Critical endpoint in the continuum limit and critical endline at N_t=6 of the finite temperature phase transition of QCD with clover fermions

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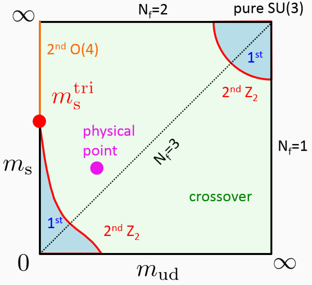
Columbia plot

nature of finite temperature phase transition of 2+1 flavor QCD at $\mu = 0$ in the plane of $m_{u,d}$ and m_s



- 1st order : small m_q region [Pisarski, Wilczek, '84]
- 1st order : heavy m_q region
- crossover : medium m_q
- 2nd order (Z_2) : boundary between 1st and crossover

At small m_q region

- crossover at the physical point
- critical end point at SU(3) flavor symmetric point, $m^{\rm sym}$, has not been determined yet
- critical end line has not been well determined yet

Previous studies for critical endpoint

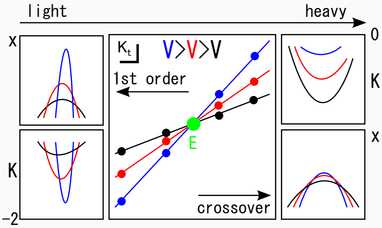
found	N_f	S_G	S_F	N_t	endpoint	ref
y	3	W	W	4	$m_{\rm PS} > 1$GeV	Iwasaki et al, '96
y	3	W	KS	4	$m_{\rm PS} \sim 290$ MeV	Karsch et al, '01
y	3	I	p4	4	$m_{\rm PS} \sim 190$ MeV	Karsch et al, '01
y	3	W	KS	4	$m_{\rm PS} = 290(20)$ MeV	Karsch et al, '04
y	3	I	p4	4	$m_{\rm PS} = 67(17)$ MeV	Karsch et al, '04
y	3	W	KS	4	$am_{\rm q} \approx 0.033$	Karsch et al, '04
y	3	W	KS	4	$am_{\rm q} = 0.0260(5)$	de Forcrand et al, '07
y	3	W	KS	4	$m_{\rm PS}/T = 1.680(4)$	de Forcrand et al, '07
y	3	W	KS	6	$m_{\rm PS}/T = 0954(12)$	de Forcrand et al, '07
n	2+1	I	stout KS	4	$m_{\rm q}/m^{\rm phy} \leq 0.07$	Endrodi et al, '07
n	2+1	I	stout KS	6	$m_{\rm q}/m^{\rm phy} \leq 0.12$	Endrodi et al, '07
y	3	I	imp. W	4,6,8	$m_{\rm PS} \sim 300$ MeV	our, '14
y	3	I	imp. W	8,10	$m_{\rm PS} \lesssim 170$ MeV	our, '17
n	3	I	HISQ	6	$m_{\rm PS} \lesssim 50$ MeV	Bazavov et al, '17
n	2+1	I	HISQ	6,8(,12)	$m_\pi \leq 80$ MeV	Ding et al, '18, '19

Endrodi et al, '07 : $m_l/m_s = m_l^{phy}/m_s^{phy}$, Ding et al, '18, '19 : $m_s = m_s^{phy}$

- we have determined the upper bound of the critical endpoint at flavor symmetric point by using $N_t = 6, 8, 10$ data and critical end line around flavor symmetric point at $N_t = 6$
- we extend our study for critical endpoint at $N_t = 12$ and critical end line away from symmetric point

Our method to determine critical endpoints

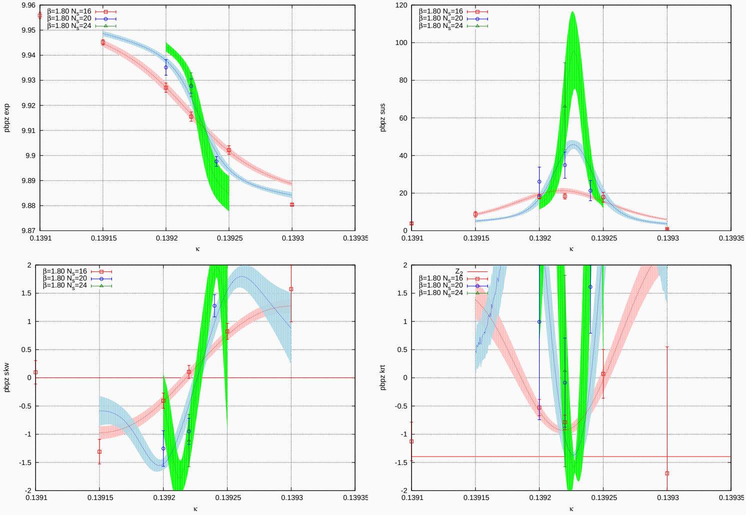
- Iwasaki gauge + NP O(a) improved Wilson fermions
- chiral condensate as a probe of transition
- transition point is determined from peak of the susceptibility, and verified by skewness 0 point
- kurtosis intersection method to determine the critical endpoint
- reweighting method to obtain more critical endpoints



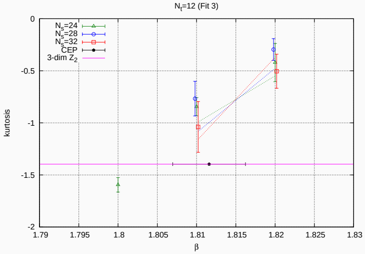
New $N_f = 3$ simulations at $N_t = 12$

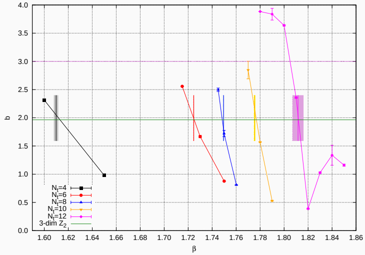
- $N_t = 12$ ($a \approx 0.12$fm), $N_l = 16, 20, 24, 28, 32$
  - $\beta = 1.78$, $\kappa = 0.1396 - 0.1397$
  - $\beta = 1.79$, $\kappa = 0.1393 - 0.1395$
  - $\beta = 1.80$, $\kappa = 0.1390 - 0.1394$
  - $\beta = 1.81$, $\kappa = 0.13895 - 0.13910$
  - $\beta = 1.82$, $\kappa = 0.13875 - 0.13895$
  - $\beta = 1.83$, $\kappa = 0.13855 - 0.13880$
  - $\beta = 1.84$, $\kappa = 0.13835 - 0.13870$
  - $\beta = 1.85$, $\kappa = 0.13815 - 0.13860$

$N_f = 3$ expectation, susceptibility, skewness and kurtosis (example)



$N_f = 3$ kurtosis intersection and critical exponet





- kurtosis intersection fitting (left figure)

$$K = \left[K_{\mathrm{E}} + AN_l^{1/\nu}(\beta - \beta_{\mathrm{E}})\right](1 + BN_l^{y_t - y_h})$$

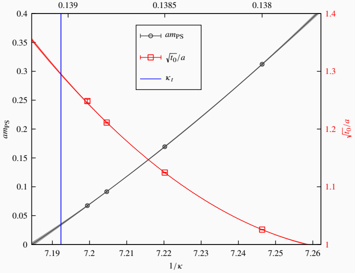
Fit	β_{E}	K_{E}	ν	A	B	$y_t - y_h$	χ^2/dof
1	1.8145(42)	−0.64(21)	0.66(56)	0.4(1.5)	0	0	0.34
2	1.7954(66)	−1.396	0.63	0.211(69)	0	0	1.34
3	1.8098(26)	−1.396	0.63	0.419(89)	−7.0(1.5)	−0.894	0.29

- determing by $b = \gamma/\nu$ of $\chi_{max} \propto N_l^b$ (right figure)

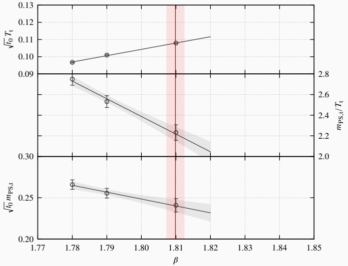
Consistent!

Critical endpoint in the physical unit, interpolation and extrapolation

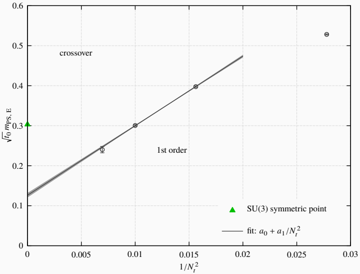
To κ_t at $\beta = 0.181$

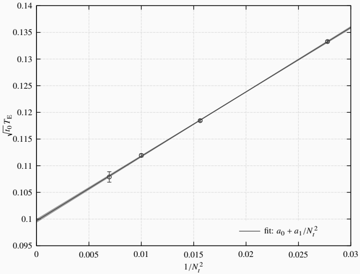


To $\beta_{\rm E}$



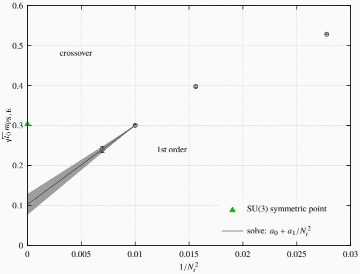
Updated critical endpoint





$\sqrt{t_0}\,m_{\mathrm{PS}} = 0.1262(57)\,,\ \chi^2/\mathrm{dof} = 0.84$

$\sqrt{t_0}\,T = 0.09968(36)\,,\ \chi^2/\mathrm{dof} = 0.34$



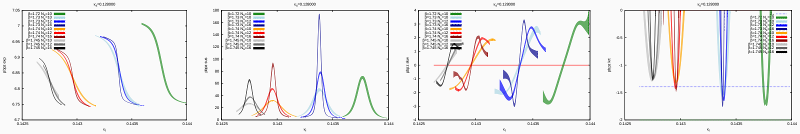
$\sqrt{t_0}\,m_{\mathrm{PS}} = 0.1262(57)(232)$

New $N_f = 2 + 1$ simulation away from symmetric point runs

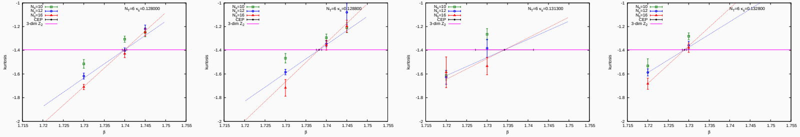
- $N_t = 6$ ($a \approx 0.19$fm), $N_l = 10, 12, 16, 20, 24$
- symmetric runs
  - $\beta = 1.715$, $\kappa = 0.140900 - 0.141100$
  - $\beta = 1.725$, $\kappa = 0.140600 - 0.140618$
  - $\beta = 1.73$, $\kappa = 0.140420 - 0.140450$
- very heavy m_s runs ($\kappa_s = 0.128$)
  - $\beta = 1.73$, $\kappa_l = 0.143365 - 0.143390$
  - $\beta = 1.74$, $\kappa_l = 0.142970 - 0.143042$
  - $\beta = 1.745$, $\kappa_l = 0.142733 - 0.142790$
- heavy m_s runs ($\kappa_s = 0.1328$)
  - $\beta = 1.72$, $\kappa_l = 0.143160$
  - $\beta = 1.73$, $\kappa_l = 0.142702 - 0.142750$
  - $\beta = 1.735$, $\kappa_l = 0.142508$

$N_f = 2 + 1$ results (example)

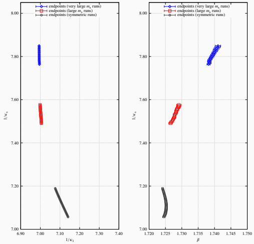
Expectation, susceptibility, skewness, kurtosis



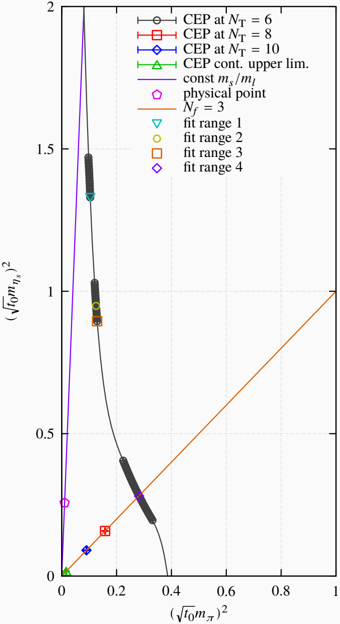
Kurtosis intersection



Critical endpoints in bare parameter plane



Critical endline at $N_t = 6$



$$m_s - m_s^{\mathrm{tric}} \sim m_l^{2/5}$$

[Rajagopal '95]

Fitting endpoints for tri-critical point

$x = (\sqrt{t_0}m_\pi)^2 \propto m_l$

$y = (\sqrt{t_0}m_{\eta_s})^2 \propto m_s$

$$y = a_0 + bx^{2/5}$$

a_0	b	χ^2/dof	range ($x <$)
6.71(8)	-13.3(2)	0.54	f. r. 1
6.44(2)	-12.62(4)	0.97	f. r. 2
6.34(3)	-12.39(6)	3.26	f. r. 3
3.28(9)	-5.1(2)	1655	f. r. 4

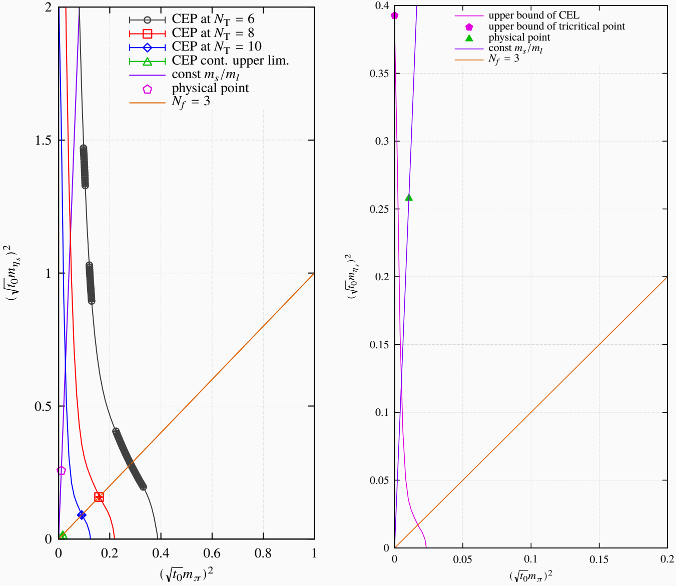
Fiting endpoints for all range with $a_0 = 6.71$

$y = bx^{2/5} + \sum_{i=0}^{3} a_i x^i$, χ^2/dof = 158

$y = bx^{2/5} + \sum_{i=0}^{4} a_i x^i$, χ^2/dof = 6.6

$y = bx^{2/5} + \sum_{i=0}^{5} a_i x^i$, χ^2/dof = 1.3

Critical endline at $N_t = 8, 10, \infty$ (estimated upper bound)



assuming same shape

Summary

We are trying to update the critical endpint in the continuum limit and the critical endline away from the SU(3)-flavor symmetric point at $N_t = 6$ and presented preliminary results for the critical end lines at $N_t = 8, 10$ and in the continuum limit with NP O(a) improved Wilson fermions. We find

- we need more zero temperature simulations to determine the critical endpoint in the continuum limit
- preliminary results in the cntinuum limit

$$m_{\mathrm{PS,E}} = 170(8)(31)\,\mathrm{MeV}$$
$$T_{\mathrm{E}} = 134(3)\,\mathrm{MeV}$$

- critical end line at $N_t = 6$ is nice agreement with $m_s - m_s^{tri} \sim m_l^{2/5}$ in small m_l region
- $m_s^{tri} \lesssim 1.5\, m_s^{phy}$ (very preliminary!!)
- tentative tri-critical scaling region : $m_{\eta_s}^2 \gtrsim 5m_\pi^2$