β dependence of the nuclear transition end points at finite quark masses

Jangho Kim Owe Philipsen Wolfgang Unger

Goethe University Frankfurt am Main

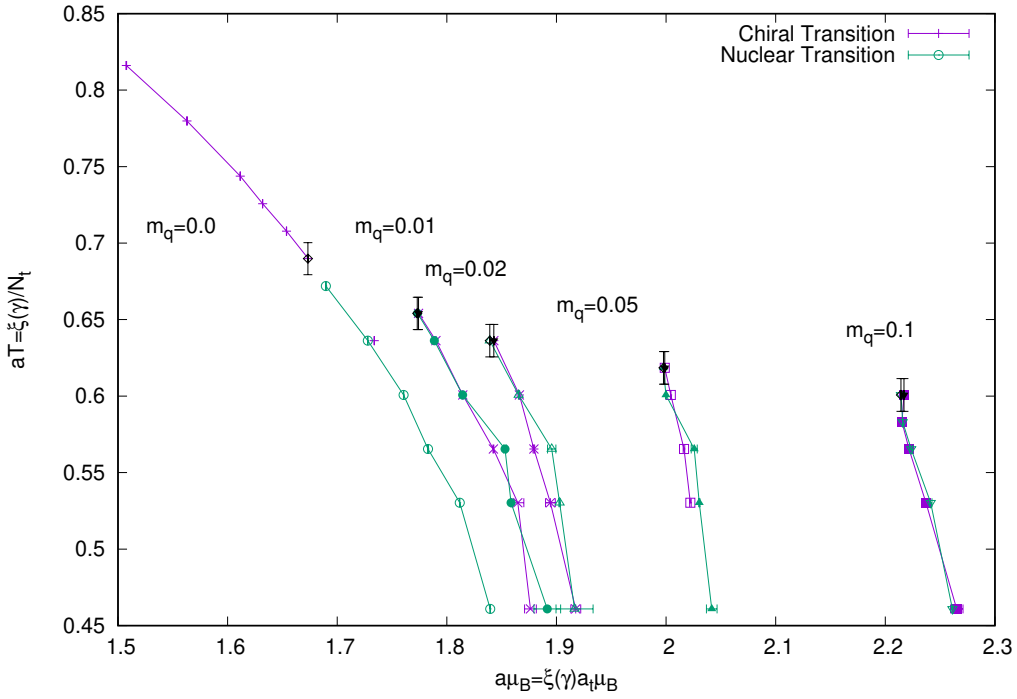
Lattice 2019, Wuhan, China, June 16-22, 2019

Outline

Previous Results I

Quark mass dependence of critical end point in the strong coupling limit

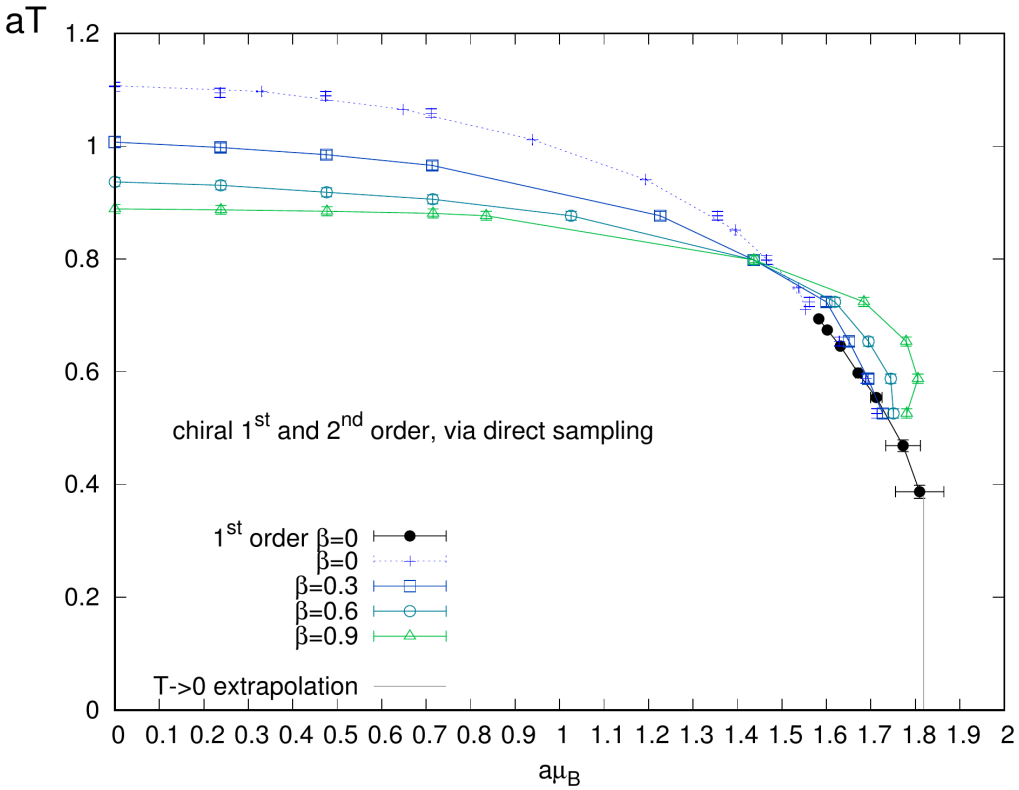
- We have recently addressed the dependence of the nuclear critical end point as a function of the quark mass m_q in the strong coupling limit ($\beta = 0$) (LATTICE2016)



Previous Results II

Inverse gauge coupling β dependence in the chiral limit

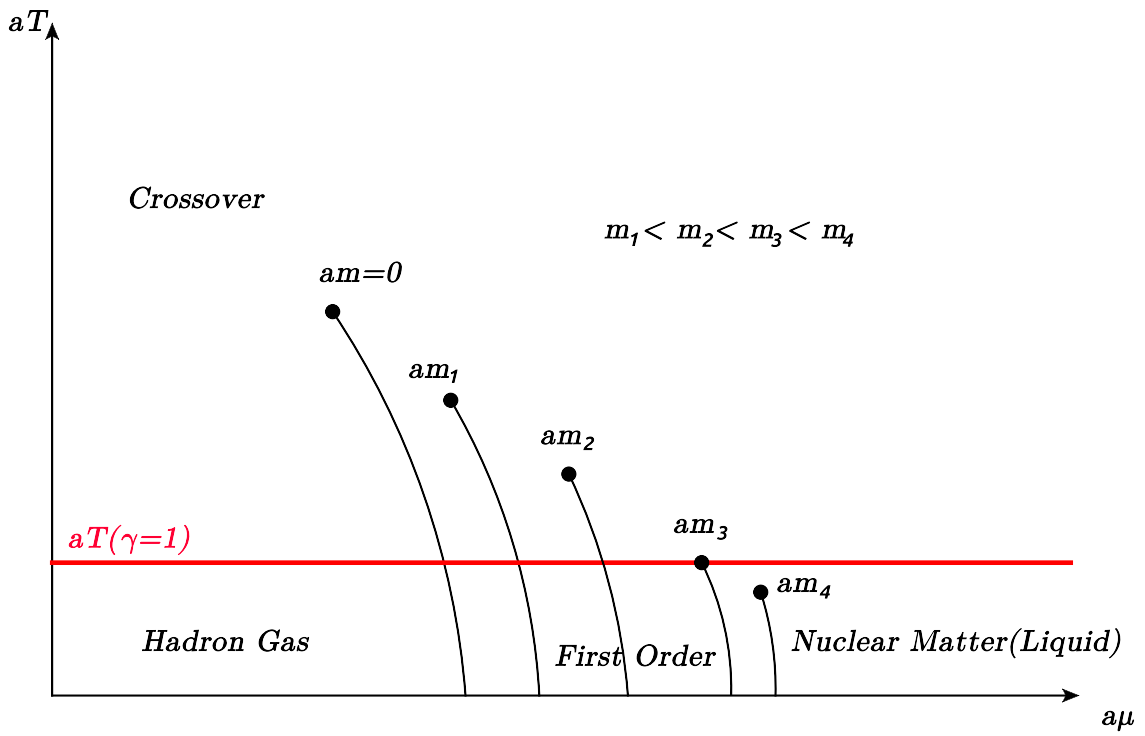
- We have presented the numerical results from the direct simulations away from the strong coupling limit.(LATTICE2017)



- Back bending occurred because we don't apply the correct anisotropic($\gamma^2 = a/a_t$).

Simulation Strategy

- Staggered fermion in dual representation
- On the isotropic lattice($\gamma^2 = a/a_t = 1$), $N_t = 4$
- aT is fixed to $aT = 0.25 = \frac{1}{N_t}$ in the strong coupling limit
- $aT = aT(\beta)$ in finite β (see Wolfgang Unger's talk).
- if am_q increases, aT_{CEP} decreases and $a\mu_{CEP}$ increases



Dual representation in the strong coupling regime

- The partition function after Grassmann integration,

$$Z(m_q,\mu,\gamma) = \sum_{\{k,n,\ell,n_p\}} \underbrace{\prod_{b=(x,\mu)} \frac{(N_c-k_b)!}{N_c!(k_b-|f_b|)!}\gamma^{2k_b\delta_{\hat{0},\hat{\mu}}}}_{\text{singlet hoppings } M_x M_y} \underbrace{\prod_x \frac{N_c!}{n_x!}(2am_q)^{n_x}}_{\text{chiral condensate } \bar{\psi}\psi}$$

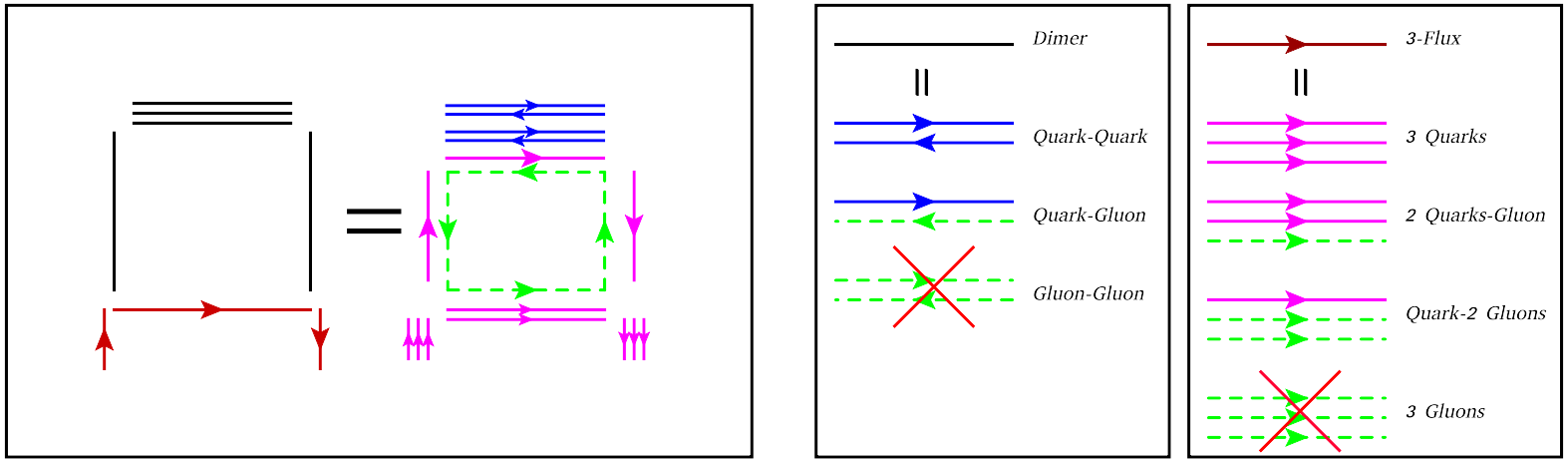
$$\times \underbrace{\prod_{\ell_3} w(\ell_3,\mu)}_{\text{triplet hoppings } \bar{B}_x B_y} \underbrace{\prod_{\ell_f} \tilde{w}(\ell_f,\mu)}_{\text{weight modification}} \underbrace{\prod_P \frac{\left(\frac{\beta}{2N_c}\right)^{n_P+\bar{n}_P}}{n_P!\bar{n}_P!}}_{\text{gluon propagation}}$$

- f_b: the number of gauge fluxes at bond b
- n_p: plaquette(counterclockwise) occupation number
- $\bar{n}_p$: plaquette(clockwise) occupation number
- $\tilde{w}(\ell_f)$: weight modification at $O(\beta^2)$

Finite β corrections

- Grassmann constraint is modified
- Purely gluonic objects are excluded

$$n_x + \sum_{\hat{\mu}=\pm\hat{0},\ldots\pm\hat{d}} \left(k_{\hat{\mu}}(x) + \frac{N_c}{2}|\ell_{\hat{\mu}}(x)| \right) = N_c + \sum_{\hat{\nu}=\pm\hat{0},\ldots\pm\hat{d}} \frac{1}{2}|f_{\hat{\nu}}(x)|$$

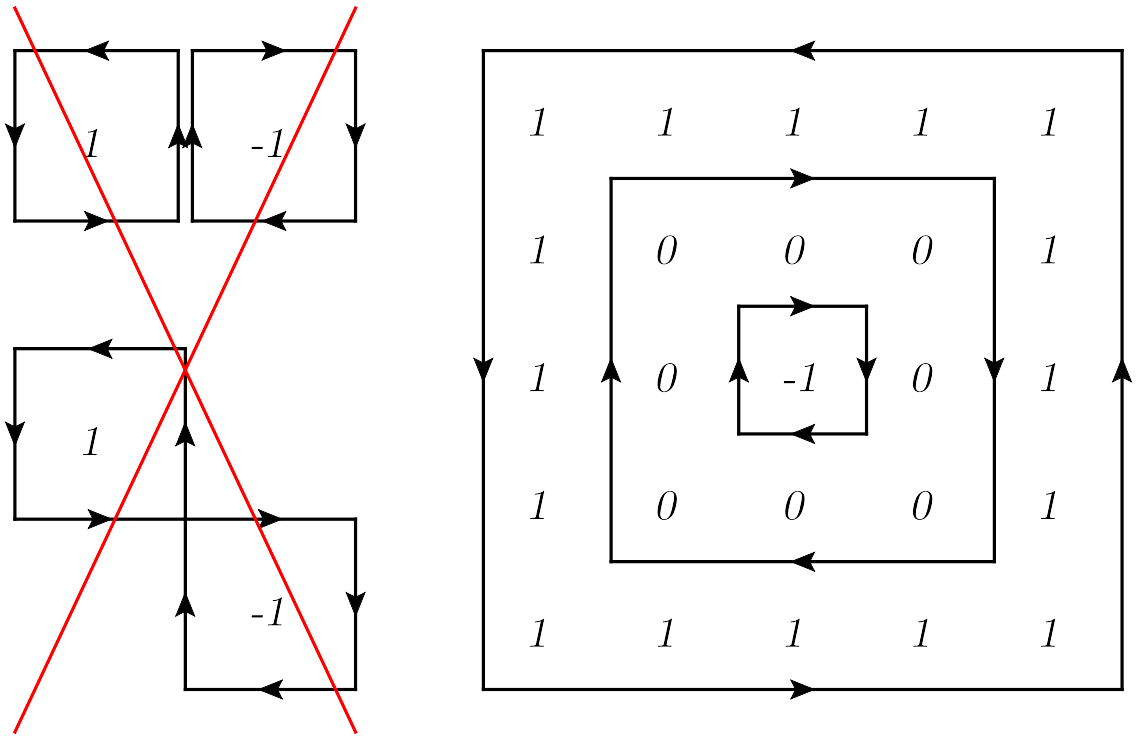


(a) At the corner of a plaquette, the color constraints is $N_C + 1$

(b) Dimer and 3-Flux can be composed of quark and gluon fluxes

Plaquette Update

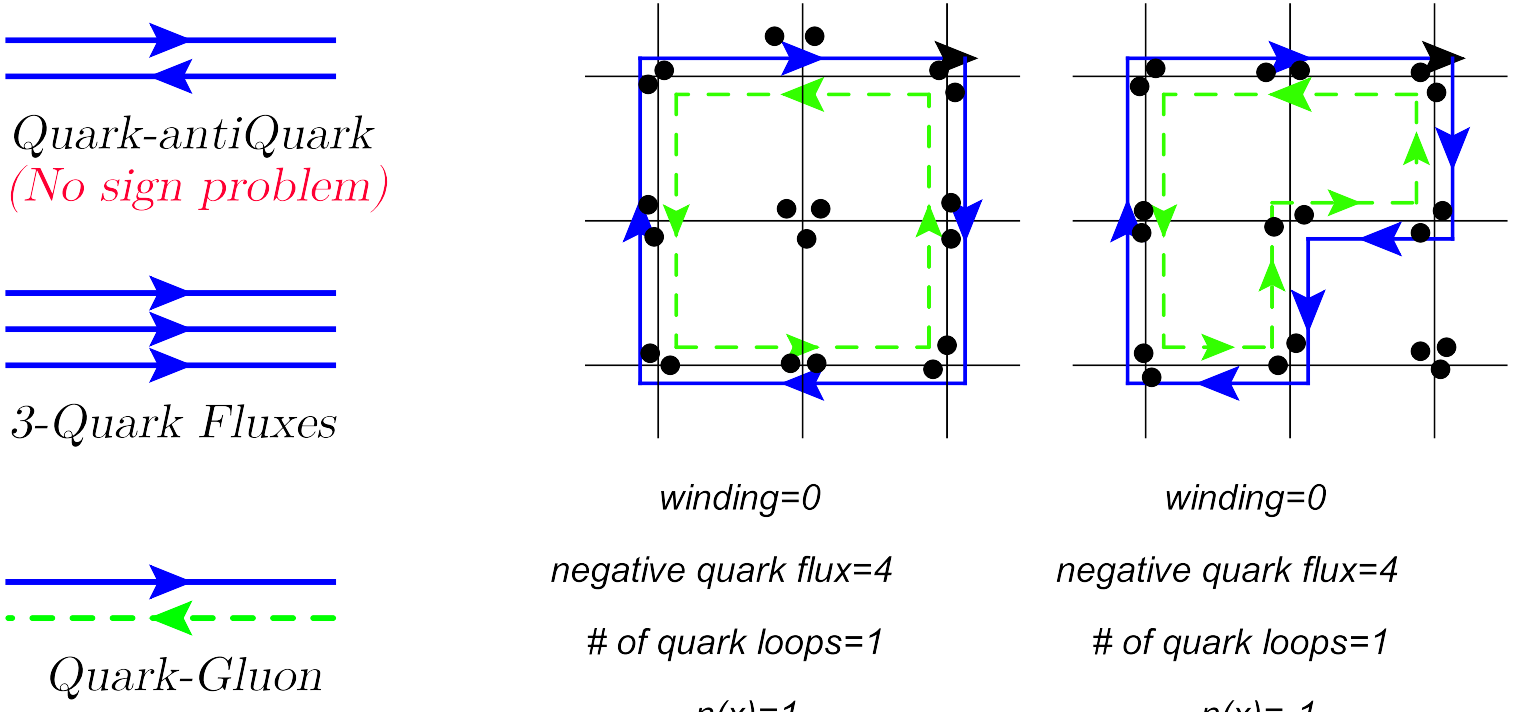
- We take into account the β corrections via plaquette occupation numbers using Metropolis update in addition to the worm algorithm.
- Two difference between adjacent plaquettes are not allowed.
- Our plaquette update completely covers $O(\beta)$ but has a few missing configurations in higher order. ($\beta <= 1$)



Induced Sign Problem ($\beta > 0$)

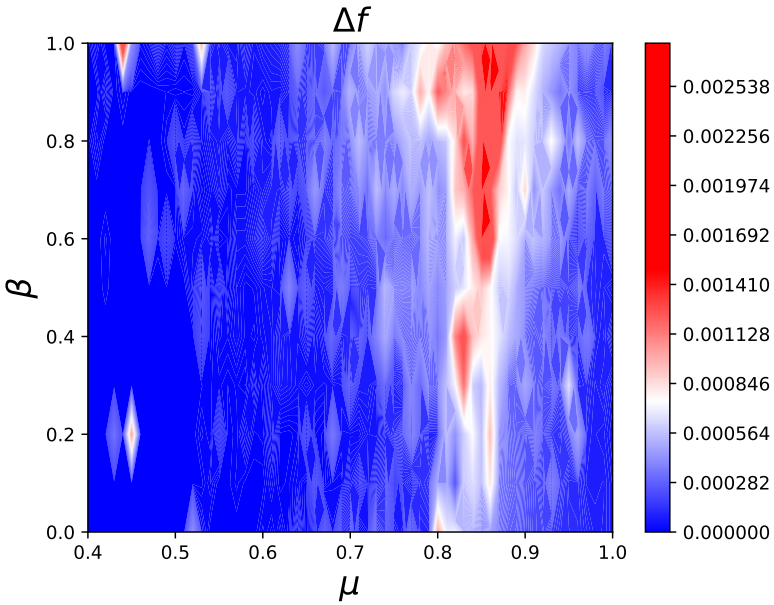
- The sign comes from the quark loop (ℓ_q).

$$\sigma(\ell_q) = (-1)^{w(\ell_q)+N_-(\ell_q)+1} \prod_{b=(x,\hat{\mu})\in\ell_q} \eta_{\hat{\mu}}(x)$$

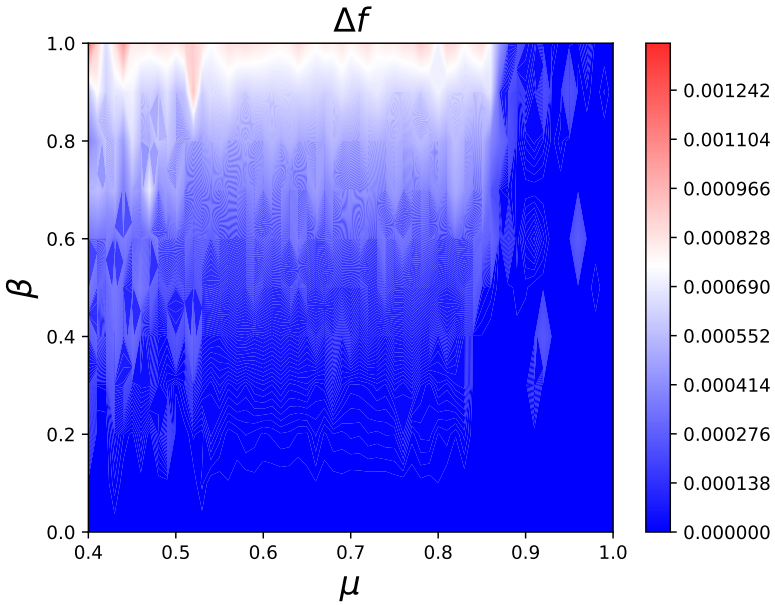


Sign Problem

- $\langle\sigma\rangle = \exp(-L^3 N t \Delta f)$
- Δf is the difference between full and sign-quenched free energy density



(c) From baryon loops

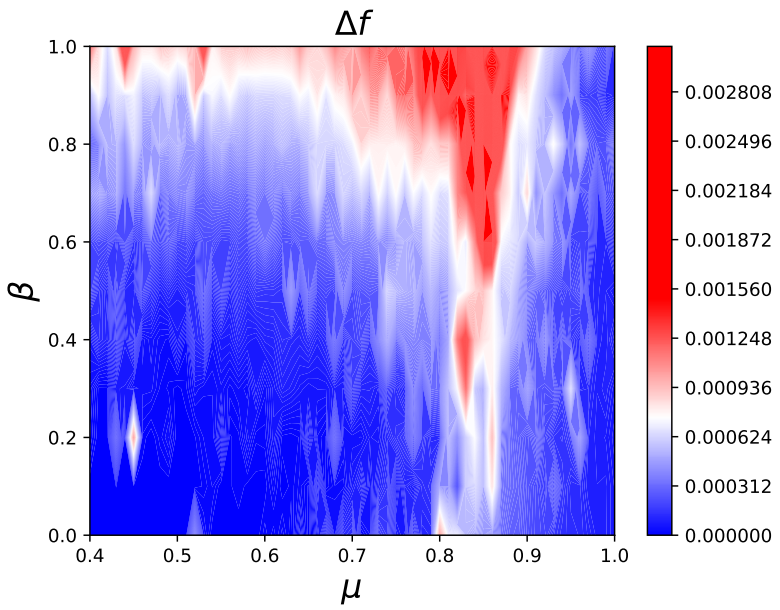


(d) $\beta > 0$

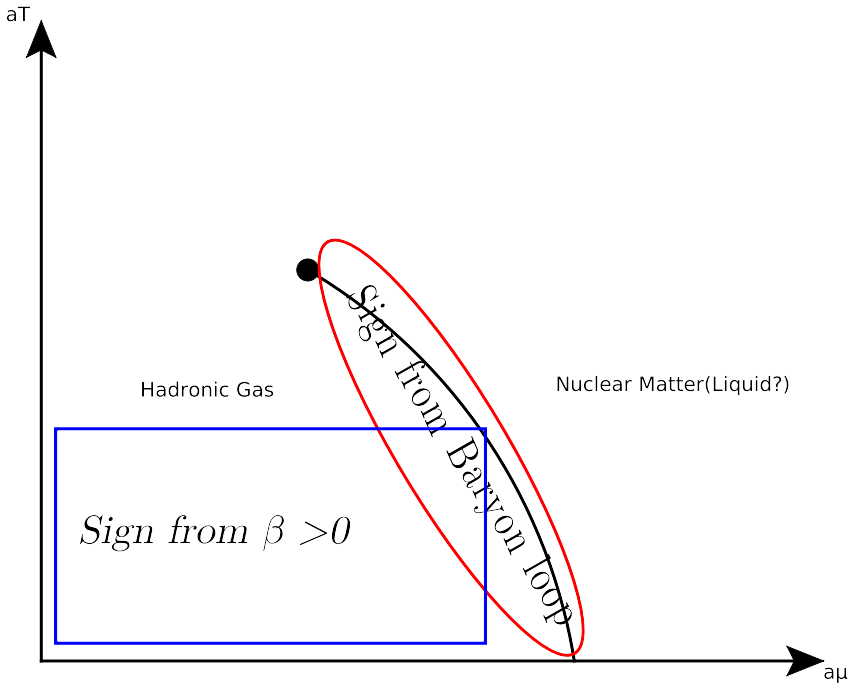
Sign Problem

Total sign problem

- $\langle \sigma \rangle = \exp(-L^3 N t \Delta f)$
- Δf is the difference between full and sign-quenched free energy density



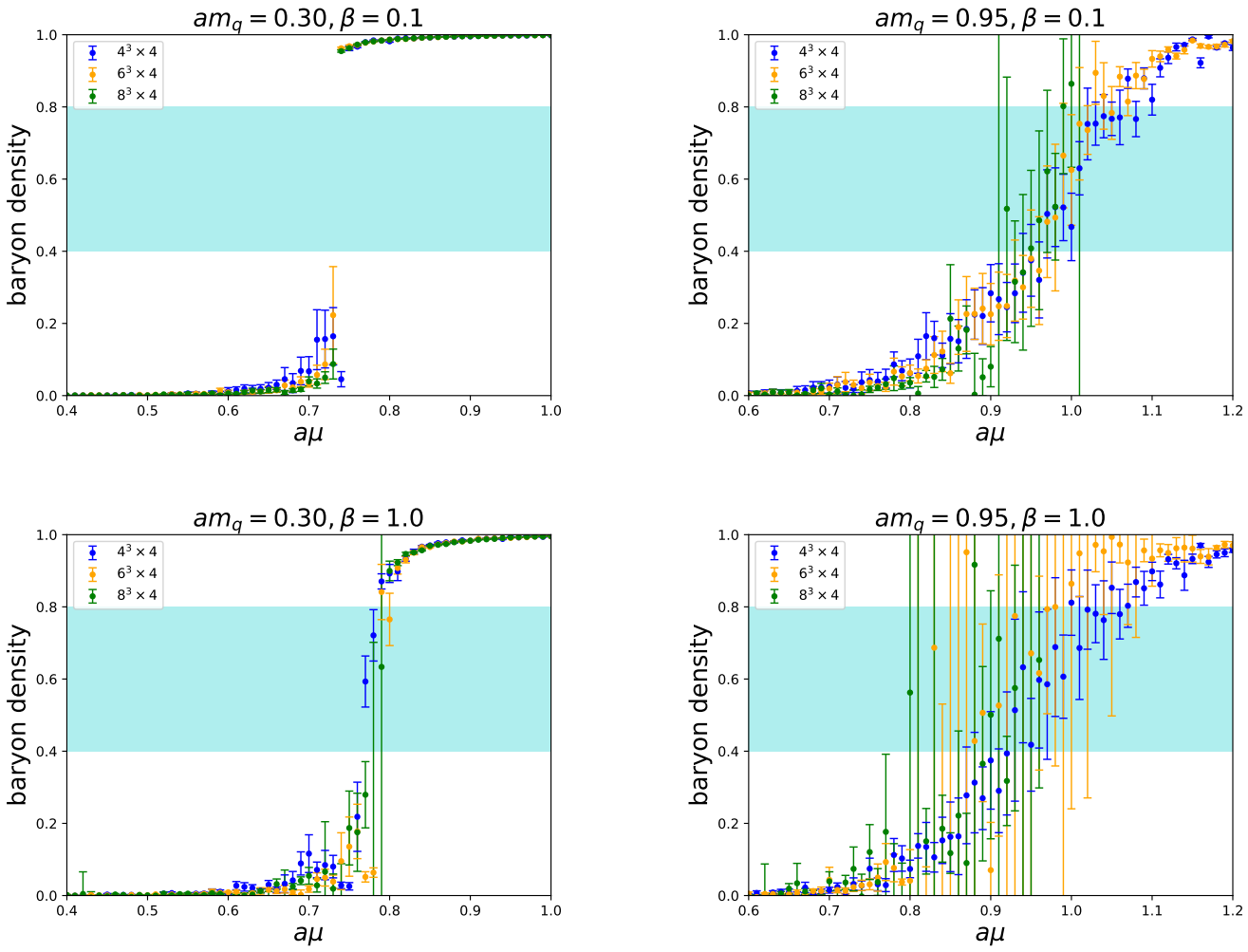
(e) Total sign



(f) Sign problems from difference sources

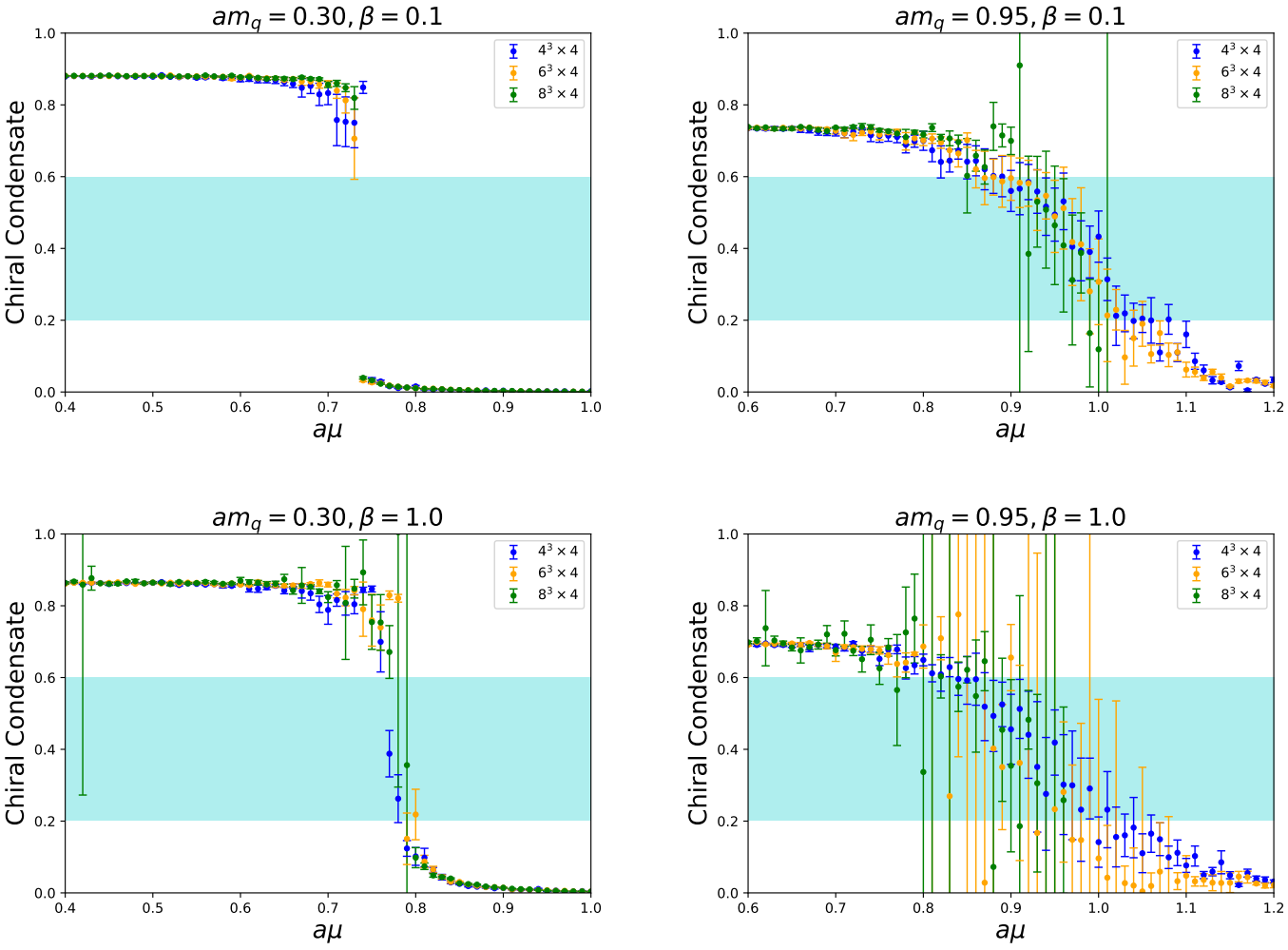
Baryon density(Preliminary)

- Determine $a\mu_c$, if data points cross the band.



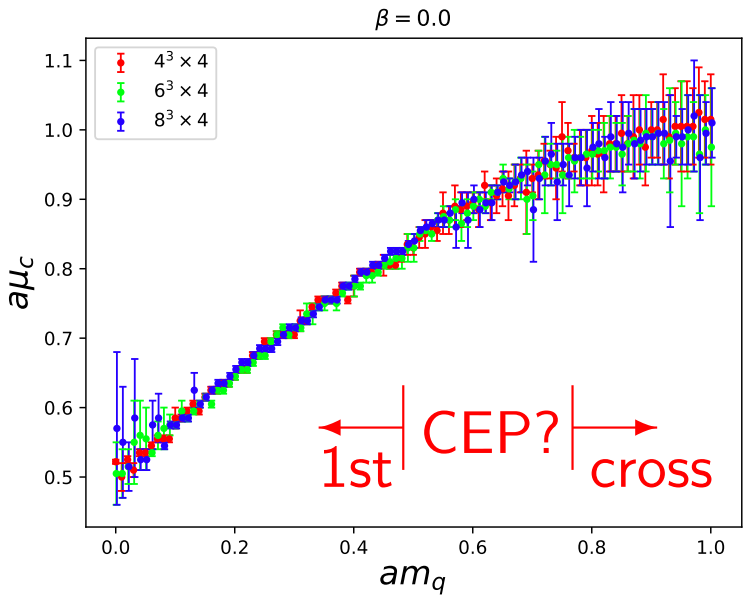
Chiral Condensate(Preliminary)

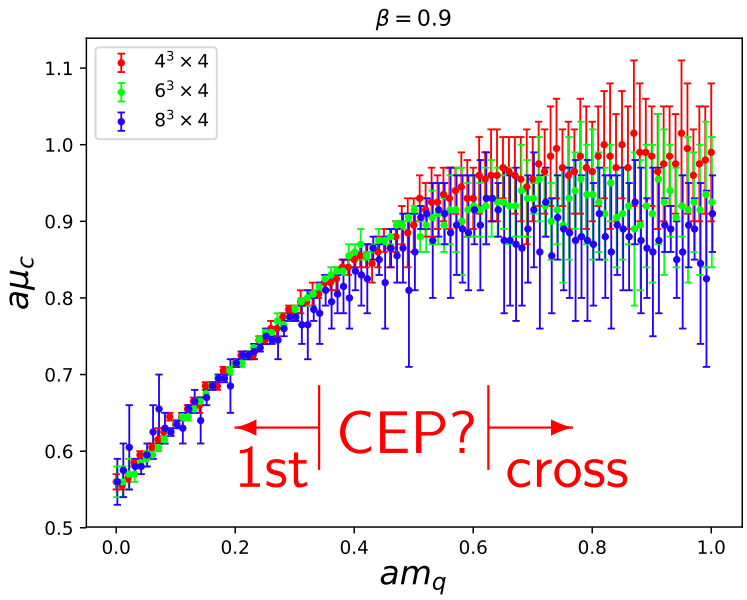
- Chiral condensates give same phase boundary with baryon density.
- No splitting between nuclear and chiral transition in finite β yet.



Results of phase boundary(Preliminary)

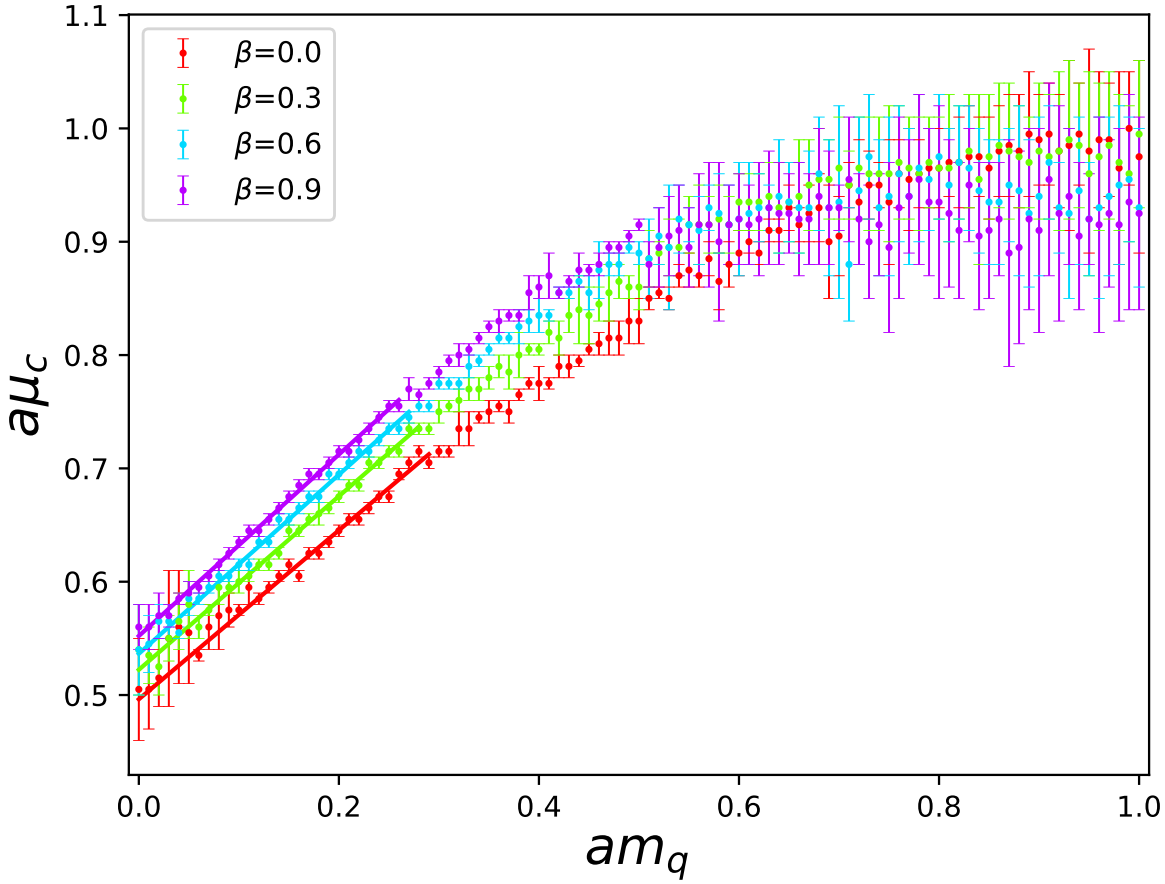
- If β increases, the critical end point looks like moving to the small m_q.
- Need more statistics to determine the critical end point.





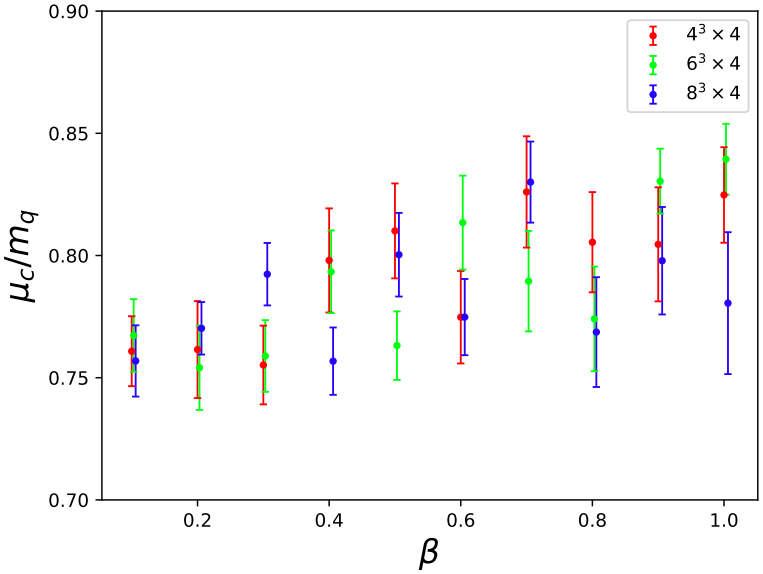
Quark mass dependence of phase boundary(Prelimin

- Phase boundary is determined from baryon density.

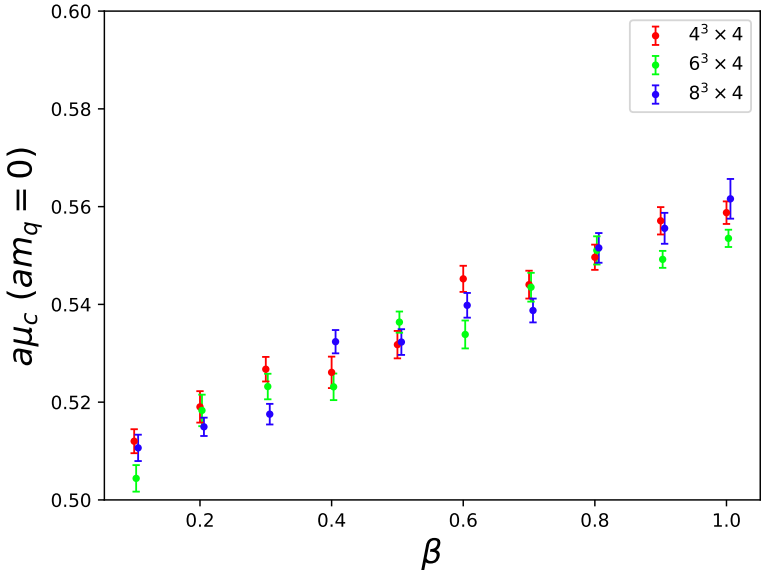


am_q dependence of $a\mu_c$(Preliminary)

- No strong β dependence on the slope(μ_c/m_q)
- $a\mu_c$ and am_q increase with β
- μ_c increases linearly with β, but very mild

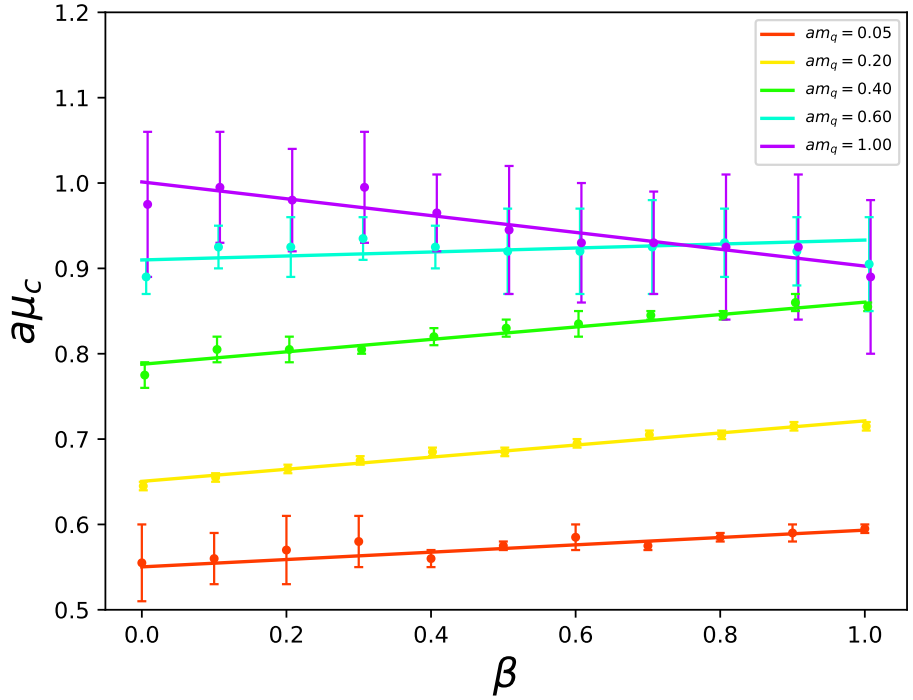


(q) μ_c/m_q

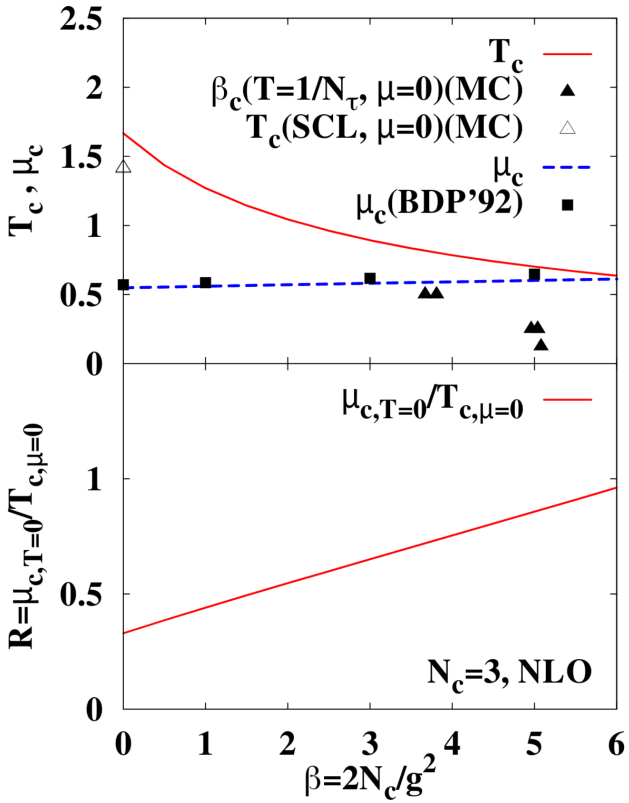


(r) μ_c at $m_q = 0$

β dependence of phase boundary(Preliminary)



(s) β dependence of $a\mu_c$



(t) All in lattice unit, $am_q = 0.05$

- Miura,Nakano,Ohnishi,Kawamoto, PRD2009
- Consistent with other group's result

Summary

- We simulate on finite β and finite quark masses.
- Very mild β dependence of $a\mu_c$.
- Outlook
  - Have to include missing plaquette configurations.
  - Need more statistics and analysis to determine the critical end point precisely.