# Warm pions in real time

Sourendu Gupta, Rishi Sharma

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# Context

- Lattice constraints on real-time phenomena at finite temperature need some knowledge of spectral densities.
- Knowledge-free methods of using spectral densities have been explored (MEM, etc) with spectacular success in some cases.
- Theoretical constraints on spectral densities largely utilize weak-coupling methods, and have increased our understanding substantially.
- Most studies to date have concentrated on high-temperature properties, namely $T > T_c$. However, the fireball produced in heavy ion collisions spend more than half its life time in the hadronic phase. So understanding the warm hadronic plasma ($T_c > T > 0$) is also important.
- This study utilizes an Effective Field Theory (EFT) approach to address the analytic continuation of lattice data in the hadronic phase.

# The EFT approach

EFTs are useful for describing physics where there are well-separated length scales. Since (approximate) chiral symmetry allows us to separate the scales of $T$ and $m_\pi$, EFTs may be expected to be useful for understanding pions and the transport of chiral charge.

The UV cutoff of such a EFT may be expected to be of the order of $T$. (Notice that $1/a > T$). Organize in powers of $q/T$; $q$ is momentum scale of correlators.

The Lagrangian of the EFT is constrained only by the symmetries of the problem; *i.e.*, all terms allowed by symmetry must be used. We use chiral symmetry, spatial rotations and CPT. No boosts: Boltzmann weight is $\exp(-E/T)$, forces choice of frame.

# Fermion mass and kinetic terms

Action for quarks

$$\mathcal{L}_4 = d^3 T_0 \overline{\psi}\psi + \overline{\psi}\partial\!\!\!/_4\psi + d^4\overline{\psi}\nabla\!\!\!\!/\,\psi$$

1. Dimensions provided by the UV cutoff of the EFT; here chosen to be $T_0 \simeq T$.
2. Note that $d^i$ are dimensionless. The combination $T_0 d^3$ is the quark mass $m_0$ (only chiral symmetry breaking term in the action).
3. At $T = 0$ full Lorentz symmetry would force $d^4 = 1$. When $d^4 \neq 1$ then there is a difference between the mass $m_0$ (pole mass) and the mass that can be deduced from the decay of static correlation functions (screening mass).

# Extending the NJL model

$$
\begin{aligned}
\mathcal{L}_6 \quad = \quad & \frac{d^{61}}{T_0^2}\left[(\overline{\psi}\psi)^2+(\overline{\psi}i\gamma_5\tau^a\psi)^2\right]+\frac{d^{62}}{T_0^2}\left[(\overline{\psi}\tau^a\psi)^2+(\overline{\psi}i\gamma_5\psi)^2\right] \\
& +\frac{d^{63}}{T_0^2}(\overline{\psi}\gamma_4\psi)^2+\frac{d^{64}}{T_0^2}(\overline{\psi}i\gamma_i\psi)^2+\frac{d^{65}}{T_0^2}(\overline{\psi}\gamma_5\gamma_4\psi)^2 \\
& +\frac{d^{66}}{T_0^2}(\overline{\psi}i\gamma_5\gamma_i\psi)^2+\frac{d^{67}}{T_0^2}\left[(\overline{\psi}\gamma_4\tau^a\psi)^2+(\overline{\psi}\gamma_5\gamma_4\tau^a\psi)^2\right] \\
& +\frac{d^{68}}{T_0^2}\left[(\overline{\psi}i\gamma_i\tau^a\psi)^2+(\overline{\psi}i\gamma_5\gamma_i\tau^a\psi)^2\right] \\
& +\frac{d^{69}}{T_0^2}\left[(\overline{\psi}iS_{i4}\psi)^2+(\overline{\psi}S_{ij}\tau^a\psi)^2\right] \\
& +\frac{d^{60}}{T_0^2}\left[(\overline{\psi}iS_{i4}\tau^a\psi)^2+(\overline{\psi}S_{ij}\psi)^2\right]
\end{aligned}
$$

10 couplings $d^{6i}$, collapse to one $(\lambda)$ in mean field theory.

# Pion correlators

Expected form of mean field theory, fixes $\lambda$. Introduce small fluctuations about the mean field (pions) using a Hubbard-Stratanovich trick

$$\psi \to \exp\left[\frac{i\pi^a\tau^a\gamma^5}{2f}\right]\psi.$$

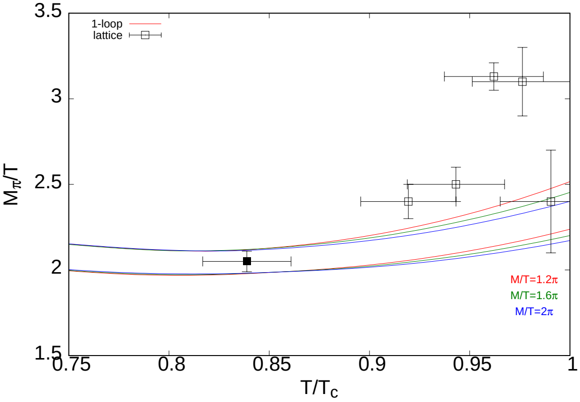
Introduce this into the fermion action, expand to second order in $\pi$.

Gives coupled pion-quark action; overcounts degrees of freedom. Integrate out the quarks. Propagator is equivalent to effective pion action

$$L_\pi = \frac{1}{2}c^2T_0^2\pi^2 + \frac{1}{2}(\partial_4\pi)^2 + \frac{1}{2}c^4(\nabla\pi)^2 + \cdots$$

Match static correlation functions (on torus) to lattice measurements to fix parameters $c^2$ and $c^4$, and through them the quark couplings $d^3$ and $d^4$. **SG and Sharma, 1710.05345**

# Euclidean predictions




Numerical approximations in prediction: $\lambda$ fitted in the chiral limit; $T_0$ without power corrections; $d^3$, $d^4$ and $\lambda$ assumed to be independent of $T$. **Lattice measurements taken from Brandt et al, 1406.5602**

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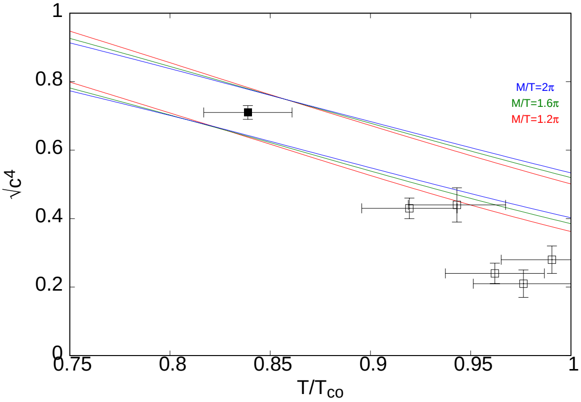


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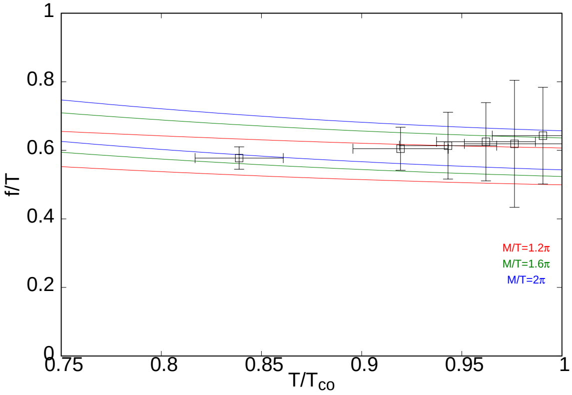


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# Analytic continuation to real time

Analytic continuation of the pion theory is straightforward: $c^2$ and $c^4$ will then have the values that they have in Euclidean. However, this captures only the long-distance part of correlators. Will shorter distance physics have no bearing on the analytic continuation?

Check by analytic continuation of the fermion theory:

1. $d^3$, $d^4$ and $\lambda$ have the values extracted by matching to lattice measurements of pion correlators. Mean field theory same.
2. Do the Hubbard-Stratanovich trick in Minkowski, and integrate out the quarks to obtain a pion EFT in Minkowski.

In the real-time Minkowski computations we use doubled fields; special care needed for causality.

**For details see SG and Sharma 1904.11265**

# Static and dynamic quantities

Loop integrals depend separately on time and space components of all momenta. Low-momentum limits depend on the order of limits. As a result the pion correlators are different; EFT supplies the spectral function.

The static correlators involve space-like external momenta $q$. IR limit obtained by taking $q_0 = 0$ first, then $\mathbf{q} \to 0$. The analogues of $c^2$ and $c^4$ have the same values as in the Euclidean computation.

Dynamical phenomena involve time-like external momentum $q$. IR limit obtained by taking $\mathbf{q} \to 0$ first and then taking the long-time limit of $q_0$. In this limit the analogues of $c^2$ and $c^4$ are quite different.

# Two masses in Minkowski theory

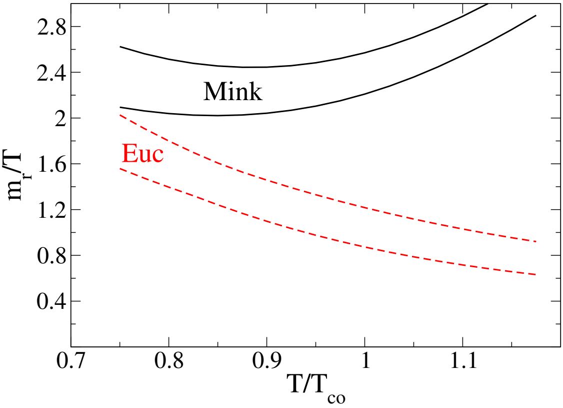
The pion rest mass is given by the mass term in the Lagrangian, $m_r^2 = c^2 T_0^2$. Also write $u_\pi^2 = c^4$.

Then, from the effective Lagrangian in Minkowski space, we find the dispersion relation

$$E = \sqrt{m_r^2 + u_\pi^2 \mathbf{q}^2} = m_r + \frac{\mathbf{q}^2}{2m_k} + \cdots\,, \qquad \text{where} \qquad m_k = \frac{m_r}{u_\pi^2}.$$

The kinetic energy term contains a different effective mass, $m_k$. This is an effect of Lorentz symmetry breaking.

# Comparing Euclidean and Minkowski




$m_\pi$ increases with $T$ for time-like momentum, decreases for space-like; $f_\pi$ decreases with $T$ for time-like momentum, flat for space-like; differences in $u_\pi$. So spectral function for the pion cannot just be a peak.

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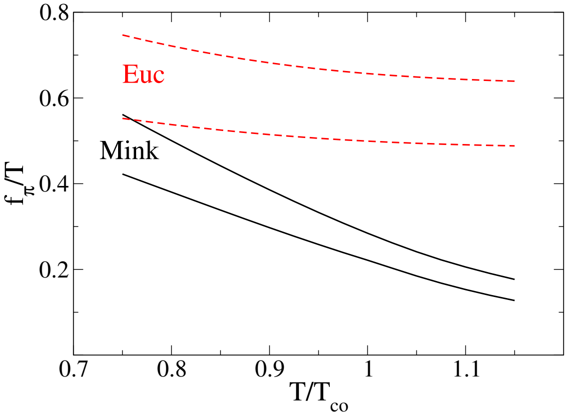


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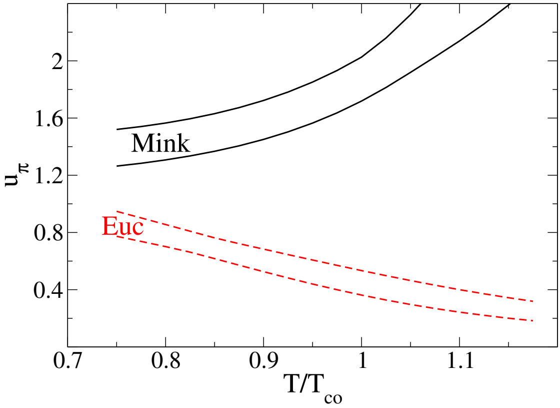


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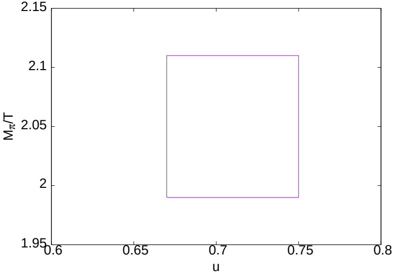
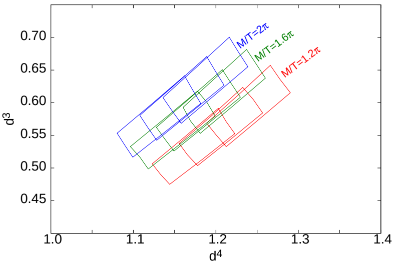
# Outline

1. Wrote down the Lagrangian of an EFT. Fixed its parameters by matching to Euclidean lattice computations of static pion correlation functions, $C(x)$. **SG and Sharma, 1710.05345**
2. Used the same Lagrangian in Minkowski. Computed pion correlation functions $C(q)$ where $q$ is an external momentum.
3. Checked that the Minkowski computation reproduces the Euclidean results when $p$ is space-like.
4. The same computation also gives $C(q)$ when $q$ is time-like. This completes the analytic continuation. **SG and Sharma 1904.11265**
5. In both regions $C(q) \simeq 1/[q_0^2 + u(T)^2\mathbf{q}^2 + m(T)^2]$, but with different $T$-dependence of $u$ and $m$. So the spectral function is not simply a peak with interesting characteristics.

# Backup: Error propagation




Input errors: lattice measurements

Output errors: EFT parameters

Covariances of lattice measurements not reported, but can be extracted. In future covariances can be propagated to the EFT parameters.