

# Lattice investigation of the phase diagram of the 1+1 dimensional Gross-Neveu model at finite number of fermion flavors

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- Phase diagram of QCD poses an interesting problem with a lot of open questions.
- Several toy models for QCD have **inhomogeneous phases in the large- $N_f$  limit**.
- In this research we focus on the **Gross-Neveu (GN) Model in 1+1 dimensions at finite number of flavors**.
- The main questions are whether the **inhomogeneous phase** still occurs and the **structure of the phase diagram**.

## Gross-Neveu Model

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# Gross-Neveu model

- The Gross-Neveu Model serves as a toy model with a crude similarity to QCD.
  - The fermion interactions interact by a **4-point fermion interaction**.
  - A **discrete chiral symmetry** is realized in the action.
  - This symmetry can be **spontaneously broken**.
- Euclidean action of the Gross-Neveu model:

$$S_E = \int d^2x \left( \bar{\psi}_f (\not{\partial} + \gamma_0 \mu) \psi_f - \frac{\lambda}{2N_f} (\bar{\psi}_f \psi_f)^2 \right).$$

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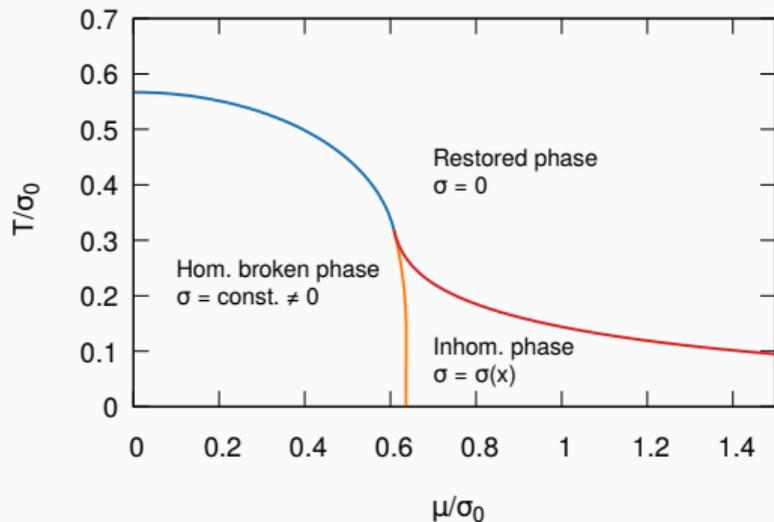
- Hubbard-Stratonovich transformation:

$$Z = \mathcal{N} \int D\psi_f D\bar{\psi}_f D\sigma \exp \left( - \int d^2x \left( \bar{\psi}_f (\not{\partial} + \gamma_0 \mu + \sigma) \psi_f + \frac{N_f}{2\lambda} \sigma^2 \right) \right).$$

- $\langle \bar{\psi}(x) \psi(x) \rangle = -\frac{N_f}{\lambda} \langle \sigma(x) \rangle \rightarrow$  from now on refer to  $\sigma$  as chiral condensate.

# Phase diagram in the large- $N_f$ limit

Revised phase diagram of the Gross-Neveu model:



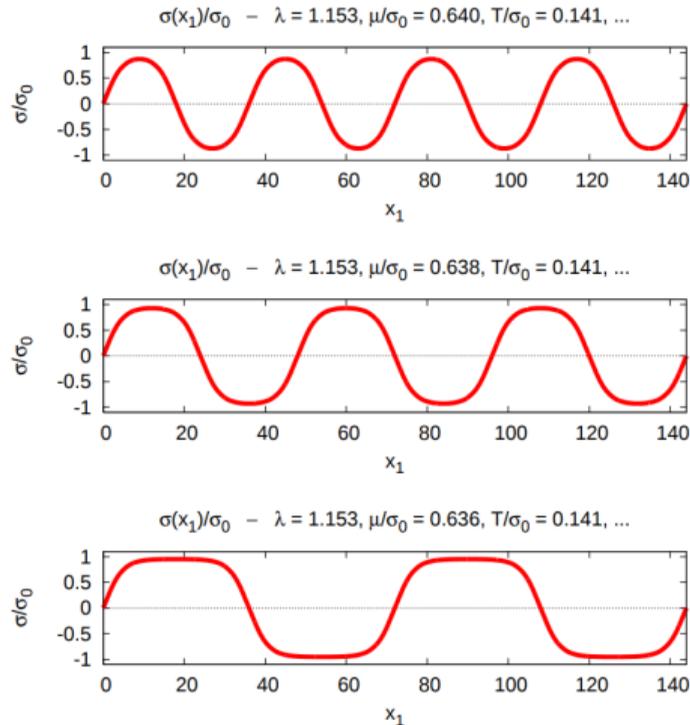
— 2nd order homogeneous phase boundary

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[O. Schnetz, M. Thies and K. Urlichs, *Annals Phys.* 314, 425 (2004)

[hep-th/0402014]]

The chiral condensate for  $T = 0.141 \sigma_0$  and different  $\mu$ :



[M. Wagner, *PoS LATTICE 2007*, 339 (2007) [arXiv:0708.2359 [hep-lat]]]

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with  $N_{\mu}$  as Lattice extent in  $\mu$  direction.

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- Scale setting is done via  $\sigma_0$  - the value of  $\sigma$  at very low temperatures and  $\mu = 0$ . All quantities are expressed in units of  $\sigma_0$ .

Inhomogeneous phase

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## Shifted field observable $\Sigma(x)$

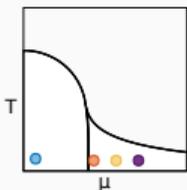
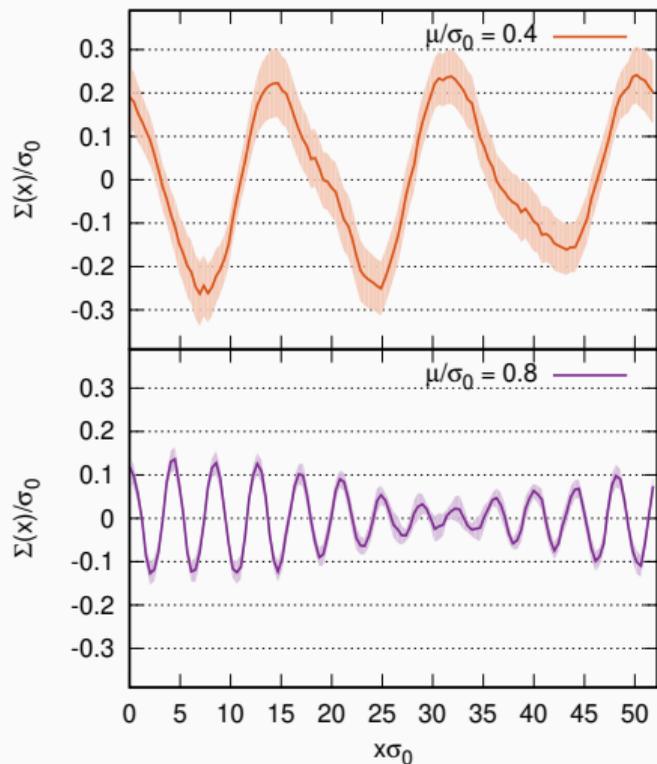
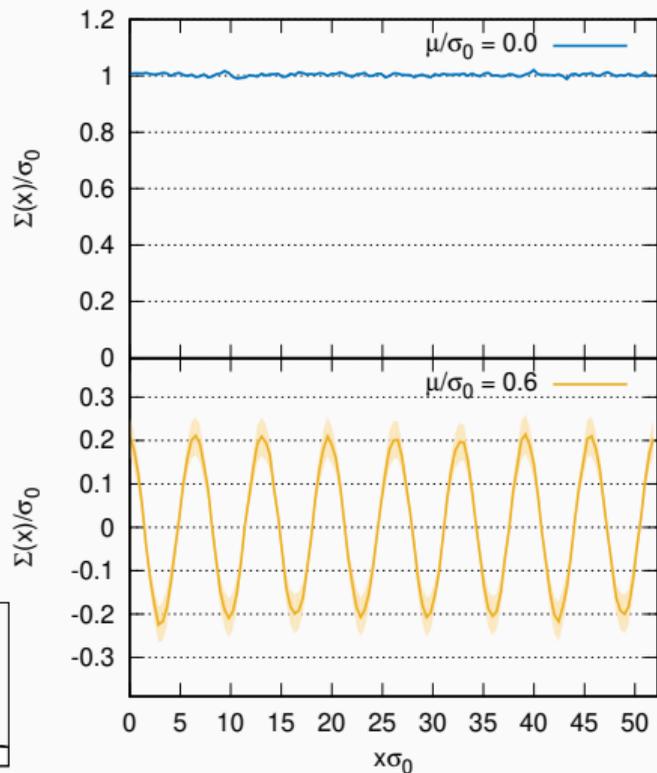
- Oscillations vanish when averaging  $\sigma(t, x)$  over configurations (Phase shift).
- Align configurations before averaging

$$\begin{aligned}\tilde{\sigma}(k) &:= \sum_{x \in \Lambda} \sigma(x) e^{-i \frac{2\pi}{N_s} x \cdot k} \\ &\rightarrow \sum_{x \in \Lambda} \sigma(x + x_{\text{shift}}) e^{-i \frac{2\pi}{N_s} x \cdot k} = e^{i \frac{2\pi}{N_s} x_{\text{shift}} \cdot k} \sum_{x \in \Lambda} \sigma(x) e^{-i \frac{2\pi}{N_s} x \cdot k} \\ &= e^{i \frac{2\pi}{N_s} x_{\text{shift}} \cdot k} \tilde{\sigma}(k).\end{aligned}$$

- Extract  $x_{\text{shift}}$  and **shift  $\sigma$ -field on every configuration individually.**
- Define shifted field observable

$$\Sigma(x) := \frac{1}{N_t} \sum_{t=0}^{N_t} \frac{1}{N_{\text{conf}}} \sum_{n=0}^{N_{\text{conf}}} \sigma_n(t, x - x_{\text{shift},n}).$$

# $\Sigma(x)$ results



Shifted field for various  $T/\sigma_0 \approx 0.031$  and  $\mu/\sigma_0$  with  $\sigma_0 = 0.408$ ,  $L = 128$  and  $N_f = 2 \cdot 4$

## Spatial correlation observable $C(x)$

- Spatial Correlation on one Configuration

$$C_n(x) := \frac{1}{V} \int_V \sigma(t, y) \cdot \sigma(t, y + x) dt dy$$

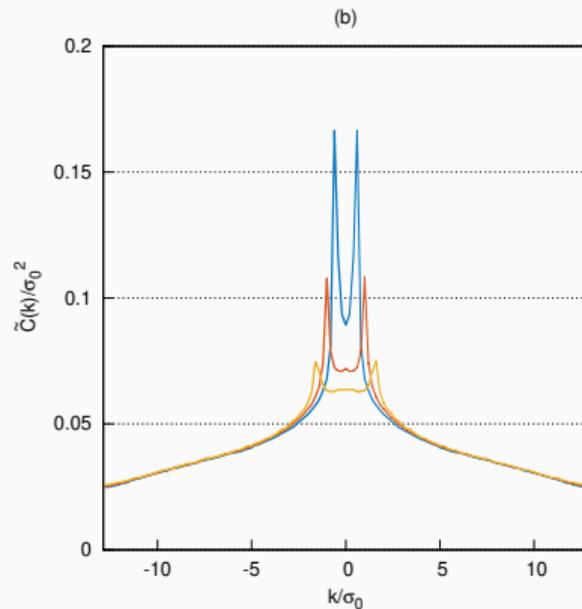
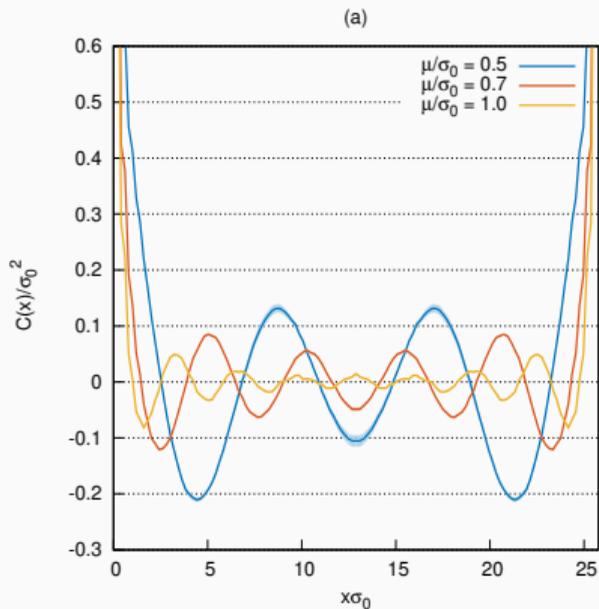
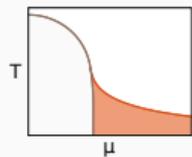
- Expectation value of the spatial correlation

$$C(x) := \frac{1}{N_{\text{conf}}} \sum_{n=0}^{N_{\text{conf}}} C_n(x)$$

- Expectation value of the Fourier transform of spatial correlation

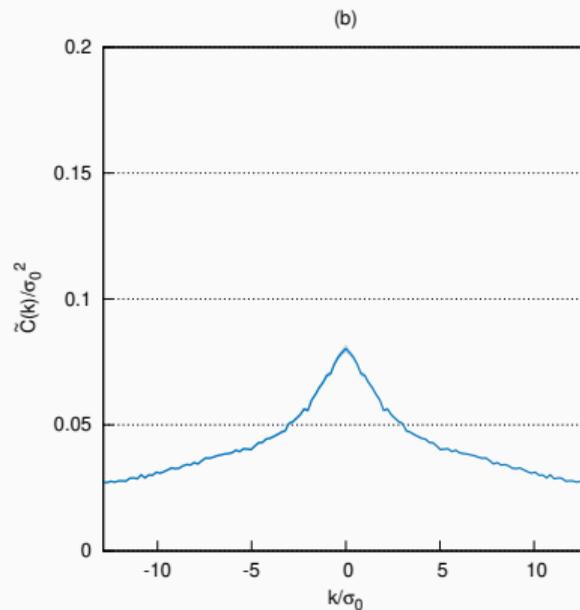
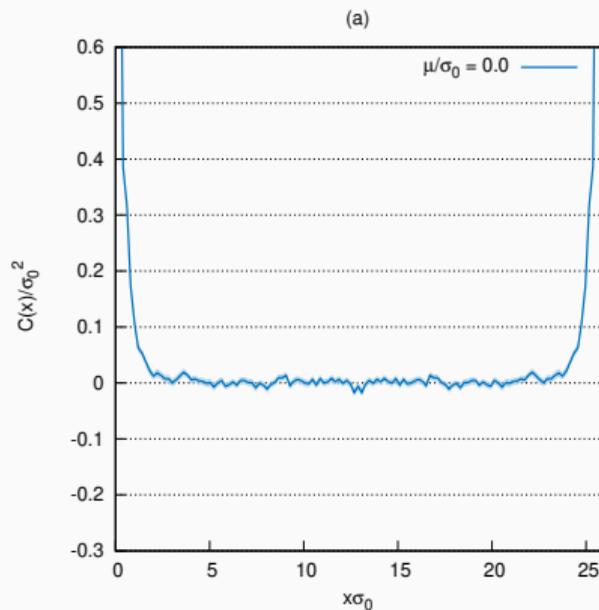
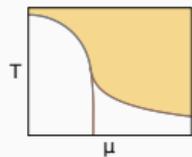
$$\tilde{C}(k) := \mathcal{F}[C](k)$$

# $C(x)$ results - inhomogeneous phase



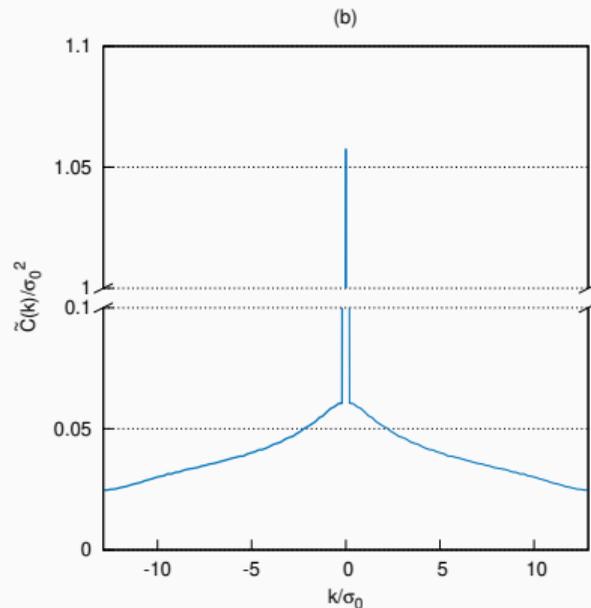
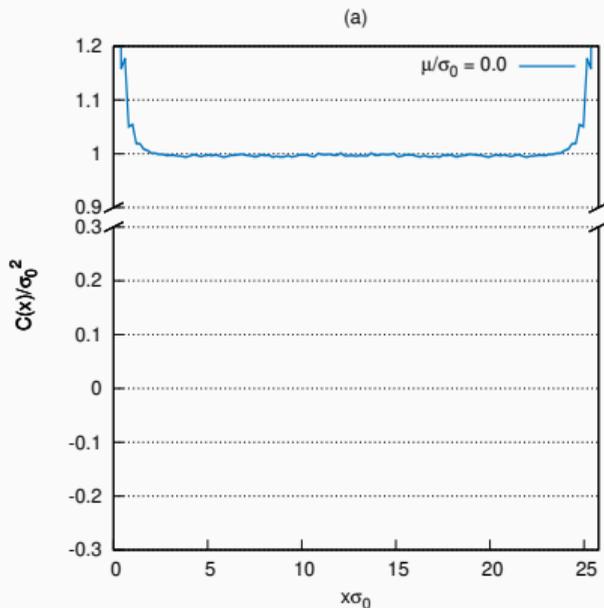
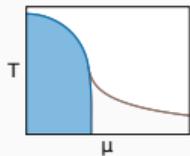
$C(x)$  in (a) and  $\tilde{C}(k)$  in (b) for the inhomogeneous phase at  $T/\sigma_0 = 0.078$  for various  $\mu/\sigma_0$  with  $\sigma_0 = 0.2013$ ,  $L = 128$  and  $N_f = 2 \cdot 4$ .

# $C(x)$ results - restored phase



$C(x)$  in (a) and  $\tilde{C}(k)$  in (b) for the restored phase at  $T/\sigma_0 = 0.621$  and  $\mu/\sigma_0 = 0$  with  $\sigma_0 = 0.2013$ ,  $L = 128$  and  $N_f = 2 \cdot 4$ .

# $C(x)$ results - homogeneously broken phase

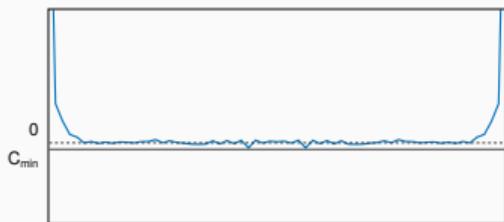


$C(x)$  in (a) and  $\tilde{C}(k)$  in (b) for the homogeneously broken phase at  $T/\sigma_0 = 0.062$  and  $\mu/\sigma_0 = 0$  with  $\sigma_0 = 0.2013$ ,  $L = 128$  and  $N_f = 2 \cdot 4$ .

- To plot a phase diagram, we need to encode the information about the phase in a single number.

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- Employ the minimum of  $C(x)$ :

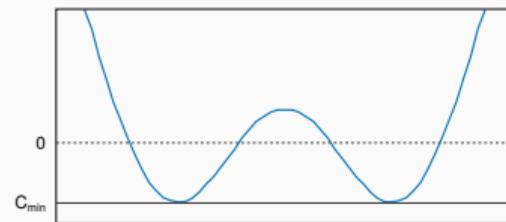
$$C_{\min} = \min_x C(x) \begin{cases} \approx 0, & \text{in restored phase} \\ > 0, & \text{in homogeneously broken phase} \\ < 0, & \text{in inhomogeneous phase} \end{cases}$$



Restored phase



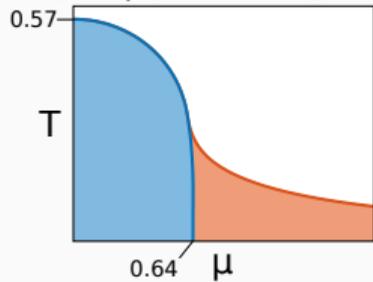
Homogeneously broken phase



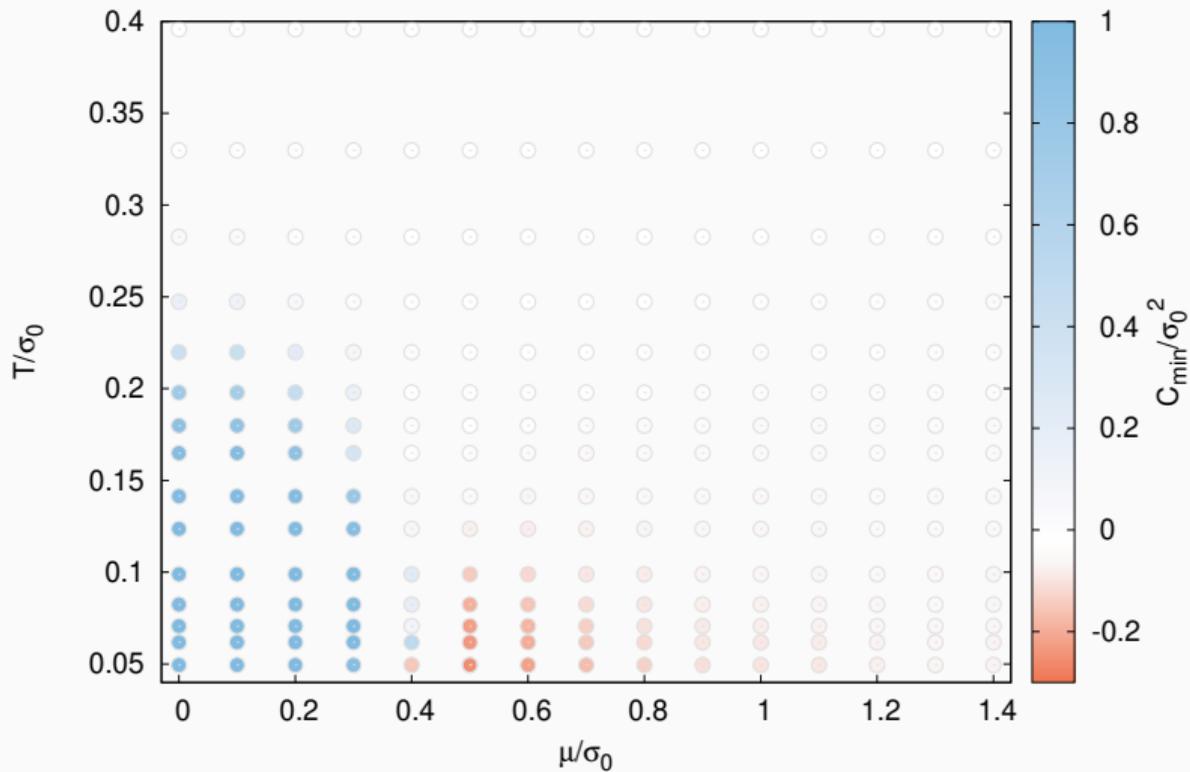
Inhomogeneous phase

# Phase diagram of $C_{\min}$

large- $N_f$  phase diagram



Phase diagram for  $C_{\min}$  with  $\sigma_0 = 0.2527$ ,  $L = 128$  and  $N_f = 2 \cdot 4$ .



# Summary and next steps

- Different ways to explore the different phases ( $\Sigma(x)$ ,  $C(x)$ ,  $C_{\min}$ ).
- The structure of finite  $N_f$  phase diagram shows similarities to that of the large- $N_f$  phase diagram.
- In the future: go to higher dimensions, explore other models.

