Warm pions in real time

Sourendu Gupta, Rishi Sharma

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Context

- Lattice constraints on real-time phenomena at finite temperature need some knowledge of spectral densities.
- Knowledge-free methods of using spectral densities have been explored (MEM, etc) with spectacular success in some cases.
- Theoretical constraints on spectral densities largely utilize weak-coupling methods, and have increased our understanding substantially.
- Most studies to date have concentrated on high-temperature properties, namely $T > T_c$. However, the fireball produced in heavy ion collisions spend more than half its life time in the hadronic phase. So understanding the warm hadronic plasma ($T_c > T > 0$) is also important.
- This study utilizes an Effective Field Theory (EFT) approach to address the analytic continuation of lattice data in the hadronic phase.
The EFT approach

EFTs are useful for describing physics where there are well-separated length scales. Since (approximate) chiral symmetry allows us to separate the scales of $T$ and $m_\pi$, EFTs may be expected to be useful for understanding pions and the transport of chiral charge.

The UV cutoff of such a EFT may be expected to be of the order of $T$. (Notice that $1/a > T$). Organize in powers of $q/T$; $q$ is momentum scale of correlators.

The Lagrangian of the EFT is constrained only by the symmetries of the problem; i.e., all terms allowed by symmetry must be used. We use chiral symmetry, spatial rotations and CPT. No boosts: Boltzmann weight is $\exp(-E/T)$, forces choice of frame.
Fermion mass and kinetic terms

Action for quarks

\[ \mathcal{L}_4 = d^3 T_0 \bar{\psi} \psi + \bar{\psi} \partial_4 \psi + d^4 \bar{\psi} \nabla \psi \]

1. Dimensions provided by the UV cutoff of the EFT; here chosen to be \( T_0 \approx T \).

2. Note that \( d^i \) are dimensionless. The combination \( T_0 d^3 \) is the quark mass \( m_0 \) (only chiral symmetry breaking term in the action).

3. At \( T = 0 \) full Lorentz symmetry would force \( d^4 = 1 \). When \( d^4 \neq 1 \) then there is a difference between the mass \( m_0 \) (pole mass) and the mass that can be deduced from the decay of static correlation functions (screening mass).
Extending the NJL model

\[ \mathcal{L}_6 = \frac{d^{61}}{T_0^2} \left[ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau^a\psi)^2 \right] + \frac{d^{62}}{T_0^2} \left[ (\bar{\psi}\tau^a\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 \right] 
+ \frac{d^{63}}{T_0^2} (\bar{\psi}\gamma_4\psi)^2 + \frac{d^{64}}{T_0^2} (\bar{\psi}i\gamma_i\psi)^2 + \frac{d^{65}}{T_0^2} (\bar{\psi}\gamma_5\gamma_4\psi)^2 
+ \frac{d^{66}}{T_0^2} (\bar{\psi}i\gamma_5\gamma_i\psi)^2 + \frac{d^{67}}{T_0^2} \left[ (\bar{\psi}\gamma_4\tau^a\psi)^2 + (\bar{\psi}\gamma_5\gamma_4\tau^a\psi)^2 \right] 
+ \frac{d^{68}}{T_0^2} \left[ (\bar{\psi}i\gamma_i\tau^a\psi)^2 + (\bar{\psi}i\gamma_5\gamma_i\tau^a\psi)^2 \right] 
+ \frac{d^{69}}{T_0^2} \left[ (\bar{\psi}iS_{i4}\psi)^2 + (\bar{\psi}S_{ij}\tau^a\psi)^2 \right] 
+ \frac{d^{60}}{T_0^2} \left[ (\bar{\psi}iS_{i4}\tau^a\psi)^2 + (\bar{\psi}S_{ij}\psi)^2 \right] \]

10 couplings \( d^{6i} \), collapse to one (\( \lambda \)) in mean field theory.
Pion correlators

Expected form of mean field theory, fixes $\lambda$. Introduce small fluctuations about the mean field (pions) using a Hubbard-Stratanovich trick

$$\psi \rightarrow \exp \left[ \frac{i \pi^a \tau^a \gamma^5}{2f} \right] \psi.$$ 

Introduce this into the fermion action, expand to second order in $\pi$.

Gives coupled pion-quark action; overcounts degrees of freedom. Integrate out the quarks. Propagator is equivalent to effective pion action

$$L_\pi = \frac{1}{2} c^2 T_0^2 \pi^2 + \frac{1}{2} \left( \partial_4 \pi \right)^2 + \frac{1}{2} c^4 (\nabla \pi)^2 + \cdots$$

Match static correlation functions (on torus) to lattice measurements to fix parameters $c^2$ and $c^4$, and through them the quark couplings $d^3$ and $d^4$. \textit{SG and Sharma, 1710.05345}
Euclidean predictions

Numerical approximations in prediction: $\lambda$ fitted in the chiral limit; $T_0$ without power corrections; $d^3$, $d^4$ and $\lambda$ assumed to be independent of $T$. Lattice measurements taken from Brandt et al, 1406.5602
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Analytic continuation to real time

Analytic continuation of the pion theory is straightforward: $c^2$ and $c^4$ will then have the values that they have in Euclidean. However, this captures only the long-distance part of correlators. Will shorter distance physics have no bearing on the analytic continuation?

Check by analytic continuation of the fermion theory:

1. $d^3$, $d^4$ and $\lambda$ have the values extracted by matching to lattice measurements of pion correlators. Mean field theory same.

2. Do the Hubbard-Stratanovich trick in Minkowski, and integrate out the quarks to obtain a pion EFT in Minkowski.

In the real-time Minkowski computations we use doubled fields; special care needed for causality.

For details see SG and Sharma 1904.11265
Static and dynamic quantities

Loop integrals depend separately on time and space components of all momenta. Low-momentum limits depend on the order of limits. As a result the pion correlators are different; EFT supplies the spectral function.

The static correlators involve space-like external momenta $q$. IR limit obtained by taking $q_0 = 0$ first, then $q \to 0$. The analogues of $c^2$ and $c^4$ have the same values as in the Euclidean computation.

Dynamical phenomena involve time-like external momentum $q$. IR limit obtained by taking $q \to 0$ first and then taking the long-time limit of $q_0$. In this limit the analogues of $c^2$ and $c^4$ are quite different.
Two masses in Minkowski theory

The pion rest mass is given by the mass term in the Lagrangian, \( m_r^2 = c^2 T_0^2 \). Also write \( u_\pi^2 = c^4 \).

Then, from the effective Lagrangian in Minkowski space, we find the dispersion relation

\[ E = \sqrt{m_r^2 + u_\pi^2 q^2} = m_r + \frac{q^2}{2m_k} + \cdots, \quad \text{where} \quad m_k = \frac{m_r}{u_\pi^2}. \]

The kinetic energy term contains a different effective mass, \( m_k \). This is an effect of Lorentz symmetry breaking.
Comparing Euclidean and Minkowski

$m_\pi$ increases with $T$ for time-like momentum, decreases for space-like; $f_\pi$ decreases with $T$ for time-like momentum, flat for space-like; differences in $u_\pi$. So spectral function for the pion cannot just be a peak.
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1. Wrote down the Lagrangian of an EFT. Fixed its parameters by matching to Euclidean lattice computations of static pion correlation functions, $C(x)$. SG and Sharma, 1710.05345

2. Used the same Lagrangian in Minkowski. Computed pion correlation functions $C(q)$ where $q$ is an external momentum.

3. Checked that the Minkowski computation reproduces the Euclidean results when $p$ is space-like.

4. The same computation also gives $C(q)$ when $q$ is time-like. This completes the analytic continuation. SG and Sharma 1904.11265

5. In both regions $C(q) \simeq 1/[q_0^2 + u(T)^2 q^2 + m(T)^2]$, but with different $T$-dependence of $u$ and $m$. So the spectral function is not simply a peak with interesting characteristics.
Backup: Error propagation

Input errors: lattice measurements

Output errors: EFT parameters

Covariances of lattice measurements not reported, but can be extracted. In future covariances can be propagated to the EFT parameters.