Conductivity of quark-gluon matter in the external magnetic field

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Heavy ion collisions — large $eB$

In non-central heavy ion collisions very strong $|e\vec{B}| \sim 10m^2_\pi$ field may emerge.
Chiral magnetic effect (CME)

K. Fukushima, D. Kharzeev, H.J. Warringa. Chiral imbalance $N_R - N_L$ creates current in the direction of external $\vec{B}$:

$$\frac{d\rho_5}{dt} = \frac{e^2}{2\pi^2} \vec{E} \cdot \vec{B}$$

CME:

1. macroscopic effect of microscopic dynamics of QCD;
2. allows probing the topological structure of $SU(3)$ gauge field;
3. non-dissipative, topologically protected;
From $\rho_5$ to $\mu_5$

Chirality-changing processes:

$$\frac{d\rho_5}{dt} = -\rho_5/\tau + \frac{e^2}{2\pi^2} \vec{E} \cdot \vec{B} \implies \rho_5 = \frac{e^2}{2\pi^2} \vec{E} \cdot \vec{B} \tau.$$

At small $\mu_5 \ll T$, $\mu_5 \ll \sqrt{eB}$, $\rho_5 = \chi(B,T)\mu_5$.

1. $T \gg \sqrt{eB}$, temperature dominates: $\chi(B,T) = T^2/3$,

2. $T \ll \sqrt{eB}$, 1st Landau level degeneracy:

$$\chi(B,T) = |eB|/2\pi^2$$

Linear response theory:

$$j^i_{\text{CME}} = \sigma^{ij}_{\text{CME}} E^j,$$

$$\sigma^{ij}_{\text{CME}} = \frac{e^4}{8\pi^4} \frac{\tau}{\chi(T,B)} B^i B^j.$$
CME observation: Dirac semimetals

▶ Experimental: Observation of the chiral magnetic effect in ZrTe$_5$, Nature physics (2016), Q. Li et al.

▶ HQMC: Lattice quantum Monte Carlo study of chiral magnetic effect in Dirac semimetals, Annals of Physics (2018), Boyda et al.

Figure: Left: $\sigma_{\text{CME}}$ within HQMC lattice simulation, right: experiment with ZrTe$_5$.
Conductivity in external magnetic field

- $\vec{E} \parallel \vec{B}$
- \[ \frac{d\rho_5}{dt} = \frac{e^2}{4\pi^2} (\vec{E}, \vec{B}) - \frac{\rho_5}{\tau}, \quad \tau \text{ — chirality-changing scattering time} \]
- $\rho_5 = \frac{e^2\tau}{4\pi^2} (\vec{E}, \vec{B})$,
- $\vec{J}_{\text{CME}} = \frac{e^2}{2\pi^2} \mu_5 \vec{B}$
- $\vec{J} = \sigma \vec{E} + \frac{e^2}{2\pi^2} \vec{B} \times \mu_5 \left( \rho_5 \sim \tau(\vec{E}, \vec{B}) \right)$
- Large magnetoconductivity $\sigma_{\parallel}$
- Classically $\delta\sigma_{\parallel} = 0$
- Observed in experiment (Weyl semimetals)
  H. Li et al., Nat. Comm. 7, 10301 (2016)

What happens in QCD?
Lattice details

- Stout smeared staggered $2 + 1$ fermions
- Physical pion $m_\pi$ and strange $m_s$ quark masses
- $T = 200, 250$ MeV
- Lattice sizes and steps:

<table>
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<th>$a$, fm</th>
<th>$L_s$</th>
<th>$N_t$</th>
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<td>0.0493</td>
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- Integral Kubo equation
  
  \[ C(\tau_i) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau_i, \omega) \rho(\omega), \quad K(\tau_i, \omega) = \frac{\cosh \omega(\beta - \tau_i/2)}{\sinh \omega \beta/2} \omega. \]

- Conductivity:
  
  \[ \frac{\sigma}{TC_{em}} = \frac{1}{6C_{em}} \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}. \]
The Backus-Gilbert method

- The method is designed for solving linear ill-defined problems with controllable regularization and systematic uncertainty.

- Define the (normalized) resolution function $\delta$ as the linear combination of adjustable coefficients $q(\bar{\omega})$:

  \[ \tilde{\rho}(\bar{\omega}) = \int d\omega \delta(\bar{\omega}, \omega) \rho(\omega), \]

  \[ \delta(\bar{\omega}, \omega) = \sum_i q_i(\bar{\omega}) K(\tau_i, \omega), \]

- Minimize the BG–functional:

  \[ H(\rho) = \lambda A(\rho) + (1 - \lambda) B(\rho), \]

  \[ A(\rho) = \int d\omega \delta(\bar{\omega}, \omega)(\omega - \bar{\omega})^2, \]

  \[ B(\rho) = \text{Var}[\rho] = q^T C q. \]

The $A$ part is the width of the resolution function (2nd moment to make $q_i$ easy to find), $B(\rho)$ — make less dependent on data (regularize). The method provides $\rho(\omega)$ and $\delta(\bar{\omega}, \omega)$ as the output!
Resolution function

The width is of order $\leq 3.5T$ (not enough $N_\tau$) to have the «delta-function». Is it possible that we underestimate the transport peak?

![Graph showing resolution function](image)

**Probably not**, characteristic transport peak width:

- $\sim 2T$ (B. Brandt et al., *Phys. Rev. D* 93, 054510 (2016));
- $\sim 4 - 5T$ (A. Amato et al., *Phys. Rev. Lett.* 111, 172001 (2013));
- $\sim 4T$ (H.-T. Ding et al., *Phys. Rev. D* 94, 034504 (2016))
Perturbative $\rho(\omega)$

$$\rho(\omega) \approx (B\omega)_{\text{small } \omega} \theta(\omega_0 - \omega) + (A\rho_{\text{UV}}(\omega))_{\text{large } \omega} \theta(\omega - \omega_0).$$

In Brandt et al. (2015) the $A \sim 1$ and $\omega_0 \sim 2$ GeV were obtained → the UV model works fine.

$$C(\tau) = A(\tau) + (-1)^\tau B(\tau).$$

$$\rho_{\text{UV}}(\omega) = C_{e/\omega} \frac{3}{4\pi^2} \omega^2 \tanh \left( \frac{\omega\beta}{4} \right)$$

$$C_e = \frac{1}{2}, \quad C_\omega = \frac{3}{2}.$$
Ultra violet reconstruction for $N_t = 96$

Figure: Left: Free case, right: interacting case.

$$(C_e + C_o)/2 \sim 1.$$
Results at $eB = 0$

At $T = 200$ MeV flat spectral function $\rightarrow$ good analysis,

At $T = 250$ MeV B. report the rise of peak at zero $\rightarrow$ possible underestimation.
Conductivity at nonzero magnetic field \( eB \neq 0 \)

Idea: consider difference \( C(t, eB) - C(t, eB = 0) \) \( \to \) avoid UV contamination, \( \delta(\tilde{\omega}, \omega) \) narrower.

The peak grows around \( \omega = 0 \), UV behavior is indeed small.
Conductivity at nonzero magnetic field $eB \neq 0$

- Linear growth observed as $eB \gg T^2$.
- The $\sigma_\perp$ decay results from the Lorentz force acting on charged particles moving in the direction of $\vec{E} \perp \vec{B}$.
- Correction due to the intermediate region is hard to estimate.
- Decrease of the slope is in agreement with high-$T$ behavior of $\tau/\chi(T, B)$.