

# Lattice computation of the quark propagator in Landau gauge at finite temperature

Paulo J. Silva, Orlando Oliveira  
Center for Physics, University of Coimbra

21 June 2019

# Outline

## 1 Introduction and Motivation

- QCD phase diagram, center symmetry and Green's functions
- Gluon propagator @ finite T

## 2 Quark propagator @ finite T

- Form factors and lattice setup
- Results
- Propagators and center symmetry

## 3 Conclusions and Outlook

# Outline

## 1 Introduction and Motivation

- QCD phase diagram, center symmetry and Green's functions
- Gluon propagator @ finite T

## 2 Quark propagator @ finite T

- Form factors and lattice setup
- Results
- Propagators and center symmetry

## 3 Conclusions and Outlook

## QCD phase diagram

- study of the phase diagram of QCD relevant e.g. for heavy ion experiments
- QCD has phase transition where quarks and gluons become deconfined for sufficiently high T
- Polyakov loop
  - order parameter for the confinement-deconfinement phase transition
  - $L = \langle L(\vec{x}) \rangle \propto e^{-F_q/T}$
  - Definition on the lattice:

$$L(\vec{x}) = \text{Tr} \prod_{t=0}^{N_t-1} \mathcal{U}_4(\vec{x}, t)$$

- $T < T_c : L = 0$  (center symmetry)
- $T > T_c : L \neq 0$  (spontaneous breaking of center symmetry)

## Center symmetry

- Wilson gauge action is invariant under a center transformation
- temporal links on a hyperplane  $x_4 = \text{const}$  multiplied by

$$z \in Z_3 = \{e^{-i2\pi/3}, 1, e^{i2\pi/3}\}$$

- Polyakov loop  $L(\vec{x}) \rightarrow zL(\vec{x})$
- $T < T_c$ 
  - local  $P_L$  phase equally distributed among the three sectors

$$L = \langle L(\vec{x}) \rangle \approx 0$$

- $T > T_c$ 
  - $Z_3$  sectors not equally populated:  $L \neq 0$

G. Endrődi, C. Gattringer, H.-P. Schadler, arXiv:1401.7228  
C. Gattringer, A. Schmidt, JHEP **01**, 051 (2011)  
C. Gattringer, Phys. Lett. **B 690**, 179 (2010)

F. M. Stokes, W. Kamleh, D. B. Leinweber, arXiv:1312.0991

# QCD Green's functions

- In a Quantum Field Theory, knowledge of all Green's functions allows a complete description of the theory
- In QCD, propagators of fundamental fields (e.g. quark, gluon and ghost propagators) encode information about non-perturbative phenomena
  - gluon propagator encodes information about confinement/deconfinement
  - quark propagator: chiral symmetry breaking
- Since propagators are gauge dependent quantities, we need to choose a gauge
  - in our works: Landau gauge

# Outline

## 1 Introduction and Motivation

- QCD phase diagram, center symmetry and Green's functions
- Gluon propagator @ finite T

## 2 Quark propagator @ finite T

- Form factors and lattice setup
- Results
- Propagators and center symmetry

## 3 Conclusions and Outlook

## Gluon propagator at finite temperature

$$D_{\mu\nu}^{ab}(\hat{q}) = \delta^{ab} \left( P_{\mu\nu}^T D_T(q_4, \vec{q}) + P_{\mu\nu}^L D_L(q_4, \vec{q}) \right)$$

- Two components:
  - transverse  $D_T$
  - longitudinal  $D_L$

$$D_{ii}^{aa}(q) = \frac{2}{V} \left\langle \text{Tr} \left[ A_i(\hat{q}) A_i^\dagger(\hat{q}) \right] \right\rangle = \delta^{aa} \left( P_{ii}^T D^T + P_{ii}^L D^L \right)$$

$$D_{44}^{aa}(q) = \frac{2}{V} \left\langle \text{Tr} \left[ A_4(\hat{q}) A_4^\dagger(\hat{q}) \right] \right\rangle = \delta^{aa} \left( P_{44}^T D^T + P_{44}^L D^L \right)$$

- Finite temperature on the lattice:  $L_t \ll L_s$

$$T = \frac{1}{aL_t}$$

# Lattice setup finite T

Temp. (MeV)	$\beta$	$L_s$	$L_t$	a [fm]	1/a (GeV)
121	6.0000	64	16	0.1016	1.943
162	6.0000	64	12	0.1016	1.943
194	6.0000	64	10	0.1016	1.943
243	6.0000	64	8	0.1016	1.943
260	6.0347	68	8	0.09502	2.0767
265	5.8876	52	6	0.1243	1.5881
275	6.0684	72	8	0.08974	2.1989
285	5.9266	56	6	0.1154	1.7103
290	6.1009	76	8	0.08502	2.3211
305	5.9640	60	6	0.1077	1.8324
305	6.1326	80	8	0.08077	2.4432
324	6.0000	64	6	0.1016	1.943
366	6.0684	72	6	0.08974	2.1989
397	5.8876	52	4	0.1243	1.5881
428	5.9266	56	4	0.1154	1.7103
458	5.9640	60	4	0.1077	1.8324
486	6.0000	64	4	0.1016	1.943

- Simulations: use of Chroma and PFFT libraries
- keep a constant (spatial) physical volume  $\sim (6.5\text{fm})^3$
- all data renormalized at  $\mu = 4\text{GeV}$

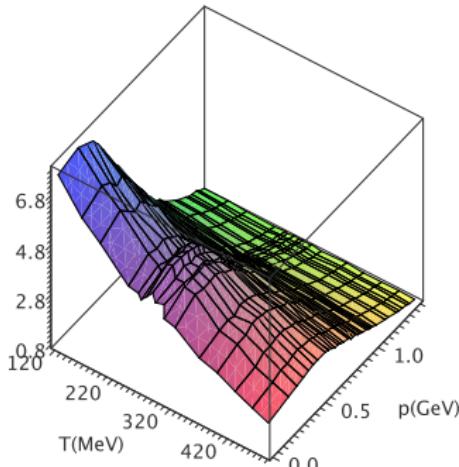
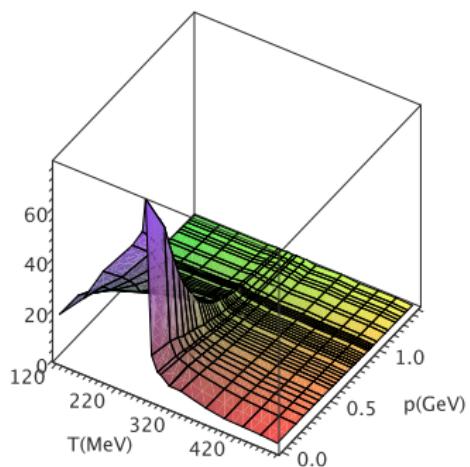
O. Oliveira, PJS, PoS(LATTICE2012)216

Acta Phys. Polon. Supp. 5 (2012) 1039

PoS(Confinement X)045



## Surface plots ( $q_4 = 0$ )



PJS, O. Oliveira, P. Bicudo, N. Cardoso, Phys.Rev. D89 (2014) 074503



# Outline

## 1 Introduction and Motivation

- QCD phase diagram, center symmetry and Green's functions
- Gluon propagator @ finite T

## 2 Quark propagator @ finite T

- Form factors and lattice setup
- Results
- Propagators and center symmetry

## 3 Conclusions and Outlook

# Quark propagator at finite temperature

- At finite T: Landau gauge quark propagator described by three form factors in momentum space

$$S(p_4, \vec{p}) = \frac{1}{i\gamma_4 p_4 \omega(p_4, \vec{p}) + i\vec{\gamma} \cdot \vec{p} Z(p_4, \vec{p}) + \sigma(p_4, \vec{p})}$$

$$S(p_4, \vec{p}) = \frac{-i\gamma_4 p_4 \omega(p_4, \vec{p}) - i\vec{\gamma} \cdot \vec{p} Z(p_4, \vec{p}) + \sigma(p_4, \vec{p})}{p_4^2 \omega^2(p_4, \vec{p}) + (\vec{p} \cdot \vec{p}) Z^2(p_4, \vec{p}) + \sigma^2(p_4, \vec{p})}$$

- $\omega$ ,  $Z$  and  $\sigma$  form factors accessed by computing traces of the propagator times  $\gamma$  matrices.
- Clover fermions, rotated sources (reduce lattice artefacts)

Sheikholeslami, Wohlert, Nucl. Phys. B259 (1985) 572



Heatlie et al, Nucl. Phys. B352 (1991) 266

# Quark propagator @ finite T — our (quenched) simulation

- Wilson gauge action
- spatial physical volume  $\sim (6.5\text{fm})^3$
- focus on the region around  $T_c \sim 270 \text{ MeV}$
- 100 configurations per ensemble
- 2 point sources (with the exception of the highest T)
- average over equivalent momenta
- cylindrical cut for momenta above 1 GeV
- we ignore data with relative errors above 50%

O. Oliveira, P. J. Silva, arXiv:1903.00263 [hep-lat]



# Lattice setup

T (MeV)	$\beta$	$L_s^3 \times L_t$	$\kappa$	$\kappa_c$	$a$ (fm)	$m_{bare}$ (MeV)	$c_{sw}$
243	6.0000	$64^3 \times 8$	0.1350	0.13520	0.1016	10	1.769
			0.1342			53	
260	6.0347	$68^3 \times 8$	0.1351	0.13530	0.09502	11	1.734
			0.1344			51	
275	6.0684	$72^3 \times 8$	0.1352	0.13540	0.08974	12	1.704
			0.1345			54	
290	6.1009	$76^3 \times 8$	0.1347	0.13550	0.08502	51	1.678
305	6.1326	$80^3 \times 8$	0.1354	0.13559	0.08077	13	1.655
			0.1348			53	
324	6.0000	$64^3 \times 6$	0.1342	0.13520	0.1016	53	1.769

Values for  $\kappa_c$  and  $c_{sw}$  computed from Luscher *et al*, Nucl. Phys. B491 (1997) 323



# Outline

## 1 Introduction and Motivation

- QCD phase diagram, center symmetry and Green's functions
- Gluon propagator @ finite T

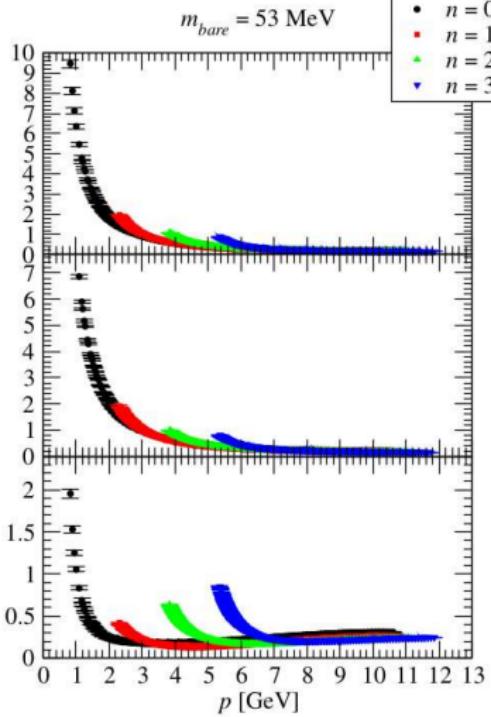
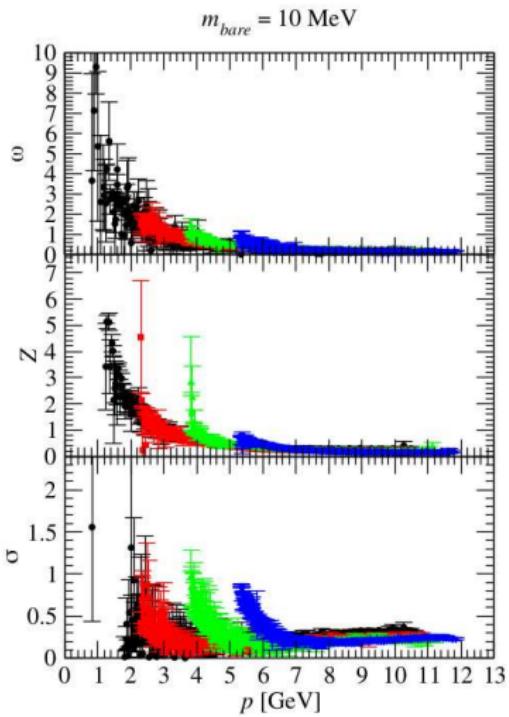
## 2 Quark propagator @ finite T

- Form factors and lattice setup
- **Results**
- Propagators and center symmetry

## 3 Conclusions and Outlook

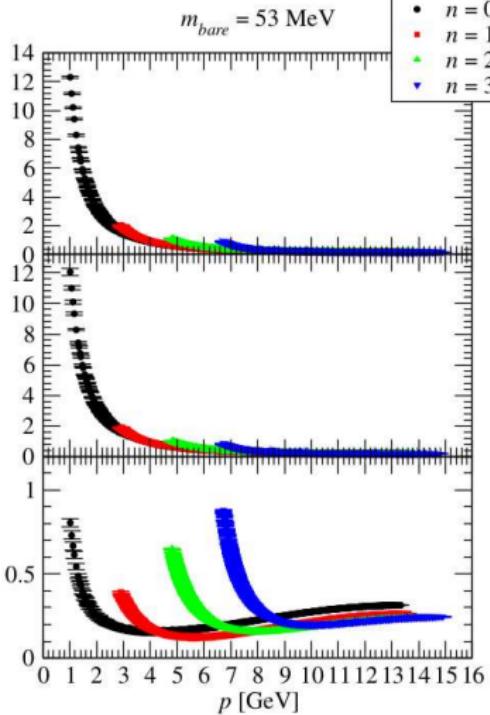
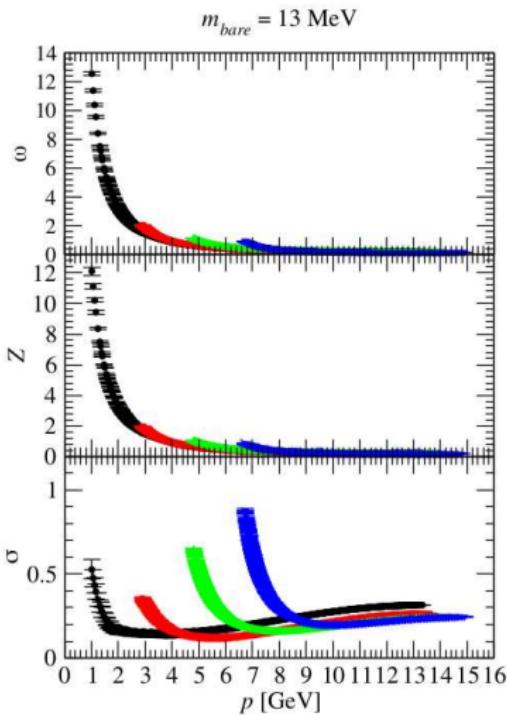
# T=243 MeV

$T = 243 \text{ MeV}$



# T=305 MeV

$T = 305 \text{ MeV}$



# continuum-like form factors

- quark wave function

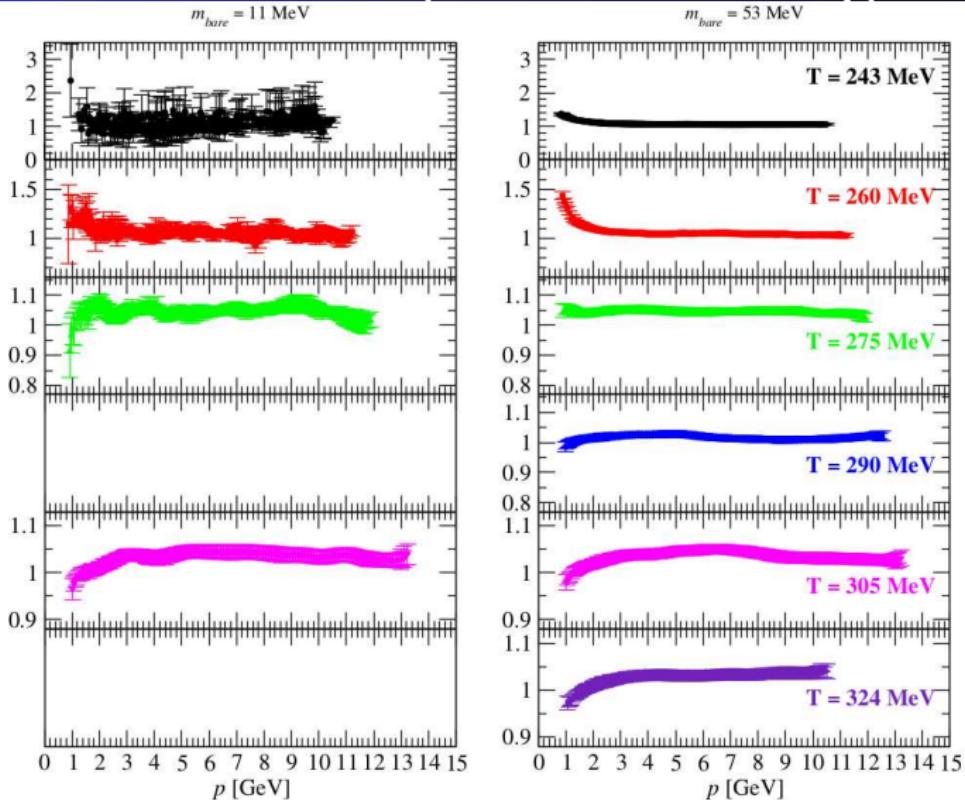
$$Z_c(p_4, \vec{p}) = \frac{Z(p_4, \vec{p})}{\omega(p_4, \vec{p})}$$

- running quark mass

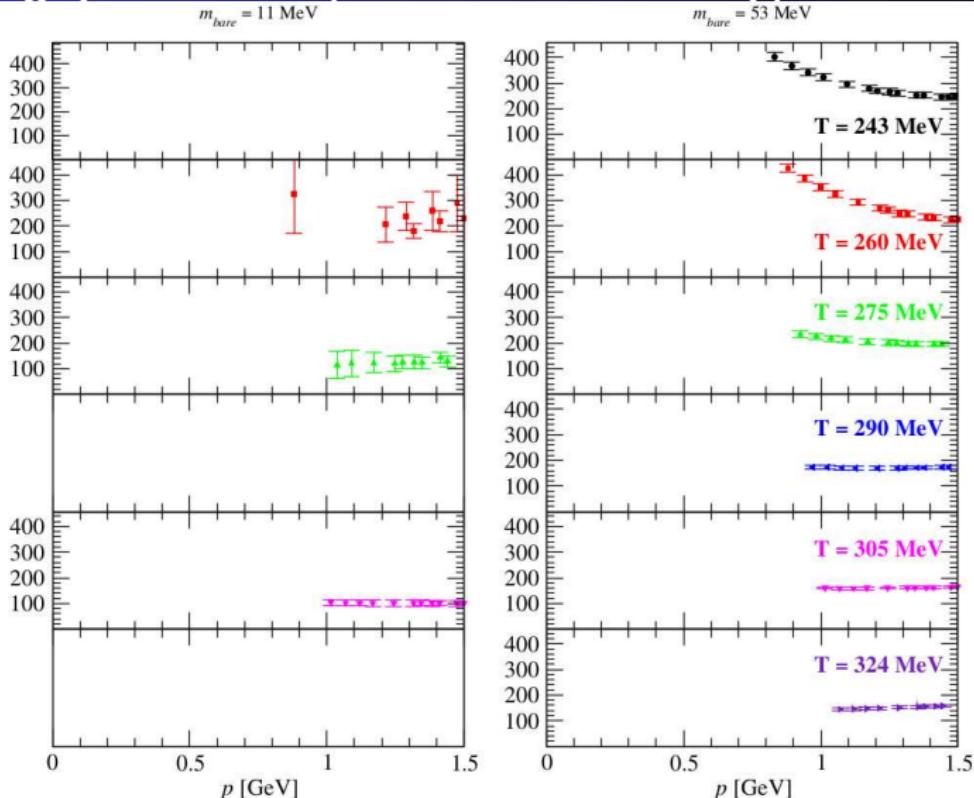
$$M(p_4, \vec{p}) = \frac{\sigma(p_4, \vec{p})}{\omega(p_4, \vec{p})}$$

- we use  $\omega$  and not  $Z$  to define the ratios  $Z_c$  and  $M$   
simulation does not allow the computation of  $Z(p_4, \vec{p} = 0)$

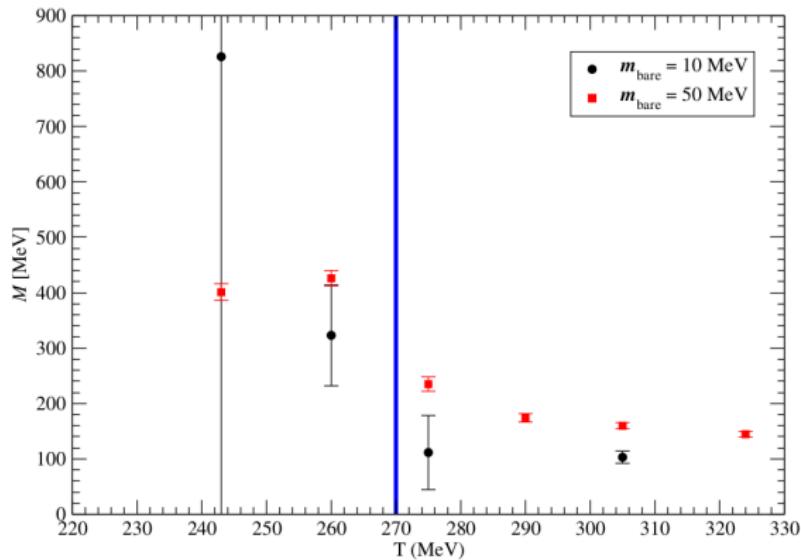
# Bare quark wave function (first Matsubara freq.)



# Running quark mass (first Matsubara freq.)



# $M(p_4, \vec{p} = 0)$ as function of $T$



$T > T_c$  — quasiparticle mass  $\sim 100 \text{ MeV}$

# Outline

## 1 Introduction and Motivation

- QCD phase diagram, center symmetry and Green's functions
- Gluon propagator @ finite T

## 2 Quark propagator @ finite T

- Form factors and lattice setup
- Results
- Propagators and center symmetry

## 3 Conclusions and Outlook

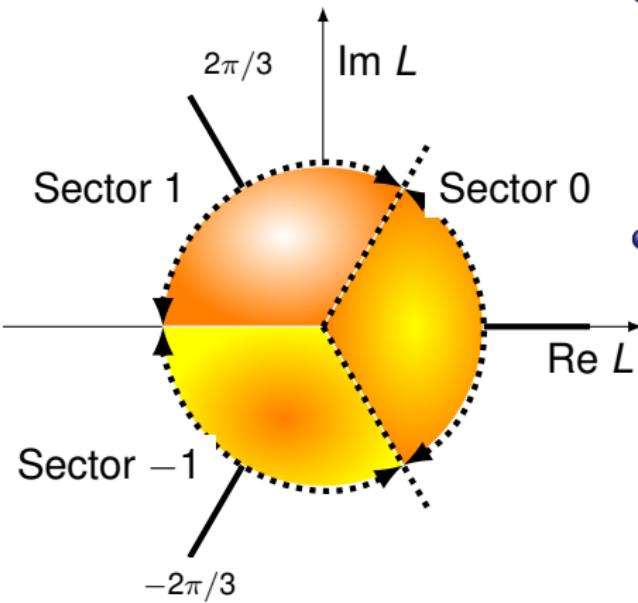
# $Z_3$ dependence

- Usually, propagators are computed such that  $\arg(P_L) < \pi/3$  ( $Z_3$  sector 0)
- what happens in the other sectors?
- Gluon propagator already studied

PJS, O. Oliveira, PRD **93** (2016) 114509

- Preliminary results for the quark propagator

# $Z_3$ dependence



- for each configuration,  
3 gauge fixings after a  $Z_3$  transformation

$$\mathcal{U}'_4(\vec{x}, t=0) = z \mathcal{U}_4(\vec{x}, t=0)$$

- configurations classified according to  $\langle L \rangle = |L| e^{i\theta}$

$$\theta = \begin{cases} -\pi < \theta \leq -\frac{\pi}{3}, & \text{Sector -1,} \\ -\frac{\pi}{3} < \theta \leq \frac{\pi}{3}, & \text{Sector 0,} \\ \frac{\pi}{3} < \theta \leq \pi, & \text{Sector 1} \end{cases}$$

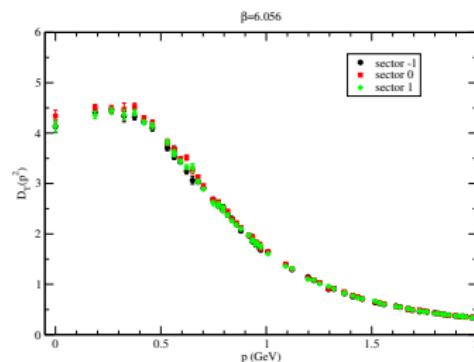
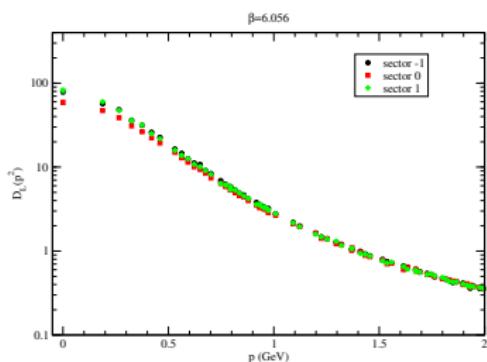
# Focus on the region around $T_c$

- spatial physical volume  $\sim (6.5\text{fm})^3$
- 100 configs per ensemble

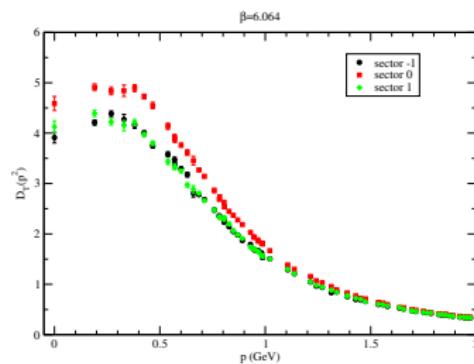
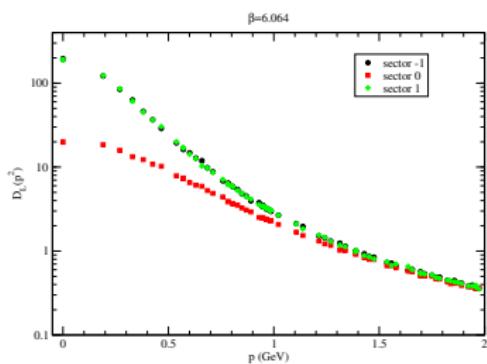
Temp. (MeV)	$L_s^3 \times L_t$	$\beta$	$a$ (fm)	$L_s a$ (fm)
265.9	$54^3 \times 6$	5.890	0.1237	6.68
266.4	$54^3 \times 6$	5.891	0.1235	6.67
266.9	$54^3 \times 6$	5.892	0.1232	6.65
267.4	$54^3 \times 6$	5.893	0.1230	6.64
268.0	$54^3 \times 6$	5.8941	0.1227	6.63
268.5	$54^3 \times 6$	5.895	0.1225	6.62
269.0	$54^3 \times 6$	5.896	0.1223	6.60
269.5	$54^3 \times 6$	5.897	0.1220	6.59
270.0	$54^3 \times 6$	5.898	0.1218	6.58
271.0	$54^3 \times 6$	5.900	0.1213	6.55
272.1	$54^3 \times 6$	5.902	0.1209	6.53
273.1	$54^3 \times 6$	5.904	0.1204	6.50

Temp. (MeV)	$L_s^3 \times L_t$	$\beta$	$a$ (fm)	$L_s a$ (fm)
269.2	$72^3 \times 8$	6.056	0.09163	6.60
270.1	$72^3 \times 8$	6.058	0.09132	6.58
271.0	$72^3 \times 8$	6.060	0.09101	6.55
271.5	$72^3 \times 8$	6.061	0.09086	6.54
271.9	$72^3 \times 8$	6.062	0.09071	6.53
272.4	$72^3 \times 8$	6.063	0.09055	6.52
272.9	$72^3 \times 8$	6.064	0.09040	6.51
273.3	$72^3 \times 8$	6.065	0.09025	6.50
273.8	$72^3 \times 8$	6.066	0.09010	6.49

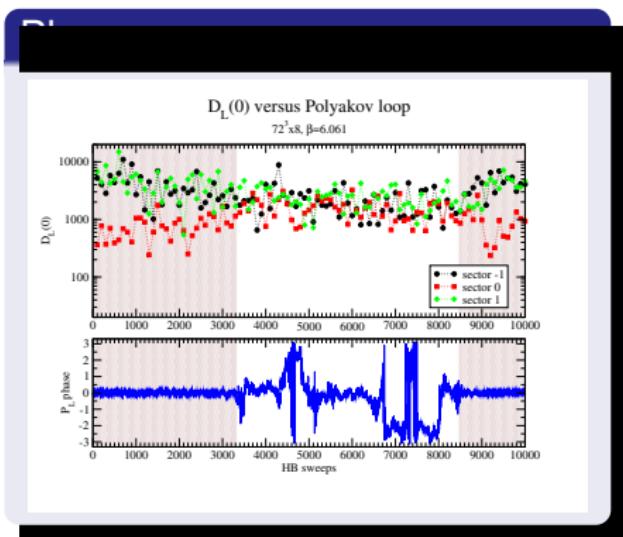
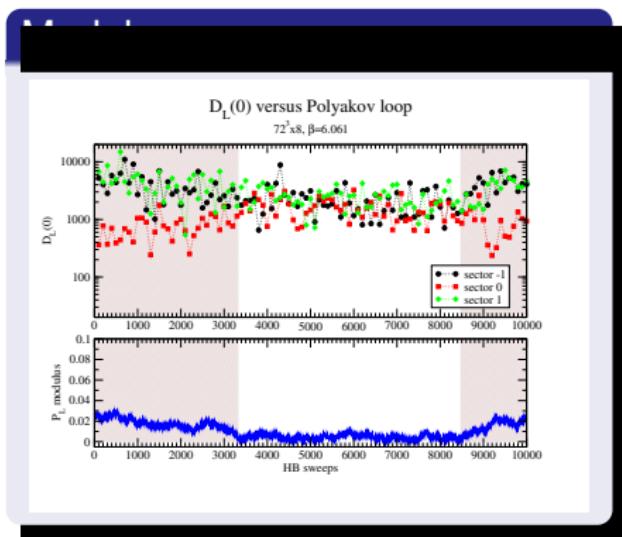
# Fine lattices, below $T_c$



# Fine lattices, above $T_c$

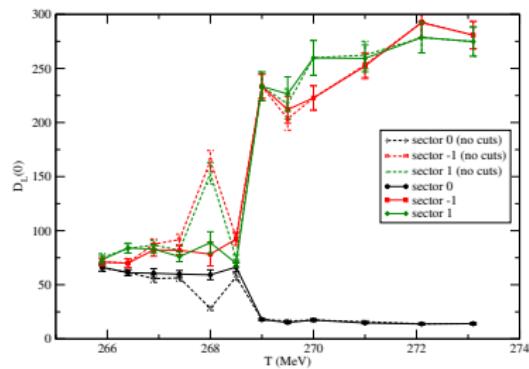


# (bare) Polyakov loop history

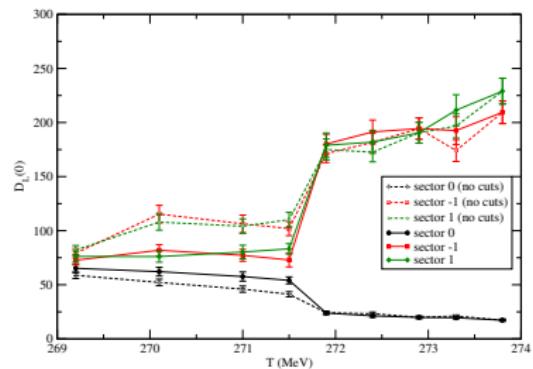


# Removing configurations in wrong phase

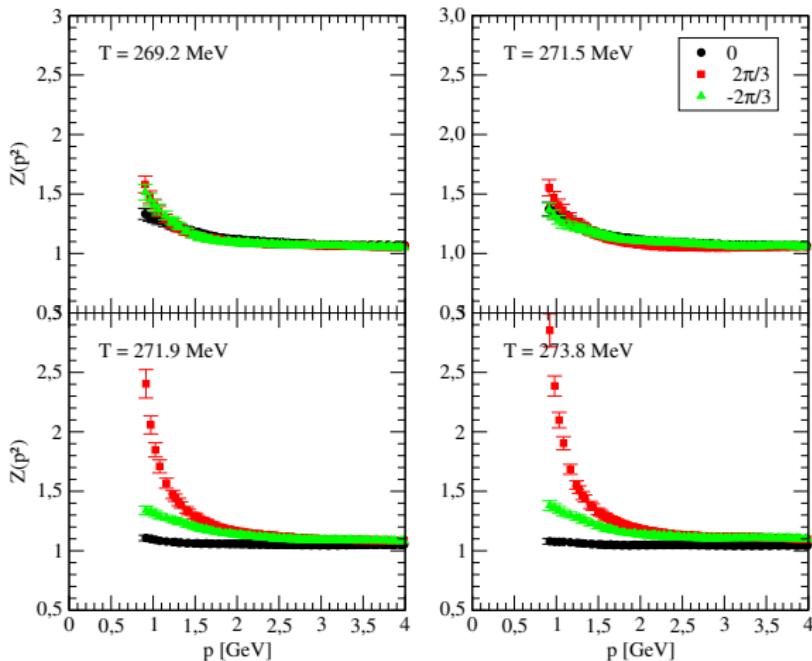
Coarse lattices



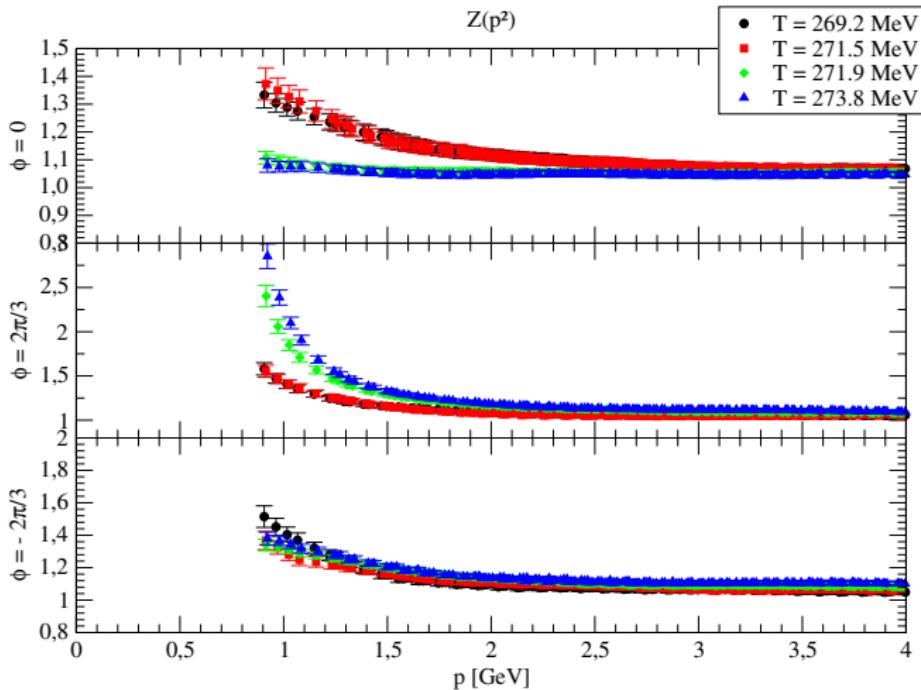
Fine lattices



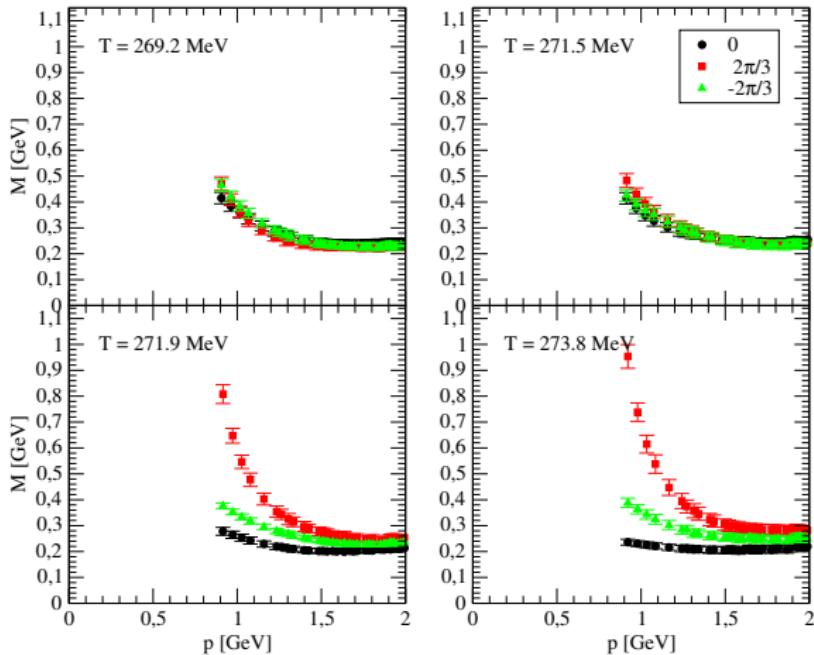
# Quark propagator — quark wave function



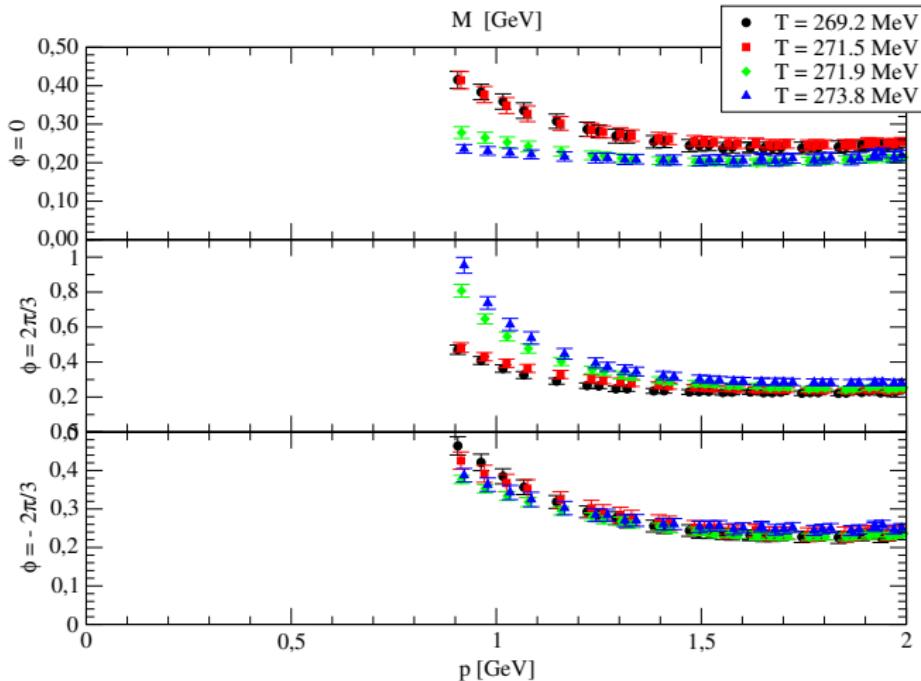
# Quark propagator — quark wave function



# Quark propagator — running mass



# Quark propagator — running mass



# Conclusions and Outlook

- Landau gauge quark propagator at finite temperature
  - form factors investigated as a function of  $T$  for the various Matsubara frequencies
- Quark wave function  $Z_c$  and running quark mass have different functional forms above and below  $T_c$
- Running quark mass suppressed for high temperatures
  - IR region: constant above  $T_c$
  - favours a free quasiparticle scenario
- $Z_3$  dependence — preliminary results
  - quark propagator sensitive to the  $Z_3$  sector
  - decoupling of sectors  $\pm 1$  not seen in the gluon case
- Outlook:
  - other temperatures
  - dynamical simulations

# Funding sources



**FCT** Fundação para a Ciência e a Tecnologia

MINISTÉRIO DA CIÊNCIA, TECNOLOGIA E ENSINO SUPERIOR

This work is funded by national funds through FCT-Fundaçāo para a Ciéncia e a Tecnologia, I.P. in the frame of the Project CERN/FIS-COM/0029/2017.

Computing time provided by the Laboratory for Advanced Computing at the University of Coimbra.

