive probes for QCD axion.
- More numerous flavour observables can be looked for axion-like particles.