High Temperature Expansion for QCD Effective Theories

Pham A. Quang

Joint work with O. Philipsen, J . Scheunert, J. Kim

Lattice 2019

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HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Ref.

M. Fromm, J. Langelage, S. Lottini, OP arXiv:1111.4953

J. Langelage, M. Neuman, OP arXiv:1403.4164

C. Domb ed. Phase transitions and critical phenomena Vol. 2-1974

F. Green, F. Karsch 1984

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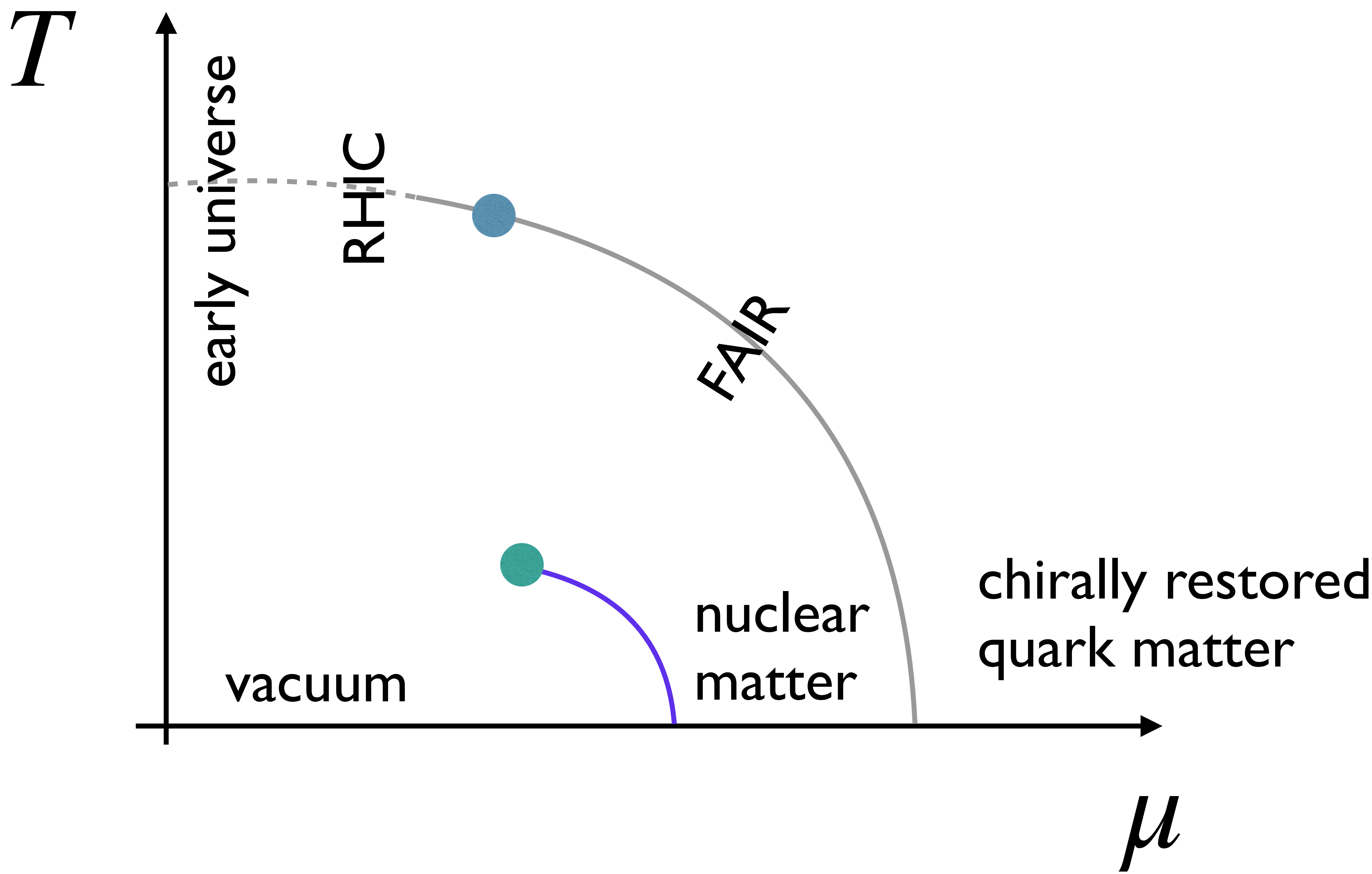
Plan

- Introduction
- QCD effective theories
- Effective YM
- High Temperature Expansion (HTE)
- Results and outlook

QCD Phase Diagram



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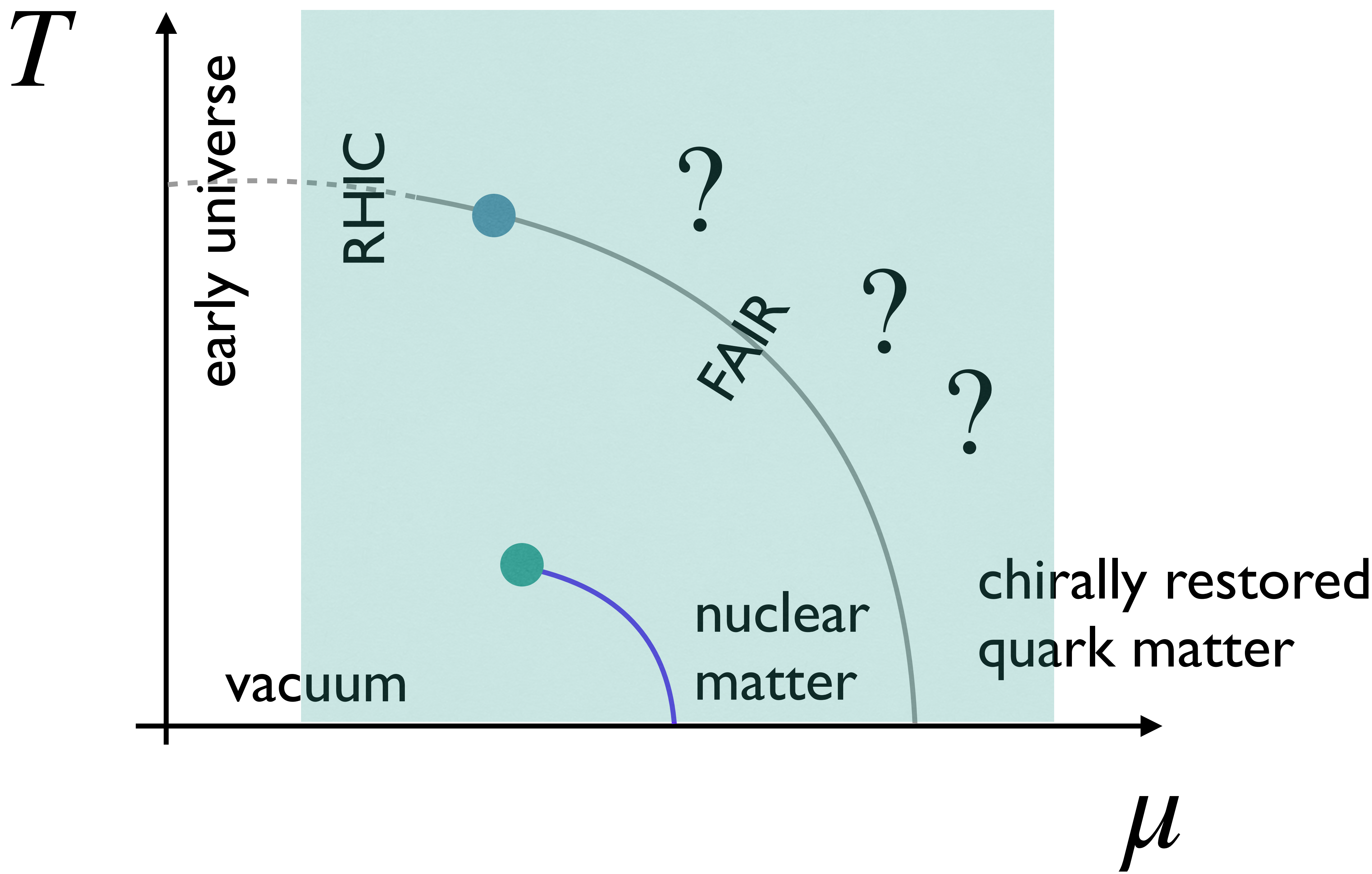

Why Effective Theories?

- At finite μ, sign problem is mild enough to simulate
- Possible to carry out computations analytically e.g. Linked Cluster or High Temperature Expansion
- Cheaper than MC

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Main focus of this talk

Why Series Expansion?

- Universality was first predicted by Series Expansion Methods
- Provides a way to study Lattice QCD at finite chemical potential (no sign problem)
- Consistent method for quantitative calculations

Lessons From Ising Model

P. Butera, M. Comi 2000

TABLE II. A comparison among recent estimates of the critical exponents γ and ν.

	This work	Series[a]	Series[b]	MC[c]	MC[d]	MC[e]	FD exp.[f]	ϵ exp.[f]
γ	1.2375(6)	1.237(2)	1.2371(4)	1.2372(17)	1.2367(20)	1.2353(25)	1.2396(13)	1.2380(50)
ν	0.6302(4)	0.6300(15)	0.63002(23)	0.6303(6)	0.6296(7)	0.6294(10)	0.6304(13)	0.6305(25)

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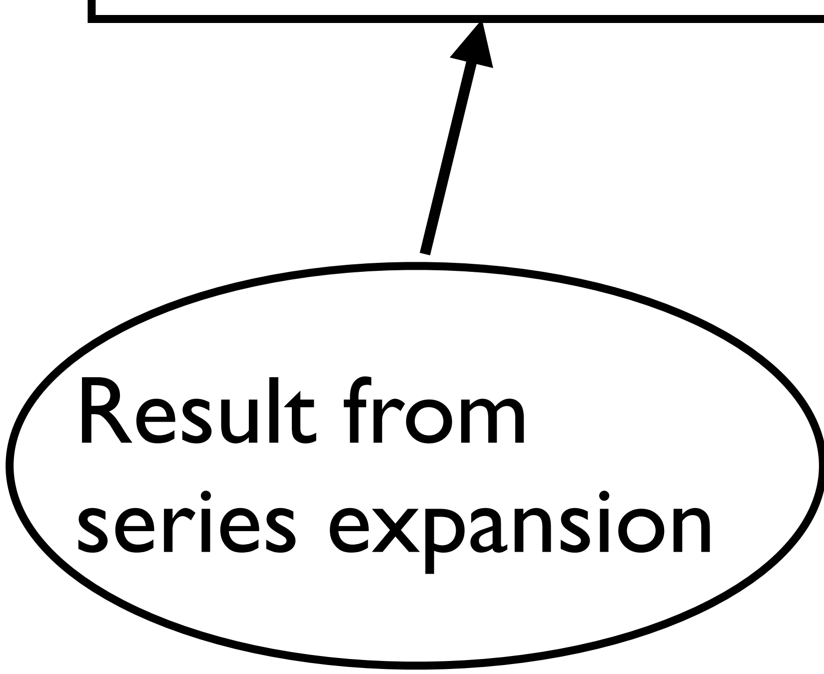

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Result from series expansion

Result from Monte Carlo

Lessons From Ising Model

$$\gamma_c = 1.237...$$

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Result from series expansion

Result from Monte Carlo

Effective Theories

The basic idea of the effective theories is integrating out all spatial links in QCD action, then the effective theories depend only on Polyakov loops

O. Philipsen, J. Langelage, S. Lottini arXiv: 1010.0951

Eg: Yang–Mills Theory

J. Glesaaen PhD thesis

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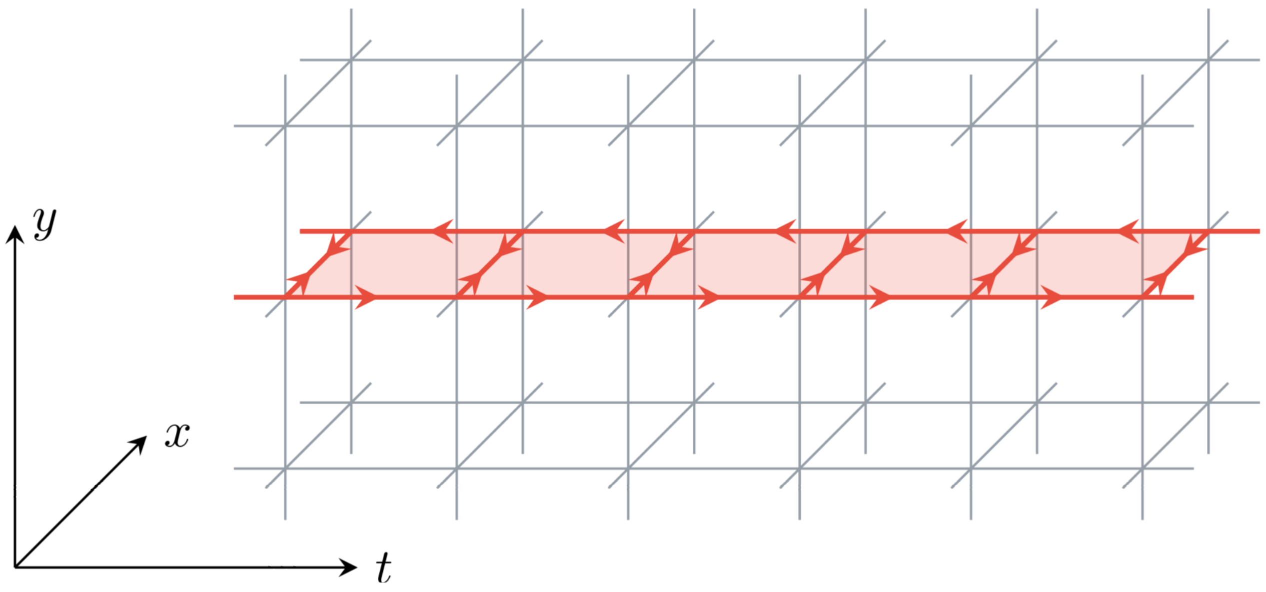
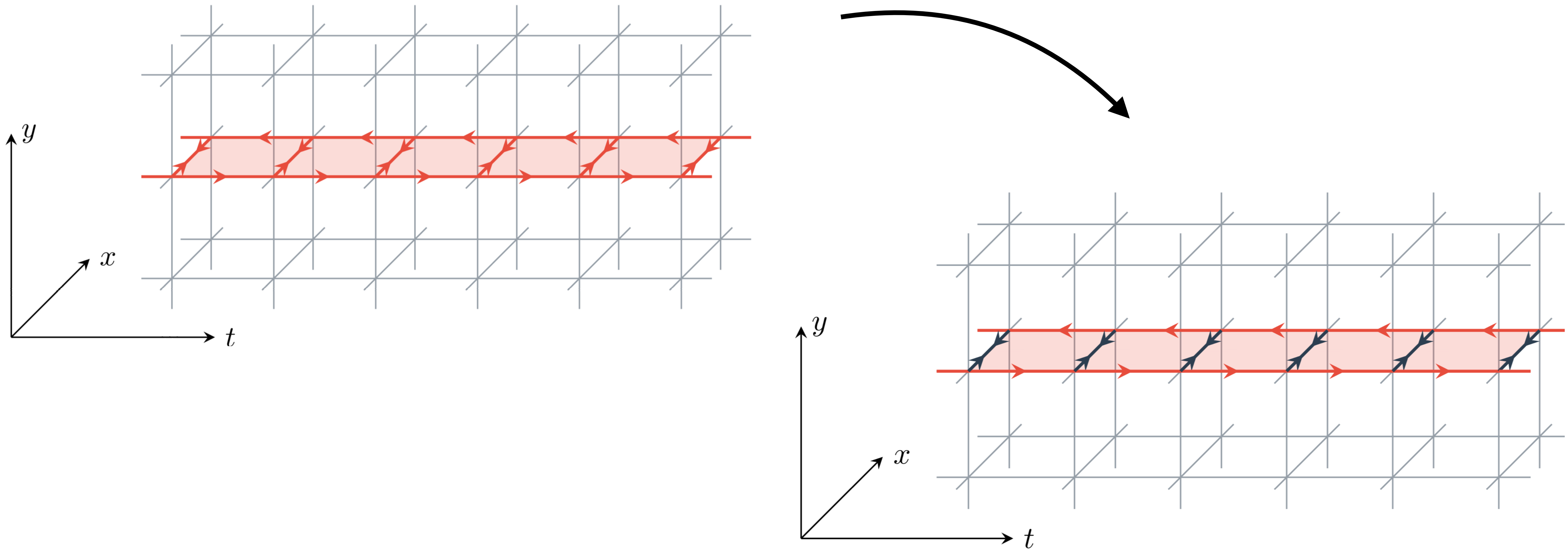


J. Glesaaen PhD thesis

Eg: Yang–Mills Theory



J. Glesaaen PhD thesis

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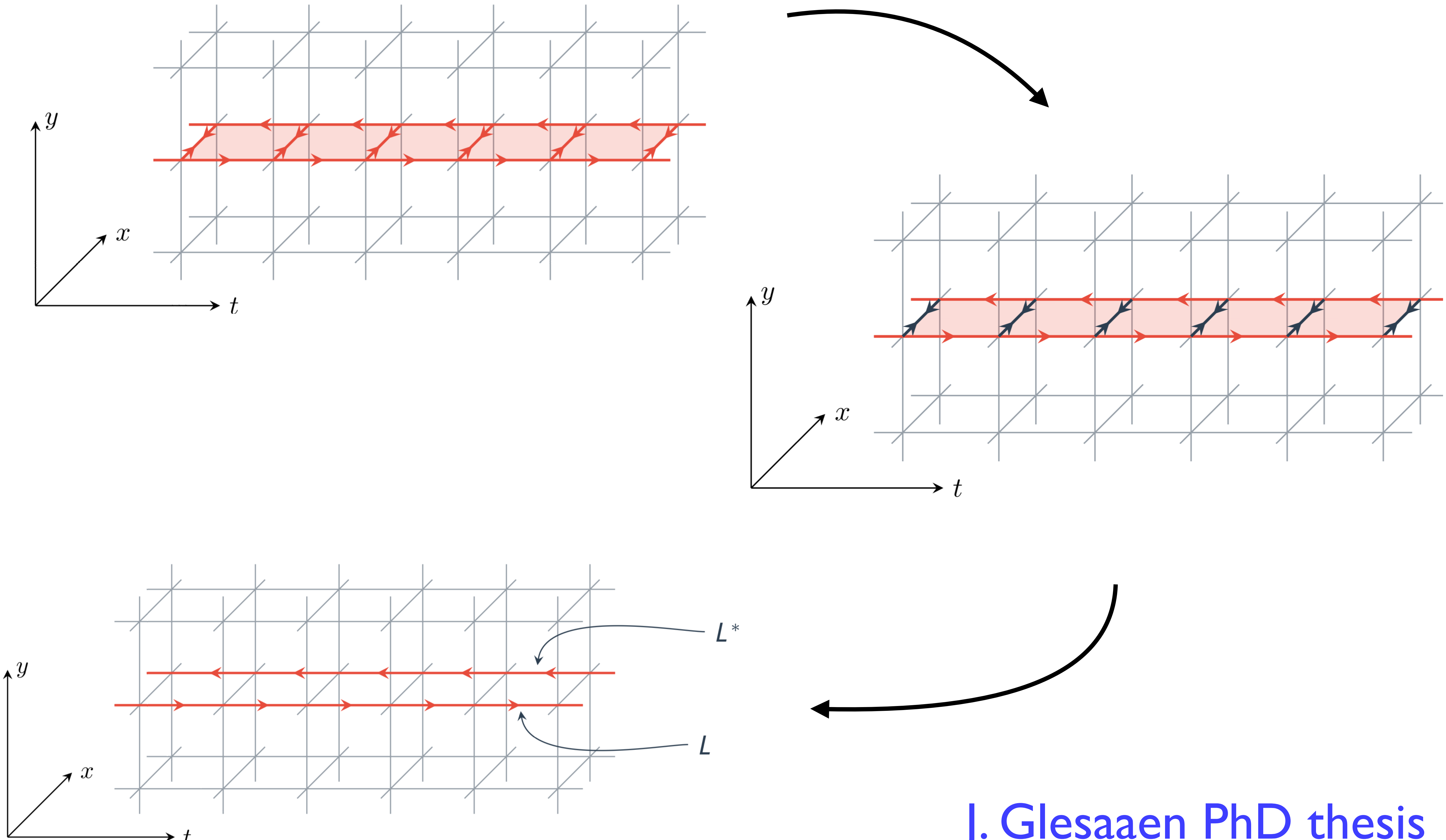

J. Glesaaen PhD thesis

Effective Yang-Mills Theory

Partition function of ET

$$Z = \int \mathrm{D}[U_0] \prod_{\langle \vec{x},\vec{y} \rangle} \Big(1 + \lambda(L_{\vec{x}} L^*_{\vec{y}} + L^*_{\vec{x}} L_{\vec{y}})\Big)$$

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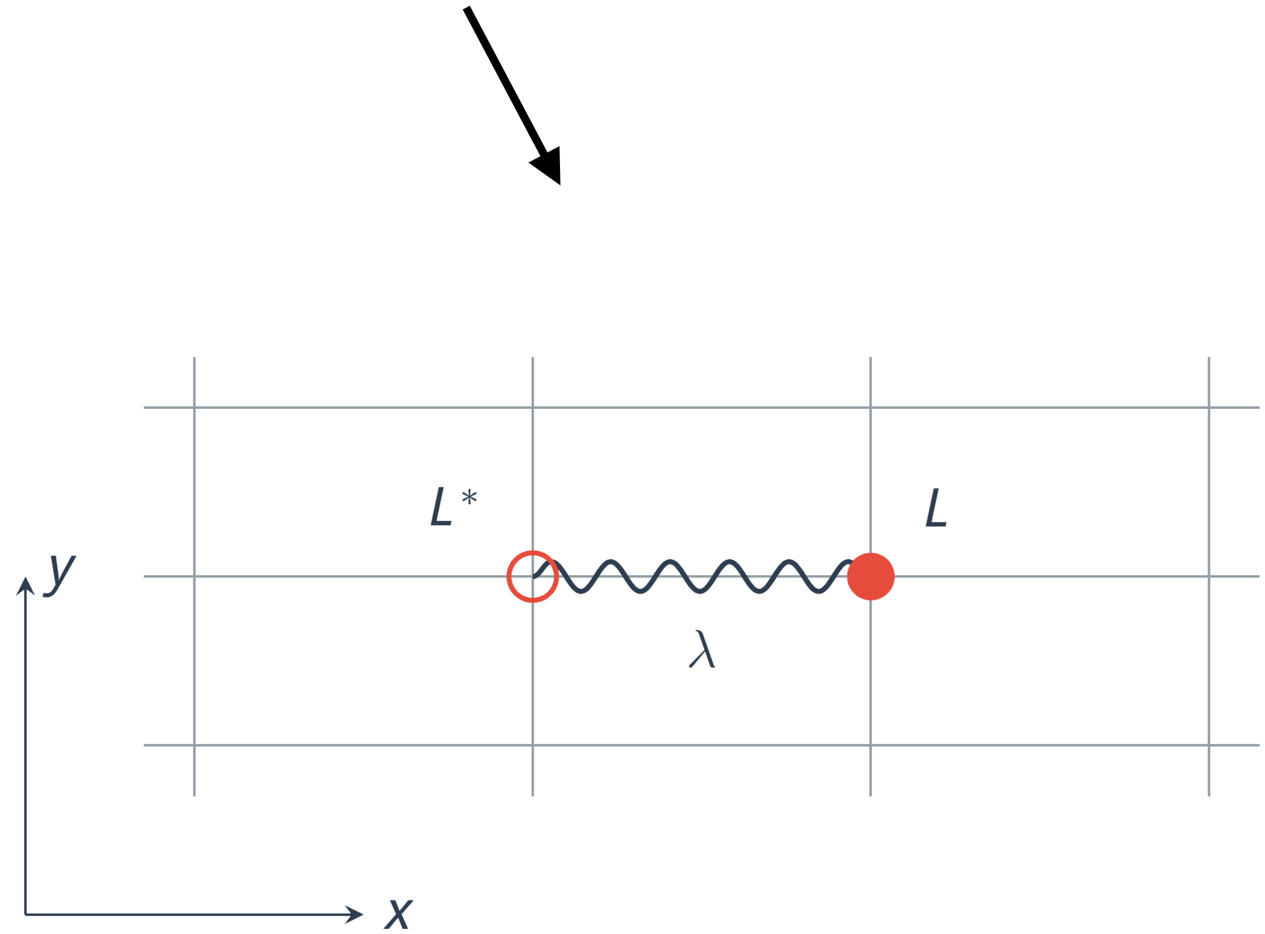

Effective Yang-Mills Theory

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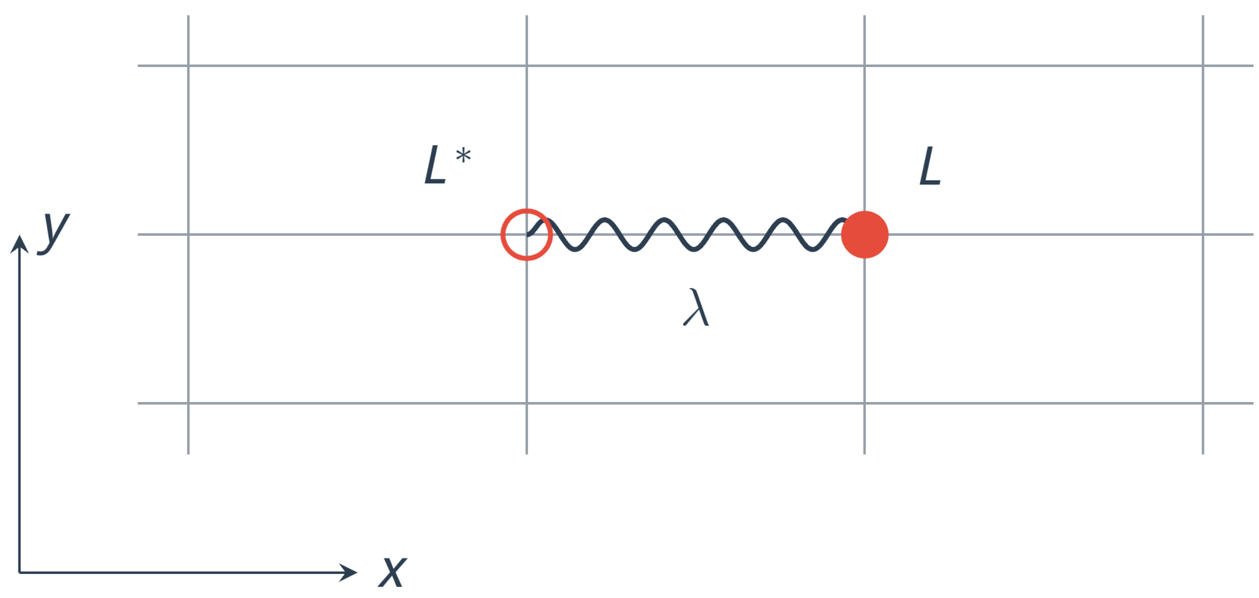
Polyakov loop

$$L_{\vec{x}} = \mathrm{tr} \prod_{t=0}^{N_t} U_0(t, \vec{x})$$



Ising–Effective Theory Analogy

Ising model

$$e^{-\beta H} = (\cosh\beta J)^{\#} \prod_{\langle i,j\rangle} (1 + s_i s_j \tanh\beta J)$$

$$Z = \sum_{\{s_i\}} e^{-\beta H}$$

$$= (\cosh\beta J)^{\#} \sum_{\{s_i\}} \prod_{\langle i,j\rangle} (1 + \tanh\beta J \, s_i s_j)$$

$$s = \frac{1}{N} \sum_{i=1}^{N} s_i$$

$$\chi_s = \frac{\partial^2 \log Z[b]}{\partial b^2}\Big|_{b=0} = V(\langle s^2\rangle - \langle s\rangle^2)$$

. . .

EYM

$$e^{-S_{\text{eff}}} = \prod_{\langle \vec{x},\vec{y}\rangle} (1 + \lambda(N_t, u)(L_{\vec{x}} L_{\vec{y}}^* + L_{\vec{x}}^* L_{\vec{y}}))$$

$$Z = \int \mathrm{D}[U_0] \prod_{\langle \vec{x},\vec{y}\rangle} \left(1 + \lambda(L_{\vec{x}} L_{\vec{y}}^* + L_{\vec{x}}^* L_{\vec{y}})\right)$$

$$L = \frac{1}{N} \sum_{\vec{x}} (L_{\vec{x}} + L_{\vec{x}}^*)$$

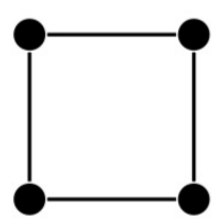
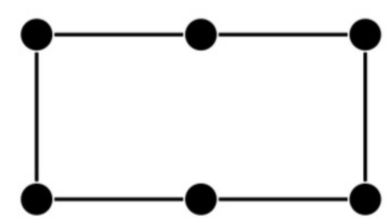
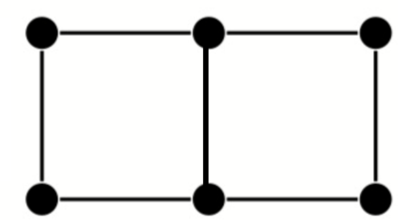
$$\chi_L = \frac{\partial^2 \log Z[J]}{\partial J^2}\Big|_{J=0} = V(\langle L^2\rangle - \langle L\rangle^2)$$

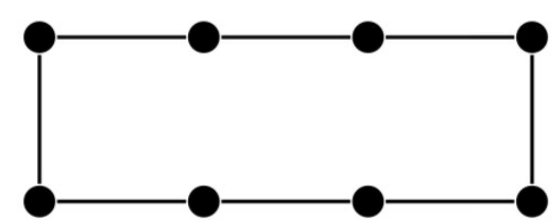
. . .

Graph Representation (EYM)

The partition function Z has a very simple expression via graph representation

$$Z[\lambda] = \sum_{n=0}^{\infty} c_n \lambda^n$$

$Z = 1 + \lambda^4$  $+ \lambda^6$  $+ \lambda^7$ 

$+ \lambda^8$  $+ \mathcal{O}(\lambda^9)$

# of bonds $\sim$ order of λ

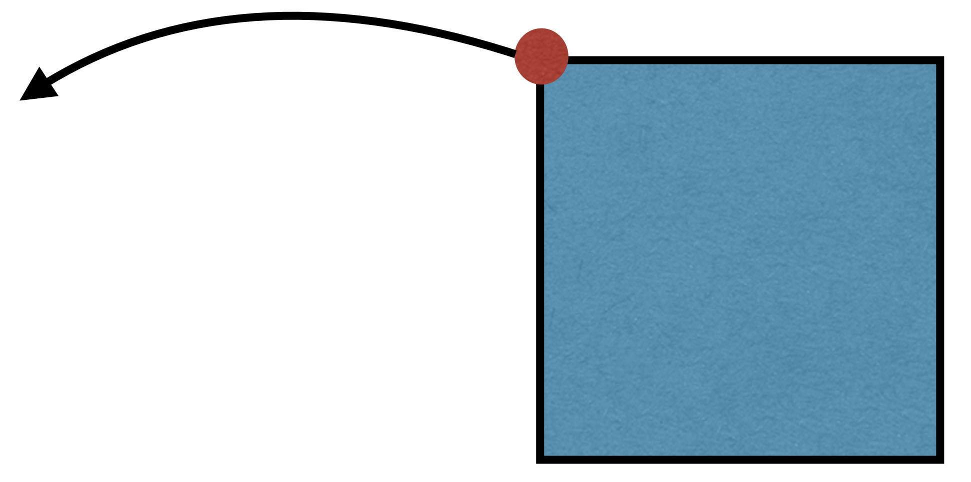
SU(3) Group Integrals

- Step 1: Group integrals at each vertex. Eg:

$$\int \mathrm{d}U_0 L^n L^{*m}$$

E.g.

$$\int \mathrm{d}U_0 \; L L^* = 1$$

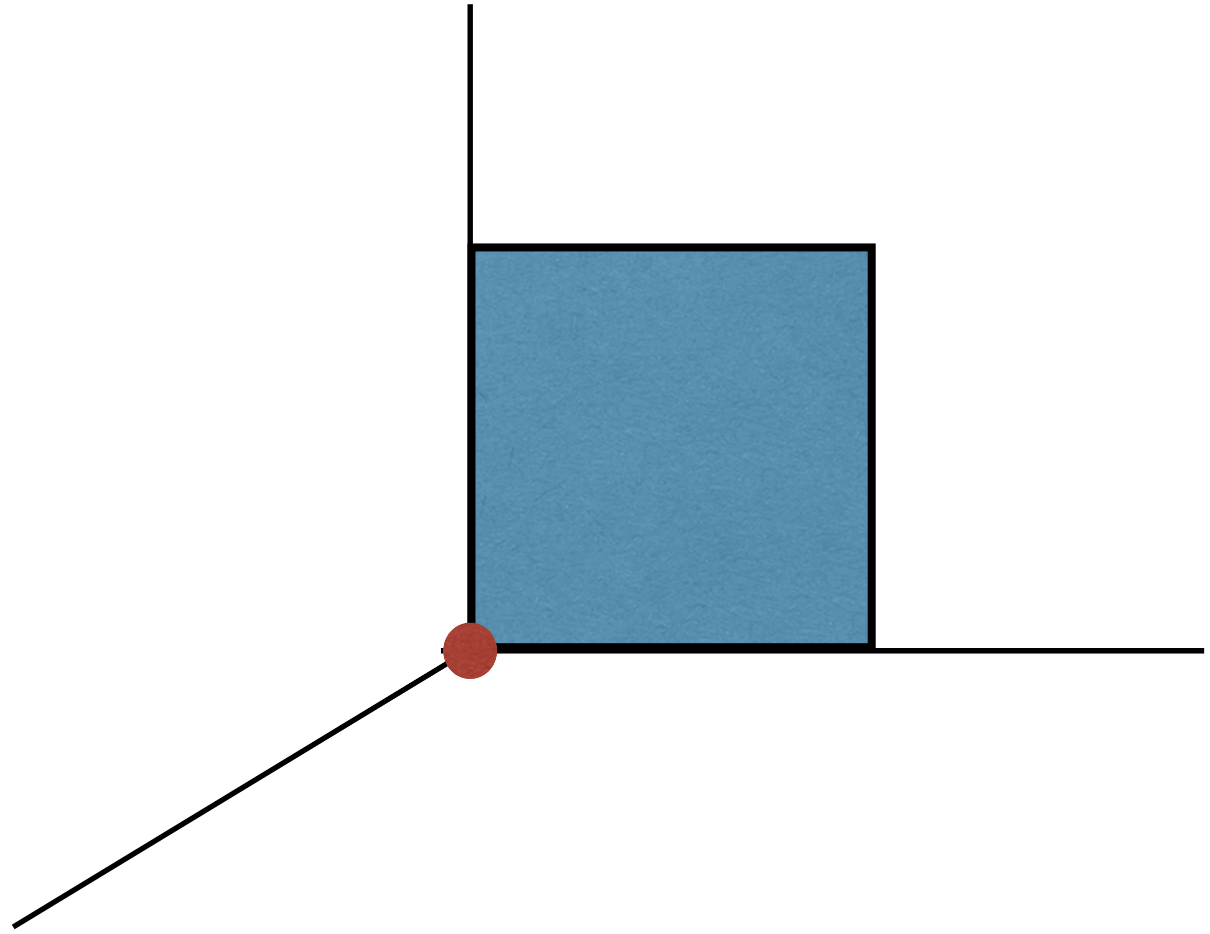


Good news: Integrals of SU(N) for general N were programmed by our group

Embedding Numbers

- Step 2: Embedding numbers ~ how many ways can one embed graphs on cubic lattice

E.g. λ^4



3 N

Lattice sites

Another good news: Graph generation can be done on computer

Polyakov Loop Susceptibility

- We define the equilibrium Polyakov loop as

$$L = \frac{1}{V}\sum_{\vec{x}}(L_{\vec{x}} + L^{\dagger}_{\vec{x}})$$

- Then the Polyakov loop susceptibility is

$$\chi_L = \frac{\partial^2 \log Z[J]}{\partial J^2}|_{J=0} = V(\langle L^2\rangle - \langle L\rangle^2)$$

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divergent when
approaching critical point $\sim \dfrac{1}{(\lambda-\lambda_c)^{\gamma}}$

Padé Approximant

We use the Padé approximant to analyse the series

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\log\chi_L \sim \frac{-\gamma}{\lambda-\lambda_c} \sim [X/Y] = \frac{a_0 + a_1\lambda + \ldots + a_X\lambda^X}{1 + b_1\lambda + \ldots + b_Y\lambda^Y}$$

Powerful technique for extracting information of poles and exponents in series

GA Baker 1961

Results

YM Effective Theory (Theory I)

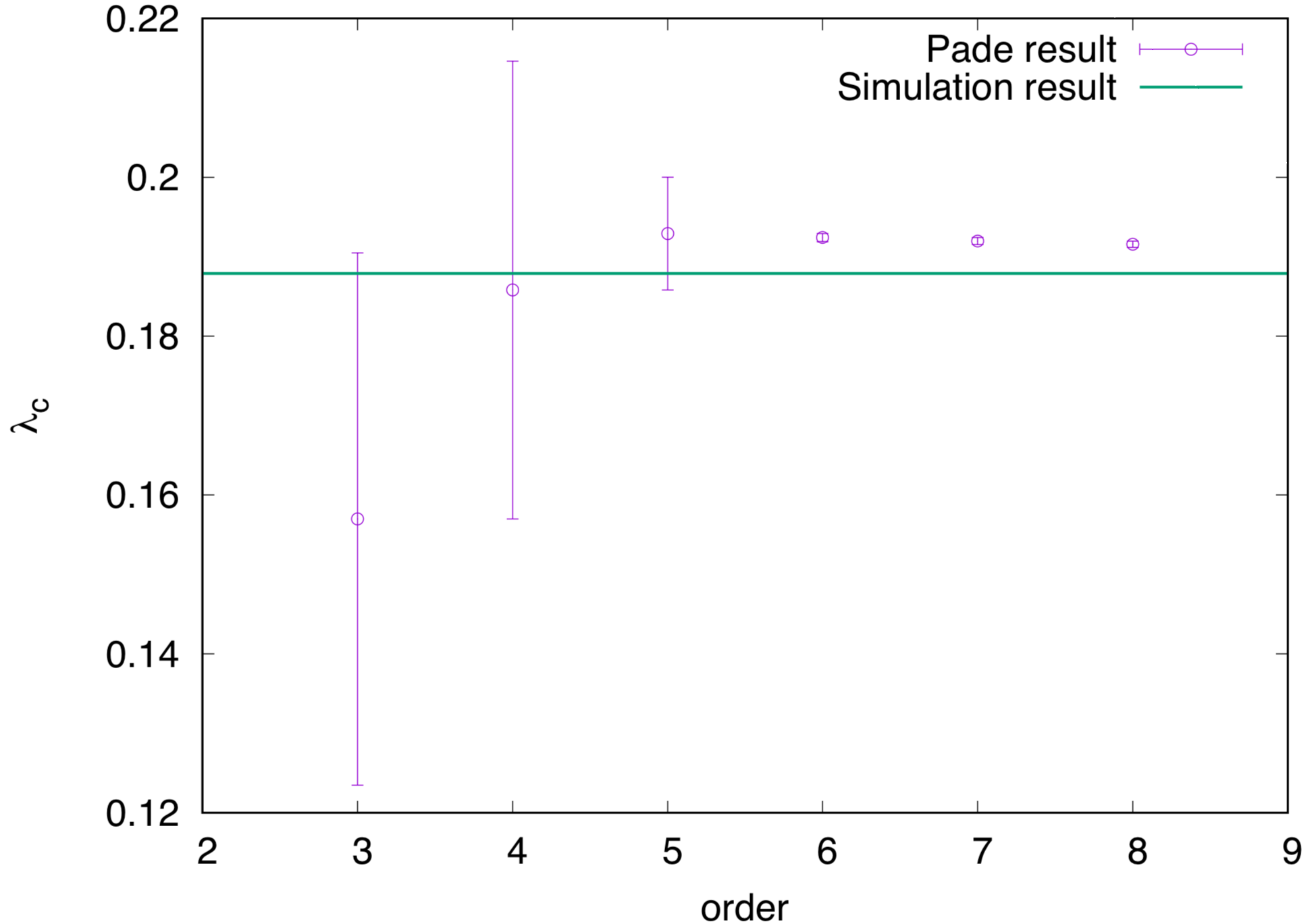
Effective Yang-Mills theory

$$S = \sum_{\langle \vec{x}, \vec{y} \rangle} \log(1 + \lambda(N_t, u)(L_{\vec{x}} L^*_{\vec{y}} + L^*_{\vec{x}} L_{\vec{y}}))$$

$$\lambda(N_t, u) = u^{N_t} \exp\left[N_t(4u^4 + 12u^5 + \ldots)\right]$$

$$u(\beta) = \frac{\beta}{18} + \ldots$$

Results of Effective YM



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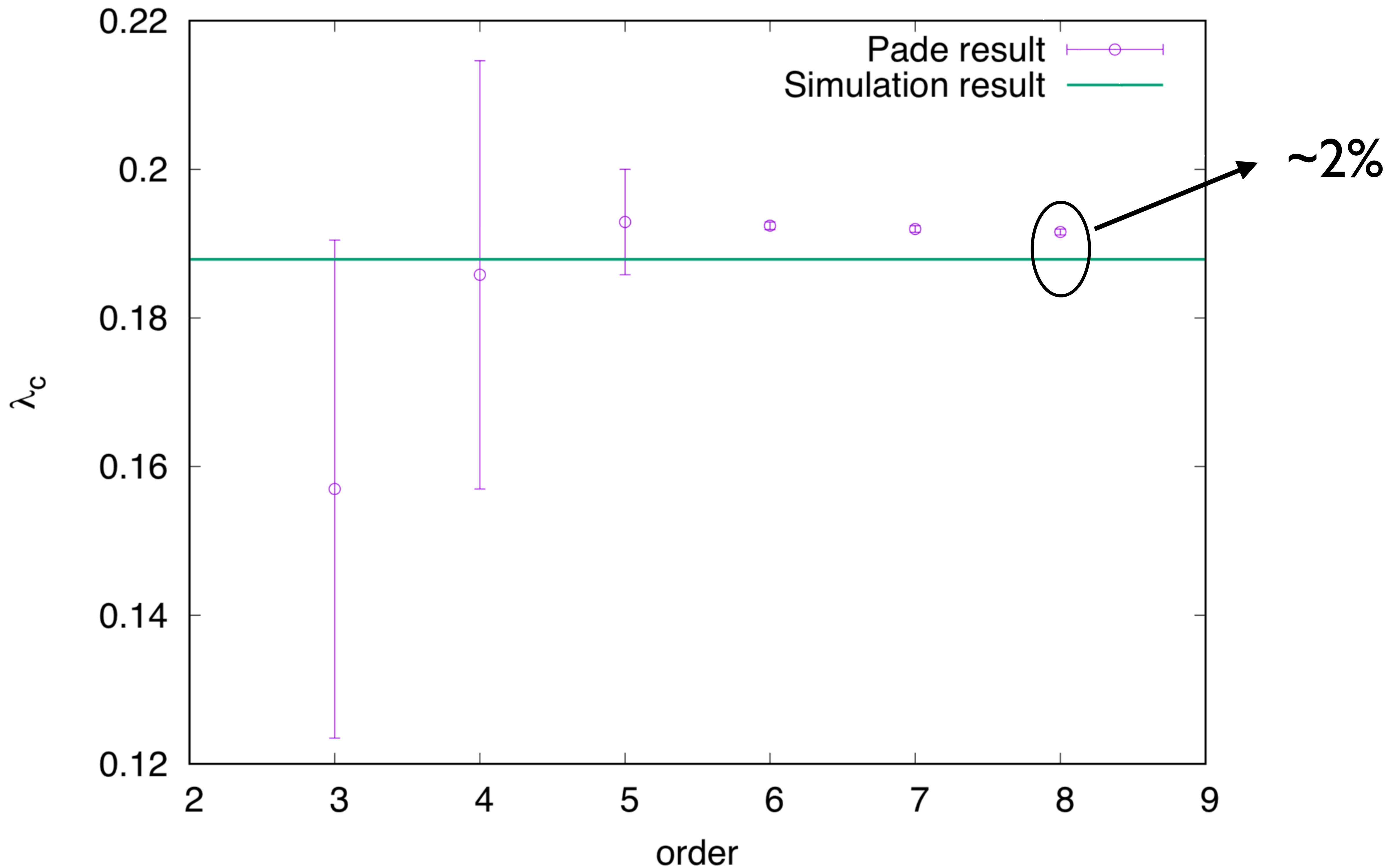
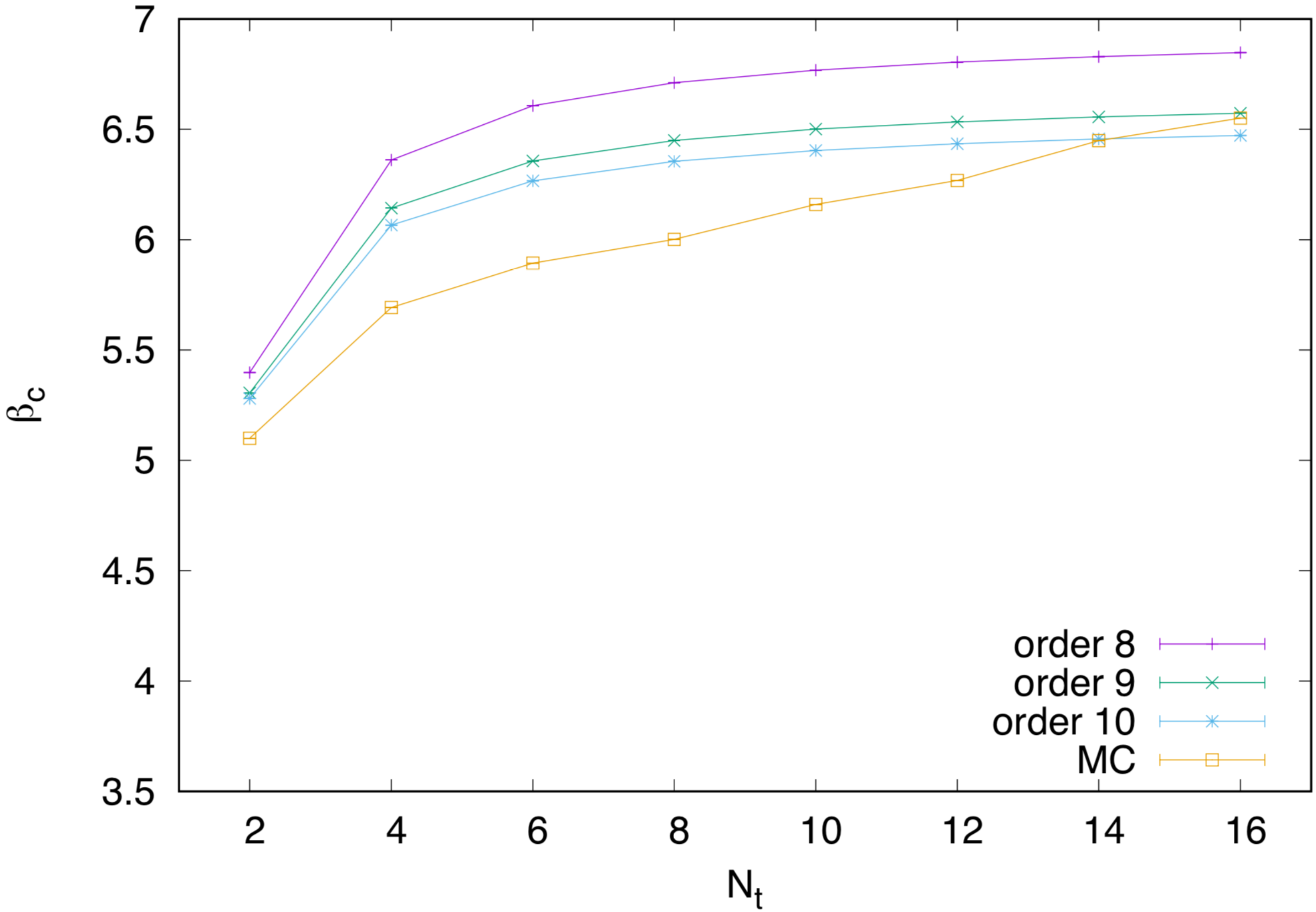


Comparison With MC



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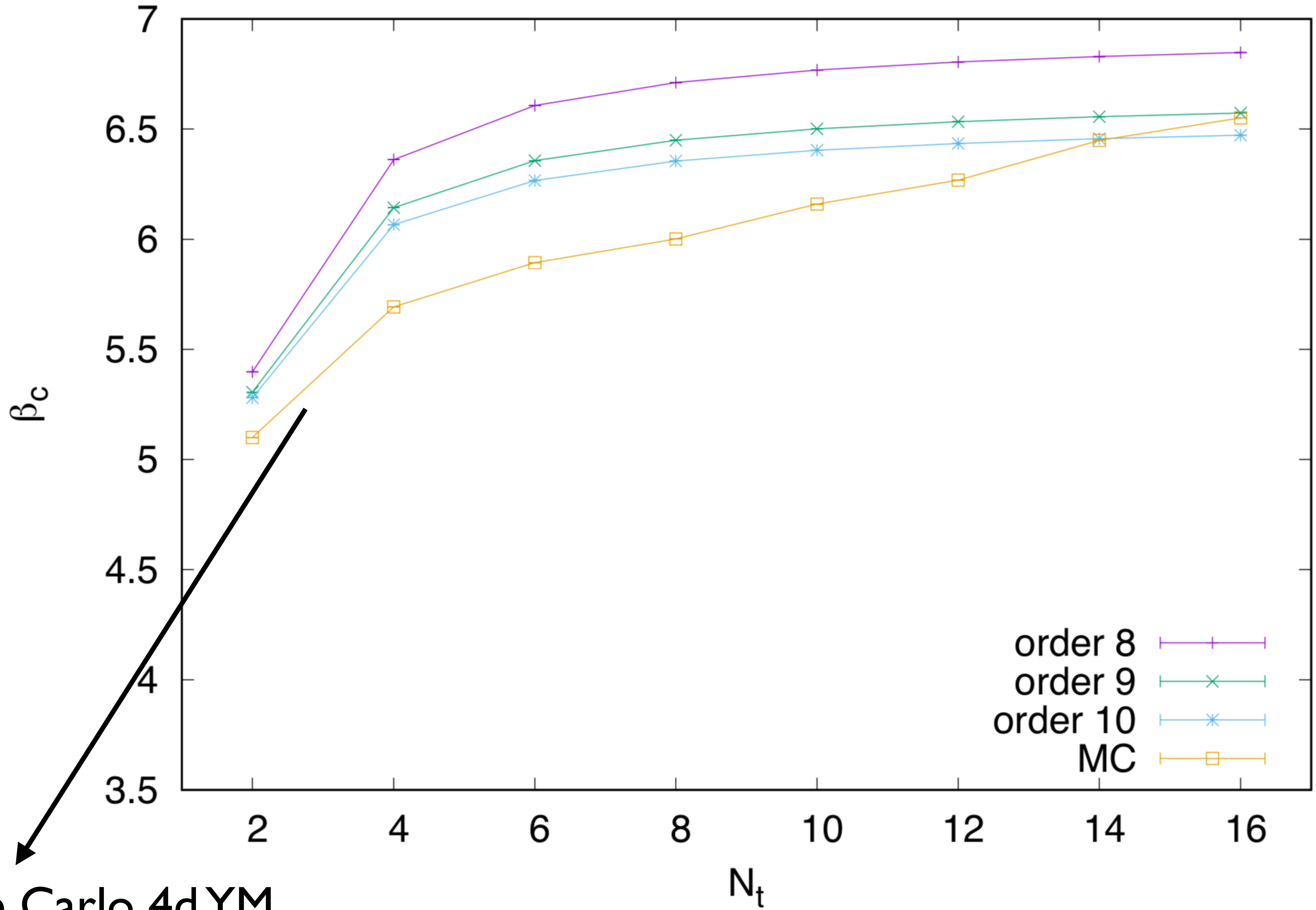
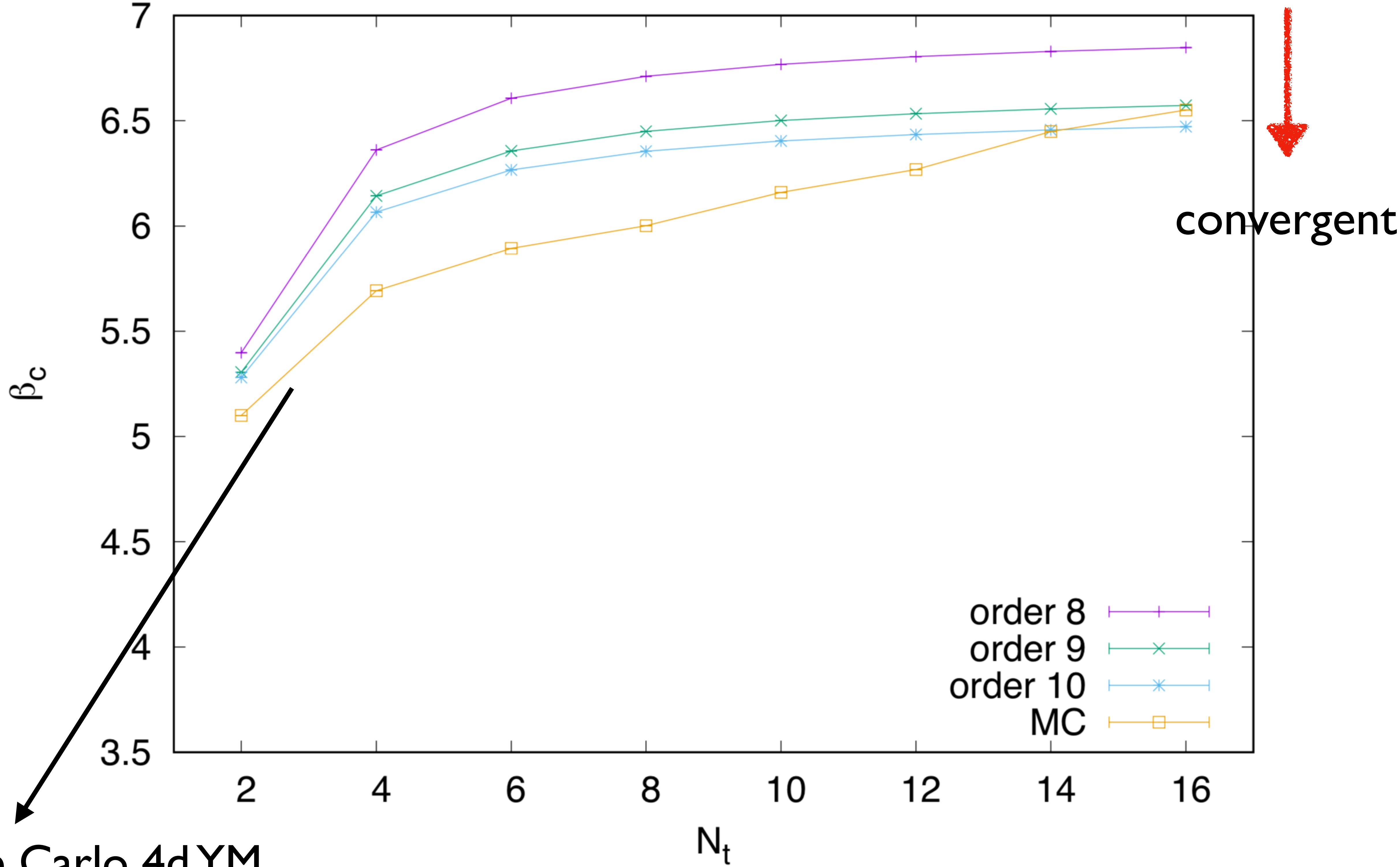


Monte Carlo 4d YM

Comparison With MC



Monte Carlo 4d YM

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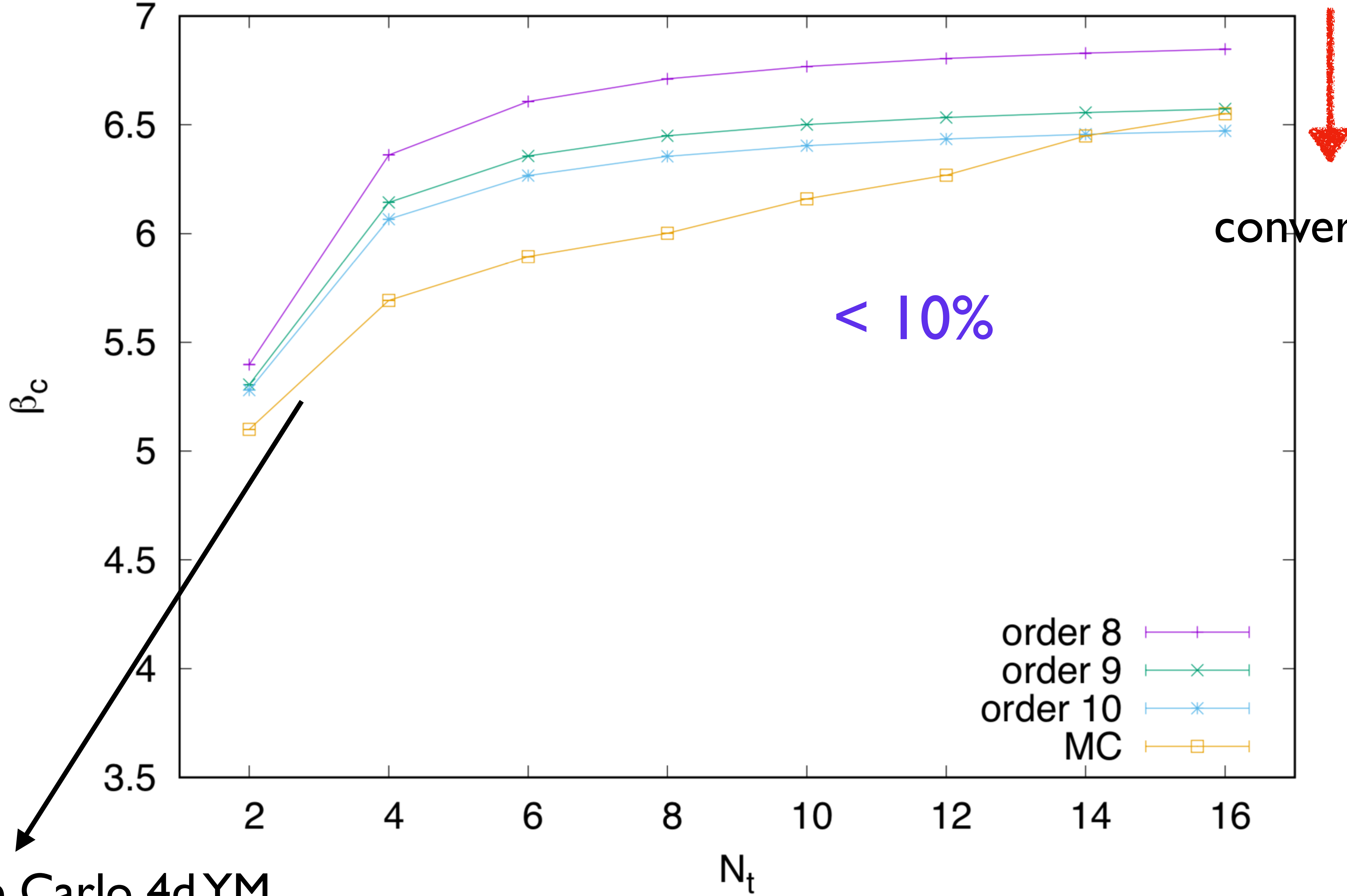


convergent

Monte Carlo 4d YM

Gauge Part of Spin Models (Theory 2)

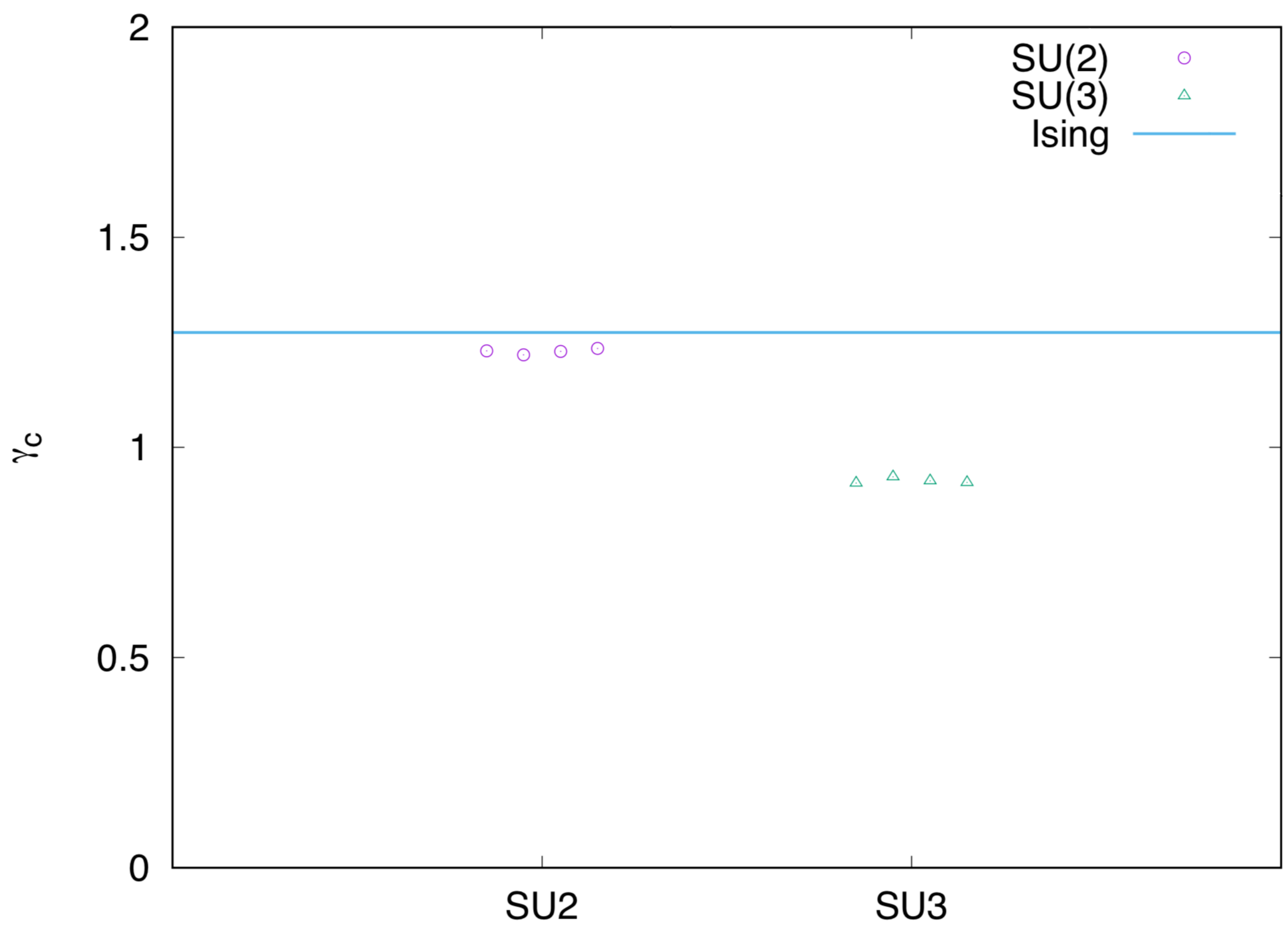
Gauge part of SU(2), SU(3) spin models

$$S_{SU(2)} = \sum_{\langle \vec{x}, \vec{y} \rangle} \lambda \, L_{\vec{x}} L_{\vec{y}}$$

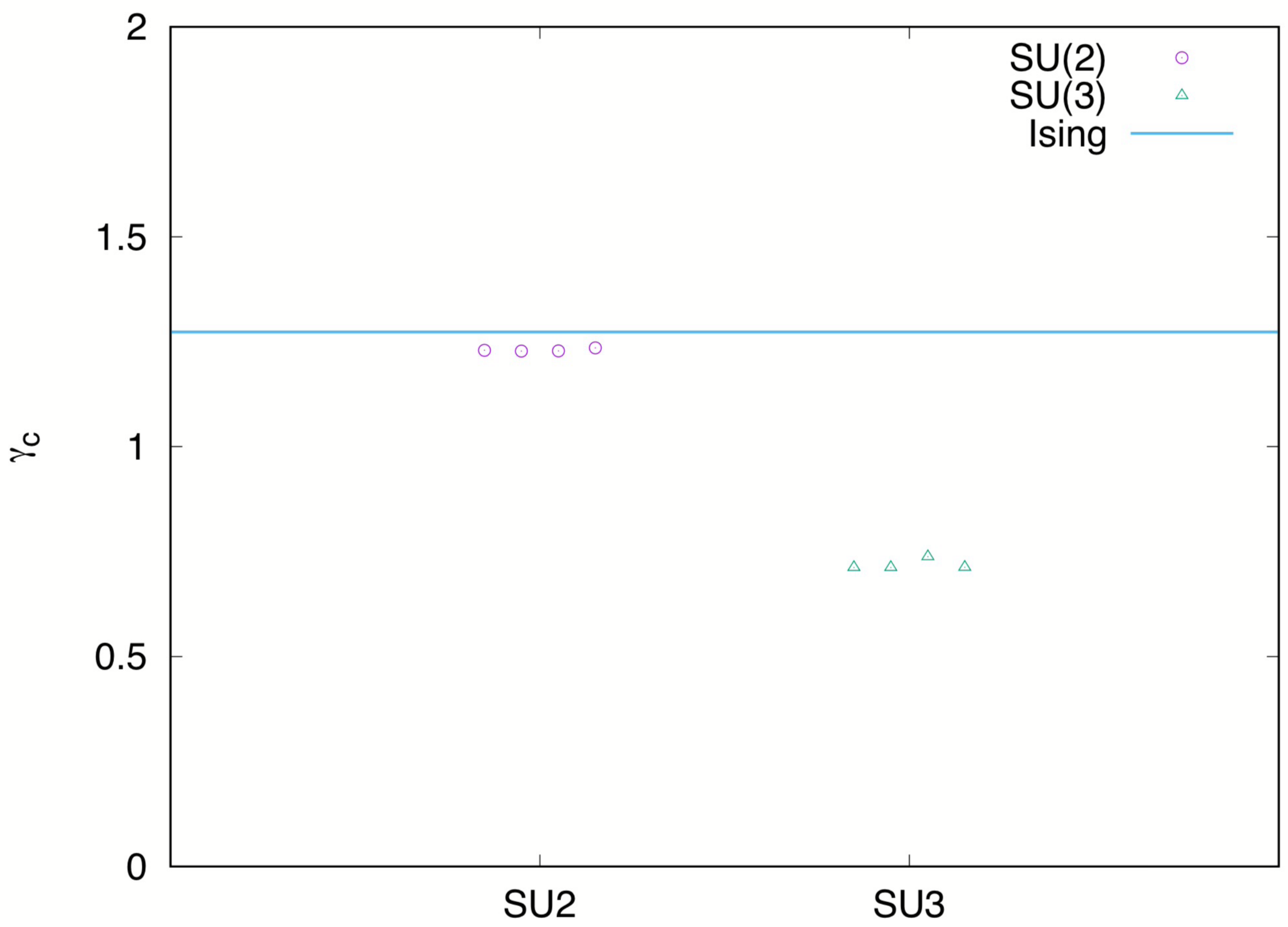
$$S_{SU(3)} = \sum_{\langle \vec{x}, \vec{y} \rangle} \lambda \, (L_{\vec{x}} L^*_{\vec{y}} + L^*_{\vec{x}} L_{\vec{y}})$$

Computed using Linked Cluster Expansion (please see our poster)

Critical Exponents



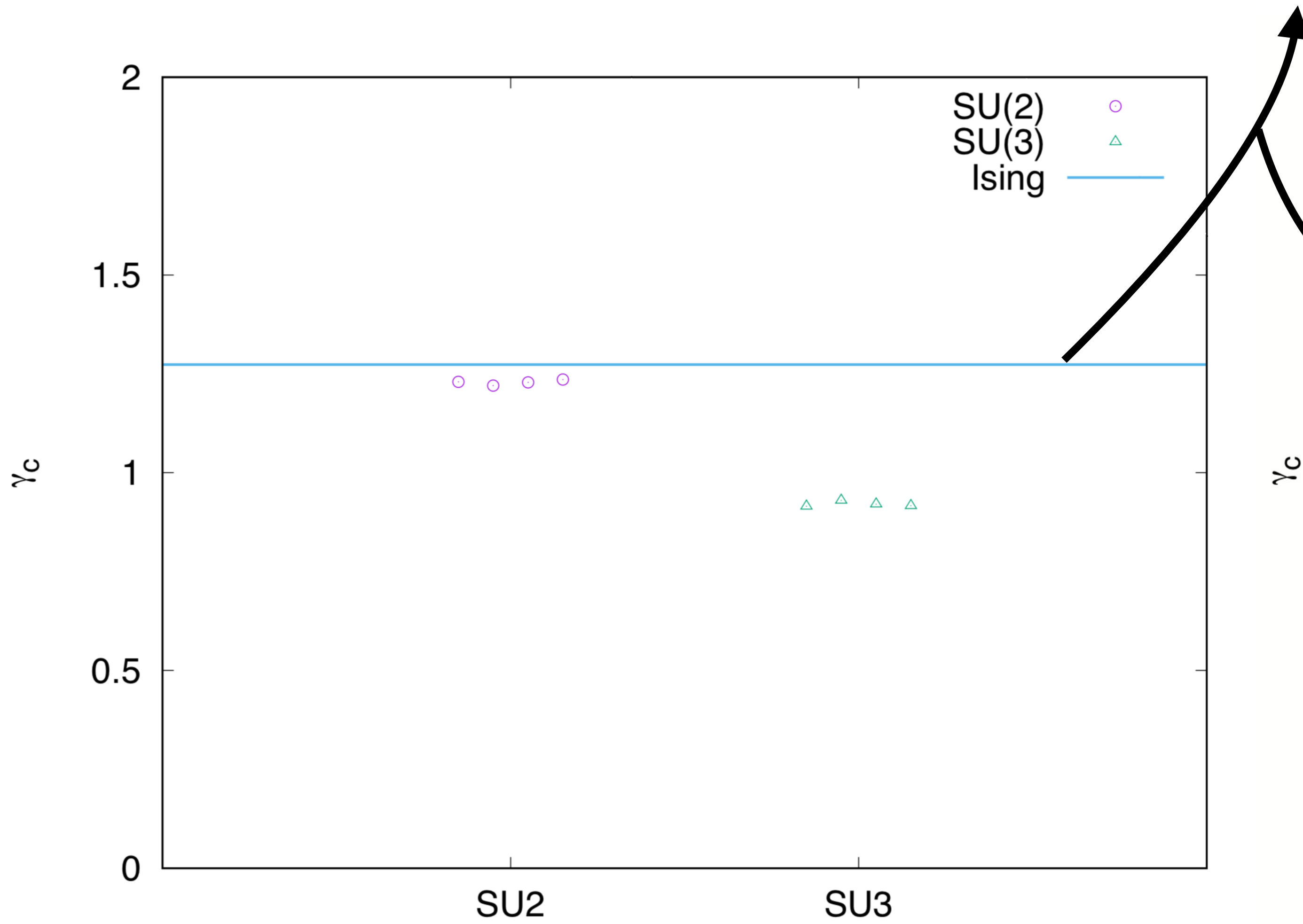
Theory 1



Theory 2

Critical Exponents

Ising ~1.23



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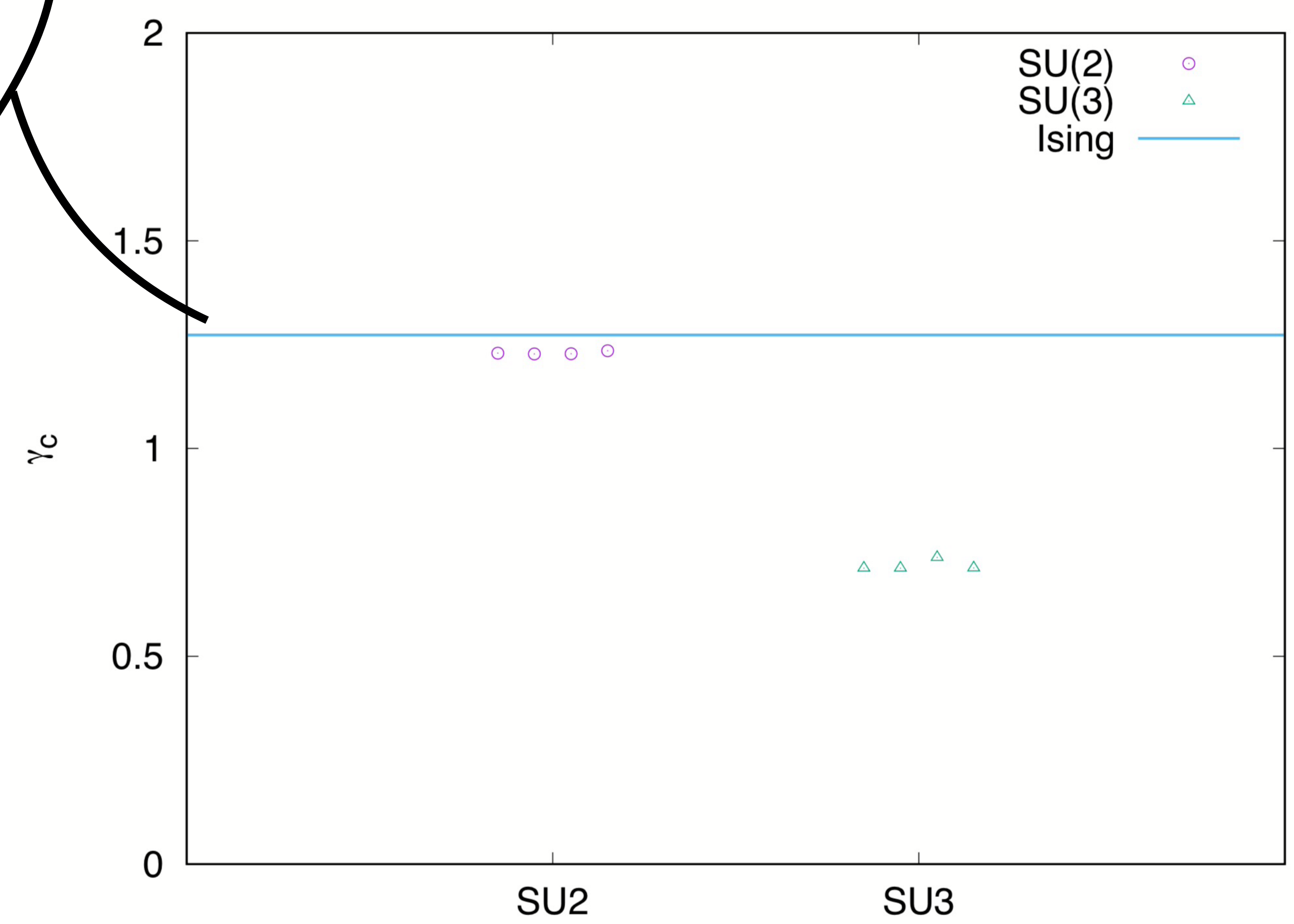

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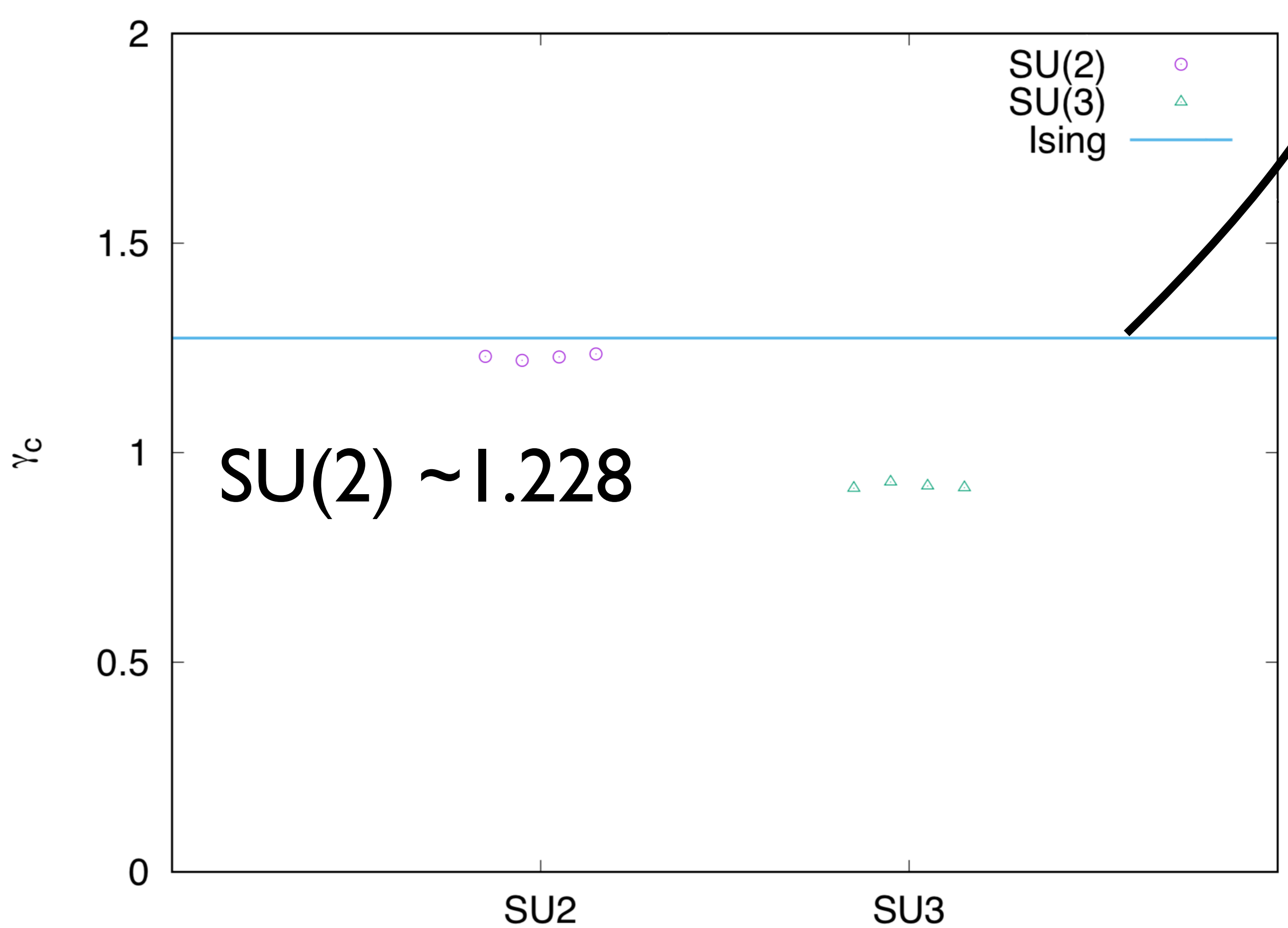
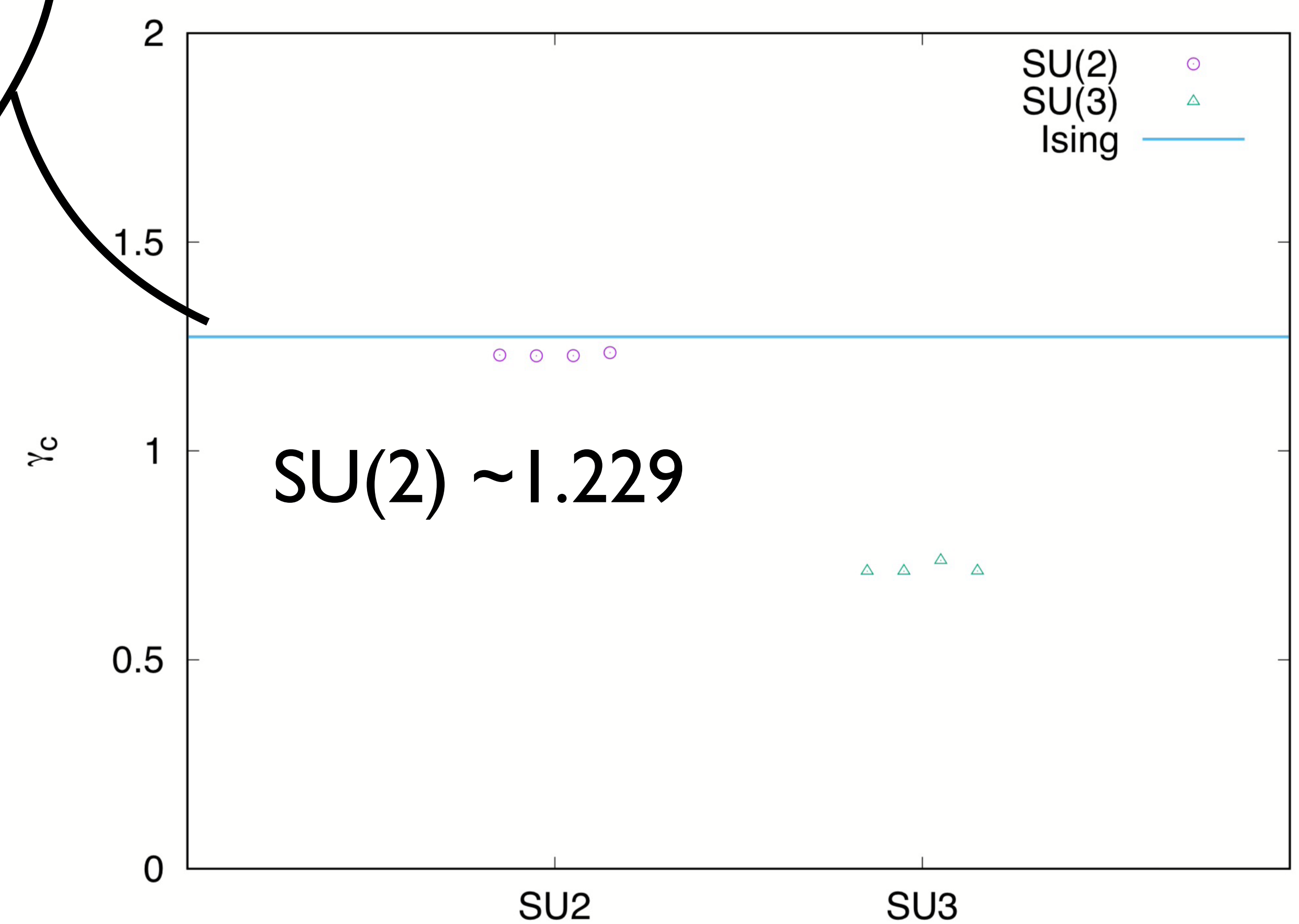


Theory 1



Theory 2

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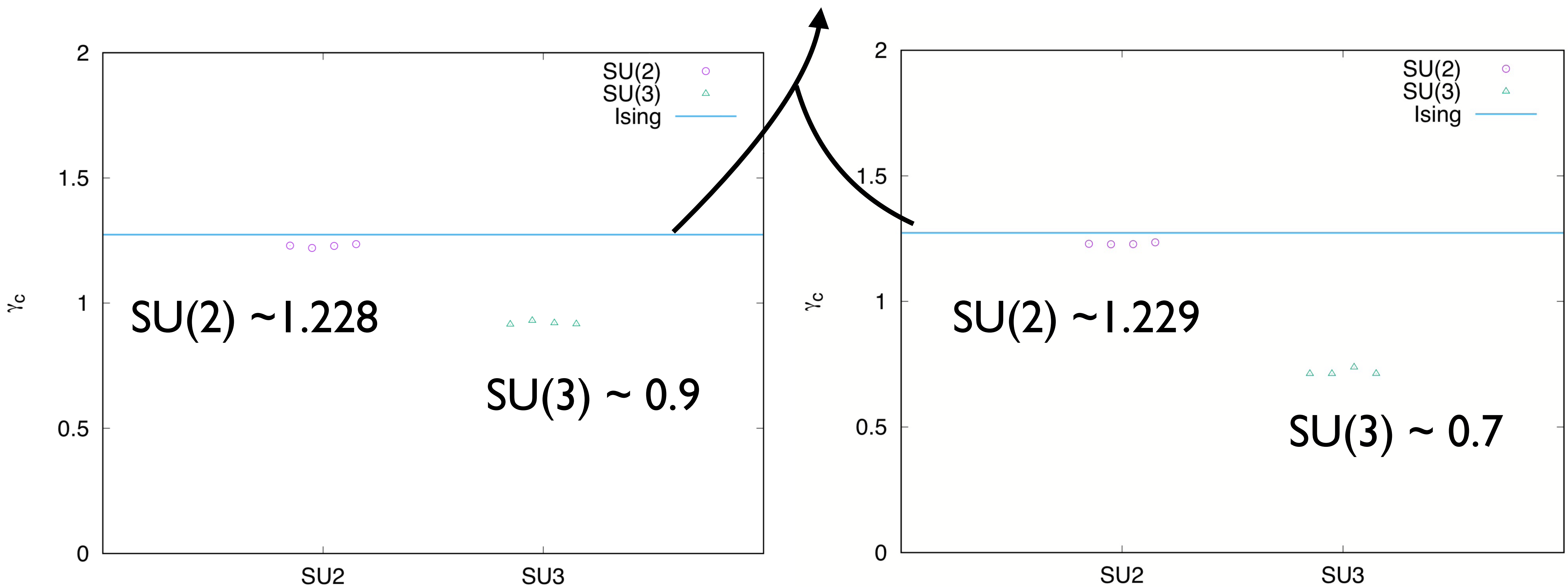

Theory 1

Theory 2

Critical Exponents

Theory 1	Theory 2	Ising

Critical Exponents

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SU(2)	γ_c^1	$\simeq$	γ_c^2	$\simeq$	γ_c^I

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and ~ γ_c of all liquid gas phase transition , and more

Critical Exponents

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and ~ γ_c of all liquid gas phase transition , and more

SU(3)	γ_c^1	$\neq$	γ_c^2	Unknow Univsersality Class ~ 1st order

Critical Exponents

Our results from series expansion approach support the following statements

- SU(2) YM shares the same universality class with Ising
- SU(3) YM theory has 1st order phase transition

A. Migdal 1975; B. Svetitsky, L. G. Yaffe 1982

Conclusion

- Analytic computations for EYM are done for the first time
- Our analytic results agree well with those from MC (within 2%)
- Physics of effective theories is not far from full theory
- Estimation of critical end points for heavy quark at finite chemical potential is in progress

Open Problems

- Computation time of next order ~ 15-20 times of the previous order
- More couplings ⟶ The need to go to higher order
- Multivariate series analysis ⟶ Higher order

Thank you!

Backup slides

With Static Quarks

$$Z = \int \mathrm{D}[U_0] \det \mathrm{D}_s[L, L^*] \prod_{\langle \vec{x}, \vec{y} \rangle} \Big(1 + \lambda(L_{\vec{x}} L^*_{\vec{y}} + L^*_{\vec{x}} L_{\vec{y}})\Big)$$

where the static quark determinant for $N_f = 1$ is

$$\det \mathrm{D}_s[L, L^*] = \prod_{\vec{x}} (1 + h_1 L_{\vec{x}} + h_1^2 L_{\vec{x}}^{*2} + h_1^3)^2 (1 + \bar{h}_1 L^*_{\vec{x}} + \bar{h}_1^2 L^2_{\vec{x}} + \bar{h}_1^3)^2$$

$$h_1 = (\kappa e^{a\mu})^{N_t}$$
$$\bar{h}_1 = (\kappa e^{-a\mu})^{N_t}$$